Modelling wave propagation using finite difference approximations

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ABSTRACT

Analyzing waves on a string is informative relative to the properties of various solutions to the wave-equation, and to the parameters used in finite difference approximations to those solutions. In addition, insight may be gained to various problems such as estimating the reflection coefficients from the cross-correlation imaging conditions.

THEORY

Consider two wave equations: the first assumes a constant velocity

$$\frac{\partial^2 P}{\partial t^2} = V^2 \frac{\partial^2 P}{\partial z^2}$$

and the second allows the velocities to vary, but is a little more complex,

$$\frac{\partial^2 P}{\partial t^2} = \frac{\partial}{\partial z}\left(V^2 \frac{\partial P}{\partial z}\right)$$

The finite difference approximations for the first derivatives were:

$$\frac{dP}{dx} \approx \frac{P(z+\delta z) - P(z-\delta z)}{2\delta z}$$

$$\frac{dP_0}{dx} \approx \frac{-P_{-3} + 9P_{-2} - 45P_{-1} + 45P_1 - 9P_2 + P_3}{60\delta x}$$

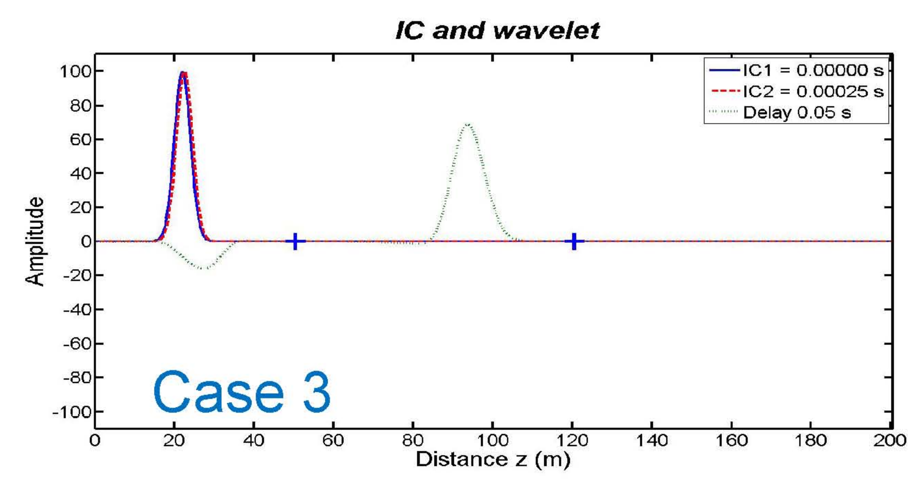
And the second derivatives:

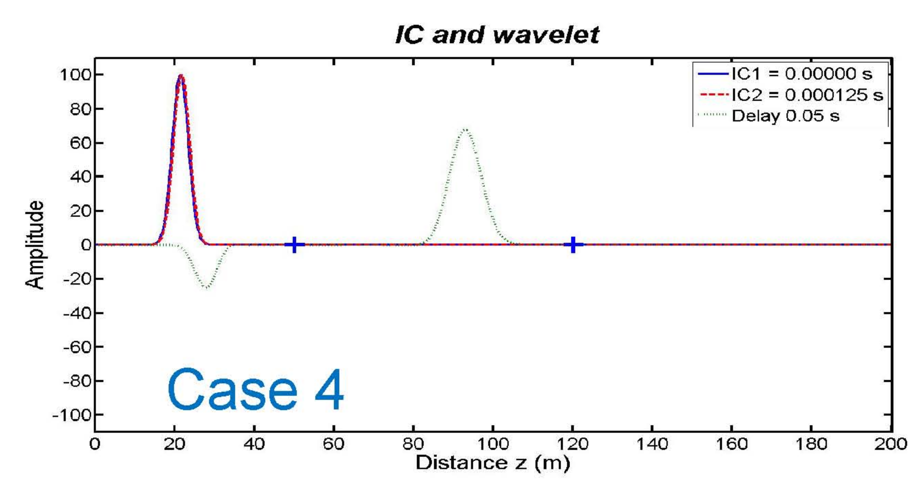
$$\frac{d^2 P_0}{dx^2} \approx \frac{P_{-1} - 2P_0 + P_1}{\delta x^2}$$

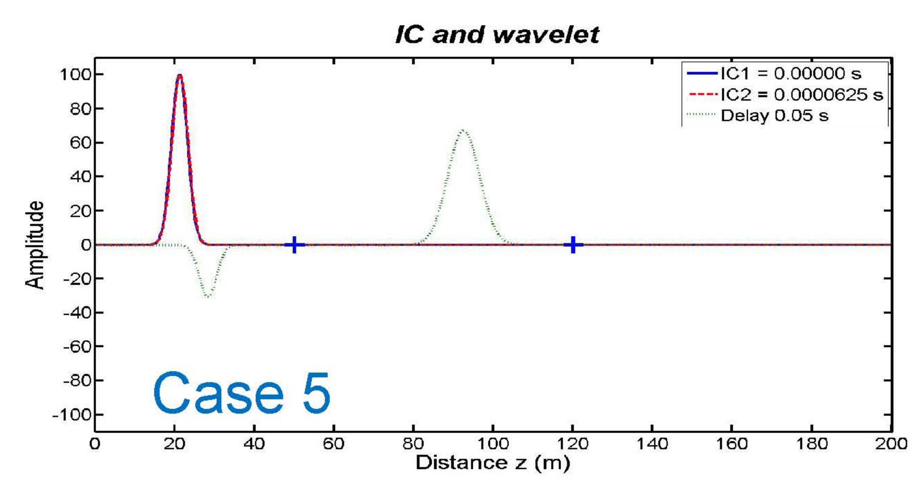
$$\frac{d^2 P_0}{dx^2} \approx \frac{2P_{-3} - 27P_{-2} + 270P_{-1} - 490P_0 + 270P_1 - 27P_2 + 2P_3}{60\delta x^2}$$

Sampling conditions by case:

```
case 3;     Dz     = 0.500;     Dt     = 0.00025;
case 4;     Dz     = 0.250;     Dt     = 0.000125;
case 5;     Dz     = 0.125;     Dt     = 0.0000625;
case 6;     Dz     = 0.0625;    Dt     = 0.00003125;
case 7;     Dz     = 0.03125;   Dt     = 0.000015625;
```



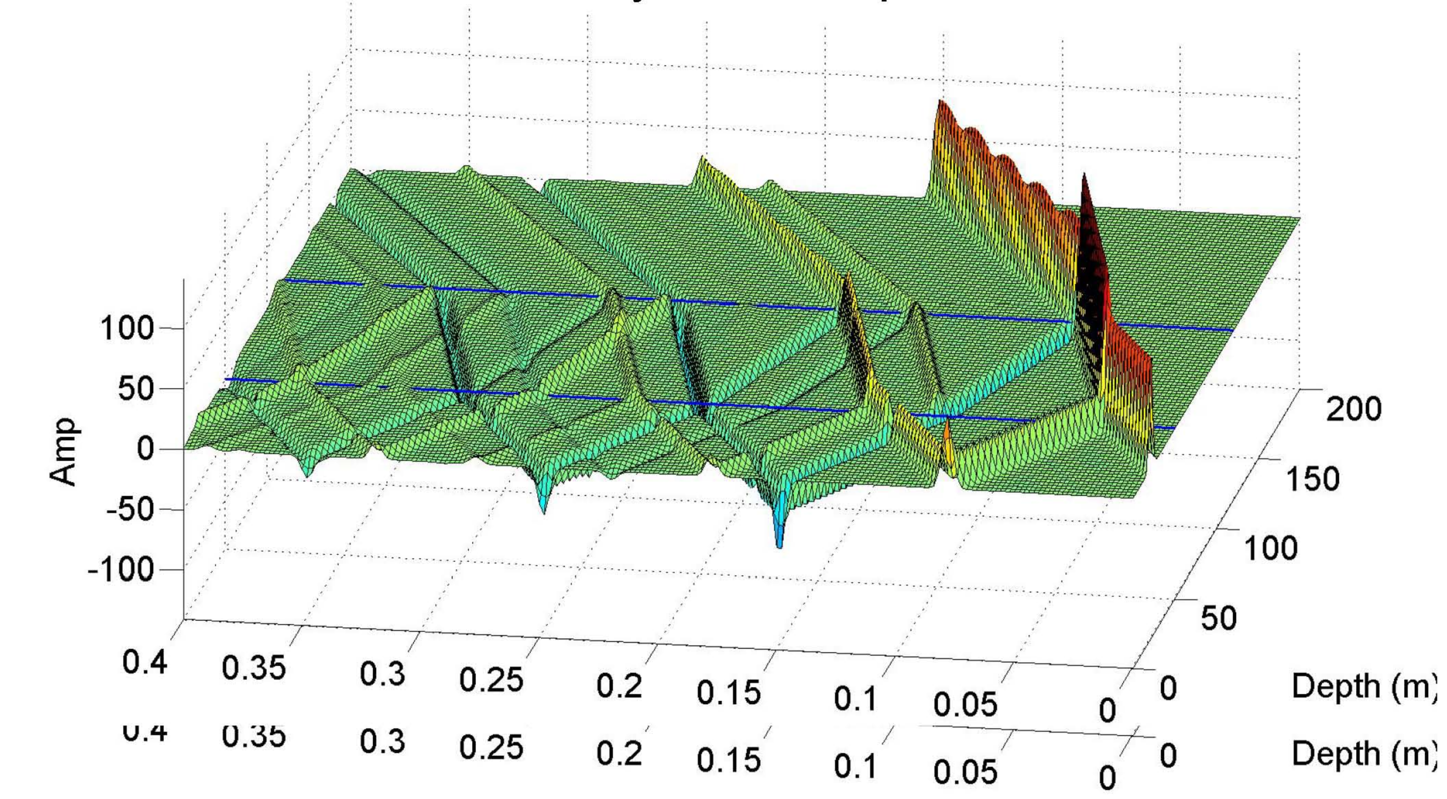




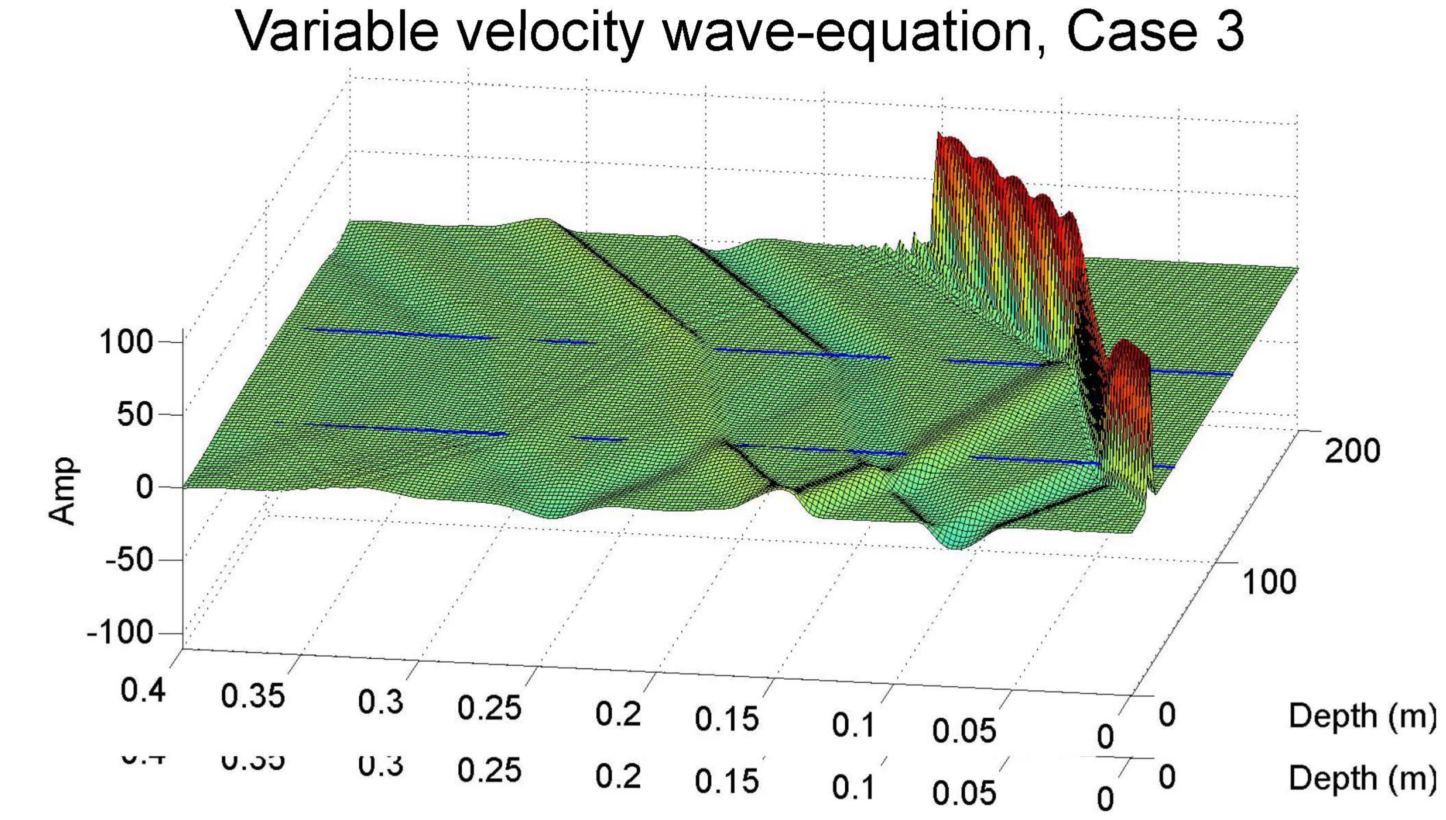
Accuracy of Trans. & Refl. Coef. by case (2/3, 1/3):

Case 3 Trans. = 0.688, Refl. = -0.160
Case 4 Trans. = 0.671, Refl. = -0.309
Case 5 Trans. = 0.671, Refl. = -0.309
Case 6 Trans. = 0.667, Refl. = -0.327
Case 7 Trans. = 0.666, Refl. = -0.333

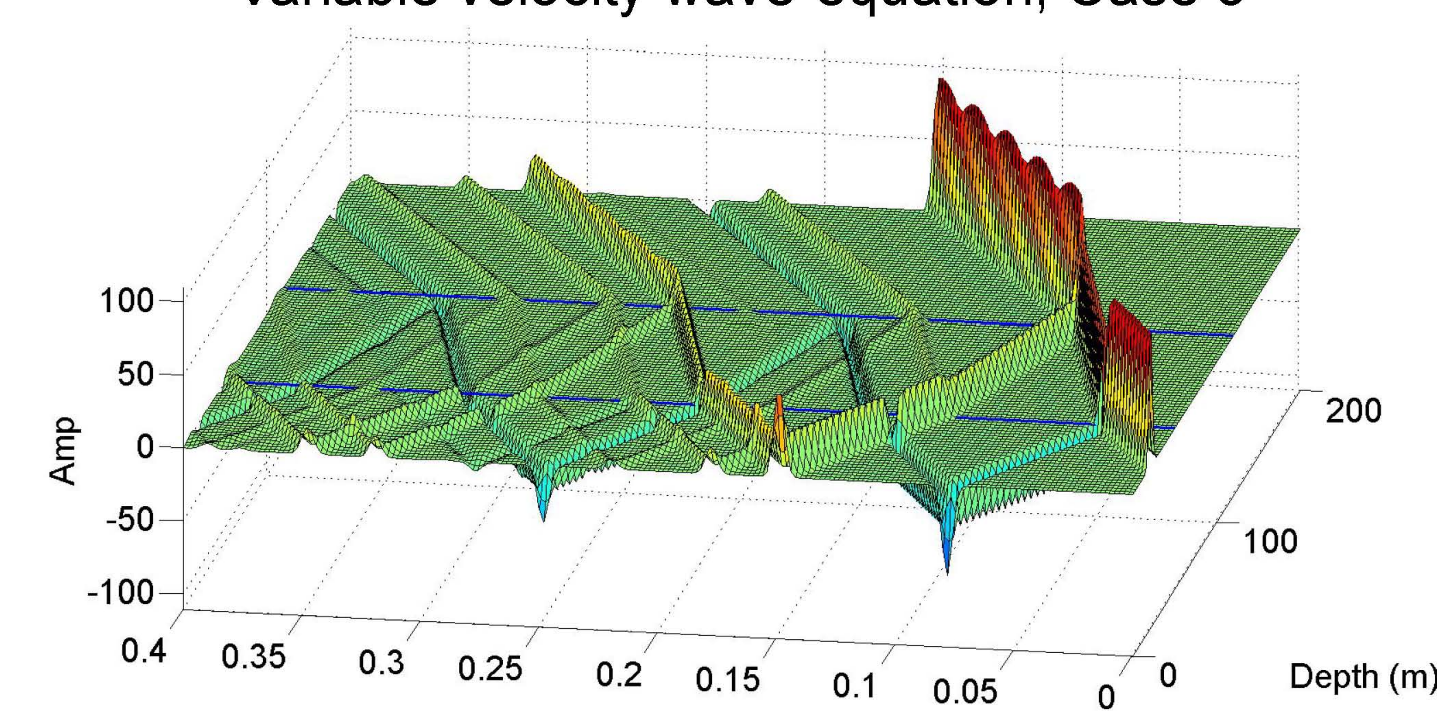
Constant velocity wave-equation, Case 5



Variable velocity wave-equation, Case 3



Variable velocity wave-equation, Case 5



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