PS and SP converted wave reflection coefficients and their application to time-lapse difference AVO

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Introduction

Multicomponent surveying has developed rapidly in both land and marine acquisition with many applications in seismology including reservoir monitoring. This raises the necessity of multicomponent 4D time-lapse analysis in a reservoir (Stewart et al. 2003). A framework has been formulated to model linear and nonlinear elastic time-lapse difference for P-P sections (Jabbari et al.,2015). The study described here focuses on applying linear and nonlinear time-lapse amplitude variation with offset methods to model the difference data for converted wave and to investigate the deference between SP and PS wave in nonlinearity.

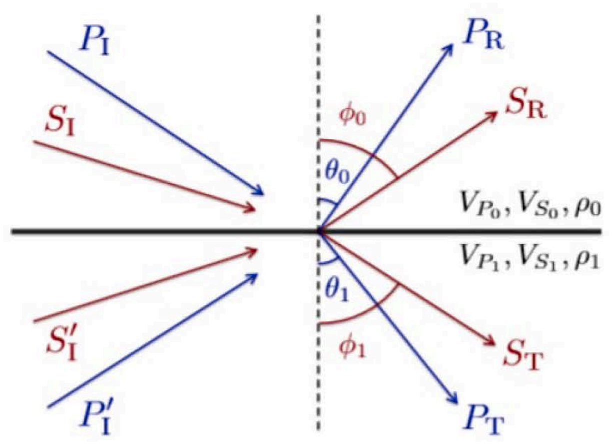

Figure 1: Displacement amplitudes associated with the Knott-Zoeppritz equations.

Theory

we consider two seismic experiments involved in a time-lapse survey, a baseline survey followed by a monitoring survey. Let V_{P_0}, V_{S_0}, ρ_0 and V_{P_x}, V_{S_x}, ρ_x be the rock properties of the cap rock and reservoir. Now lets consider a P wave and an S wave which are impinging on the boundary of a planar interface between the two elastic media; cap rock overlying the reservoir. Amplitudes of reflected and transmitted P and S waves are calculated through setting the boundary conditions in the Zoeppritz equations which can be rearranged in matrix form e.g. (Keys, 1989):

$$P\begin{bmatrix}R_{PP}\\R_{PS}\\T_{PP}\\T_{PS}\end{bmatrix} = b_P\ ,\ S\begin{bmatrix}R_{SS}\\R_{SP}\\T_{SS}\\T_{SP}\end{bmatrix} = b_S, \quad (1)$$

P, S, b_P, and b_S are matrices and vectors which their elements are functions of P incident angle, S incident angle, and elastic parameters which are defined as:

$$A_x = \frac{\rho_x}{\rho_0},\ B = \frac{V_{S_0}}{V_{P_0}},\ B^{-1} \equiv \frac{V_{P_0}}{V_{S_0}},\ C_x = \frac{V_{P_x}}{V_{P_0}},\ D_x = \frac{V_{S_x}}{V_{P_0}},\ E \equiv \frac{V_{P_x}}{V_{S_0}},\ F \equiv \frac{V_{S_x}}{V_{S_0}}. \quad (2)$$

Reflection coefficients for both converted waves are determined by forming an auxiliary matrix P_{PS} by replacing the second columns of P with b_P , and then forming another auxiliary matrix S_{SP} by replacing the second columns of S with c_S:

$$R_{PS}(\theta) = \frac{det(P_{PS})}{det(P)}\ R_{PS}(\phi) = \frac{det(S_{SP})}{det(S)}. \quad (3)$$

R_{PS} and R_{SP} for the baseline and monitor surveys are calculated using the method explained above, where rock properties for cap rock are the same, but reservoir properties change from V_{P_b}, V_{S_b}, ρ_b (replace $x = b$ in Equation 2) at the time of the baseline survey to V_{Pm}, V_{Sm}, ρ_m (replace $x = m$ in Equation 2) at the time of the monitor survey.

$$\begin{aligned}\Delta R_{PS}(\theta) &= R^m_{PS}(\theta) - R^b_{PS}(\theta)\\ \Delta R_{SP}(\phi) &= R^m_{SP}(\phi) - R^b_{SP}(\phi).\end{aligned} \quad (4)$$

We have considered two groups of perturbation parameters (Stolt and Weglein, 2012). The first group expresses the perturbation in the baseline survey. The second group expresses the time-lapse perturbation.

$$b_{VP} = 1 - \frac{V^2_{P_0}}{V^2_{P_b}},\ b_{VS} = 1 - \frac{V^2_{S_0}}{V^2_{S_b}},\ b_\rho = 1 - \frac{\rho_0}{\rho_b}. \quad (5)$$

Theory continued

$$a_{VP} = 1 - \frac{V^2_{P_b}}{V^2_{P_m}},\ a_{VS} = 1 - \frac{V^2_{S_b}}{V^2_{S_m}},\ a_\rho = 1 - \frac{\rho_b}{\rho_m}. \quad (6)$$

Re-defining elastic parameters in terms of perturbation parameters, Equation 4 can be calculated and then expanded in first and second order for all six perturbations, $\sin^2\theta$, and $\sin^2\phi$.

$$\begin{aligned}\Delta R_{PS}(\theta) &= \Delta R^{(1)}_{PS}(t) + \Delta R^{(2)}_{PS}(\theta) + \Delta R^{(3)}_{PS}(\theta) + ...\\ \Delta R_{SP}(\phi) &= \Delta R^{(1)}_{SP}(\phi) + \Delta R^{(2)}_{SP}(\phi) + \Delta R^{(3)}_{SP}(\phi) + ...\end{aligned} \quad (7)$$

Results

The linear, second, and third order terms for time-lapse difference data for PS and SP converted wave are calculated and can be found in the context of the report. In this section, we examine the derived linear and nonlinear difference time-lapse AVO terms for PS and SP converted wave qualitatively with numerical examples. In the first example, the data used by Landrø (2001) are applied. The exact difference data are compared with the calculated linear and higher order approximations in Figure 2 and Figure 3. Results are also compared for the higher contrast in seismic parameters in the reservoir after the production.

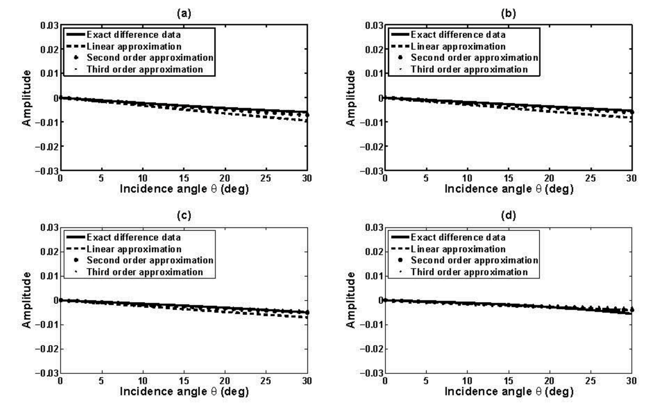

Figure 2: ΔR_{PS} for the exact, linear, second, and third order approximation. Elastic incidence parameters: $V_{P0} = 1900m/s$, $V_{S0} = 995m/s$ and $\rho_0 = 1.95g/cc$; Baseline parameters:$V_{P_{BL}} = 2066m/s$, $V_{S_{BL}} = 1075m/s$ and $\rho_{BL} = 2.1300g/cc$. a: +13 %, -2 %, and +4 %, b: +16 %, -3 %, and +5 %, c: +20 %, -4 %, and +6 %, d: +25 %, -6 %, and +8 % changes in P-wave and S-wave velocities and density respectively in the reservoir after production.

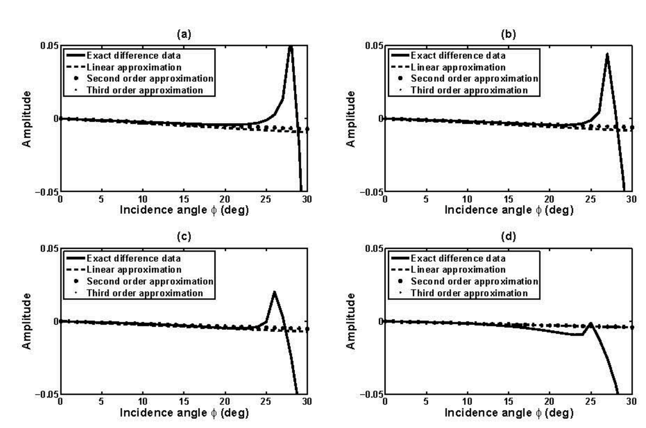

Figure 3: ΔR_{SP} for the exact, linear, second, and third order approximation. Elastic parameters as in Figure 2. a: +13 %, -2 %, and +4 %, b: +16 %, -3 %, and +5 %, c: +20 %, -4 %, and +6 %, d: +25 %, -6 %, and +8 % changes in P-wave and S-wave velocities and density respectively in the reservoir after production.

For the second example, we used data by Veire (2006). We examined our formulation and compared them with the exact difference data for the P- and S-wave velocities and density changes of 15 %, 11 %, and 1 % respectively, and also for higher contrasts (Figure 4 and Figure 5). The second and third order time-lapse AVO approximations are always in better agreement with the exact difference data, especially for higher contrasts in seismic parameters. More importantly the third order approximation emphasizes on the difference between ΔR_{PS} and ΔR_{SP} by following the same trend as the exact difference in each case as in Figure 2-5.

Results continued

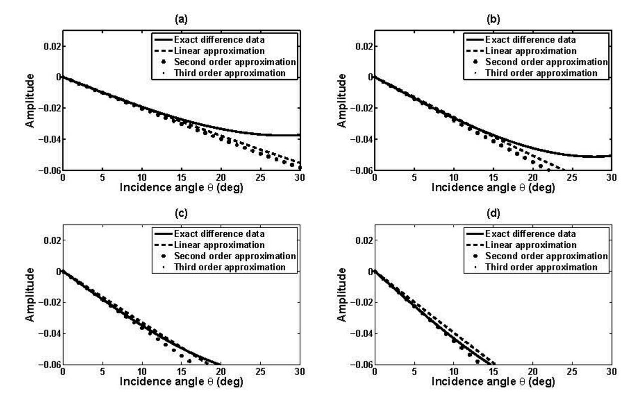

Figure 4: ΔR_{PS} for the exact, linear, second, and third order approximation. Elastic parameters: Elastic parameters: $V_{P0} = 2000m/s$, $V_{S0} = 1000m/s$ and $\rho_0 = 2.000g/cc$; Baseline parameters:$V_{P_{BL}} = 1900m/s$, $V_{S_{BL}} = 1100m/s$ and $\rho_{BL} = 1.950g/cc$; and b. Data used by (Veire, 2006). a: +15 %, +11 %, and +1 %, b: +20 %, +15 %, and +2 %, c: +25 %, +20 %, and +3 %, d: +30 %, +25 %, and +4 % changes in P- and S-wave velocities and density respectively in the reservoir after production.

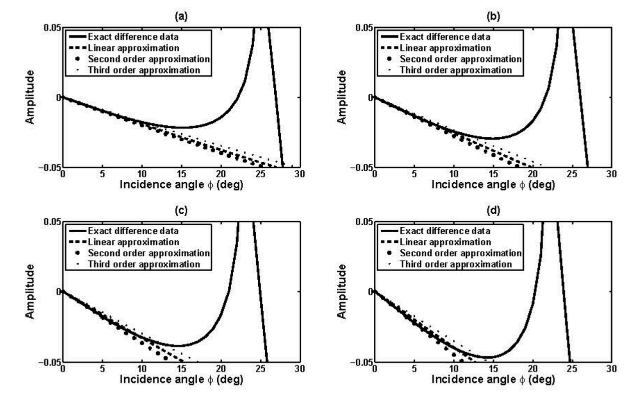

Figure 5: ΔR_{SP} for the exact, linear, second, and third order approximation.Elastic parameters as in Figure 6. a: +15 %, +11 %, and +1 %, b: +20 %, +15 %, and +2 %, c: +25 %, +20 %, and +3 %, d: +30 %, +25 %, and +4 % changes in P- and S-wave velocities and density respectively in the reservoir after production.

Conclusions

Jabbari et al. (2015) have showen that adding the higher order terms in ΔR_{PP} to the linear approximation for difference time-lapse data increases the accuracy of the ΔR_{PP}. In this study we focused on the difference between ΔR_{PS} and ΔR_{PS} for SP and PS converted wave. The results showed that, including higher order terms in ΔR for converted wave improves the accuracy of approximating time-lapse difference reflection data, particularly for large contrast cases. Comparing linear, second , and third order terms for ΔR_{PS} and ΔR_{SP} indicates as we are moving toward higher order approximations; ΔR_{PS} and ΔR_{SP} are different. This confirms the difference between exact ΔR_{PS} and ΔR_{SP} which does not show up in the linear approximation case.

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