Multiparameter seismic full-waveform inversion in fractured media: anisotropic parameter estimation and cross-talk suppression

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Summary

We consider the estimation of the elastic constants of a fractured medium, using multi-parameter FWI and modeling naturally fractured reservoirs as equivalent anisotropic media. Multi-parameter FWI remains exposed to a range of challenges, one of which being the parameter cross-talk problem resulting from overlap of Fr é chet derivative wavefields. Parameter cross-talk is strongly influenced by the form of the scattering patterns for each parameter, i.e., the relative strength of incoming to outgoing waves from a volume scattering element as a function of opening angle. In the numerical section, we illustrate the analytic and numerical scattering patterns of different elastic constants in HTI media for parameter cross-talk analysis. We also analyze the role of multi-parameter approximate Hessian in suppressing cross-talk. We apply Gauss-Newton multi-parameter FWI on several numerical examples to verify multi-parameter Hessian's role in suppressing parameter cross-talk.

Principle of FWI

The objective function of FWI is formulated as a L-2 norm:

$$\Phi(\mathbf{m}) = \frac{1}{2}\sum_{\mathbf{x}_s}\sum_{\mathbf{x}_g}\sum_{\omega}\|\Delta\mathbf{d}(\mathbf{x}_g,\mathbf{x}_s,\omega)\|^2$$

The gradient can be constructed by cross-correlating the forward modelled wavefield with the back-propagated residual wavefield with adjoint-state method:

$$\mathbf{g}(\mathbf{x}) = \sum_{\mathbf{x}_g}\sum_{\mathbf{x}_s}\sum_{\omega}\Re\left(\omega^2 f_s(\omega)\, G(\mathbf{x},\mathbf{x}_s,\omega)\, G(\mathbf{x}_g,\mathbf{x},\omega)\,\Delta\mathbf{d}^*(\mathbf{x}_g,\mathbf{x}_s,\omega)\right)$$

The search direction is obtained by solving the Newton linear system:

$$\mathbf{H}_k\Delta\mathbf{m}_k = -\mathbf{g}_k$$

where **H** is the Hessian matrix, which represents the second-order partial derivative of the misfit function with respect to the model parameter:

$$\mathbf{H} = \frac{\partial^2\Phi}{(\partial\mathbf{m})^2} = \sum_{\mathbf{r}_s}\sum_{\mathbf{r}_g}\sum_{\omega}\Re\left(\mathbf{J}^\dagger\mathbf{J}^* + \left(\frac{\partial\mathbf{J}}{\partial\mathbf{m}}\right)^\dagger\Delta\mathbf{d}^*(\mathbf{r}_g,\mathbf{r}_s,\omega)\right)$$

The first term indicates the Gauss-Newton Hessian. The gradient suffers from geometrical spreading and finite-frequency effects. The Gauss-Newton Hessian can compensate the geometrical spreading and remove the finite-frequency effects. In multi-parameter FWI, the gradient also suffers from parameter cross-talk problem, in this paper, we will reveal the role of Gauss-Newton Hessian in decoupling different physical parameters.

Scattering Patterns and Parameter Cross-talk

Identical or nearly identical variations are one of the key mechanisms of parameter cross-talk in FWI. The interaction of the incident wavefield with the model perturbation serves as the "virtual source" or "secondary scattered source". The scattering pattern of the "virtual source" governs the amplitude variations of Fr é chet derivative wavefields as a function of scattering angle.

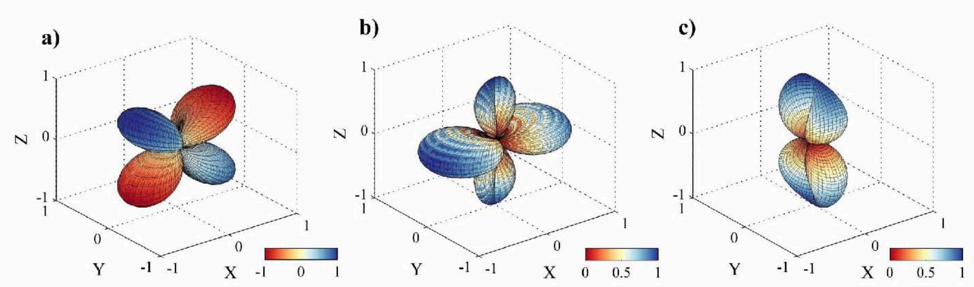

Figure 1. P-P (a), P-SV (b) and P-SH (c) scattering patterns due to c55.

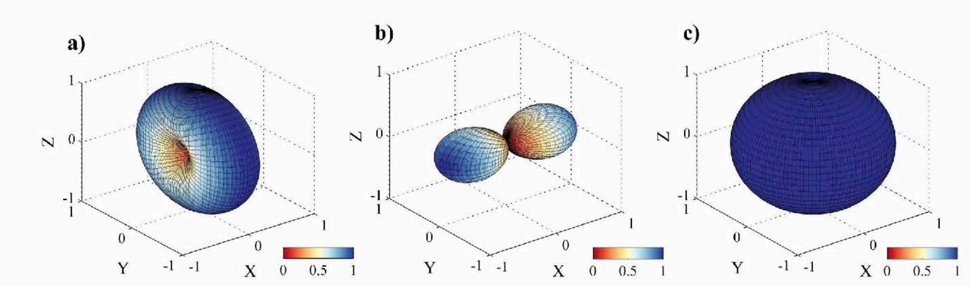

Figure 2. P-P scattering patterns due to c33 (a), c11 (b) and c13 (c).

Multi-parameter Hessian

Multi-parameter Hessian in multi-parameter FWI has a block structure and it carries more information than mono-parameter Hessian. Considering a 2D subsurface model with NxNz nodes and Np physical parameters are assigned to describe the properties of each node. The multi-parameter Hessian is a NzNxNp X NzNxNp square and symmetric matrix with Np diagonal blocks and Np(Np-1) off-diagonal blocks. Hence, the multi-parameter approximate Hessian **H** for inversing the 4 elastic constants in 2D HTI media has 16 block sub-matrices (Np=4):

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{3333} & \mathbf{H}_{3355} & \mathbf{H}_{3311} & \mathbf{H}_{3313} \\ \mathbf{H}_{5533} & \mathbf{H}_{5555} & \mathbf{H}_{5511} & \mathbf{H}_{5513} \\ \mathbf{H}_{1133} & \mathbf{H}_{1155} & \mathbf{H}_{1111} & \mathbf{H}_{1113} \\ \mathbf{H}_{1333} & \mathbf{H}_{1355} & \mathbf{H}_{1311} & \mathbf{H}_{1313} \end{bmatrix}$$

The multi-parameter Gauss-Newton Hessian measures the correlation of the partial derivative wavefields with respect to different physical parameters:

$$\tilde{\mathbf{H}}_{\mathbf{m}_1\mathbf{m}_2}(\mathbf{r},\mathbf{r}') = \frac{\partial\mathbf{u}^\dagger}{\partial\mathbf{m}_1(\mathbf{r})}\frac{\partial\mathbf{u}^*}{\partial\mathbf{m}_2(\mathbf{r}')}$$

where when $\mathbf{m}_1=\mathbf{m}_2$, it indicates the diagonal blocks, when $\mathbf{m}_1\neq\mathbf{m}_2$, it indicates the off-diagonal blocks. The off-diagonal blocks measure the correlations of different physical parameters.

Numerical Experiments

In this numerical example, we examine the analytic and numerical scattering patterns of the elastic constants for parameter cross-talk analysis.

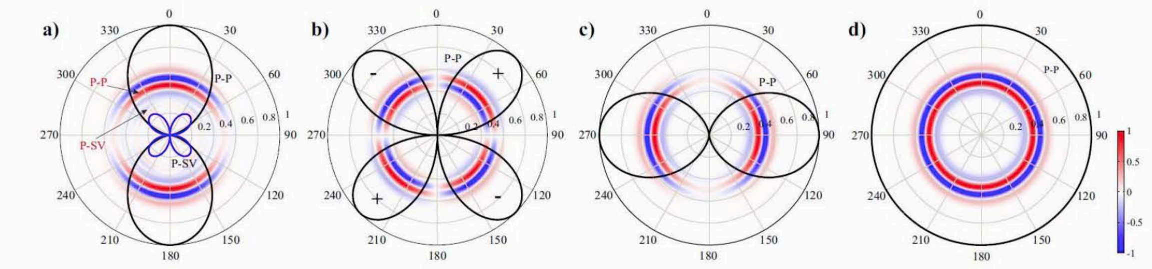

Figure 3. Analytic vs. numerical results of the scattering patterns for the elastic constants in 2D HTI media. (a), (b), (c) and (d) show the scattering patterns due to elastic constants c_{33}, c_{55}, c_{11} and c_{13} respectively.

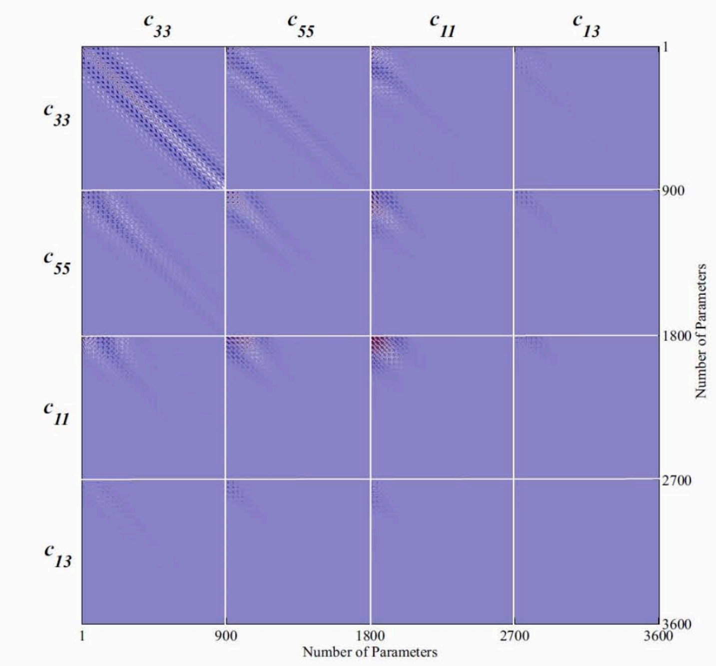

Figure 4. The multi-parameter approximate Hessian for elastic constants c_{33}, c_{55}, c_{11} and c_{13} respectively.

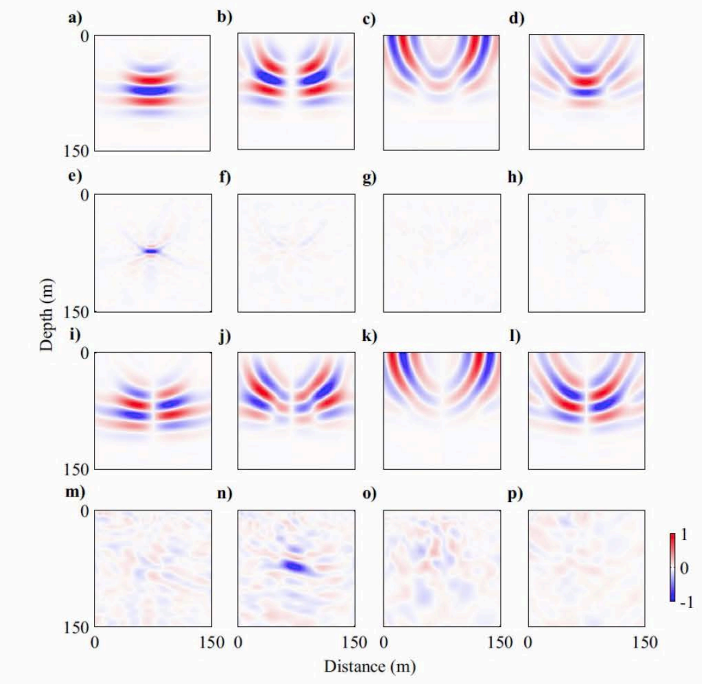

Figure 5. (a), (b), (c) and (d) show the gradient updates for the 4 elastic constants with $\Delta\mathbf{d}_{33}$. (e), (f), (g) and (h) show the corresponding Gauss-Newton updates. (i), (j), (k) and (l) show the gradient updates for the 4 elastic constants with $\Delta\mathbf{d}_{55}$. (m), (n), (o) and (p) show the corresponding Gauss-Newton updates.

Conclusions

In this research, we are trying to inverse the elastic constants in HTI media using full-waveform inversion method. We analyze the parameter cross-talk difficulty in multi-parameter FWI. Then, the role of multi-parameter approximate Hessian in suppressing the parameter cross-talk is revealed. In the numerical section, we illustrate the analytic and numerical scattering patterns of the elastic constants. We also present numerical examples to show that the multi-parameter Gauss-Newton Hessian can mitigate parameter cross-talk problem.

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