Joint inversion of PP- wave and PSV- wave data for P- wave and S- wave inverse quality factors at different frequencies — Synthetic test

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Abstract

We demonstrate a method to estimate P-wave and S-wave velocities, density, and inverse quality factors from PP- wave and PSV- wave seismic data at different frequencies. We first employ the extension of Mavko- Jizba squirt relations for all frequencies to calculate P- wave and S- wave velocities and quality factors to find how the velocity and quality factor change with frequency in seismic frequency range, and then we derive the approximate expressions of PP- wave and PSV- wave reflection coefficients in terms of P-wave and S-wave impedances, density, and inversion quality factors to discuss how the real and imaginary parts of PP- wave and PSV- wave reflection coefficients vary with frequency and the angle of incidence. Base on the derived approximate reflection coefficients, we propose a method to use seismic data to predict elastic properties (P- wave and S- wave impedances, and density) and inverse quality factors. Singular value decomposition (SVD) algorithm is used to solve the inversion problem, and initial models are utilized to constraint the inversion. We illustrate the inversion method with synthetic data, and we also test the robustness of the inversion method.

Theory and method

1 The expression of complex stiffness perturbation

The propagation factors for the elastic and viscoelastic media are respectively given by Aki and Richards (2002) as

$$\exp\left[i\left(kx-\omega t\right)\right]=\exp\left[i\omega\left(\frac{x}{V_e}-t\right)\right]$$

$$\exp\left[i\left(Kx-\omega t\right)\right]=\exp\left[\frac{-\omega x}{2V(\omega)Q(\omega)}\right]\exp\left[\frac{i\omega x}{V(\omega)}-i\omega t\right]$$

The complex P-wave and S-wave velocities are given by

$$V_{\mathrm{P\_visco}}=V_{\mathrm{P}}(\omega)\left[1-\frac{i}{2Q_{\mathrm{P}}(\omega)}\right]\qquad V_{\mathrm{S\_visco}}=V_{\mathrm{S}}(\omega)\left[1-\frac{i}{2Q_{\mathrm{S}}(\omega)}\right]$$

we use the extension of Mavko-Jizba squirt model for all frequencies (Dvorkin et al., 1995; Mavko et al., 2009) to generate the P-wave and S-wave velocities and quality factors variations with frequency and rock property (porosity and fluid).

Z value is 0.08, pore aspect ratio is 0.1, and the effective bulk modulus of dry rock at very high pressure is 50 GPa.

The complex stiffness parameters are given by

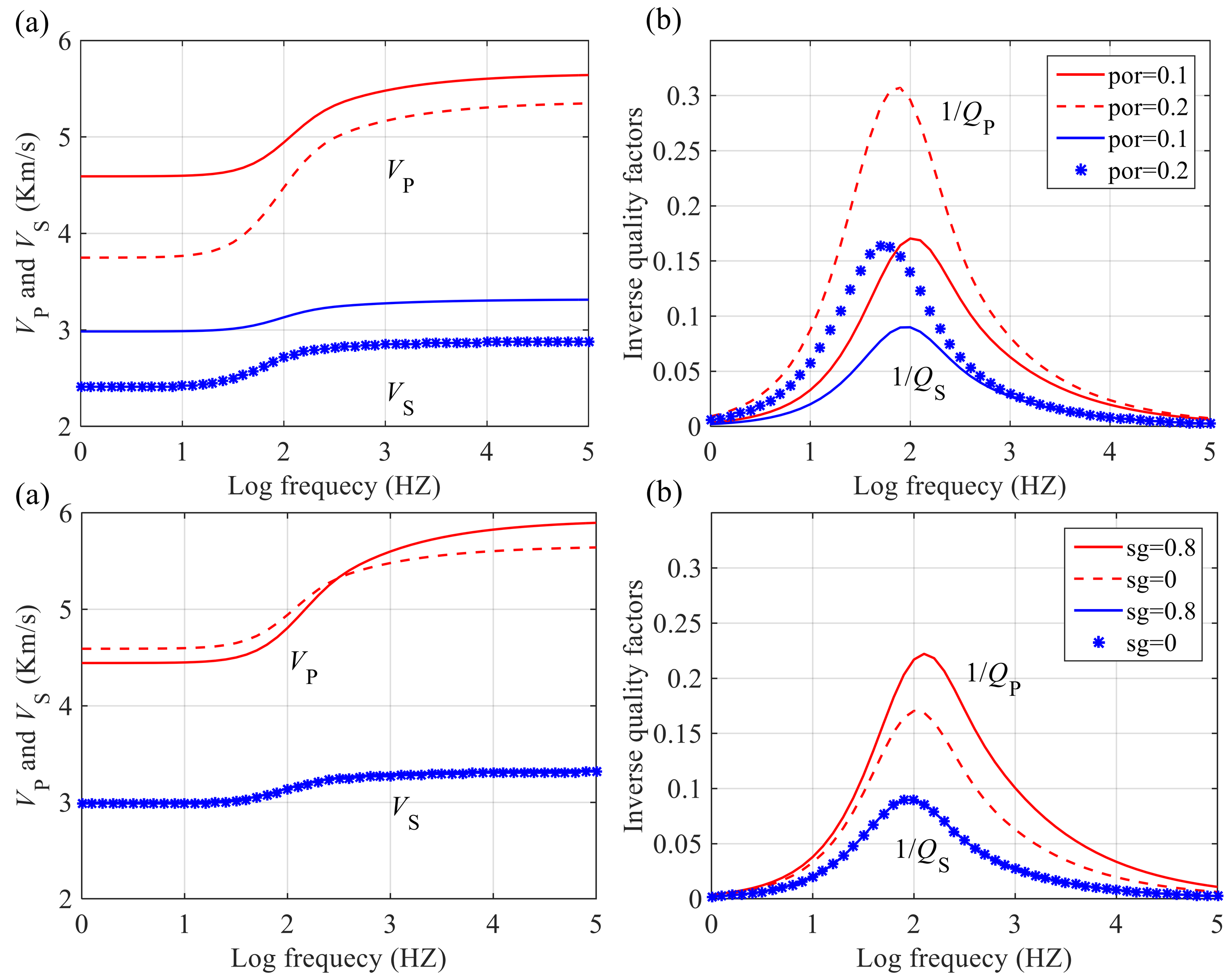
$$C_{33\_visco}=\rho\left[V_{\mathrm{P}}(\omega)\right]^2\left[1-\frac{i}{2Q_{\mathrm{P}}(\omega)}\right]^2\approx C_{33}(\omega)\left[1-\frac{i}{Q_{\mathrm{P}}(\omega)}\right]$$

$$C_{55\_visco}=\rho\left[V_{\mathrm{S}}(\omega)\right]^2\left[1-\frac{i}{2Q_{\mathrm{S}}(\omega)}\right]^2\approx C_{55}(\omega)\left[1-\frac{i}{Q_{\mathrm{S}}(\omega)}\right]$$

The perturbations of complex stiffness parameters are expressed as

$$\Delta C_{33\_visco}=\Delta C_{33}(\omega)\left[1-\frac{i}{Q_{\mathrm{P}}(\omega)}\right]+C_{33}(\omega)\frac{\Delta Q_{\mathrm{P}}(\omega)}{\left[Q_{\mathrm{P}}(\omega)\right]^2}i$$

$$\Delta C_{55\_visco}=\Delta C_{55}(\omega)\left[1-\frac{i}{Q_{\mathrm{S}}(\omega)}\right]+C_{55}(\omega)\frac{\Delta Q_{\mathrm{S}}(\omega)}{\left[Q_{\mathrm{S}}(\omega)\right]^2}i$$



2 Linearized PP-wave and PSV-wave reflection coefficients

Under the assumption of Born integral and stationary phase, Shaw and Sen (2004) derived the linearized equations to calculate PP-wave and PSV-wave reflection coefficients

$$R_{\mathrm{PP}}(\theta)=\frac{1}{4\rho\cos^2\theta}S,\qquad R_{\mathrm{PSV}}(\varphi)=\frac{\sin\theta}{2\rho\cos\varphi\sin(\theta+\varphi)}S$$

Combining the perturbations of stiffness parameters and scattering function, we derive the approximate reflection coefficients

$$\begin{aligned}R_{\mathrm{PP}}(\theta,\omega)&=\sec^2\theta R_{\mathrm{P}}(\omega)-8g\sin^2\theta R_{\mathrm{S}}(\omega)+\left(4g\sin^2\theta-\tan^2\theta\right)R_{\mathrm{D}}\\&-\sec^2\theta\left[R_{\mathrm{P}}(\omega)-\frac{1}{2}R_{\mathrm{D}}-\frac{1}{4}\frac{\Delta Q_{\mathrm{P}}(\omega)}{Q_{\mathrm{P}}(\omega)}\right]\frac{i}{Q_{\mathrm{P}}(\omega)}\\&+4g\sin^2\theta\left[2R_{\mathrm{S}}(\omega)-R_{\mathrm{D}}-\frac{1}{2}\frac{\Delta Q_{\mathrm{S}}(\omega)}{Q_{\mathrm{S}}(\omega)}\right]\frac{i}{Q_{\mathrm{S}}(\omega)}\end{aligned}$$

$$\begin{aligned}R_{\mathrm{PSV}}(\varphi,\omega)&=-4\frac{\sin\theta}{\cos\varphi}\left(\sqrt{g}\cos\theta\cos\varphi-g\sin^2\theta\right)R_{\mathrm{S}}(\omega)\\&+\frac{\sin\theta}{\cos\varphi}\left(2\sqrt{g}\cos\theta\cos\varphi-2g\sin^2\theta-1\right)R_{\mathrm{D}}\\&+2\frac{\sin\theta}{\cos\varphi}\left(\sqrt{g}\cos\theta\cos\varphi-g\sin^2\theta\right)\left[2R_{\mathrm{S}}(\omega)-R_{\mathrm{D}}-\frac{1}{2}\frac{\Delta Q_{\mathrm{S}}(\omega)}{Q_{\mathrm{S}}(\omega)}\right]\frac{i}{Q_{\mathrm{S}}(\omega)}\end{aligned}$$

Conclusion and Discussion

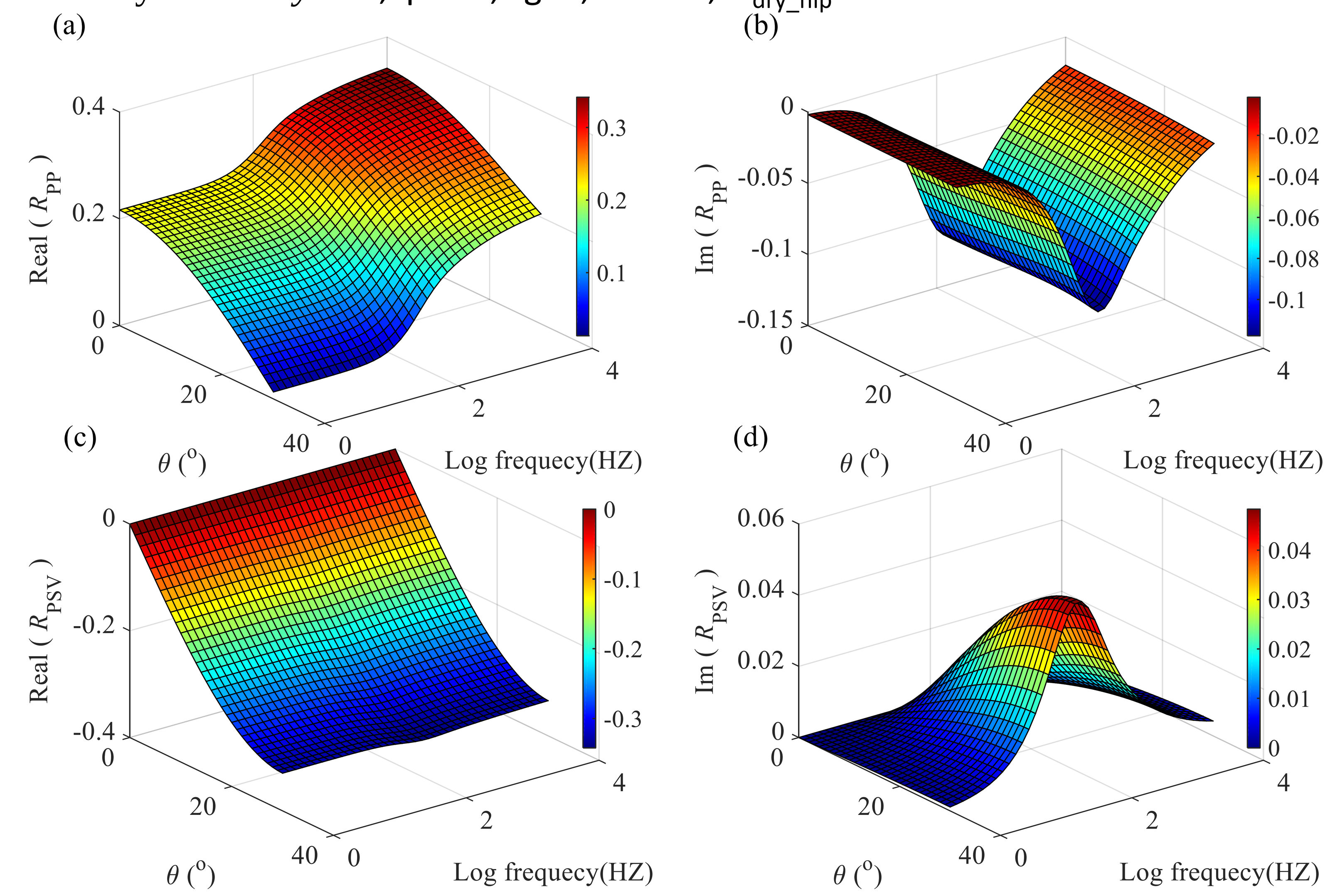
1. We derive the approximate expressions of PP- wave and PSV- wave reflection coefficients in terms of P-wave and S-wave impedances, density, and inversion quality factors.

2. We demonstrate a method to estimate inverse quality factors from PP and PSV data, and synthetic tests may verify the joint inversion method.

3. For real data, more details about the inversion should be discussed.

Layer1: V_{P1}=2898m/s, V_{S1}=1290m/s, ρ_1=2.425, $1/Q_{P1}$=0, $1/Q_{S1}$=0

Layer2: $Vclay$=0.1, ϕ=0.1, sg=1, Z=0.08, K_{dry_hip}=50GPa



3 Joint inversion of PP and PSV data for inverse quality factors

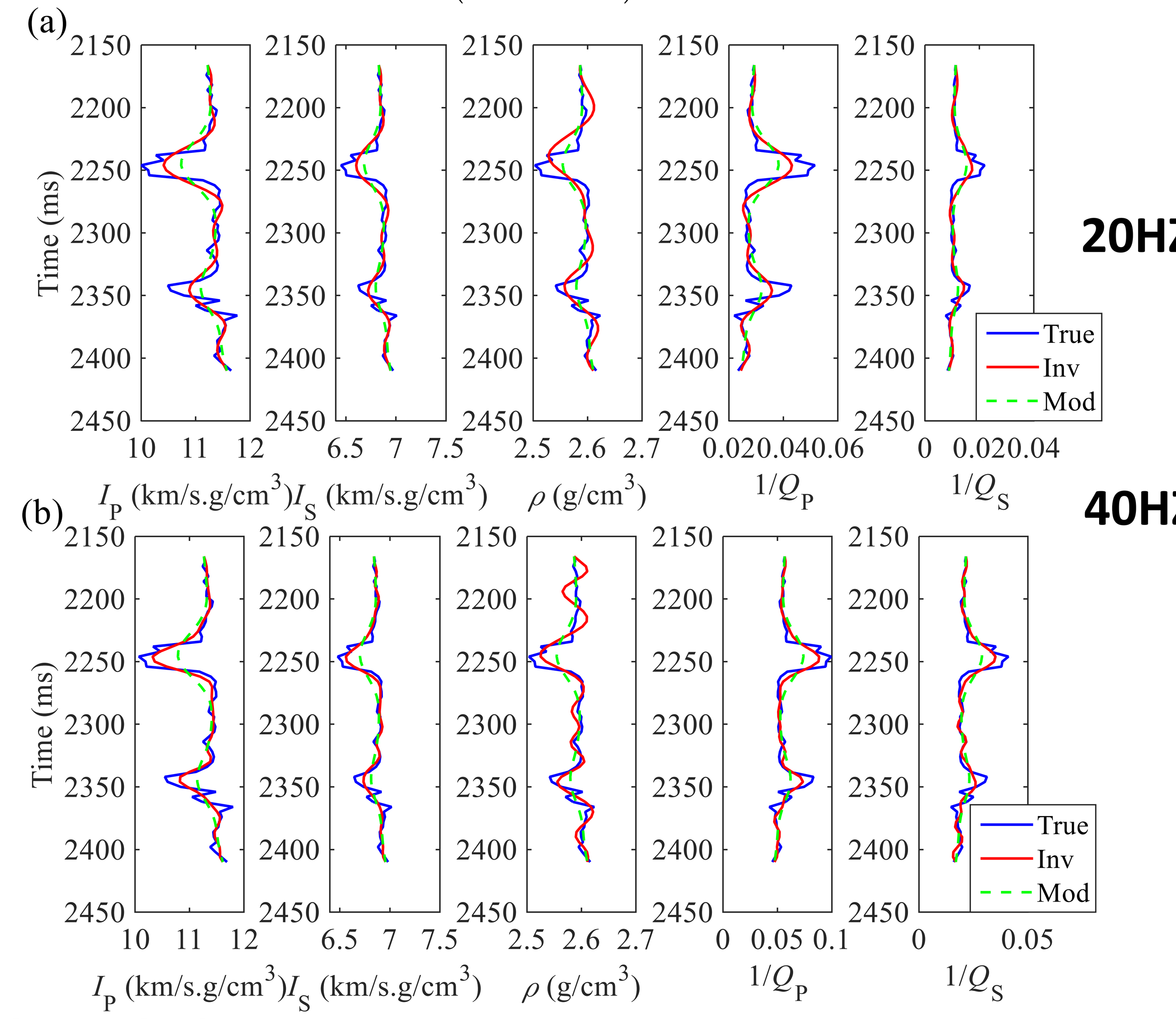
We assume that the media, which overlie and underlie the hydrocarbon reservoir, are approximately elastic ($1/Q_{P1}\approx 0$, $1/Q_{S1}\approx 0$).

Seismic data are generated by using the convolution model.

$$\begin{bmatrix}S_{\mathrm{PP}}(\theta,\omega)\\S_{\mathrm{PSV}}(\varphi,\omega)\end{bmatrix}=\begin{bmatrix}A(\theta)&B(\theta)&C(\theta)&D(\theta)&E(\theta)\\0&H(\varphi)&K(\varphi)&0&L(\varphi)\end{bmatrix}[R]$$

Joint inversion of PP and PSV data (the damped least squares method)

$$\mathbf{m}=\mathbf{m}_{\mathrm{init}}+\mathbf{V}\left(\Lambda^{\mathrm{T}}\Lambda+\varepsilon^2\mathbf{I}\right)^{-1}\Lambda^{\mathrm{T}}\mathbf{U}^{\mathrm{T}}\left(\mathbf{d}-\mathbf{G}\mathbf{m}_{\mathrm{init}}\right)$$



Acknowledgement

We thank the sponsors of CREWES for their support. We gratefully acknowledge support from NSERC (Natural Science and Engineering Research Council of Canada). We also thank the support from SINOPEC Key Lab of Multi-Component Seismic Technology.