Instantaneous Frequency Estimation using Weighted Least Squares Tikhonov Regularization with Box Constraints

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Abstract

We begin with a brief introduction into the concept of instantaneous frequency, where we use the complex analytical signal to obtain the time dependent signal phase, and differentiate the phase with respect to time to obtain the instantaneous frequency. We reformulate our problem into a weighted least squares Tikhonov regularization with box constraints, and explain the advantages of the added terms in helping to smooth and stabilize our results. We conclude with some simulation results and compare the proposed algorithm results with a well-established time-frequency distribution method.

Theory

Gabor's analytical signal $z(t)$ for a real signal $f(t)$ and its Hilbert transform $H[f(t)]$.

$$z(t)=f(t)+H[f(t)]=f(t)+ig(t). \tag{1}$$

We can use the analytical siganl to find the instantaneous angular frequency $\omega(t)$.

$$\omega(t)=\frac{d\phi(t)}{dt}=\frac{f(t)g'(t)+g(t)f'(t)}{(g(t))^2+(f(t))^2}. \tag{2}$$

Reformulating (2) into matrix format.

$$A\omega=b.\quad where\quad A=\|f\|^2+\|g\|^2\quad b=f\bullet(\frac{\Delta g}{\Delta t})-g\bullet(\frac{\Delta f}{\Delta t}) \tag{3}$$

For most real data, matrix A is ill-conditioned (high ratio between smallest and highest eigenvalue), and the measurement vector b is highly contaminated with noise. To solve these problems we use three approaches.

1. Weighted Least Squares Approach:

We reformulate our problem into a weighted least squares minimization problem with the following general solution.

$$A^T C_b A\ \hat{\omega}=A^T C_b b \tag{4}$$

$$C_b=diag[\ \mathrm{var}iance(\ noise\ )\ ] \tag{5}$$

2. Least Squares Tikhonov Regularization Approach:

We add a regularization term and reformulate our problem to obtain the following solution.

$$(A^T A+C_x)\ \hat{\omega}=A^T b \tag{6}$$

$$C_x=\alpha\ W,\ \ Where\quad \alpha=[\frac{\mathrm{var}\ (noise)}{(\Delta t)^2}]^{1/3},\qquad W=\ \ Second\ Order\ Difference\ Matrix \tag{7}$$

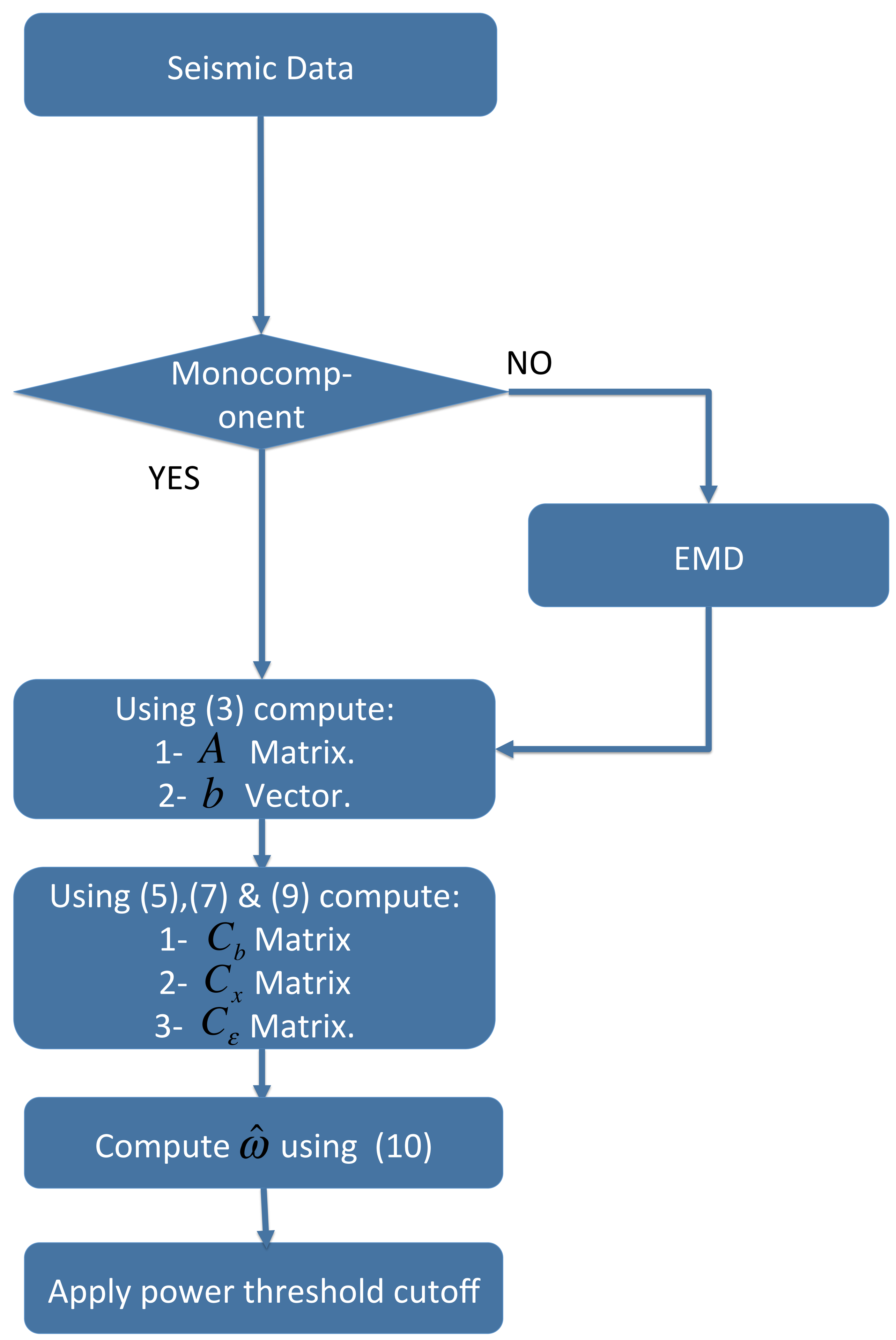
3. Box Constraints

Adding a box constraint as quadratic term with a penalty matrix C_ε as follows.

$$(A^T A+C_\varepsilon)\ \hat{\omega}=A^T b+C_\varepsilon\varpi \tag{8}$$

$$C_\varepsilon=\varepsilon\ diag[\ \frac{(\omega_{\max}-\omega_{\min})}{4}\ ],\quad \varpi=\frac{(\omega_{\max}+\omega_{\min})}{2}. \tag{9}$$

We obtain the optimal value for ε by solving the problem in a loop until the constraints are satisfied.



Combining all three approaches

By combing all three approaches we obtain a weighted least squares Tikhonov regularization with box constraints minization problem, with the following solution,

$$(A^T C_b A+C_x+C_\varepsilon)\ \hat{\omega}=A^T C_b b+C_\varepsilon\varpi \tag{10}$$

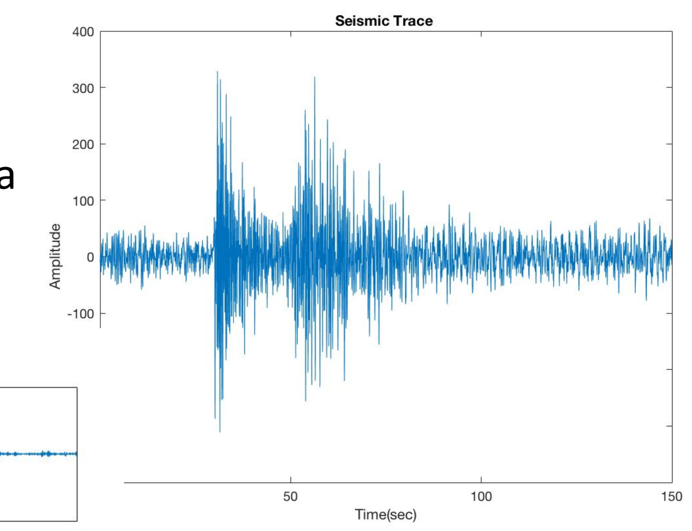
Conclusion

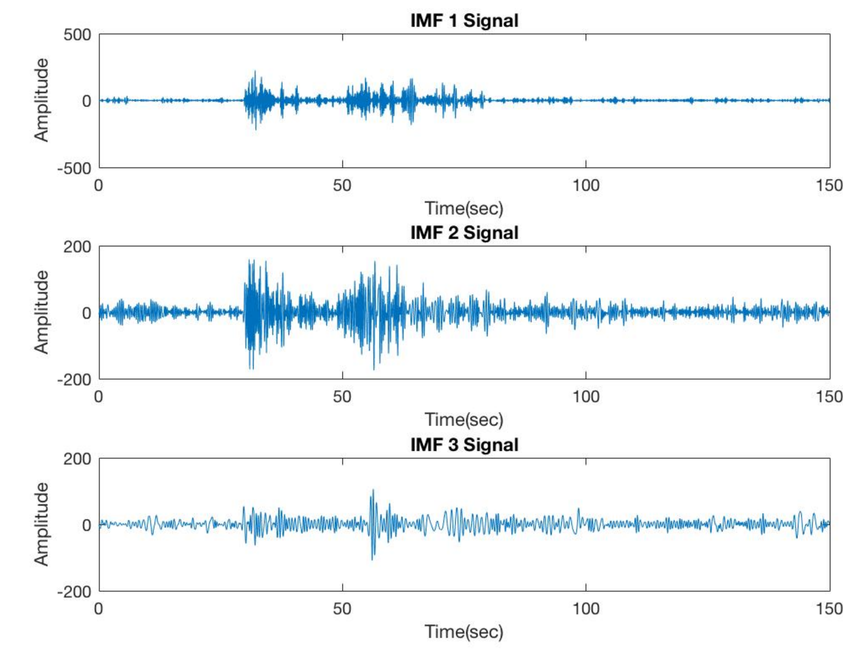
In this report, we toke a simple least squares minimization problem and enhanced it using three methods.

1. Weighted least squares.
2. Tikhonov regularization.
3. Box Constraints.

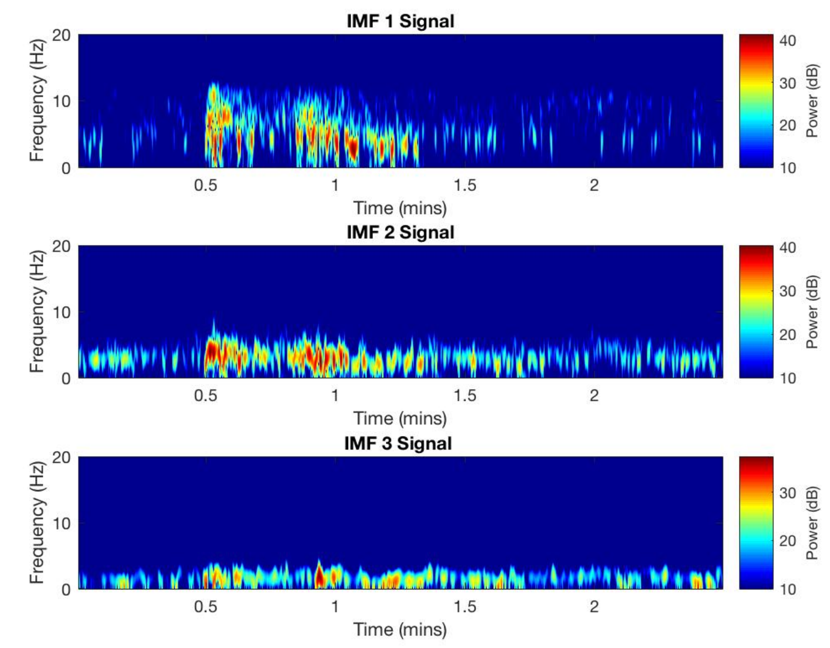
Using the proposed algorithm we were able to take quarry blast data that was heavily corrupted by noise and estimate the instantaneous frequency and obtain an accurate pick for the P and S-wave onset.

Seismic trace from quarry blast data



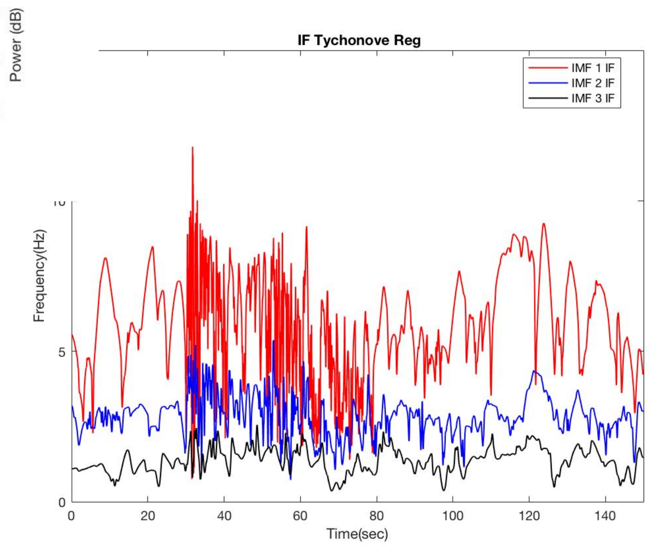


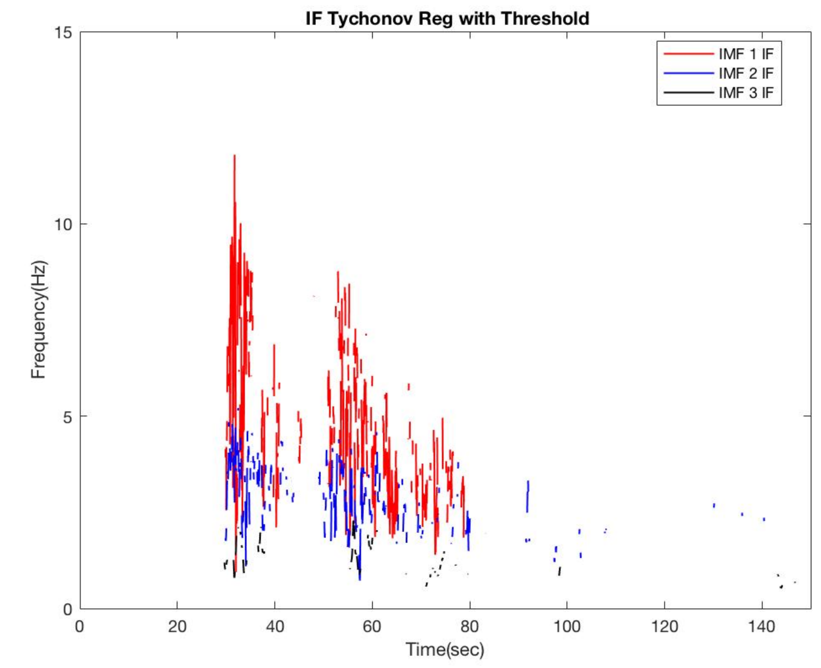
Empirical Mode Decomposition (EMD)
Producing three Intrinsic Mode Functions (IMF) signals



Spectrogram for each IMF signal

Instantaneous Frequency (IF) Estimation





Instantaneous Frequency estimation after applying power threshold cutoff

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