Linearized reflection coefficient and reflectivity modeling in fractured and attenuative reservoirs

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Abstract

Based on the complex linear slip theory, we derive complex stiffness parameters in terms of fracture weaknesses and induced attenuation factor under the assumption of the host rock being elastic and isotropic. Incorporating with the attenuative crack model, we relate the induced attenuation factor to fracture properties (fracture density and aspect ratio) and fluid parameters (fluid bulk modulus and viscosity), and study how fracture density and water saturation affect the variation of the induced factor in seismic frequency range (1-100 Hz). Using perturbations in the complex stiffness parameters, we derive a complex linearized reflection coefficient involving the induced attenuation factor and fracture weaknesses. The accuracy of the derived reflection coefficient is confirmed by comparing the result calculated using the extended reflectivity method and that computed using the derived equation. We finally use the derived linearized reflection coefficient to obtain the seismic reflection response for the case of fractured reservoirs with different values of fracture density and water saturation. We conclude that the attenuation factor is applicable to distinguishing between oil-bearing and water-bearing reservoirs, and seismic response difference induced by fracture density and water saturation increases with the incidence angle.

Theory and Method

1. Stiffness matrix related to induced attenuation

Based on the linear slip theory (Schoenberg and Douma, 1988; Schoenberg and Sayers, 1995), Chichinina (2006) proposed a complex stiffness matrix for a homogeneous and isotropic host rock with a set of parallel fractures whose normals parallel to the x_1-axis

$$\widetilde{\mathbf{C}} = \begin{bmatrix} M(1-\widetilde{\Delta_N}) & \lambda(1-\widetilde{\Delta_N}) & \lambda(1-\widetilde{\Delta_N}) & 0 & 0 & 0 \\ \lambda(1-\widetilde{\Delta_N}) & M(1-\chi^2\widetilde{\Delta_N}) & \lambda(1-\chi\widetilde{\Delta_N}) & 0 & 0 & 0 \\ \lambda(1-\widetilde{\Delta_N}) & \lambda(1-\chi\widetilde{\Delta_N}) & M(1-\chi^2\widetilde{\Delta_N}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu(1-\widetilde{\Delta_T}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu(1-\widetilde{\Delta_T}) \end{bmatrix}, \quad (1)$$

where $M = \lambda + 2\mu$, $\chi = \lambda/M = 1-2g$, $g = \mu/M$, λ and μ are Lamé constants of the homogeneous isotropic and elastic host rock, and $\widetilde{\Delta_N}$ and $\widetilde{\Delta_T}$ are the complex normal and tangential fracture weaknesses

$$\widetilde{\Delta_N} = \Delta_N - i\Delta_N^I, \quad \widetilde{\Delta_T} = \Delta_T - i\Delta_T^I, \quad (2)$$

where Δ_N and Δ_T, and Δ_N^I and Δ_T^I are the real and imaginary parts of the complex fracture weaknesses, respectively.

Carcione(2000) presented the attenuation factor $1/Q$ for each stiffness parameter of an anisotropic and attenuative medium

$$1/Q = \mathrm{Im}(\widetilde{C}_{mn})/\mathrm{Re}(\widetilde{C}_{mn}), \quad (3)$$

where Re and Im denote the real and imaginary parts of the stiffness parameter, and $\widetilde{C}_{mn}$ represents the element of the complex stiffness matrix. The complex normal and tangential fracture weaknesses are expressed as a function of the attenuation factors

$$\widetilde{\Delta_N} = \Delta_N - i\frac{1}{Q_N}(1-\Delta_N), \quad \widetilde{\Delta_T} = \Delta_T - i\frac{1}{Q_T}(1-\Delta_T). \quad (4)$$

Hudson et al. (1996) proposed an effective model to calculate stiffness matrix for an elastic solid with thin, penny-shaped ellipsoidal cracks, which involves two important parameters

$$\widetilde{U}_{11} = \frac{16}{3(3-2g)}\frac{1}{1+\Psi(\omega)}, \quad \widetilde{U}_{33} = \frac{4}{3(1-g)}\frac{1}{1+\Upsilon(\omega)}. \quad (5)$$

Theory and Method

In the case of fluid saturated cracks, Ψ and Υ are expressed as

$$\Psi(\omega) = \frac{4}{\pi}\frac{i\omega\eta_f}{\mu}\frac{1}{3-2g}, \quad \Upsilon(\omega) = \frac{1}{\pi}\frac{aK_f}{c\,\mu}\frac{1}{1-g}\frac{1}{1+3(1-i)J/(2c)}, \quad (6)$$

where c/a is the fracture aspect ratio, K_f is the bulk moduli of fluid, η_f is the fluid viscosity, ω is the angular frequency, and the quantity J is related to the host rock permeability P_m, the host rock porosity ϕ_h, the fluid viscosity η_f and the bulk modulus of the fillings K_f

$$J = \sqrt{\frac{\omega\phi_h K_f P_m}{2\eta_f}}. \quad (7)$$

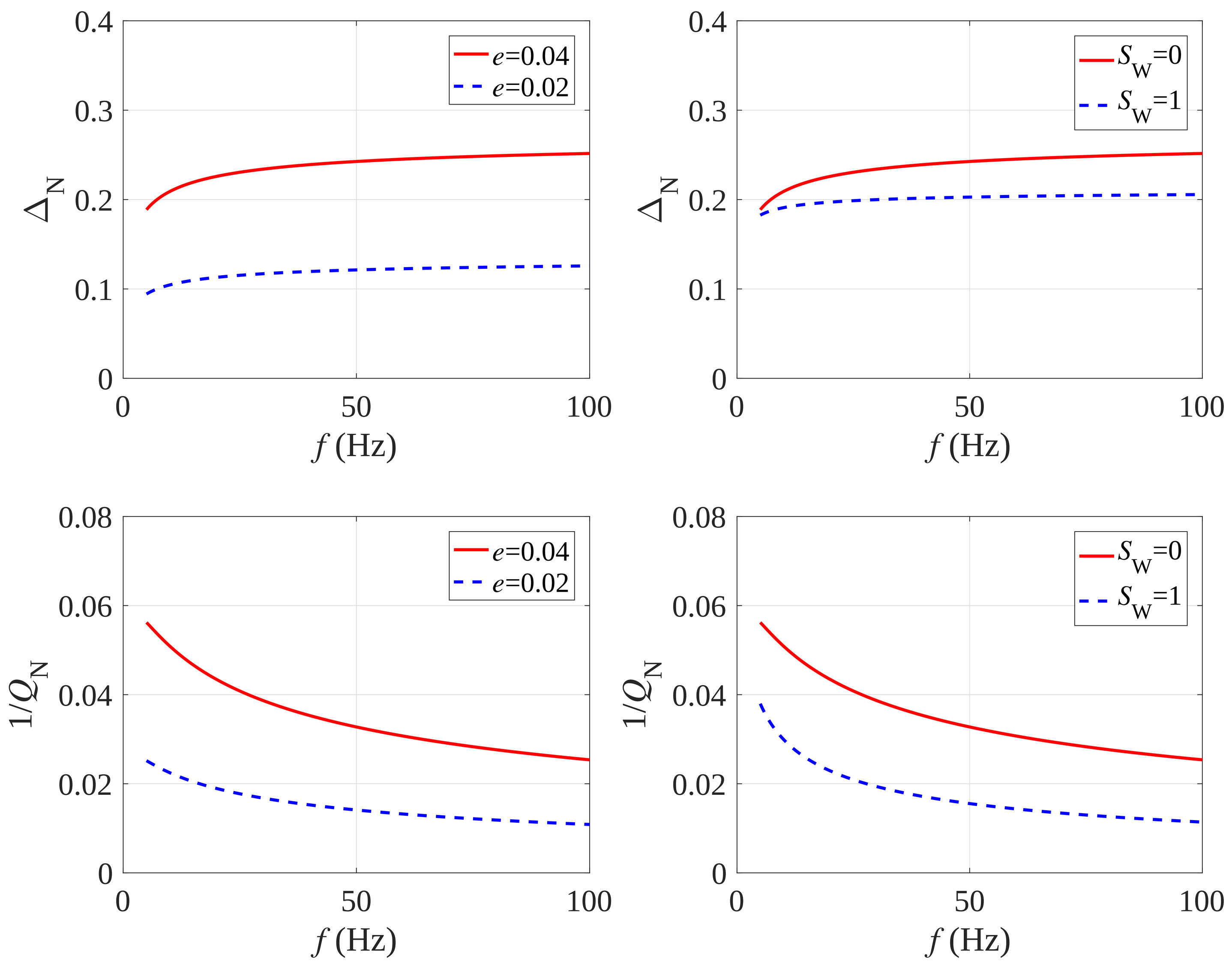

Fig 1. Variation of the normal fracture weakness and the attenuation factor with frequency given different values of fracture density and water saturation. The fracture aspect ratio $c/a = 0.005$, the host rock permeability $P_m = 0.01\mathrm{md}$, and the host rock porosity $\phi_h = 0.02$.

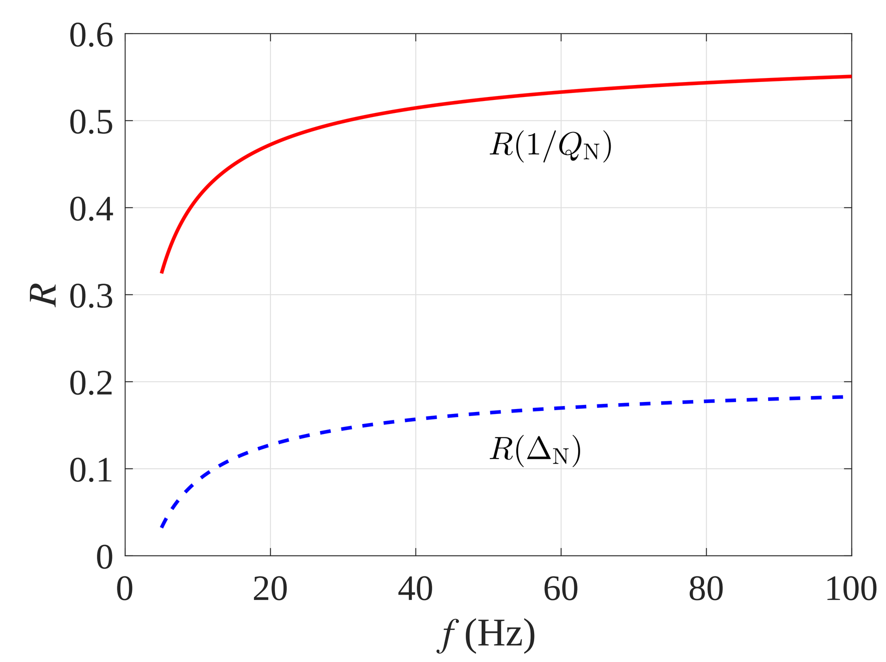

Fig 2. Absolute value of relative difference between water and oil saturated rocks for the normal fracture weakness and the attenuation factor.

2.Linearized reflection coefficient derived in terms of the attenuation factor

The relationship between PP-wave reflection coefficient R_{PP} and the scattering function S involving the perturbation in stiffness is given by (Shaw and Sen, 2004,2006)

Theory and Method

$$\begin{aligned} R_{PP} =& \frac{1}{4\rho\cos^2\theta}S \\ =& a_M(\theta)R_M + a_\mu(\theta)R_\mu + a_\rho(\theta)R_\rho \\ &+ a_{\Delta_N}(\theta,\varphi)\Delta_N + a_{\Delta_T}(\theta,\varphi)\Delta_T + a_{Q_N}(\theta,\varphi)\frac{i}{Q_N}, \end{aligned} \quad (8)$$

where

$$\begin{aligned} &a_M(\theta) = \frac{1}{2\cos^2\theta}, a_\mu(\theta) = -4g\sin^2\theta, a_\rho(\theta) = 1 - \frac{1}{2\cos^2\theta}, \\ &a_{\Delta_N}(\theta,\varphi) = -\frac{1}{4\cos^2\theta}\left[1-2g\left(\sin^2\theta\sin^2\varphi + \cos^2\theta\right)\right]^2, \\ &a_{\Delta_T}(\theta,\varphi) = -g\tan^2\theta\cos^2\varphi\left(\sin^2\theta\sin^2\varphi - \cos^2\theta\right), \\ &a_{Q_N}(\theta,\varphi) = -a_{\Delta_N}(\theta,\varphi), \end{aligned} \quad (9)$$

Numerical modeling

We proceed to forward modeling for PP-wave reflection coefficient using the derived linearized reflection coefficient in the case of different values of fracture density, water saturation and fluid viscosity. P- and S-wave moduli and density of the upper layer are $M = 60\mathrm{GPa}$, $\mu = 15\mathrm{GPa}$ and $\rho = 2.2\mathrm{g/cm^3}$, and permeability and porosity of the host rock are $P_m = 0.001\mathrm{md}$ and $\phi_h = 0.02$. We use a 35 Hz Ricker wavelet to generate seismic profiles for oil-bearing fractured and attenuative carbonate rocks with different values of fracture density.

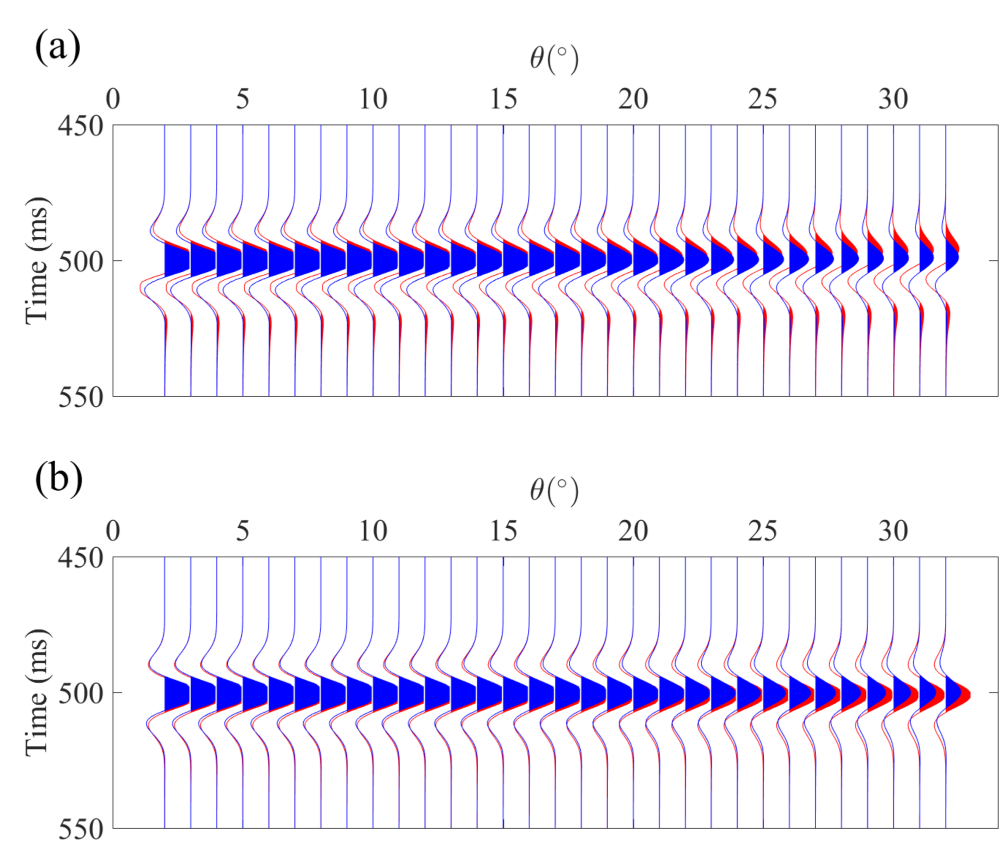

Fig 3. Seismic profiles generated for oil-bearing fractured and attenuative rocks.

Conclusions

We verify the capability for distinguishing oil-bearing and water-bearing rocks using the attenuation factor; We derive a complex linearized reflection coefficient involving the induced attenuation factor and fracture weaknesses, which is applicable to the calculation of PP-wave reflection coefficient in the case of the incidence angle being less than 30°; We conclude that seismic response difference induced by fracture density and water saturation increases with the incidence angle.

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