# Elastic modeling and reverse time migration

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## Abstract

In an elastic medium, reverse time migration (RTM) uses the velocity-stress method for the wavefield propagation and a numerical seismic source to create shot and receiver wavefields required in the imaging conditions. This essay discusses different ways to introduce a seismic source in the elastic modeling process. The seismic source in the elastic reverse time migration/RTM is normally expressed by a combination of wavelet functions in spatial and time dimensions. In most occasions, pure P/S wave sources are preferred. This paper shows that the typical way to introduce the source in RTM is not a pure P/S wave source. A better approach to approximate pure P/S modes is to use a plane source. Then we use elastic modeling process to generate P&S shot records. Then we use acoustic RTM and least squares RTM to migrate these elastic data, using P and S velocities in different experiments. In both cases, the reflectivity models obtained from RTM are very poorly defined, but the LSRTM produces a clean image because of its capability to filter out events that can not be properly predicted by the modeling operator.

## Numerical seismic source in elastic modeling

1. Numerical seismic source simulation in velocity-stress method

The velocity-stress method is a way to compute wavefield propagation in the elastic medium where stress tensor $\sigma_{xx}$, $\sigma_{xz}$, $\sigma_{zz}$, and velocities $v_x$,$v_z$ are computed by the finite difference in a staggered grid.

Typically the pure P wave source is simulated as follows:

$$\begin{bmatrix} f_x \\ f_z \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xz} \\ \sigma_{zx} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_z \end{bmatrix}, \quad (1)$$

where $f_x$, $f_z$ are the stress components (surface forces), $\sigma_{xx}$, $\sigma_{xz}$, $\sigma_{zz}$ are the components of the stress tensor, $n_x$, $n_z$ are the unit direction of the force. A pure P wave source stress tensor is commonly set as

$$\begin{bmatrix} f_x \\ f_z \end{bmatrix} = \begin{bmatrix} wlt & 0 \\ 0 & wlt \end{bmatrix} \begin{bmatrix} \sin(\theta) \\ \cos(\theta) \end{bmatrix}, \quad (2)$$

where wlt is the wavelet. If the stress tensor satisfies the equations above, the value of stresses is the same in all directions. But in physics, the seismic source is an external body force instead of a stress, which could be a source of inaccuracy.

An improvement on this approach is as follows: a seismic source acts on a velocity medium as an external force, so if there is a seismic source, equations of velocity-stress method shall change to

$$\rho\frac{\partial^2 u_x}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} + f_x, \qquad \rho\frac{\partial^2 u_z}{\partial t^2} = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} + f_z, \quad (3)$$

where $f_z$ and $f_z$ are external forces of source in x and z direction.

2. Dilatation and rotation separation

The P/S wave components can be calculated by Helmholtz decomposition,

$$u_p = \nabla \cdot u = \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}, \qquad u_s = \nabla \times u = \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right)e_y, \quad (4)$$

where $u_p$ is dilatation and $u_s$ is rotation, $e_y$ is the direction of the curl decided by right-hand rule, which is perpendicular to the x-z surface.

For a pure P wave source the $u_s$ component is zero and for a pure S wave source, the $u_p$ component is zero. A source is a pure P wave source if:

$$\frac{\partial u_x}{\partial z} = \frac{\partial u_z}{\partial x}, \quad (5)$$

which means that for a pure P wave source, the change of $u_x$ in z-direction must be the same as the change of $u_z$ in the x-direction.

## Numerical seismic source in elastic modeling

In the following tests, the first order derivatives are approximated by two points finite difference method. In order to test whether the P/S wave contains S/P wave component or not, the divergence and curl in the wavefields are computed at the start of the propagation, during which only source points have displacement and all the others are zero.

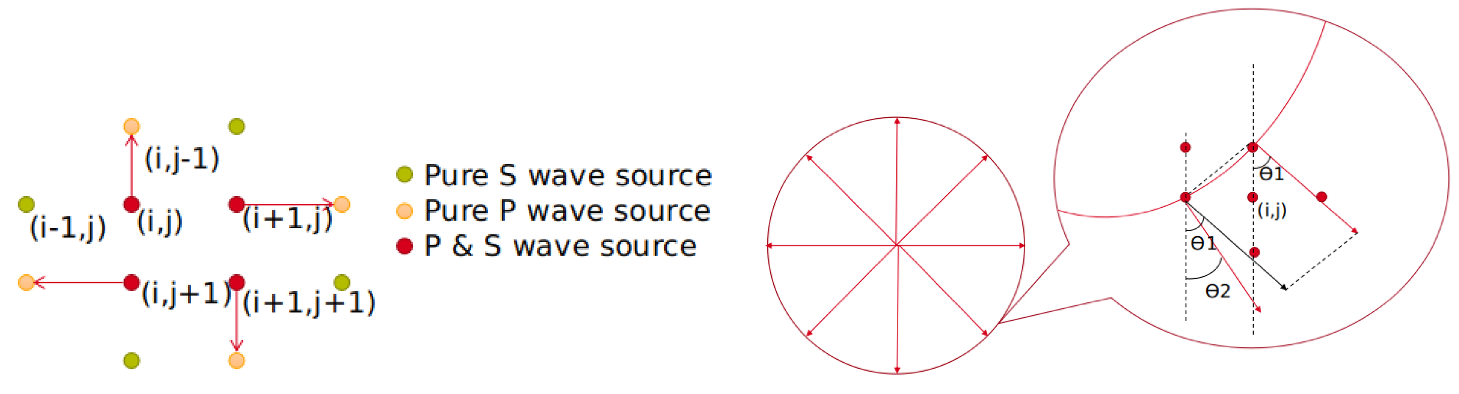



Fig 1. P/S wave components for initial guess of P/S wave source .

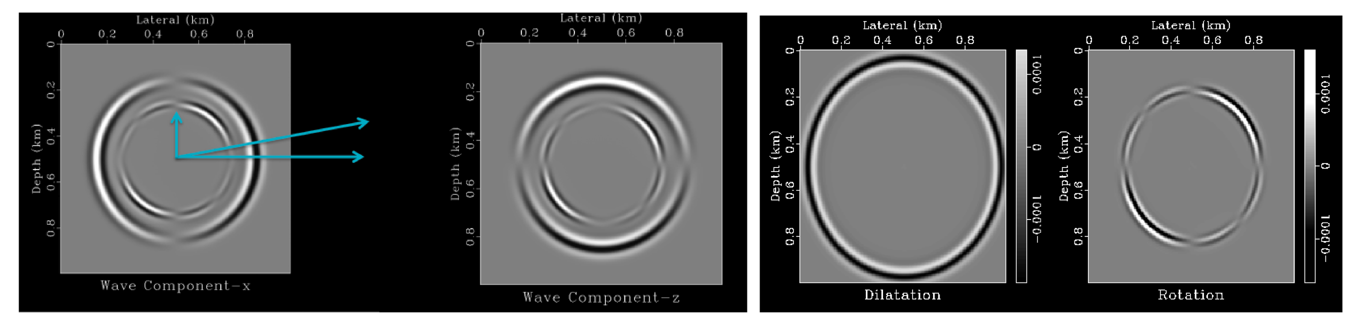



Fig 2. Velocity and P/S component for initial guess of pure P source in equation 2.

Figure 2 shows that Helmholtz decomposition can separate P and S wave perfectly.

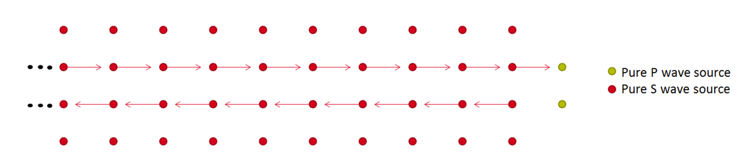



Fig 3. An example of plate source.

Figure 3 shows an improved seismic source. It is a plate source that can produce pure S wave. But P wave exists on the two ends of the plate.

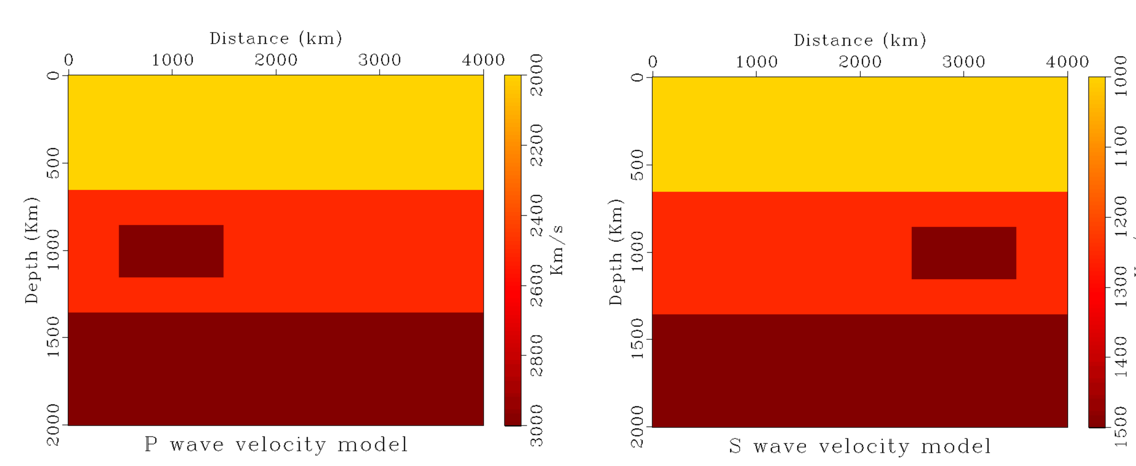



Fig 4. P and S wave velocity model.

We generated several elastic shot records using Finite-Difference method in staggered grid.

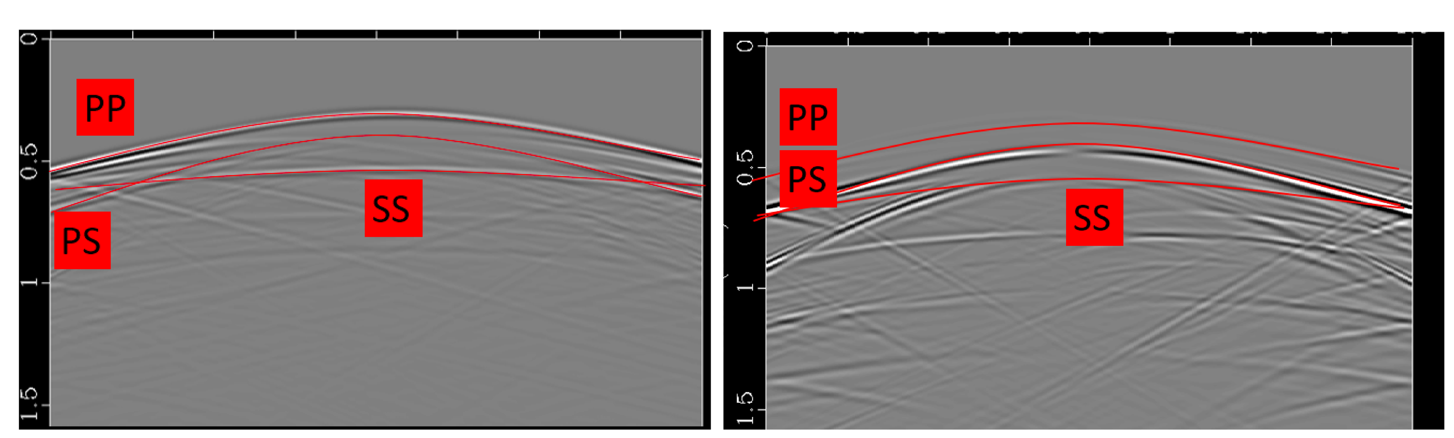
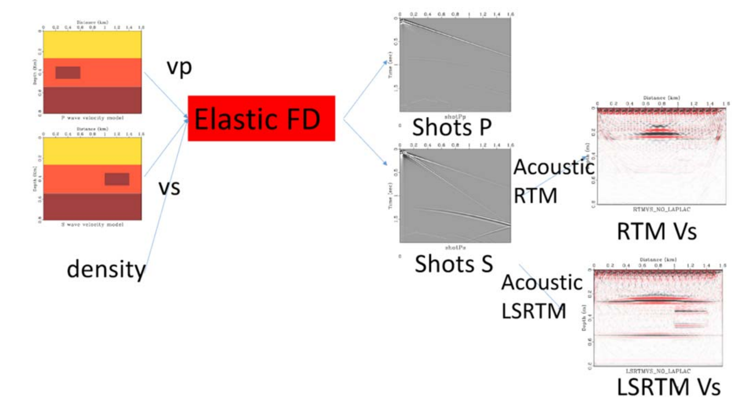



Fig 5. P&S shot records from elastic modeling.

In Figure 5 we see the P&S wave record produced by reflection waves. According to equation 4, the Helmholtz decomposition could separate P&S wave components provided from elastic modeling.

## Reverse time migration

Because of the high computational cost of elastic RTM and LSRTM shown in chart below, we use the acoustic RTM and LSRTM for records generated from elastic modeling. We created vp and vs model in Figure 4 in order to distinguish the effects of P and S wave,




| cost | 2D | 3D |
|---|---|---|
| Acoustic RTM | 1 | |
| Acoustic LSRTM | 20 | |
| Elastic RTM | 5 | 9 |
| Elastic LSRTM | 100 | 180 |

Fig 6. Workflow of RTM and cost comparison for 9 iterations

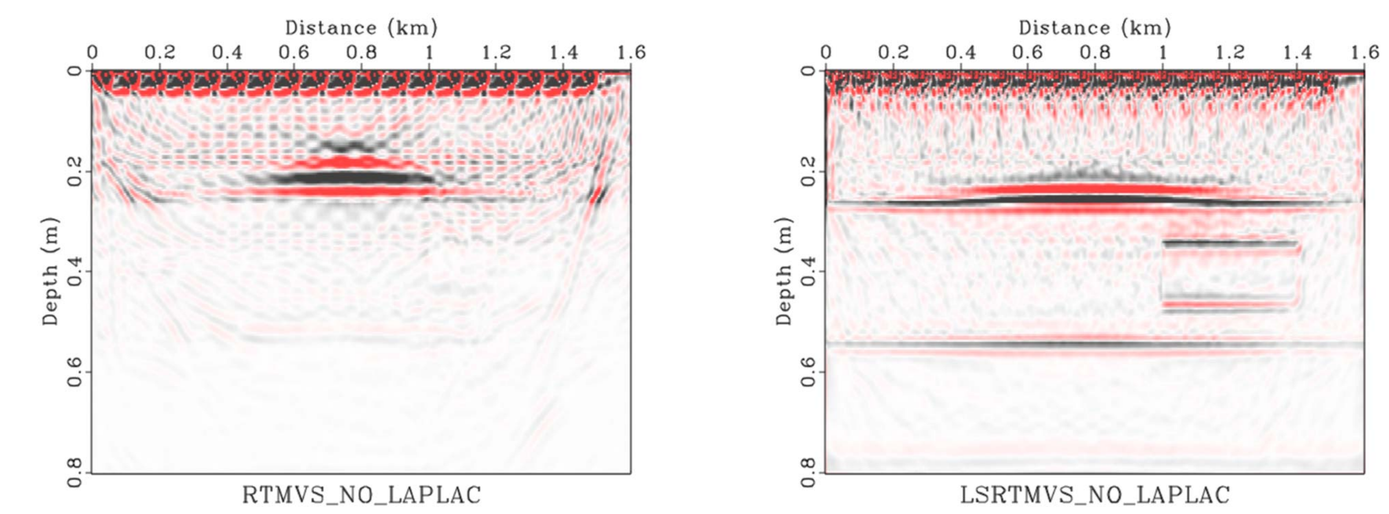



Fig 7. The RTM and LSRTM outputs from rotation shot records. They both contain PS and SS images. PP waves exists on the top.

It can be seen that least squares improve the image significantly.

## Conclusions

In an elastic media, the velocity-stress method is commonly utilized to compute various wavefields, which requires a source that preferentially should be a pure P/S source. The usual and simplest method to create a pure P/S wave source is not precise. One possible reason is that the seismic source is created by an external body force instead of stress, which is a surface force. Another reason is that a point source is not accurate to create a pure P/S source compared with a plate source according to the definition of Helmholtz decomposition. The plate source satisfies the condition that there's no dilatation for pure S wave source and no rotation for pure P wave source. But it fails at the two ends of the plate. Helmholtz decomposition is applied in the shot record to separate P and S wave component from velocities gathered from the geophone. It can't separate dilatation and rotation perfectly but the residual can be ignored in the rotation wavefield. To investigate the energy content of the synthetic data, on each case, we use acoustic RTM and LSRTM to calculate the reflectivity images of the elastic medium. As an interesting consequence of these tests, we can see that for the elastic shot records that contain information of vp, vs, and density, S wave acoustic LSRTM could filter the unwanted velocity if the correct velocity model is given.

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