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Waveform inversion combining one-way and two-way wave-equation migration

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Summary

Growing out from FWI, iterative modelling, migration, and inversion (IMMI) considers waveform inversion as a cyclical process of the migration and standard inversion. In IMMI, any type of depth migration is available, which gives greater convenience to waveform inversion. In this paper, we examine IMMI in the absence of well log data. we introduce how to choose impedance inversion algorithms in IMMI for different depth migration algorithms. In our research, the one-way depth migration algorithm used is phase shift plus interpolation (PSPI) migration, and the two-way depth migration algorithm used is reverse time migration (RTM). Built on this, we develop a combined IMMI method which uses the one-way depth migration and the two-way depth migration sequentially in IMMI. To do comparisons between FWI, IMMI using PSPI migration, IMMI using RTM, and the combined IMMI method, two numerical examples are used. The comparisons show that IMMI using RTM and using PSPI are better than FWI, and the best wave to implement waveform inversion in the absence of well log data is the combined IMMI method.

FWI

The L2 norm of data residual:

$$E(\mathbf{m}) = \frac{1}{2}\delta\mathbf{u}^T\delta\mathbf{u} \qquad (1)$$

Linearized formula:

$$E(\mathbf{m}_0+\Delta\mathbf{m}) = E(\mathbf{m}_0)+\frac{\partial E(\mathbf{m}_0)}{\partial \mathbf{m}}\Delta\mathbf{m} \qquad (2)$$

$$\frac{\partial}{\partial\mathbf{m}} \curvearrowright \quad \frac{\partial E(\mathbf{m})}{\partial\mathbf{m}} = \frac{\partial E(\mathbf{m}_0)}{\partial\mathbf{m}} + \frac{\partial^2 E(\mathbf{m}_0)}{\partial\mathbf{m}^2}\Delta\mathbf{m} \qquad (3)$$

The model perturbation:

$$\Delta\mathbf{m} = -\mathbf{H}^{-1}\mathbf{g} \Longrightarrow \Delta\mathbf{m} = -\mu\mathbf{g} \qquad (4)$$

Hessian's inversion is hard to calculate

Step length is calculated by line search

IMMI

1. Prepare the data
2. Build initial background model as a very smooth migration model
3. Create synthetic seismic data with the current model and the geometry of the real seismic data using the current wavelet estimate
4. Migrate the data difference with a prestack depth migration (any depth migration is avaible)
   = reflectivity residual
5. Convert the migrated stack into a velocity update
   = impedance inversion, line search
6. Update both velocity model

For a one-way wave-equation migration

$$\Delta\ln s \approx -2R_i \Longrightarrow \Delta s = s_0\exp\left\{F^{-1}\left\{\left(-2\frac{\tilde{R}(\omega)}{j\omega}\right)\right\}\right\} - \bar{s} \qquad (5)$$

For a two-way wave-equation migration

$$R_i = -\frac{s_{i+1}-s_i}{s_{i+1}+s_i} = -\frac{\Delta s}{2s} \Longrightarrow \Delta s = -2sR_i \qquad (6)$$

Difference between FWI and IMMI

FWI

$$\Delta s_{FWI}(\mathbf{x}) = \mu\sum_{r=1}^{ng}\sum_{s=1}^{ns}\int_0^{t_{max}} dt\left[2s(\mathbf{x})\frac{\ddot{u}_{syn}(\mathbf{x},t;\mathbf{x}_s)\delta u(\mathbf{x},t;\mathbf{x}_s)}{u_{syn}(\mathbf{x},t;\mathbf{x}_s)u_{syn}(\mathbf{x},t;\mathbf{x}_s)+\lambda I_{max}}\right] \qquad (7)$$

IMMI using PSPI migration

$$\Delta s_{PSPI}(\mathbf{x}) = \mu\sum_{r=1}^{ng}\sum_{s=1}^{ns} imp\left(\int_0^{t_{max}} dt\left[\frac{u_{syn}(\mathbf{x},t;\mathbf{x}_s)\delta u(\mathbf{x},t;\mathbf{x}_s)}{u_{syn}(\mathbf{x},t;\mathbf{x}_s)u_{syn}(\mathbf{x},t;\mathbf{x}_s)+\lambda I_{max}}\right]\right) \qquad (8)$$

IMMI using RTM

$$\Delta s_{RTM}(\mathbf{x}) = \mu\sum_{r=1}^{ng}\sum_{s=1}^{ns}\left(\int_0^{t_{max}} dt\left[-2s(\mathbf{x})\frac{u_{syn}(\mathbf{x},t;\mathbf{x}_s)\delta u(\mathbf{x},t;\mathbf{x}_s)}{u_{syn}(\mathbf{x},t;\mathbf{x}_s)u_{syn}(\mathbf{x},t;\mathbf{x}_s)+\lambda I_{max}}\right]\right) \qquad (9)$$

The combined IMMI

The first several iterations: $\Delta s_{PSPI}(\mathbf{x})$

The remaining iterations: $\Delta s_{RTM}(\mathbf{x})$

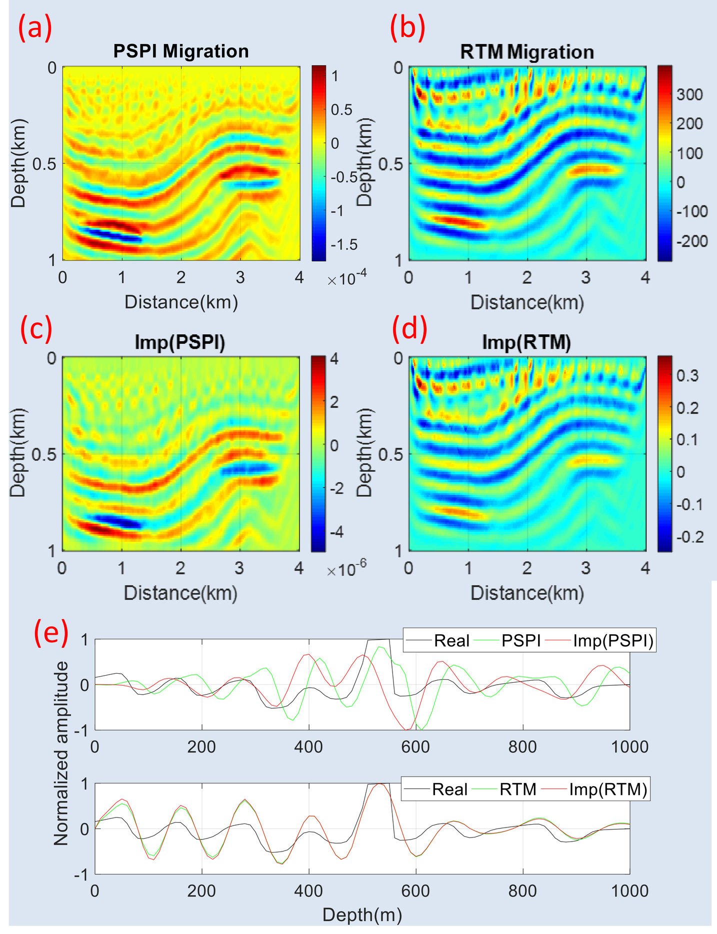


FIG. 1.
(a)The result of PSPI migration of the data residual,
(b) the result of RTM of the data residual,
(c) the impedance inversion result of (a),
(d) the impedance inversion result of (b),
(e) are the results of (c) and (d) at trace 3km.

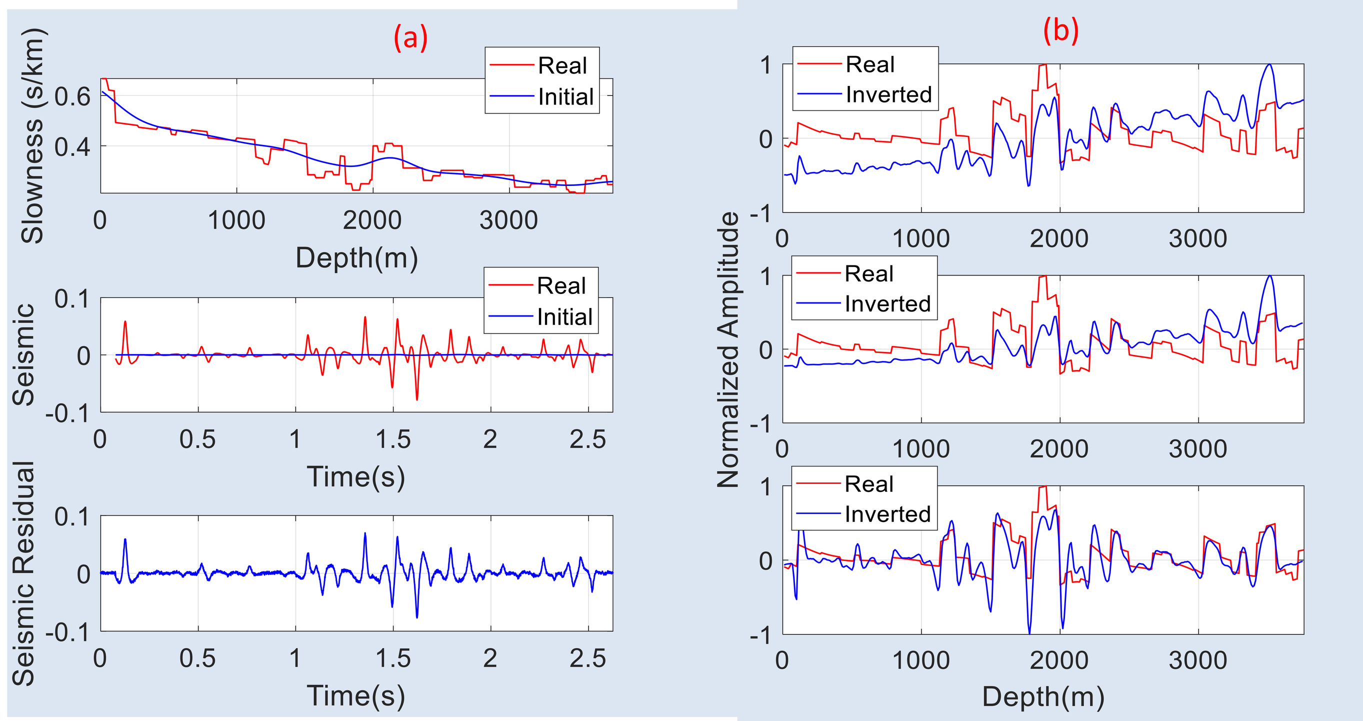
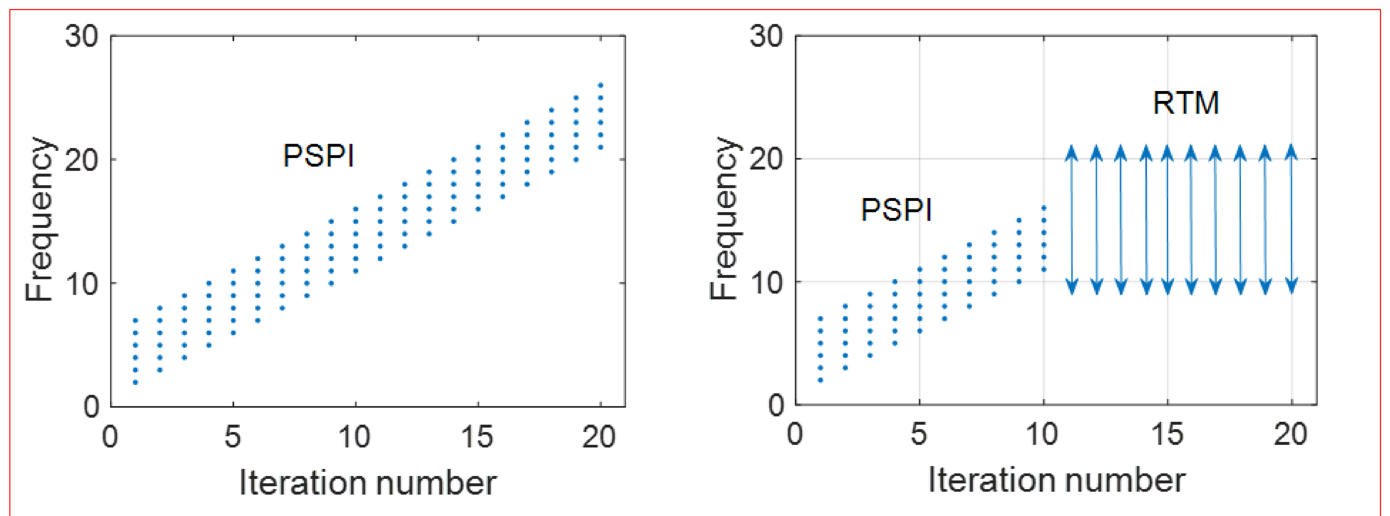


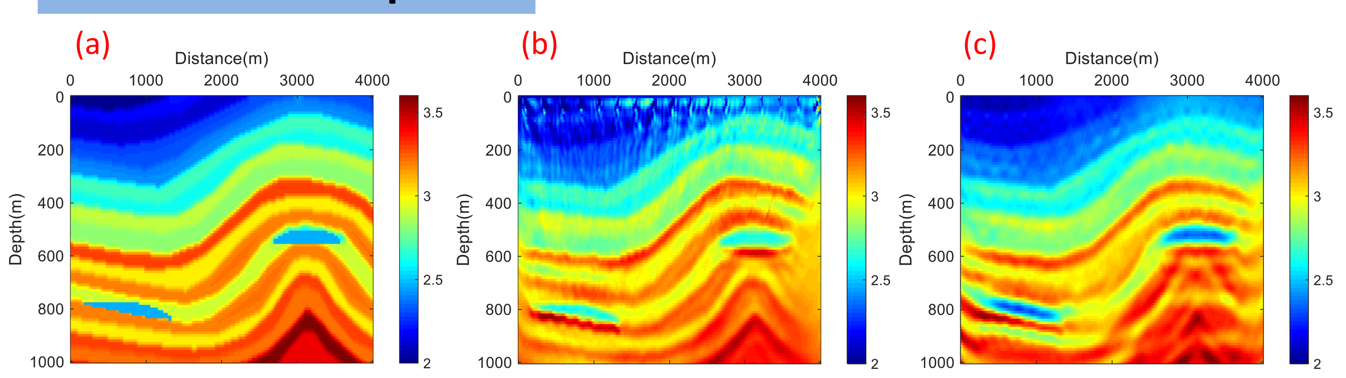
FIG. 2.
(a) The 1D model and synthetic seismic data.
(b) Trace integration results in depth, time and frequency domain from up to down, respectively.



The multiscale strategy for IMMI using PSPI migration

The strategy in frequency for the combined IMMI

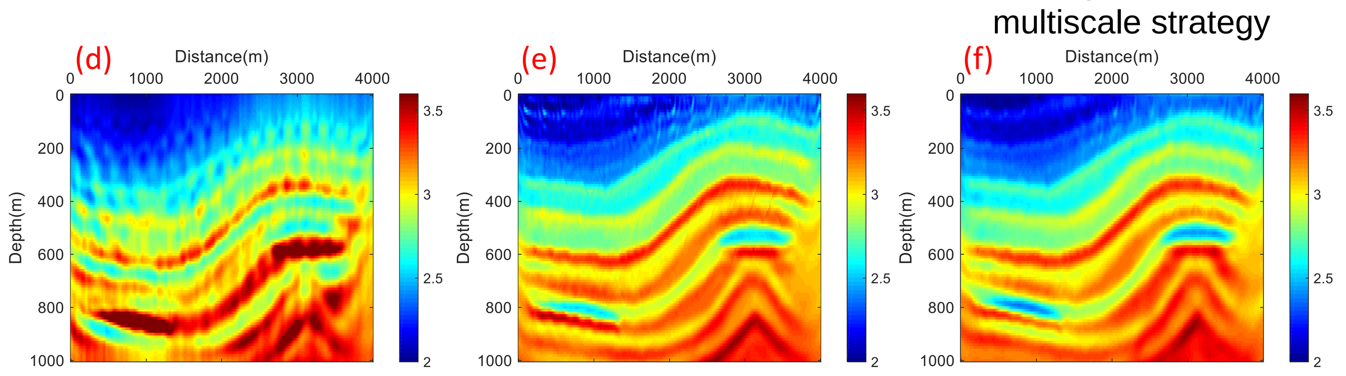
Numerical examples 1



(a) The true model

(b) The result of FWI

(c) The result of IMMI using PSPI migration and the multiscale strategy

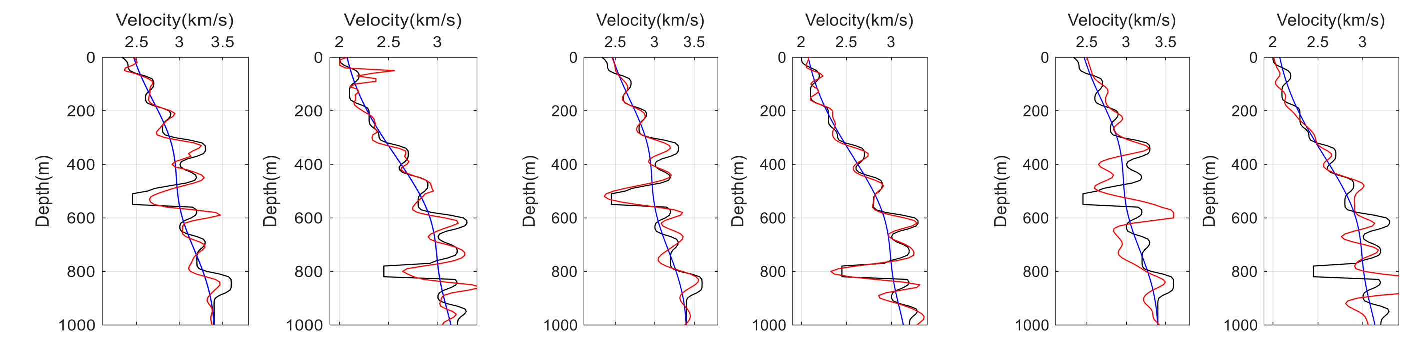


(d) The result of IMMI using PSPI migration but no use of the multiscale strategy

(e) The result of IMMI using RTM

(f) The result of the combined IMMI

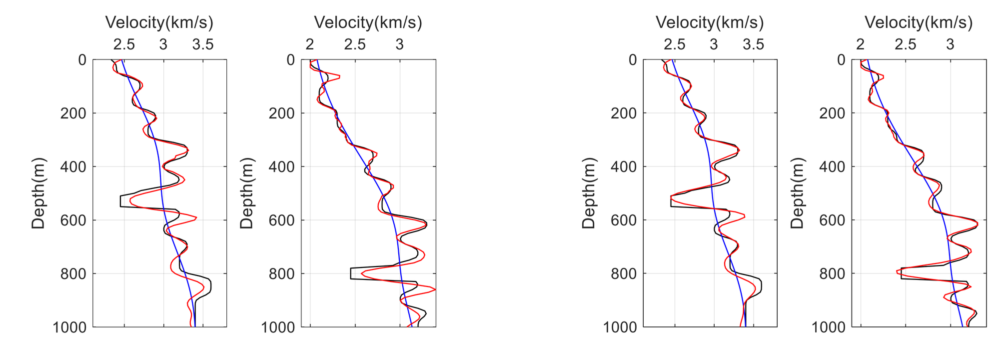
The results at traces 860m and 3160 m of (b)-(f)



Results of (b)

Results of (c)

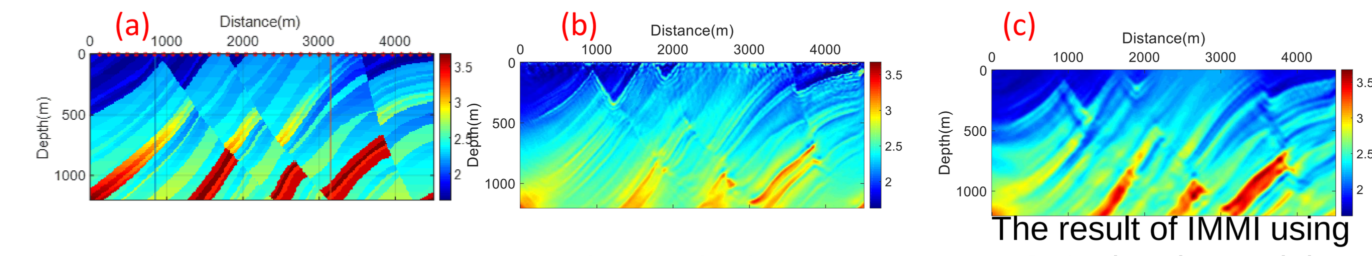
Results of (d)



Results of (e)

Results of (f)

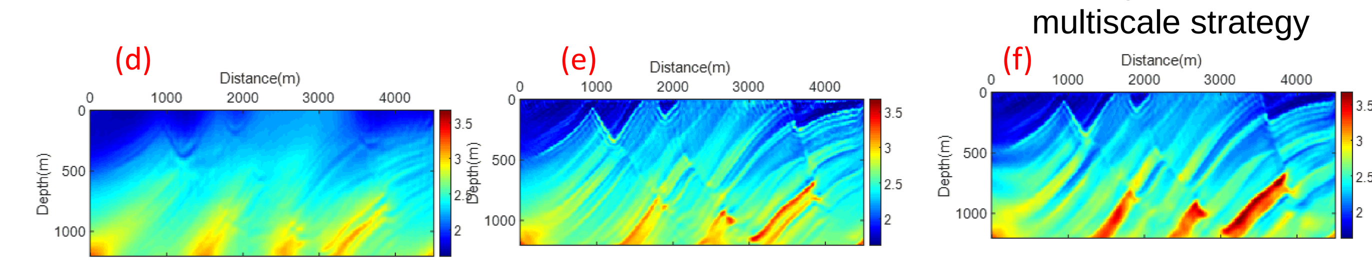
Numerical examples 2



(a) The true model

(b) The result of FWI

(c) The result of IMMI using PSPI migration and the multiscale strategy



(d) The result of IMMI using PSPI migration but no use of the multiscale strategy

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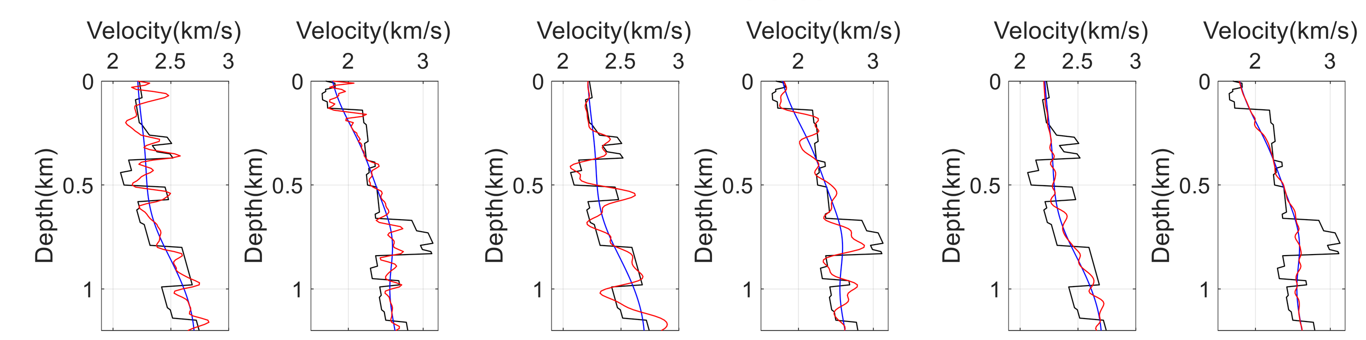
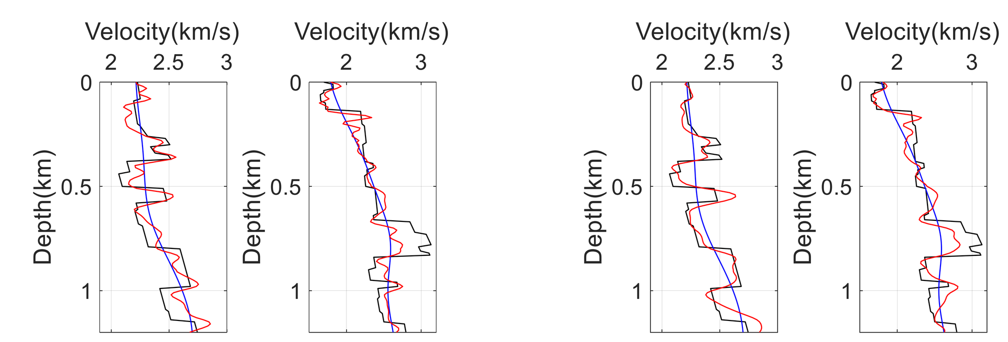


Results of (b)

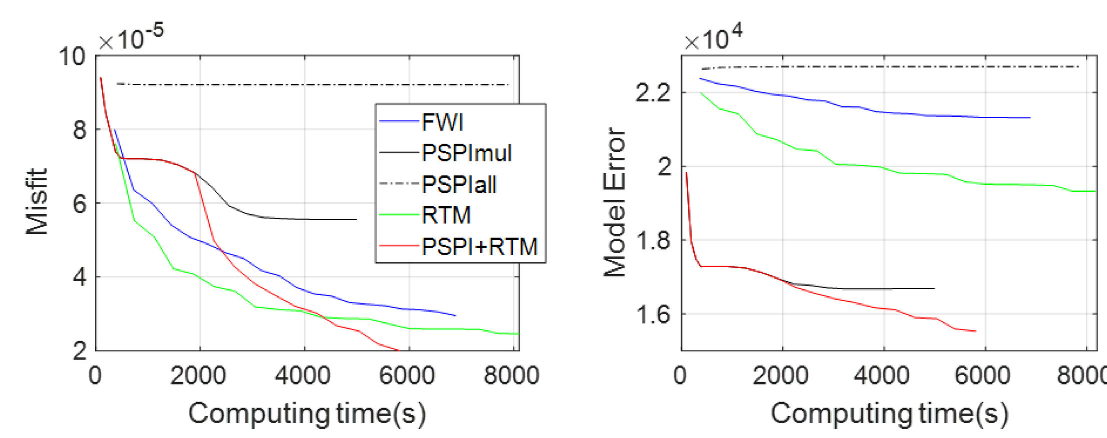
Results of (c)

Results of (d)



Results of (e)

Results of (f)



Data misfit versus computing time

Model error versus computing time

Conclusion

- The combined IMMI is the fastest and can provides the best inverted model
- either FWI or IMMI using RTM cannot obtain a good near surface inversion
- IMMI using RTM is better than FWI
- IMMI using PSPI migration can lower the model error faster than FWI and IMMI using RTM for the complicate model, but it fails to give clear edges of faults and only works for the multiscale strategy

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