

# Migration with surface and internal multiples

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## Abstract

- ▶ Reverse time migration (RTM) and least-squares reverse time migration (LSRTM) of surface multiples give a more extensive illumination than primaries.
- ▶ Full-wavefield migration (FWM) of total wavefields can provide more details in the image compared with using primary only.

## Methods

### 1. Reverse time migration (RTM) with surface multiple

In RTM with the first-order surface multiple, we implement cross-correlation imaging condition

$$\text{Image}(x, z) = \sum_{t=1}^{t_{\max}} P_F(x, z, t) * M_B(x, z, t) \quad (1)$$

### 2. Least-squares reverse time migration (LSRTM) with surface multiple

To improve image resolution and suppress artifacts, we use LSRTM and the L2 norm objective function

$$f(\mathbf{m}) = \frac{1}{2} \|\mathbf{Lm} - \mathbf{M}\|^2 \quad (2)$$

The gradient of this objective function is

$$\mathbf{g} = \mathbf{L}^T [\mathbf{Lm} - \mathbf{M}] = \mathbf{L}^T \Delta \mathbf{d} \quad (3)$$

### 3. Full-wavefield migration (FWM) with surface and internal multiple

Forward modeling in FWM obtains downgoing and upgoing wavefield respectively

$$\vec{P}^+(z_m) = \sum_{n < m} \mathbf{W}(z_m, z_n) [\vec{S}^+(z_n) + \delta \vec{S}(z_n)], \quad (4)$$

$$\vec{P}^-(z_m) = \sum_{n > m} \mathbf{W}(z_m, z_n) \delta \vec{S}(z_n), \quad (5)$$

The objective function in FWM

$$J = \sum_{\omega} \|\mathbf{P}_{\text{obs}} - \mathbf{P}_{\text{mod}}\|_2^2 + f(\mathbf{R}) \quad (6)$$

The gradients with respect to the above reflectivity coefficient:

$$\mathbf{C}^U(z_m) = [\Delta \mathbf{P}^-(z_m)] [\mathbf{P}^+(z_m)]^H \quad (7)$$

$$\Delta \mathbf{P}^-(z_m) = [\mathbf{W}(z_0, z_m)]^H [\mathbf{P}_{\text{obs}}(z_0) - \mathbf{P}_{\text{mod}}(z_0)] \quad (8)$$

Imaging condition in FWM gives

$$\Delta \mathbf{R}(z_m) = \text{diag} \left( \sum_{\omega} \mathbf{C}(z_m) \right) + f'(\mathbf{R}(z_m)) \quad (9)$$

Next, we can update the reflectivity matrix as follow:

$$\mathbf{R}_i(z_m) = \mathbf{R}_{i-1}(z_m) + \alpha \Delta \mathbf{R}_{i-1}(z_m) \quad (10)$$

## Results

### Example 1: RTM with the first-order surface multiple

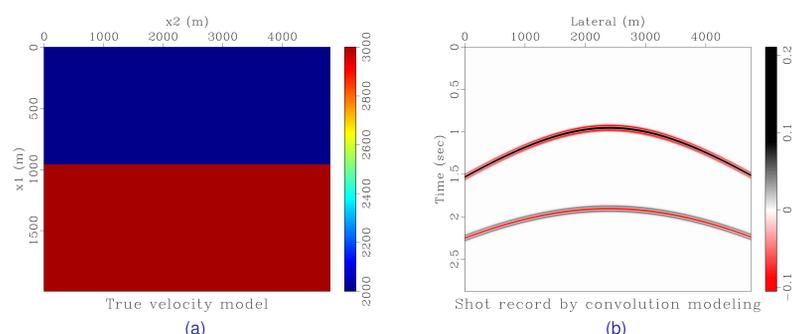


Figure 1: (a) True horizontal-layered velocity model. (b) Synthetic shot record.

RTM of the first-order surface multiple (Figure 2b) provides a wider illumination, precisely, between offset 0-1000 meters and after 4000 meters. Figure 2b also highlights fewer artifacts near the shot compared with Figure 2a which uses primary wave.

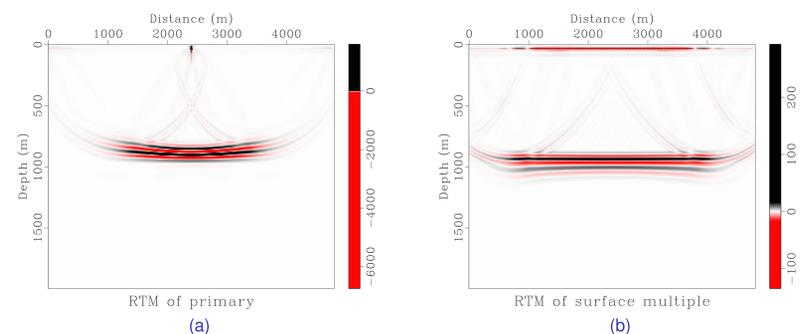


Figure 2: (a) Reverse time migration of primary reflection. (b) Reverse time migration of surface multiple.

### Example 2: LSRTM with the first-order surface multiple

Least-squares reverse time migration of surface multiple (Figure 4b) enhances the amplitude of the upper-side dipping event as well as some flanks information at the bottom reflector. Also, it improves the image resolution compared with Figure 4a.

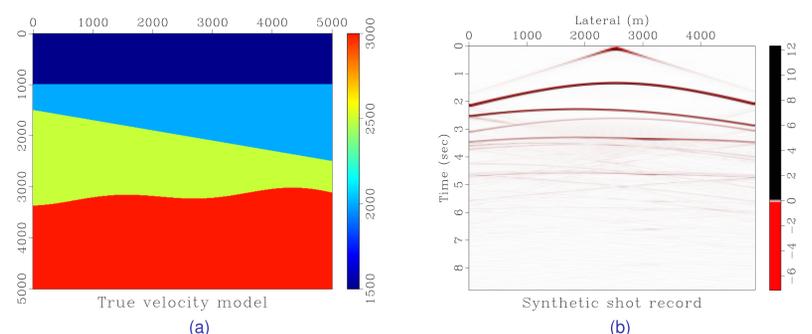


Figure 3: (a) True velocity model. (b) Synthetic shot record.

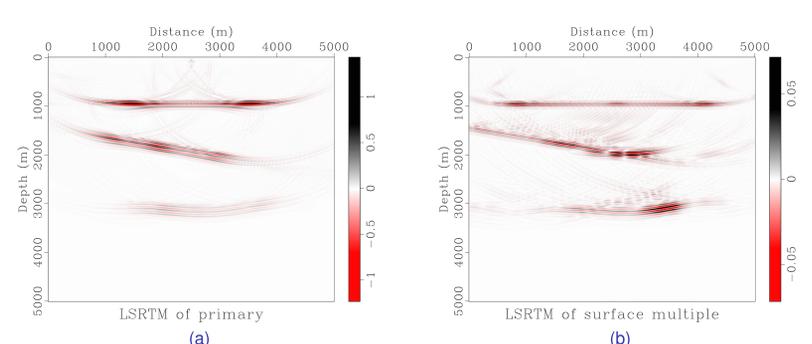


Figure 4: (a) LSRTM with primary. (b) LSRTM with surface multiple.

### Example 3: FWM with surface and internal multiples

Forward modeling in FWM (Figure 6a) can predict correctly not only

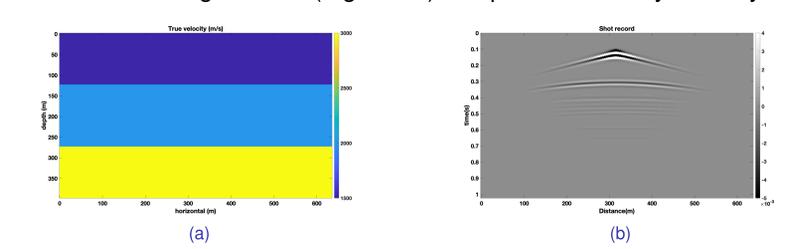


Figure 5: (a) True velocity model. (b) Observed data.

primary wave but also different orders of surface and internal multiples which are corresponding to the observed data (Figure 5b) generated from finite-difference method.

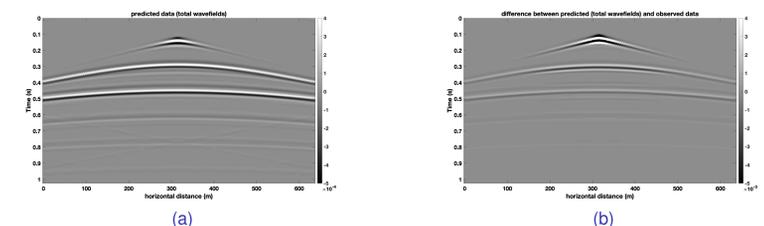


Figure 6: (a) Predicted data by full wavefield. (b) Difference between Figure 5(b) and Figure 6(a).

For Figure 7a, we only simulate impulsive wavelet on the surface to create primary reflections. Figure 7b indicates that full wavefield modeling can predict multiple events compared with Figure 7a; this additional information can improve reflectivity coefficient amplitude.

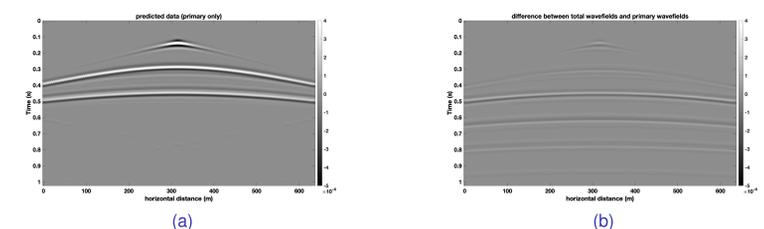


Figure 7: (a) Predicted data by primary wavefield. (b) Difference between Figure 6(a) and Figure 7(a).

Full wavefield migration results (black line in Figure 8b) is more accurate and close to the true reflectivity coefficient values compared with applying primary only (orange line in Figure 8b). Precisely, at depth 270 meters, FWM improves reflectivity coefficient amplitude and is 6.9% larger than primary migration and 34% larger than the initial model.

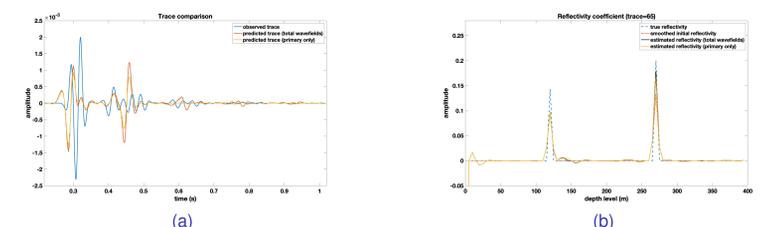


Figure 8: (a) Trace No. 64 comparison. (b) Reflectivity coefficient comparison.

## Conclusions

- ▶ RTM and LSRTM with the first order surface multiple can enhance the illumination and signal-to-noise ratio in the image compared with primary wave.
- ▶ FWM can predict and use surface and internal multiples, recover reflectivity coefficient amplitude.
- ▶ Migration of multiples can be a useful complement to improve the image quality for an accurate geological interpretation.

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