Trans-Dimensional multimode surface wave inversion of DAS data at CaMI-FRS

Luping Qu*, Jan Dettmer and Kris Innanen
*luping.qu1@ucalgary.ca

1 ABSTRACT

Due to the limited research on the combined utilizations of multimodal phase velocity in surface wave dispersion inversion, this study implements a trans-Dimensional surface wave dispersion inversion by jointly using the multimodal phase velocity of the Rayleigh wave, and applies it to surface Distributed Acoustic Sensing (DAS) data. The joint principle of multiple modes combination in a stochastic sense is explained. A thorough spectral analysis and error estimations on DAS data are displayed and a new mode separation method called dispersion compensation is adopted for clear dispersion curves picking. city dispersion curves are extracted from the densely sampled DAS data, and then utilized for a multimode phase velocity trans-Dimensional inversion. Underground information inferred by the phase velocity are demonstrated, better results using higher mode Rayleigh wave phase velocity are shown. Tests are carried out on synthetic models and field DAS data in Containment and Monitoring Institute-Field Research Station. Results of synthetic models are consistent with the theoretical expectations, and results of real data is in excellent agreement with known geology features. A better characterization of shallow area is revealed compared with other research results.

2 THEORY

Trans-Dimensional SWD Inversion

Based on Bayes' rule, the posterior probability density is defined as

$$P(\mathbf{m}|\mathbf{d}) = \frac{P(\mathbf{d}|\mathbf{m})\,P(\mathbf{m})}{P(\mathbf{d})}. \tag{1}$$

In trans-Dimensional inversions, model parameter is treated as unknown, and integrated over in a hierarchical Bayesian sense. With the incorporation of variant model parameter number *k*, the posterior can be transformed into

$$P(k,\mathbf{m}_k|\mathbf{d}) = \frac{P(k)\,P(\mathbf{d}|k,\mathbf{m}_k)\,P(\mathbf{m}_k|k)}{\sum_{k'\in\mathcal{K}}\int_{\mathcal{G}} P(k')\,P(\mathbf{d}|k',\mathbf{m}'_{k'})\,P(\mathbf{m}'_{k'}|k')\,d\mathbf{m}'_{k'}} \tag{2}$$

Correspondingly, a similar Metropolis Hasting acceptance criterion from current model $\mathbf{m}_k$ to a proposed model $\mathbf{m}_k$' is

$$\alpha = \min\left[1, \frac{P(k',\mathbf{m}'_{k'})}{P(k,\mathbf{m}_k)}\frac{P(d|k',\mathbf{m}'_{k'})}{P(d|k,\mathbf{m}_k)}\frac{Q(k,\mathbf{m}_k|k',\mathbf{m}'_{k'})}{Q(k',\mathbf{m}'_{k'}|k,\mathbf{m}_k)}|\mathbf{J}|\right]. \tag{3}$$

The acceptance criterion for state exchange of a chain pair is

$$\alpha_{PT} = \min\left[1, \left\{\frac{P(d|k',\mathbf{m}'_{k'})}{P(d|k,\mathbf{m}_k)}\right\}^{\beta_i-\beta_j}\right]. \tag{4}$$

Multimode likelihood formulation

the likelihood of the model that meets both the fundamental and higher modes of phase velocity dispersion curves is the product of the model likelihoods which fit all those dispersion curves, expressed as

$$L(\mathbf{m}) = \prod_{i=1}^{S}\frac{1}{\sqrt{(2\pi)^{N_i}|\mathbf{C}_{\mathbf{d}i}|}}\exp\left(-\frac{1}{2}\mathbf{r}_i^T\mathbf{C}_{\mathbf{d}i}^{-1}\mathbf{r}_i\right). \tag{5}$$

Here, i is the index for different dispersion curves with a total number of *S*. *r* is the data residuals. $\mathbf{C_d}$ is the data covariance matrix.

In addItion, a mode separation method named dispersion compensation is adopted to conduct mode separation for DAS data,

whereby the fundamental mode is extended, and the higher modes are decoupled.

3 SIMULATION TESTS

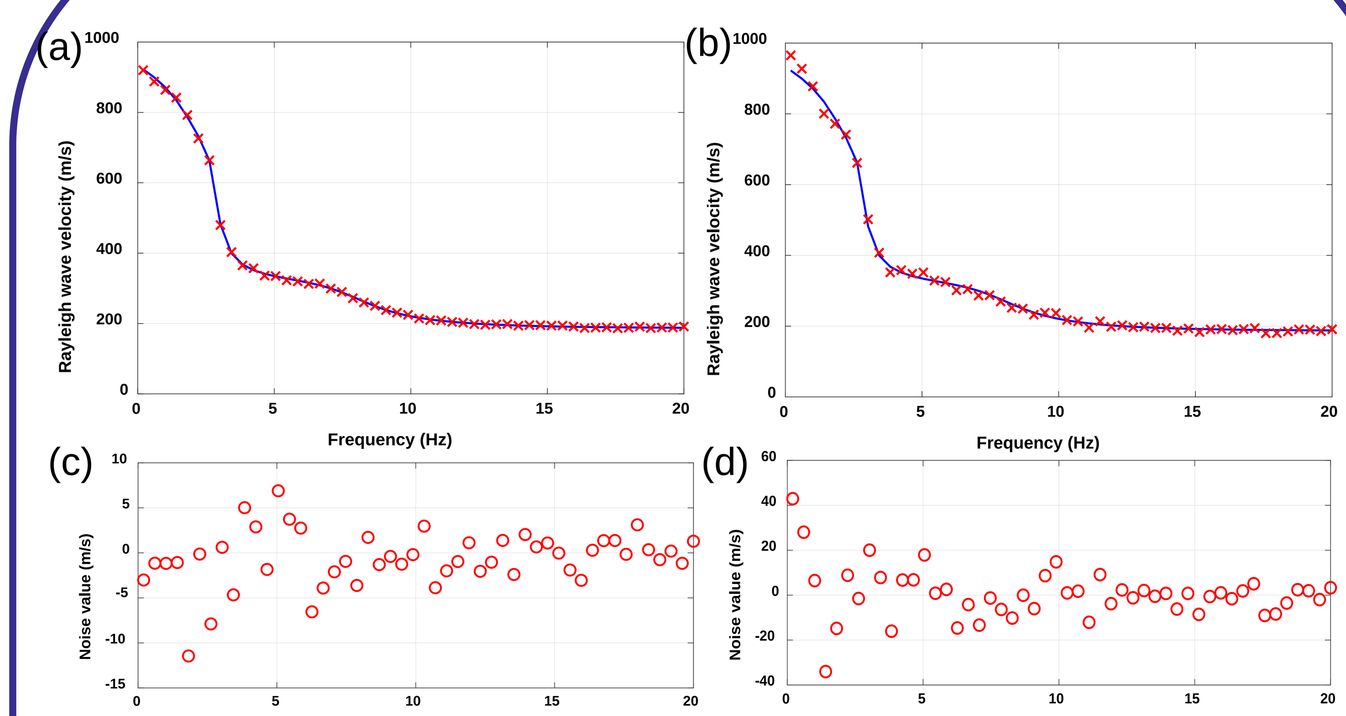

Fig 1. Synthetic data for a four-layered model with random noise. (a) Observing data with 1% data noise, (c) is the corresponding noise. (b) Observing data with 3 % data noise, (d) is the corresponding noise.

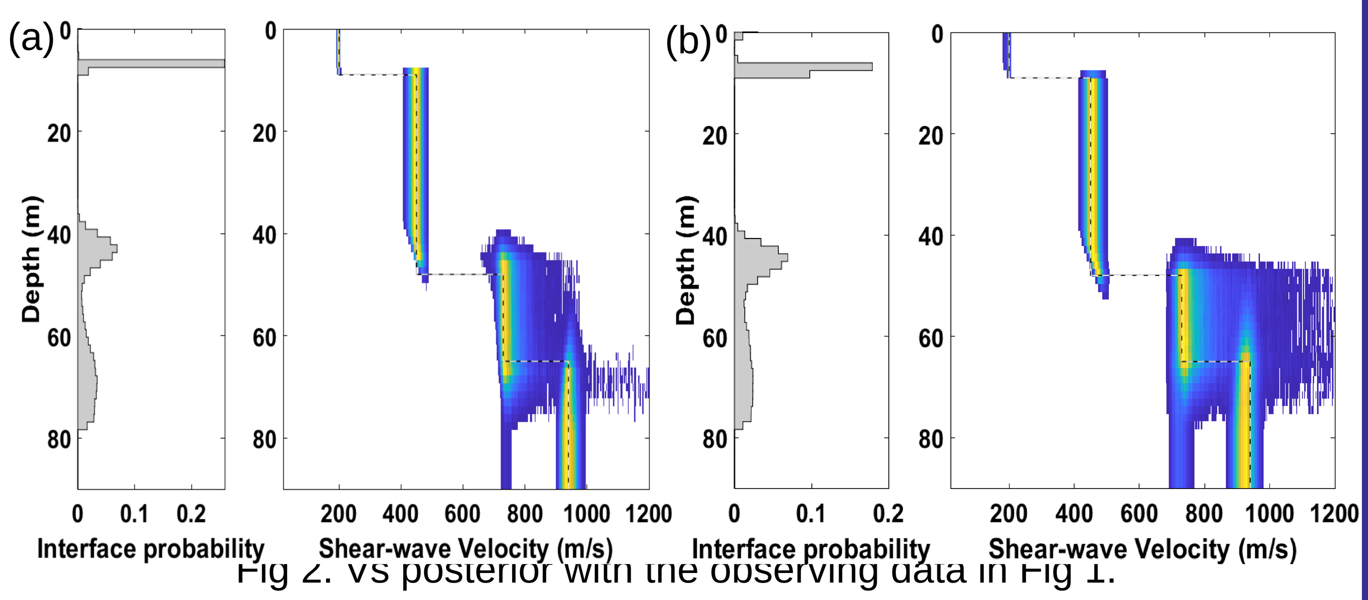

Fig 2. Vs posterior with the observing data in Fig 1.

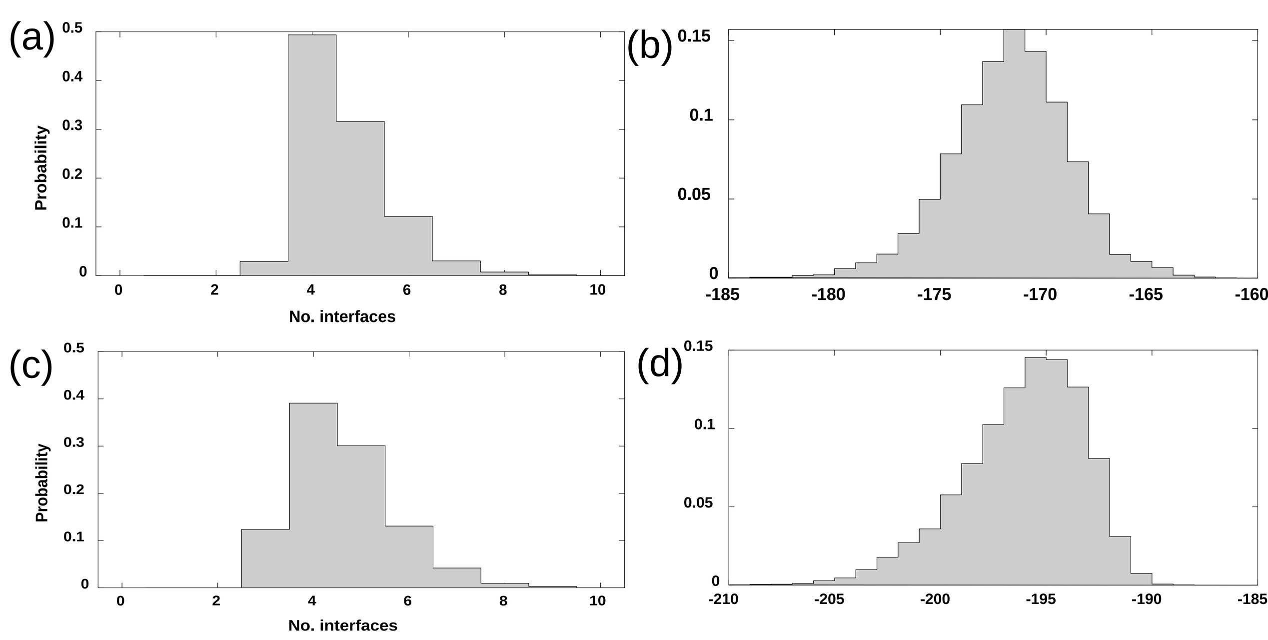

Fig 3. (a) Layer number posterior distribution for the data with 1% random noise, and (b) is the misfit distribution. (c) Layer number posterior distribution for the data with 3% random noise, and (b) is the misfit distribution.

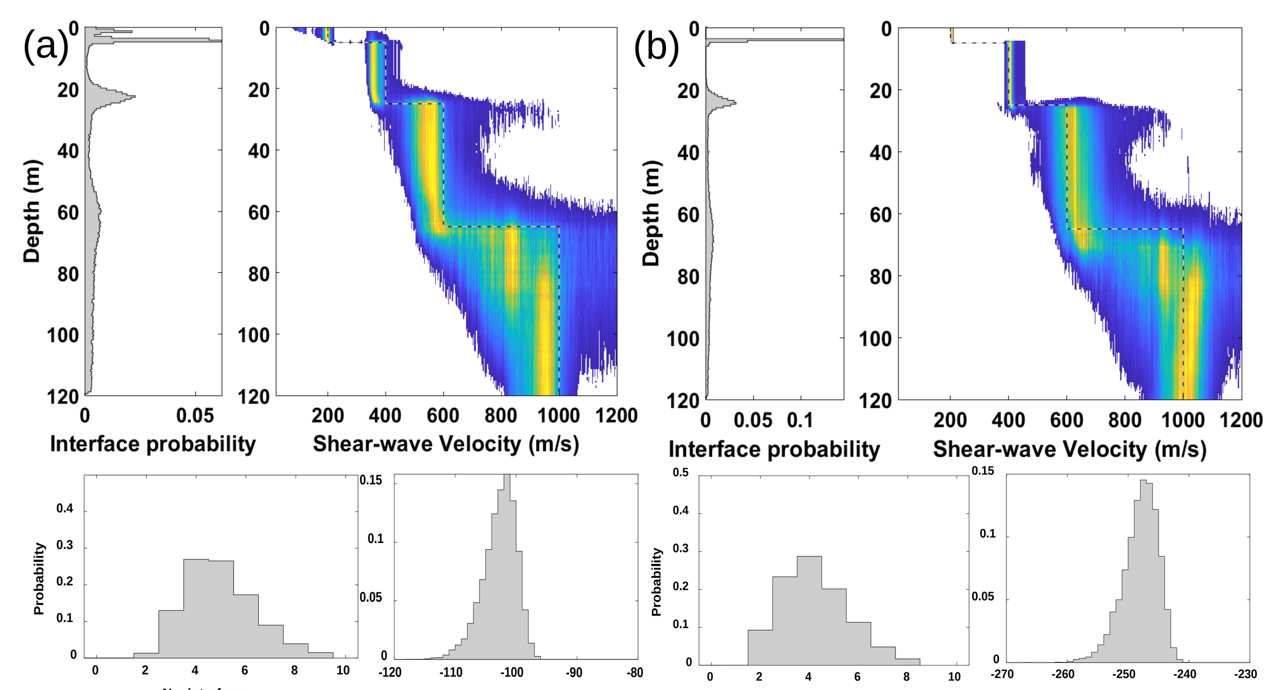

Fig 4. Inversion results for fundamental mode and multimode. (a) Fundamental mode inversion results. (d) Multimode inversion results.

4 DAS DATA PROCESSING

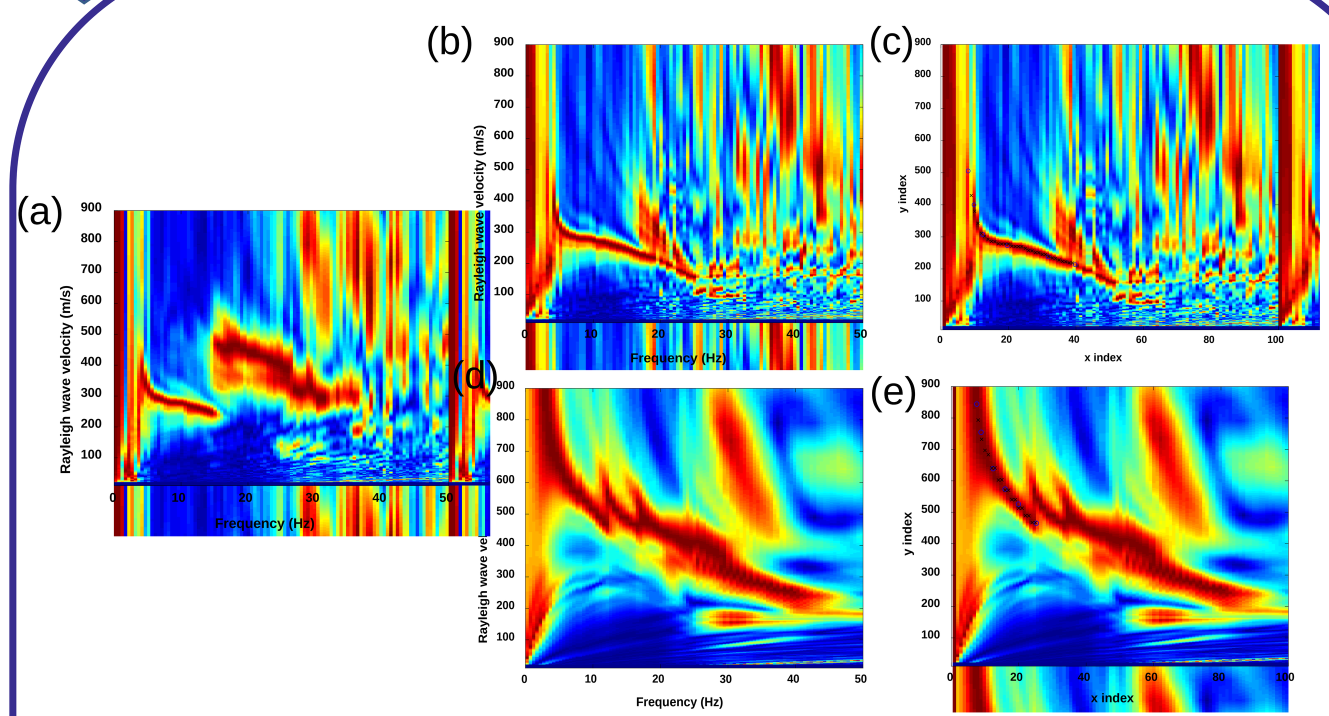

Fig 5. Mode separation using dispersion compensation. (a) Original spectrum. (b) Fundamental mode after separation, and (c) is fundamental mode picking.(d) Higher mode after separation and (e) is higher mode picking.

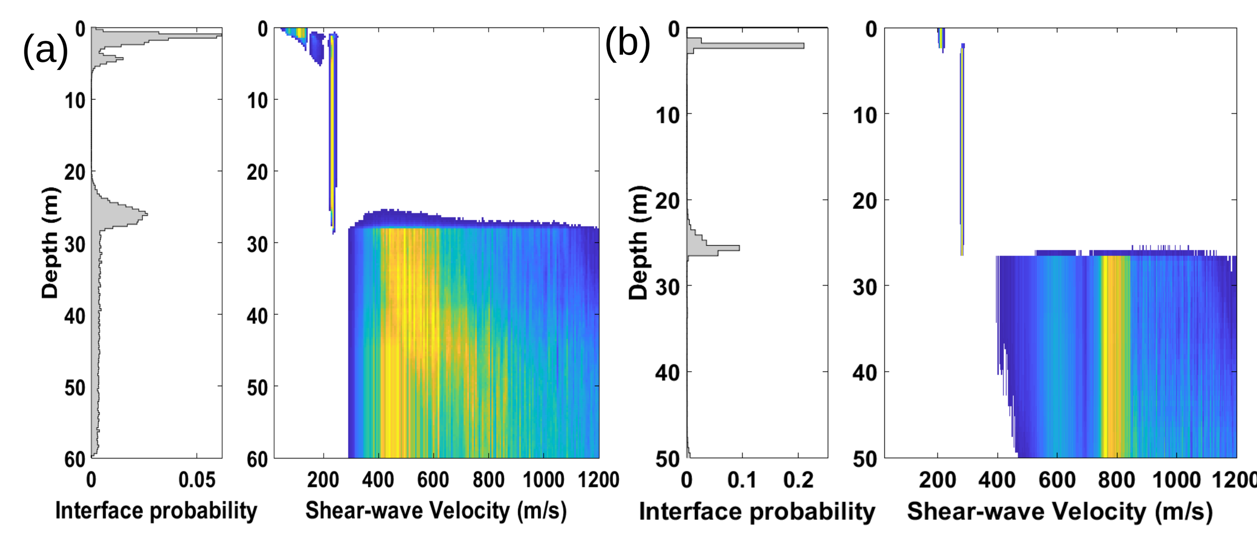

Fig 6. Vs posterior. (a) Fundamental mode inversion result. (b) Multimode inversion result.

5 DISCUSSION

We found solution non-uniqueness when we apply the inversion algorithm on certain synthetic models. The concrete conflicts are reflected in additional lower velocity layer appearing in our inversion results and models can not match with the true model appearing repeatedly in the PPD. We believe, this reveals the trans-Dimensional inversion is quite "intelligent", it explores all the possibilities that could match the dispersion curves we observed. As the forward modeling is not sensitive to low velocity zone, models with low velocity zones can be a possible model generating same dispersion curve. And since Rayleigh wave is more sensitive to root mean square (RMS) shear wave velocity due to its intrinsic characteristic, trade off models with same RMS shear wave velocity can also be the solutions. The results shows all the models that match with certain dispersion curve, including more general posterior. We have to acknowledge the information contained in the dispersion curves is limited. In order to obtain unique solution, additional data like travel time, ellipticity or priors should be incorporated as constraints. In addition, more efficient parallel tempering strategy that allow the chain block rapidly jump out of a local minimum should be explored.

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