3-D f-k Filtering

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ABSTRACT

Seismic sections or slices resulting from the processing of 3-D data can often be enhanced by the application of various filters. This study investigates the design and use of 3-D f-k filters operating on the post-stack data volume. In particular the results of a one-pass true 3-D filter are compared with those realized from two perpendicular passes of a 2-D filter. In principle, given a specific velocity (dip) reject range, the basic two-pass 2-D filter and a one-pass 3-D filter have a different response. In fact, filtered synthetic data show that the two-pass filters do not provide axially-symmetric results; greater dips are passed at 45° to the filtering directions. The one-pass 3-D filter does provide axially symmetric results. This study also compares a mean and median implementation of the 3-D f-k filter. The application of the mean filter results in a more smoothed image than use of the median-based filter. In cases where 3-D edges (faults) should be preserved or where the data are particularly noisy, the median design may be more desirable.

INTRODUCTION

The acquisition and processing of 3-D seismic data is an expensive but increasingly common procedure (Robertson, 1989). Remarkably accurate and useful geologic images have been generated from these data. The quality of these images is due, in part, to the more realistic assumptions made in processing (e.g. 3-D Earth and wave propagation). In addition, if some further knowledge about the subsurface is known, say that the range of dipping beds is limited, then it may be desirable to reject any events on the seismic sections which have dips greater than this limited amount. Velocity or f-k filters have proven to be quite useful in this regard for 2-D enhancements. Two-dimensional f-k filters have been known and used for some time (Embree et al., 1963; Yilmaz, 1987). While 3-D filtering concepts have also been presented (Burg, 1964; Hubral, 1972), they do not appear to have been extensively developed. It seems reasonable, however to attempt to use a 3-D data volume for noise attenuation just as a 3-D data volume is useful for migrating diffractions. This study analyses the design of a basic conical, dip rejection filter. Can we use a two-pass 2-D f-k filter or should we use a one-pass 3-D filter? We also introduce the concept of a 3-D median f-k filter. Instead of multiplying and summing filter coefficients and data points to find the output point, the median filter selects the f-k weighted median point as the output of the moving data volume. The standard or mean f-k filter is compared to the median filter on field 3-D seismic data.

METHODS

We conceive of a 3-D filter which will reject all events (linear, planar) that have dips outside a certain design range (Figure 1). This filter in the time-space domain (t, x, y) or frequency-wavenumber domain (w, k_x, k_y) will again, symmetrically reject events with low velocities (say less than V_L).

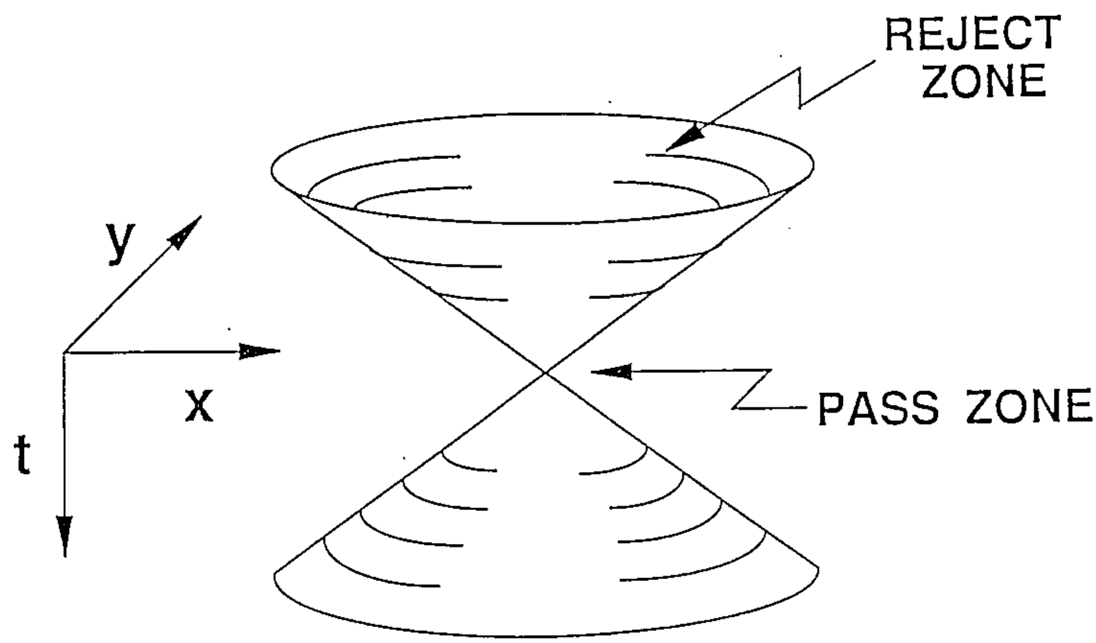

FIG. 1. Conical pass and reject regions of a 3-D filter.

One pass versus two pass

In the frequency domain, a true 3-D filter F would have an expression as

$$S(w,k_x,k_y) = F(w,k_x,k_y)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(t,x,y)e^{-i(wt+k_x x+k_y y)}dtdxdy$$

$$\text{where } F(w,k_x,k_y) = \begin{cases} 1 & w/\left(k_x^2+k_y^2\right)^{1/2} > V_L \\ 0 & w/\left(k_x^2+k_y^2\right)^{1/2} \leq V_L \end{cases} .$$

but a basic two-pass 2-D filter would have an expression as

$$S'(w,k_x,k_y) = F'''(w,k_y)\int_{-\infty}^{\infty}\left[F''(w,k_x)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(t,x,y)e^{-i(wt+k_xx)}dtdx\right]e^{-ik_yy}dy$$

$$\text{or } S'(w,k_x,k_y) = F'(w,k_x,k_y)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(t,x,y,)e^{-i(wt+k_xx+k_yy)}dtdxdy$$

where $F'(w,k_x,k_y) = F''(w,k_x)\cdot F'''(w,k_y)$,

$$F''(w,k_x) = \begin{cases} 1 & w/k_x > V_L \\ 0 & w/k_x \leq V_L \end{cases} ,$$

$$F'''(w,k_x) = \begin{cases} 1 & w/k_y > V_L \\ 0 & w/k_y \leq V_L \end{cases} .$$

But F′ has a pyramidal pass zone while F has a conical pass zone (Figures 2a and 2b).

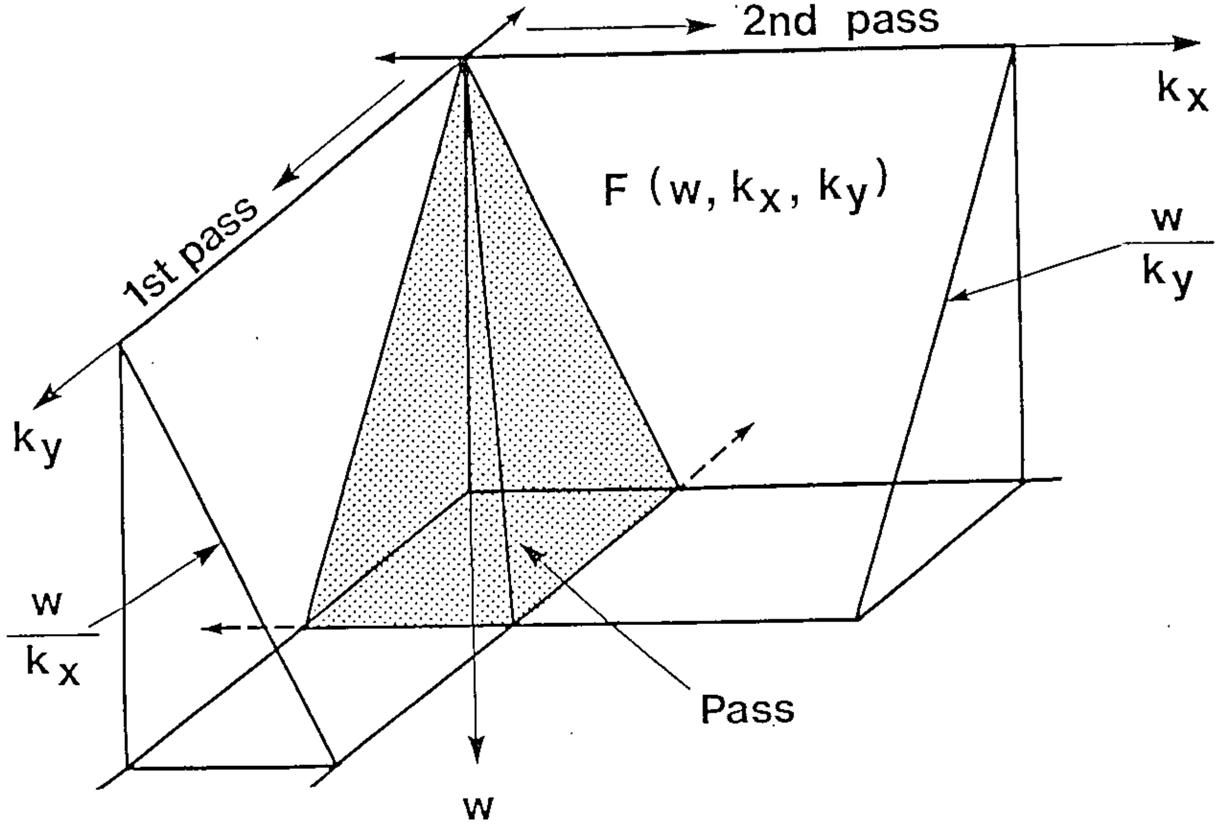

FIG. 2. (a) Schematic diagram in f-k space of a two-pass 2-D filter,

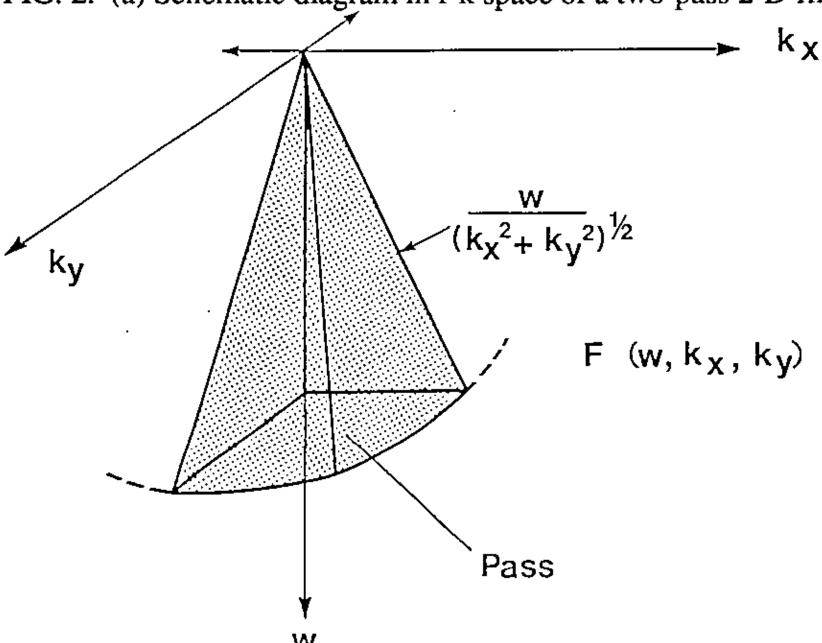

FIG. 2. (b) Schematic diagram in f-k space of a one-pass 3-D f-k filter.

To test these theoretical results, we built a 3-D computer model with a hemispherical anomaly (Figure 3a). The maximum dip of the flanks of the hemisphere is 8ms/trace. The f-k filter is designed to reject dips greater than 4ms/trace. The time-space domain 3-D operator can be generated by the inverse transform of the appropriate conical zone in the frequency domain. Axial symmetry in the frequency domain implies axial symmetry in the time domain (see Appendix A). Furthermore, although the 3-D time domain response of the conical pass zone is axially symmetric it is not the same as the 2-D f-k filter (see Figures 8a and 8b). Filtering of a 3-D hemisphere model confirms that indeed there are different pass properties of these filters (Figures 3b and 3c). The two-pass filter allows events with larger dips to remain at 45° to the horizontal (x) axis. This leads to an asymmetric pass zone. The one-pass filter rejects high dips in a symmetric manner.

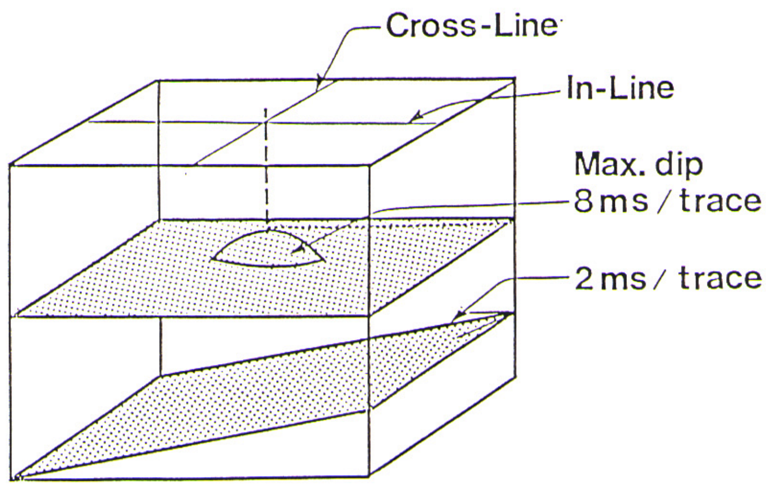

FIG. 3. (a) 3-D model geometry with a hemispherical anomaly,

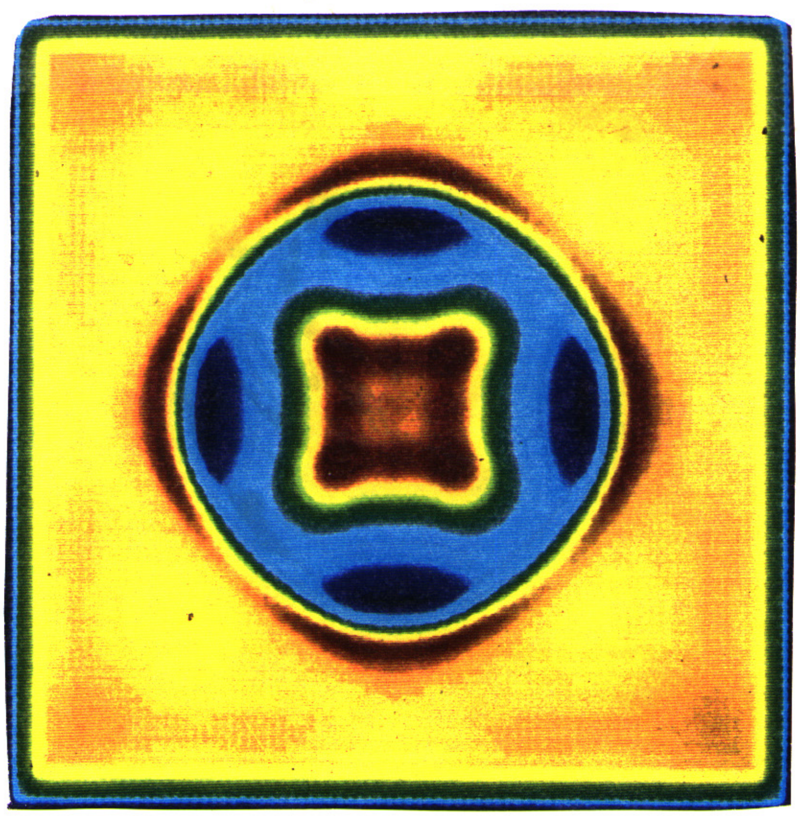

FIG. 3. (b) Amplitude of hemispherical event after two-pass filtering. The light colored zone shows part of the reject area,

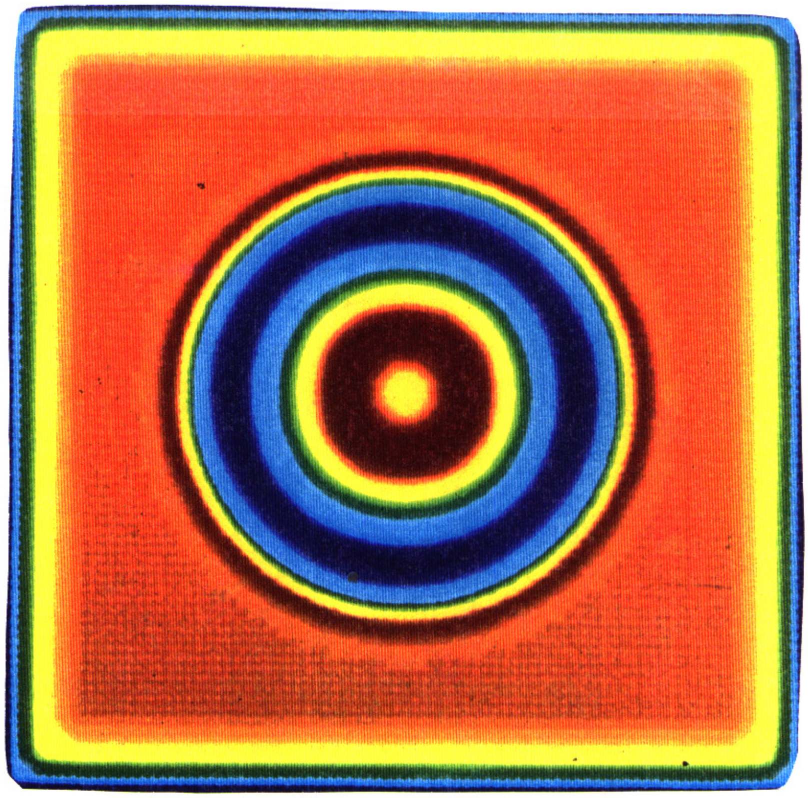

FIG. 3. (c) Amplitude of same event after one-pass filtering. Note the symmetric rejection.

The median concept

If the 3-D seismic volume displays real faults or noisy glitches then it is known that running average (mean) filters smear out these anomalous events; median filters tend not to do so (Claerbout and Muir, 1973; Stewart, 1985). Thus if there are edges in the data to be preserved or if the data are particularly noisy, median filters may be more effective than mean filters. Figure 4a (from Stewart, 1985) shows the 1-D median filtering concept for a 5-point filter. Weighting values or filter coefficients can also be incorporated into the median estimation process. A more detailed discussion of the meaning of the weighting is given in Appendix B. This is shown schematically in Figure 4b. The median concept can be applied to a 3-D f-k filtering process as follows (see Figure 4c):

i) compute the 3-D time domain response of the dip filter, (from inverse transforming the appropriate zone in the frequency domain)

ii) select a moving box of data to be filtered,

iii) attach a filter coefficient to each data point in the box,

iv) order the data in ascending value, keeping the associated filter coefficients with the data, (if the filter coefficient is neagtive, make it positive and switch the sign of the data points)

v) find the cumulative half-power point of the filter coefficients,

vi) select the data point associated with the half power point of the filter as the output point of this box, (or equivalently, repeat the data points by their filter coefficient values, order and select the median)

vii) repeat this process for a moving box centered at all points in the data volume.

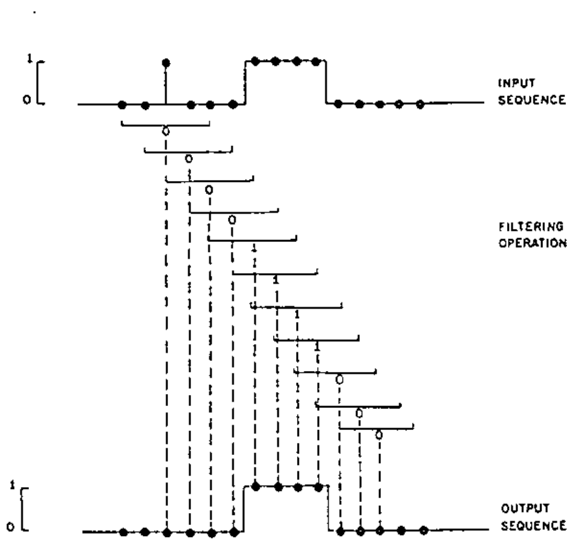

FIG. 4. (a) Schematic diagram of the 1-D median filtering process with a 5-point window,

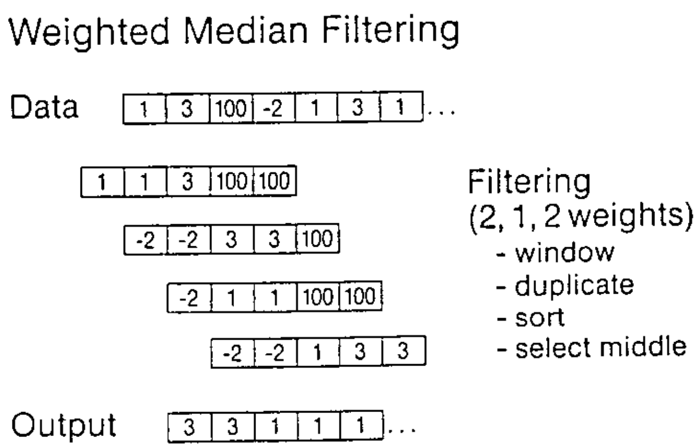

FIG. 4. (b) schematic diagram of the use of weights or filter coefficients in the median filter,

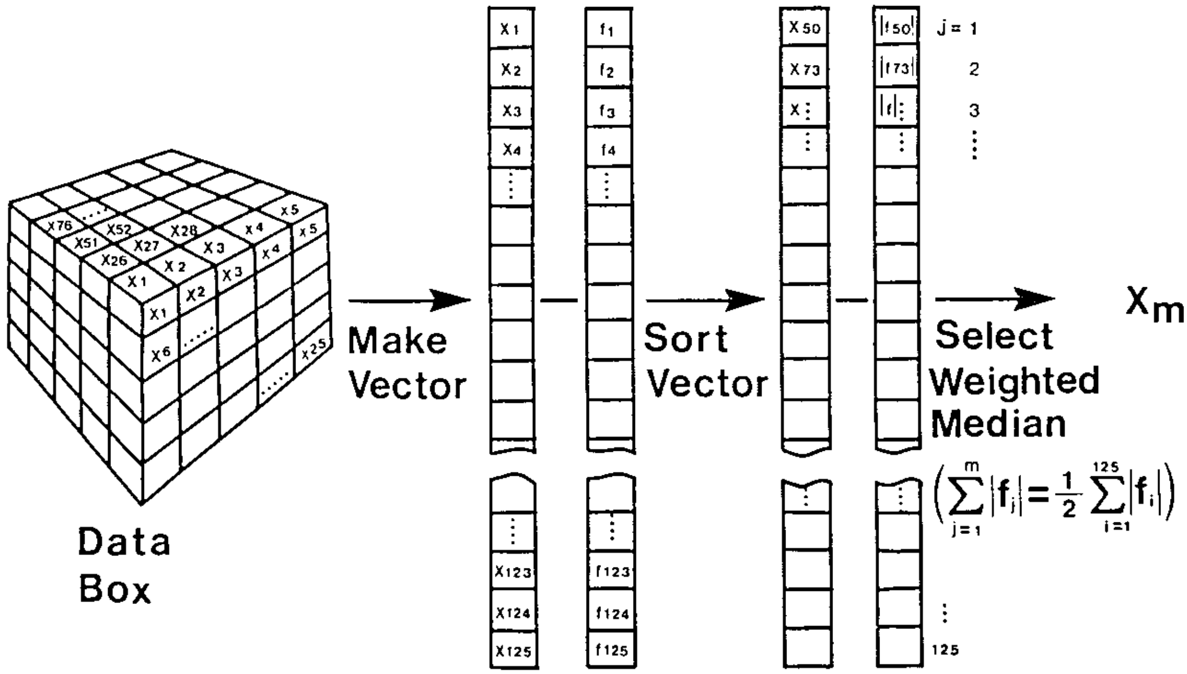

FIG. 4. (c) Schematic diagram of the 3-D median filter process with filter coefficients.

RESULTS

A 3-D seismic survey was conducted in Northern Alberta over a 7.3 km by 3.1 km area (Figure 5). Due to poor surface conditions the resulting seismic data volume was contaminated with a great deal of noise. The prospective zone (around 1.3 s) was associated with faulted carbonates. The migrated data volume was filtered using 3-D mean and median f-k processes. The results are compared here on just a diagonal seismic section from the data volume. A diagonal line from the 3-D volume is shown in Figure 6a. We note the very noisy near surface expression. The resulting diagonal line from the mean f-k filtered volume is shown in Figure 6b. while much more clear than the original data, some might say that an artificial coherency (worminess) has been produced. The median filter has also enhanced coherency but local character appears to be retained in the data (Figure 6c). Time slices of the data volume show the effect of the two filters in a more dramatic manner. A slice of the raw data at a time of 1338 ms is shown in Figure 7a. A slice of the 3-D f-k filtered volume at the same time is displayed in Figure 7b. We note the considerable improvement in the continuity of the plan-view reflections. The 3-D median f-k filtered slice is shown in Figure 7c. We now also see improved continuity in the reflectors, but local and small features appear better preserved than in the Figure 7b (eg. in the lower left corner).

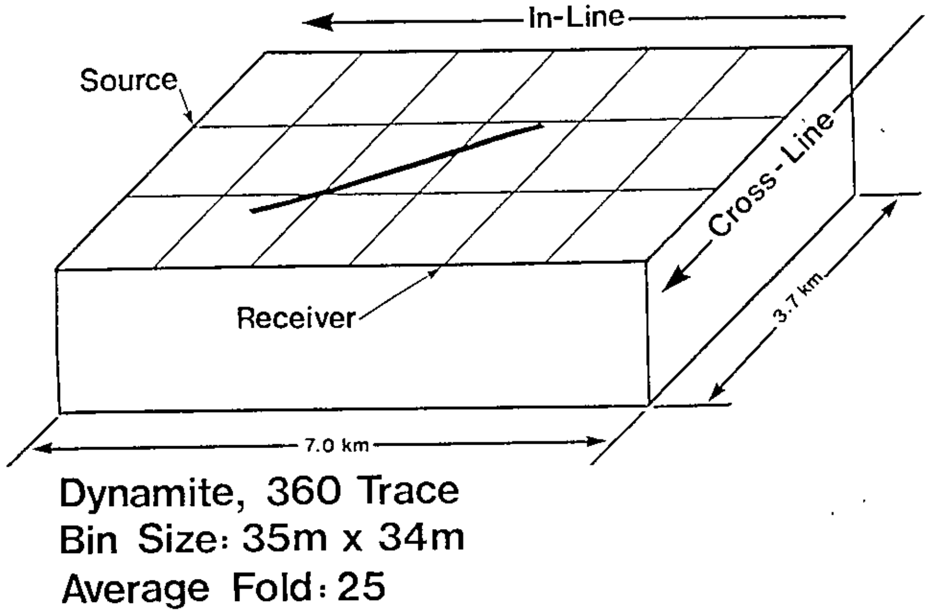

FIG. 5. Geometry of N. Alberta 3-D data seismic survey.

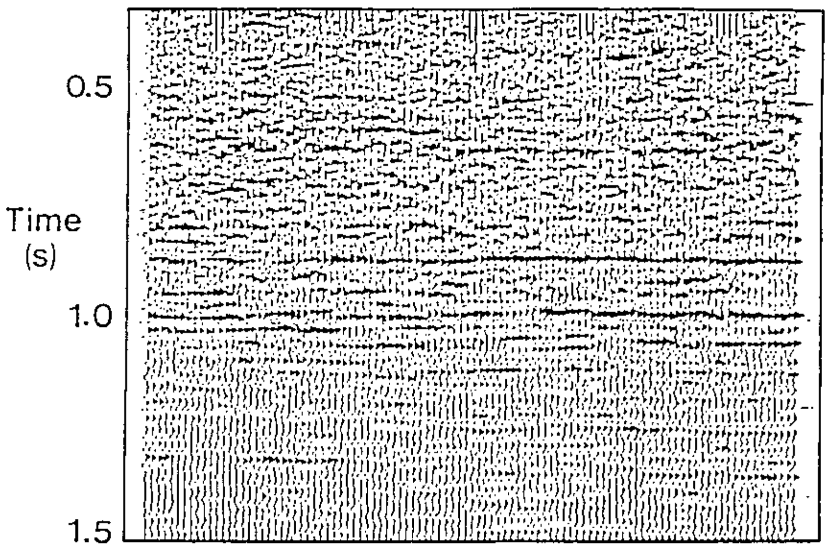

FIG. 6. (a) Diagonal line from N. Alberta 3-D data volume,

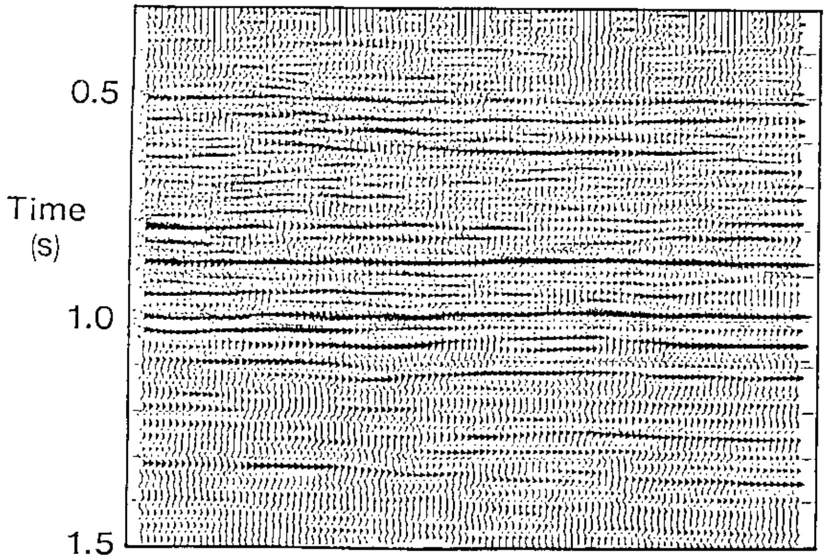

FIG. 6. (b) Same line extracted from the 3-D volume after mean 3-D f-k filtering,

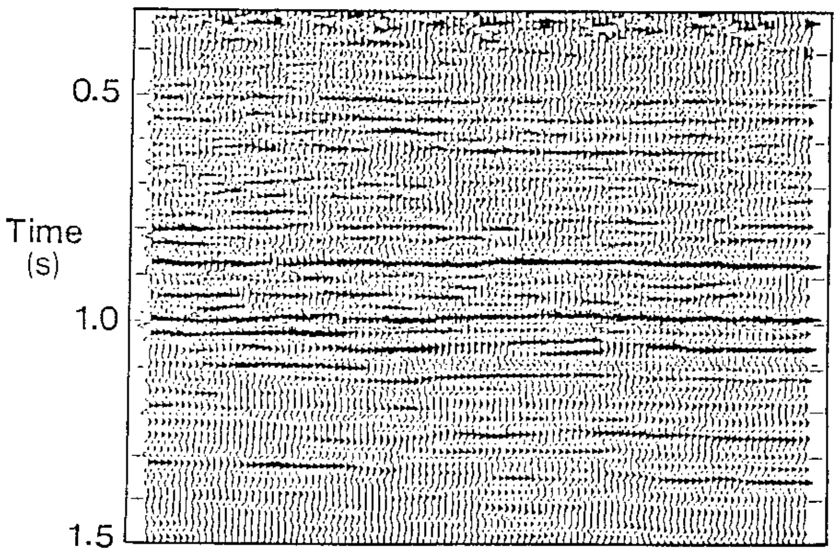

FIG. 6. (c) Same line extracted from the 3-D volume after median 3-D f-k filtering.

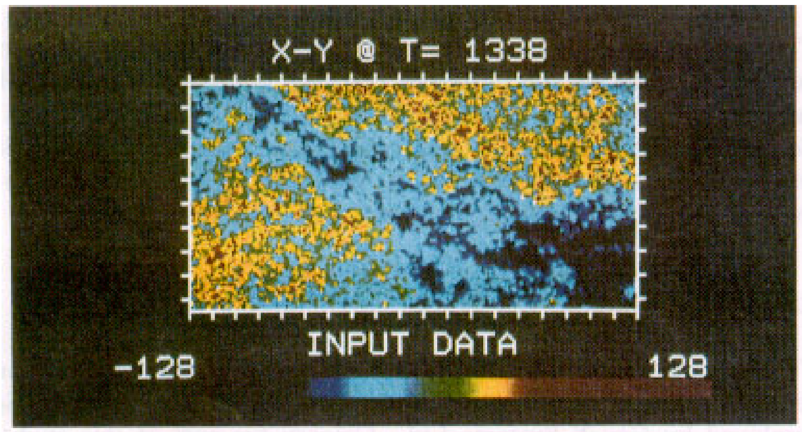

FIG. 7. (a) Time slice of 3-D data volume at 1.338s through the top of the faulted carbonate of interest,

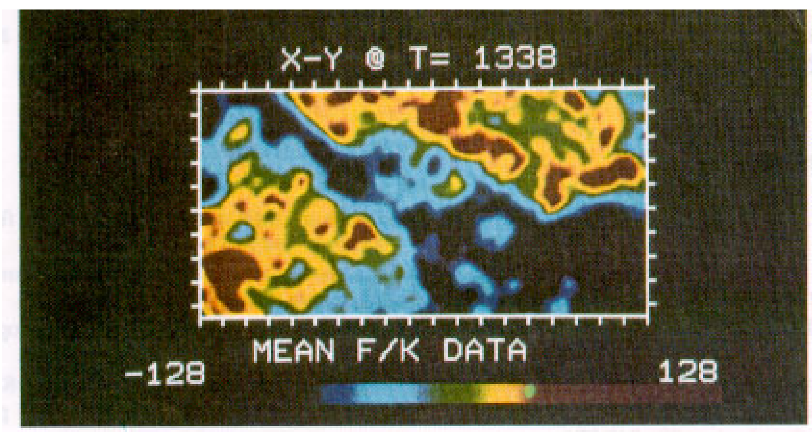

FIG. 7. (b) Time slice of the 3-D volume after conventional f-k filtering,

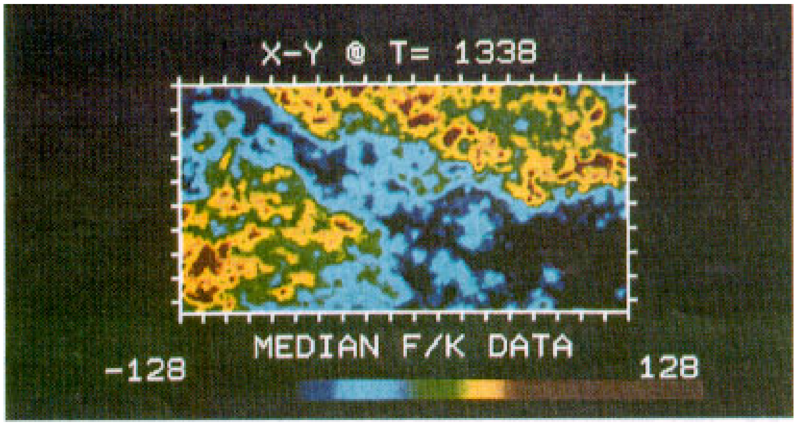

FIG. 7. (c) The same time slice after median F-k filtering.

CONCLUSIONS

Three-dimensional filters are useful in increasing the coherency of seismic data sections and slices. Basic one-pass 3-D filters have a different response than two-pass 2-D filters. One-pass filters provide the symmetric pass region that is likely desirable. The median concept can be extended to 3-D filters and appears to reduce noise effectively without inordinate smearing of local anomalies.

ACKNOWLEDGEMENTS

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APPENDIX A

To create a 3-D time domain operator it seems intuitively correct to time axially rotate a 2-D operator by 2 π to sweep out a 3-D volume. However, one must first establish that axial symmetry in the frequency domain (such as the cone filter in figure 2b) represents axial symmetry in the time domain (Mesko, 1984). Further, it must be shown that the 2-D time domain dip reject coefficients obtained from the inverse transform is equivalent to an axial slice of the true 3-D time domain operator.

The inverse transform of the 3-D filter F is

$$f(t,x,y) = \int_{-\infty}^{\infty}\int\int F(w,k_r)e^{i(wt+k_x x+k_y y)}\,dw\,dk_x\,dk_y$$

or by writing the spatial components as a dot product and representing the filter F as symmetric

$$f(t,x,y,) = \int_{-\infty}^{\infty}\int\int F(w,k_r)e^{i(wt+\bar{k}_r\cdot\bar{r})}\,dw\,dk_r\,dr$$

$\bar{k}_r = k_x + k_y$, $\bar{r} = x+y$ where $|k_r| = \sqrt{k_x^2+k_y^2}$ and $|r| = \sqrt{x^2+y^2}$

then $\bar{k}_r\cdot\bar{r} = |k_r||r|\cos\alpha$ and $dk_x dk_y = k_r dk_r d\alpha$ so

$$f(t,x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(w,k_r)k_r e^{iwt}\left[\int_0^{2\pi} e^{i(k_r r\cos\alpha)}d\alpha\right]dk_r\ dw$$

Recall the integral formulation of a zero order Bessel function

$$2\pi J_0(u) = \int_0^{2\pi} e^{iu\cos\alpha}d\alpha$$

Thus we see the right hand side of the final equation is independent of α . Axial symmetry in the frequency domain implies axial symmetry also in the time domain.

$$f(t,x,y,) = f(t,r) = 2\pi \int_{-\infty}^{\infty} \int F(w,k_r)k_r\, e^{iwt} J_0(rk_r)\, dk_r dw$$

Secondly, we compare this expression with the integral equation of the inverse 2-D transform of the same filter.

$$f(t,R) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(w,k_R)\, e^{i(wt + k_R R)}\, dk_R dw$$

One can clearly see that the integration over the Bessel function $J_0\ (rk_r)$ with respect to k_r is not equivalent to the integration of the same filter in the 2-D case. This indicates that the axial slice of the 3-D time domain operator is not equivalent to the 2-D operator. The filter time domain response of these two equations is demonstrated in figures 8 a) and 8 b)for a dip reject filter of 4 ms/trace.

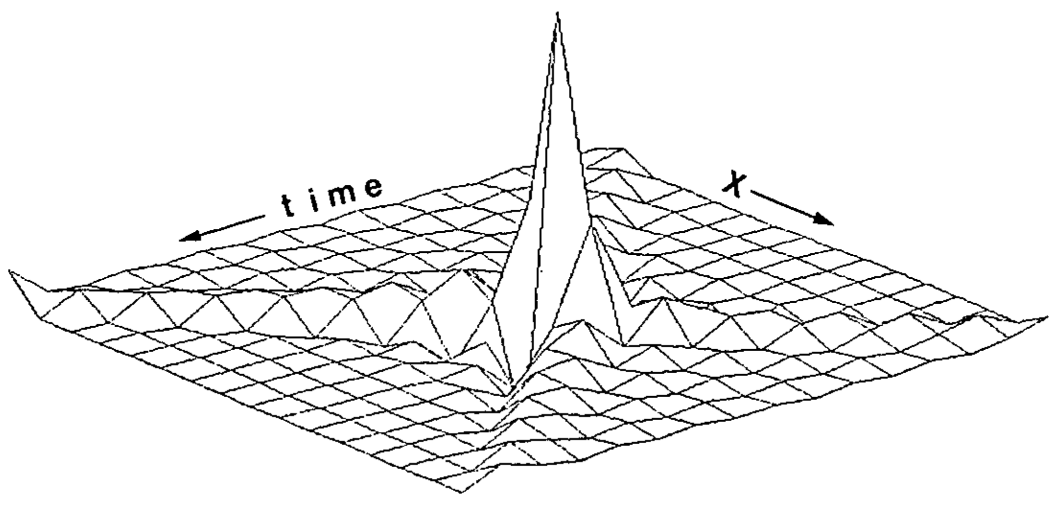

FIG 8. (a) 2-D time domain operator

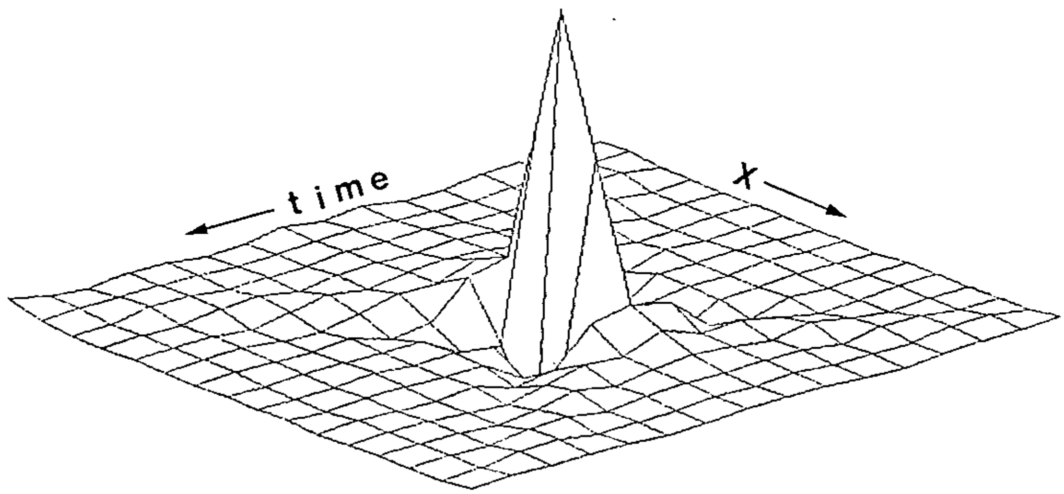

FIG 8. (b) Axial slice of 3-D time domain operator

APPENDIX B

WEIGHTED MEDIANS

If in selecting the median of a group of numbers, some numbers were considered more significant (or reliable) than others, then it might be desirable to bias the median selection toward these values: Certain numbers would have greater weight in the estimation procedure. A way to do this is to repeat the more important numbers some multiplicity of times, order this augmented sequence and select the middle value (Claerbout and Muir, 1973). This corresponds to minimizing a new function M, defined with respect to the weighted median x_m :

$$M = \sum_{i=1}^{N} w_i |x_i - x_m| \quad , \qquad (1)$$

where x_i are the data values,
N is the number of original data values,
w_i are the positive - valued weights of x_i.

If we take the derivative of M with respect to x_m, then M is minimized when

$$\sum_{i=1}^{N} w_i \cdot \text{sgn}\ (x_i - x_m) = 0 \quad ,$$

$$\text{where sgn } a_i = \begin{cases} 1 & a_i > 0 \\ -1 & a_i < 0 \end{cases} ,$$

or when x_m is the middle value of the augmented sequence.

A weighted median filter extracts the median point from an augmented window of data values, outputs this value, then moves to the next data window, outputs the median, etc. Convolutional filters, on the other hand produce an output point by multiplying the weight (filter coefficients) by the data values and summing these products (then possibly normalizing): They find a scaled mean of the filter coefficient-data products. A convolutional filter would use a window with assigned position weights to calculate the appropriate mean, then slide the window to a new set of points find the product mean, and so on. The basic reason to use the convolutional filter weights with the median concept is to develop a hybrid filter which will display desirable properties of both mean and median processes (Stewart, 1985).

Claerbout and Muir (1973) defined the function to be minimized for the weighted median by equation (1) with $w_i = |f_i|$, where f_i are the filter coefficients (weights) associated with data points x_i. Negative weights are not admitted into the estimation process. However, some filters (e.g. band-pass, f - k) have negative coefficients from which they derive their special properties. The questions arise then: What is the meaning of negative weights? How can they be admitted into a median selection? In convolutional filtering the coefficient-data product could be negative due to a negative coefficient and positive data value or vice versa. By analogy, I define a new function M_W to be minimized which will give a meaning to negative weights in the selection of a weighted median value x_W. In this case the data values are multiplied by the sign of their associated filter coefficients and weighted according to the magnitude of the coefficients:

$$M_w = \sum_{1=i}^{N} |f_i| \left|(\operatorname{sgn} f_i) \cdot x_i - x_w\right| , \tag{2}$$

where x_i are the data values,
N is the number of data values,
f_i are the filter coefficients
(weights) associated with x_i.

To minimize M_W take the derivative with respect to x_w:

$$\frac{\partial M}{\partial x_w} = \sum_{i=1}^{N} |f_i| \frac{\partial}{\partial x_w} \left|(\operatorname{sgn} f_i) \cdot x_i - x_w\right| , \tag{3}$$

$$= \sum_{i-1}^{N} |f_i| \frac{\partial |\gamma_i|}{\partial x_w} ,$$

where $\gamma_i = (\operatorname{sgn} f_i) \cdot x_i - x_w$,

$$= \sum_{i=1}^{N} |f_i| \frac{\partial}{\partial x_w} (\operatorname{sgn} \gamma_i) \cdot \gamma_i ,$$

$$= -\sum_{i-1}^{N} |f_i| \operatorname{sgn} \gamma_i , \qquad \text{with} \frac{\partial \operatorname{sgn} \gamma_i}{\partial x_w} = 0 , \qquad \gamma_i \neq 0 .$$

Then for the minimum set

$$\sum_{i=1}^{N} |f_i| \left[(\operatorname{sgn} f_i) \cdot x_i - x_w \right] = 0. \tag{4}$$

So the weighted median is found by signing the data values according to their associated filter weight signs, augmenting the data values by their associated weight magnitude, ordering and finding the middle value. The weighted median point x_K can be equivalently determined by finding the data point corresponding to the half cumulative weight point:

$$\sum_{j=1}^{K} |f_j| = \frac{1}{2} \sum_{i=1}^{N} |f_i| \ , \tag{5}$$

where j is the index of the ordered data values and their corresponding filter coefficients, K is the number of data-associated filter coefficients that must be added to equal the half cumulative magnitude of filter coefficients.

While the basic median procedure selects an actual data point, the signed, weighted selection is only guaranteed to select an actual absolute value; the output point could be the negative of an actual point.

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