Improving converted-wave (P-S) moveout estimation

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ABSTRACT

The moveout characteristic of converted-wave (P-S or S-P) seismic data differs subtly, yet significantly, from the moveout characteristic of unimodal (P-P or S-S) seismic data. Converted-wave velocity analyses and moveout corrections that the use the standard hyperbolic approximation can be problematic, primarily when data with offset-to-depth ratios are of 1:1.5 or more are used. Improved traveltime estimation can be achieved by adding another degree of freedom into the analysis, for example by using higher-ordered traveltime equations. However, in doing so, simplicity in the analysis is lost, and more effort is required for accurate seismic data processing.

In this paper we present a new formula that combines the compactness of the standard equation (currently the industry workhorse) with the accuracy of higher-ordered equations. It is derived both empirically and analytically and is shown to be in the form of a time-shifted hyperbola---a curve which others already have shown better fits the traveltime curve of unimodal data. The performance of the new equation is illustrated with a number of practical examples.

INTRODUCTION

For a horizontally stratified medium, the normal-moveout curve of converted-wave (P-S or S-P) seismic data is different than that of conventionally reflected (P-P or S-S) seismic data. The standard hyperbolic NMO formula, has limited application to converted-wave NMO correction and root-mean-square velocity estimation, particularly when data with high offset-to-depth ratios are used.

Improved converted-wave moveout estimates are given when an extra degree of freedom is allowed into the velocity search, for example by using higher-order versions of Taner and Koehler's (1969) series. Tessmer and Behle (1988) derived a power series, similar to that of Taner and Koehler's (1969), specifically for the converted-wave case. Improved results are also given by higher-order versions of their series. This improved accuracy, however, is at the cost of efficiency because procedures such as velocity analysis and inverse moveout application are complicated when done using such higher-order expressions.

In this paper we present a new formula for converted-wave moveout estimation that is in the form of the equation of a shifted hyperbola . We develop the equation from the standard equation and prove it analytically using the exact equations, in parametric form, for converted-wave traveltime and offset. Because the equation is compact it is efficient. With the use of some practical examples, we show that the new formula is not just efficient, but is more accurate than the standard equation, even when data with high offset-to-depth ratios are processed.

ESTIMATING P-S MOVEOUT: ACCURACY VERSUS EFFICIENCY

The simplest formula for P-S moveout estimation and correction is the hyperbolic approximation, referred to in this paper as the standard equation, and which is equivalent to Taner and Koehler's (1969) power series truncated to two terms, shown below:

$$t^2 = t_0^2 + \frac{x^2}{v^2}, \tag{1}$$

where t is the offset travel time, t_0 is the zero-offset travel time, x is the source-to-receiver offset, and v is the medium velocity for an equivalent single-layer medium. For data that have moderate offsets, it is accurate and v is equivalent to the P-S root-mean-square velocity (Tessmer and Behle, 1988; Iverson et al., 1989). However, as shown in Figure 1, the equation progressively overestimates converted-wave moveout with increasing offset, affecting, in particular, data from shallow depths, where the offset-to-depth ratios are high (ie. greater than 1.5:1).

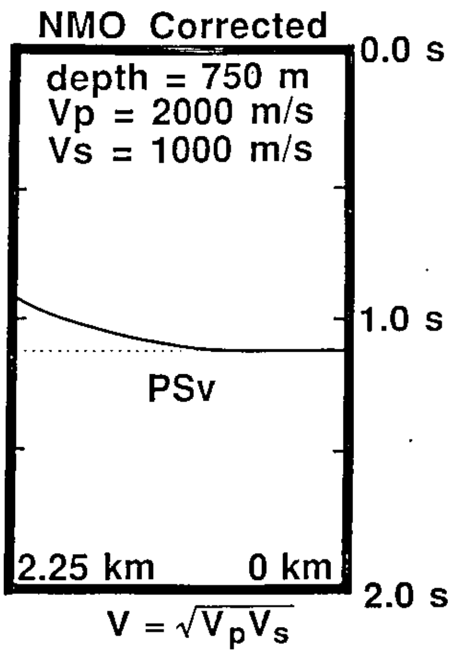


Figure 1. PS event for a single layer, NMO corrected using model velocities and standard equation.

As is shown below, improved results are given by higher ordered equations eg. higher ordered versions of Taner and Koehler's series or Tessmer and Behle's series, or by the equation of Castle (1988), who derived a time-shifted hyperbola equation which he showed to be equivalent to higher-ordered versions of Taner and Koehler's series.

Thus, in estimating converted-wave traveltime, the problem with these methods is that a choice must be made between using an equation that is easy to use, but only gives good results to an offset-to-depth ratio of about 1:1.5, as with the standard equation, or using an equation that is accurate even for large offset-to-depth ratios (3:1 for third-order truncations), but is cumbersome to use. The next section gives the derivation of a new equation that is both accurate and compact.

P-S TRAVELTIME EQUATION

The familiar conventional travel time equation (Equation 1) provides the starting point for developing the new equation. Taking the square root of (1), expanding that root using the binomial series, and truncating the result to two terms results in:

$$t = t_0 + \frac{x^2}{2t_0v^2}, \tag{2}$$

which is the parabolic approximation for normal moveout. Since (1) and it's approximation (2) progressively overestimate moveout with increasing offset, t_0 in the denominator of the moveout term of (2) is replaced by t:

$$t = t_0 + \frac{x^2}{2tv^2}. \tag{3}$$

Each term of (3) is multiplied by t to give the penultimate expression:

$$t^2 = t_0t + \frac{x^2}{2v^2}. \tag{4}$$

This in turn is solved using the quadratic formula to give the final result, which is in the form of a time-shifted hyperbola:

$$t = \frac{t_0}{2} + \sqrt{\frac{t_0^2}{4} + \frac{x^2}{2v^2}}. \tag{5}$$

Appendix (A) gives an analytical derivation of (4). There it is proven that t_0 and v of (5) are the converted-wave zero-offset time and root-mean-square velocity, respectively.

EXAMPLES

Three examples illustrating the efficacy of the new equation and other methods are presented: traveltime estimation error with offset for a horizontal single layer model; error with offset for a horizontal multilayered model based on VSP measurements, and; a comparison of rms velocity estimates obtained by stacking velocity analyses of synthetic data and their resulting NMO-corrected gathers.

Figures 2(A) and 2(B) show that for a horizontal single layer model with exact velocities known (compressional-to-shear velocity ratio of 2.0) the new equation more accurately estimates converted-wave moveout than the standard equation and gives results comparable to other, higher-ordered, formulae. For these figures, P-S traveltime estimation error was determined by subtracting the traveltime calculated using known velocities and the standard, new, Castle (1988), and Tessmer and Behle (1988) (three-term truncation) formulae from the true traveltime, which was determined by raytracing. For comparison purposes, error is plotted as a fraction of the zero-offset traveltime, rather than as absolute error in milliseconds, and offset is expressed in terms of offset-to-depth ratios. Thus if the zero offset traveltime were one second, the maximum of the standard equation (Curve A, Figure 2(a)) would be -170 milliseconds.

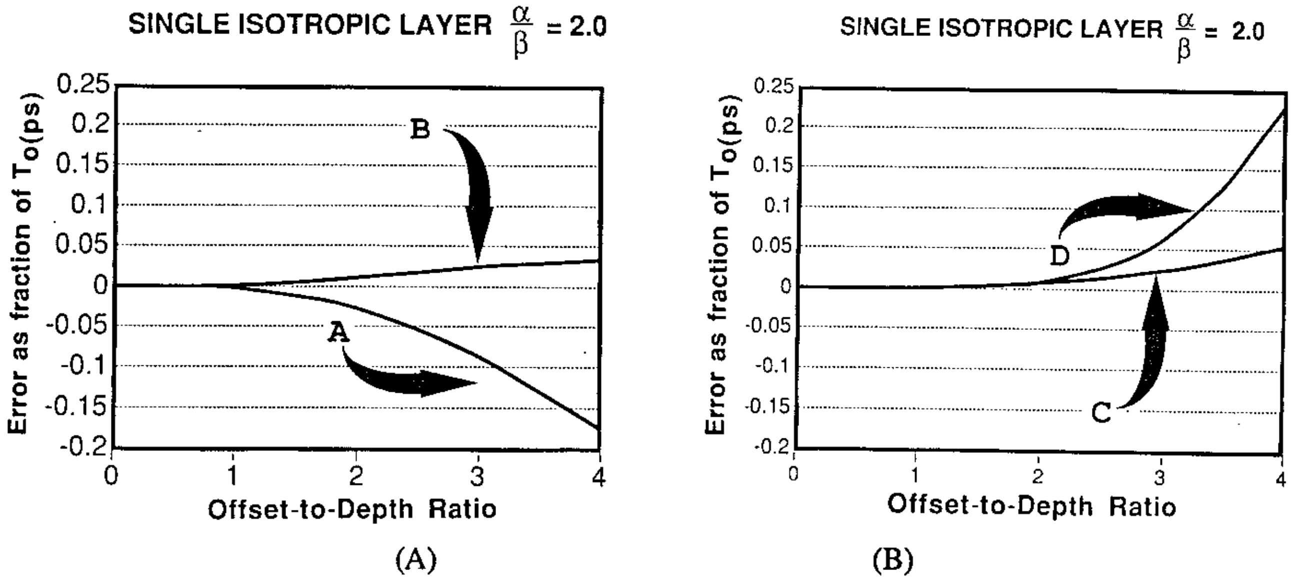


Figure 2. P-S traveltime estimation error with offset (where error is defined as the raytraced time less the formula time - exact input velocities used). (A) Curve A - standard equation, Curve B - new formula; (B) Curve C - equation of Castle, 1988, Curve D - formula of Tessmer and Behle, 1988 (3-term truncation).

A more realistic velocity model with multiple horizontal layers and varying compressional-to-shear velocity ratios (Figure 3) that was based on VSP measurements made in southern Alberta was used in a similar experiment. The true traveltime trajectory for the Pekisko event (fourteenth layer) was determined by ray tracing and then NMO corrected using the exact input velocities and the standard equation, the equation of Castle, and the new equation. These results are shown in Figure 4. It can be seen from this figure that the new equation gives accurate results for multi-layered models, even to the far offsets.

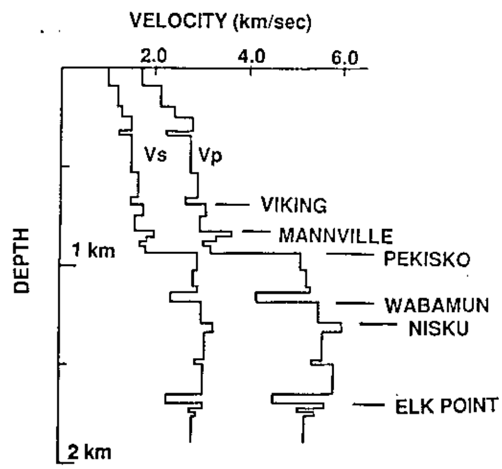


Figure 3. Southern Alberta velocity profile.

The standard and new formulae were used in stacking-velocity analyses that included varying ranges of offsets, on a single event (Mannville) created using the velocity model shown in Figure 3. Representative moveout-corrected shot gathers are shown in Figures 5 (A) and 5 (B). They show the improvement in event flattening and RMS velocity estimation that is given when the new formula is used on large-offset data (offset-to-depth 2:1 or more).

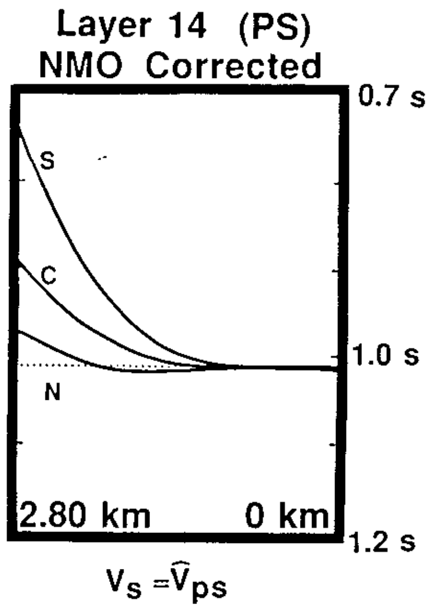
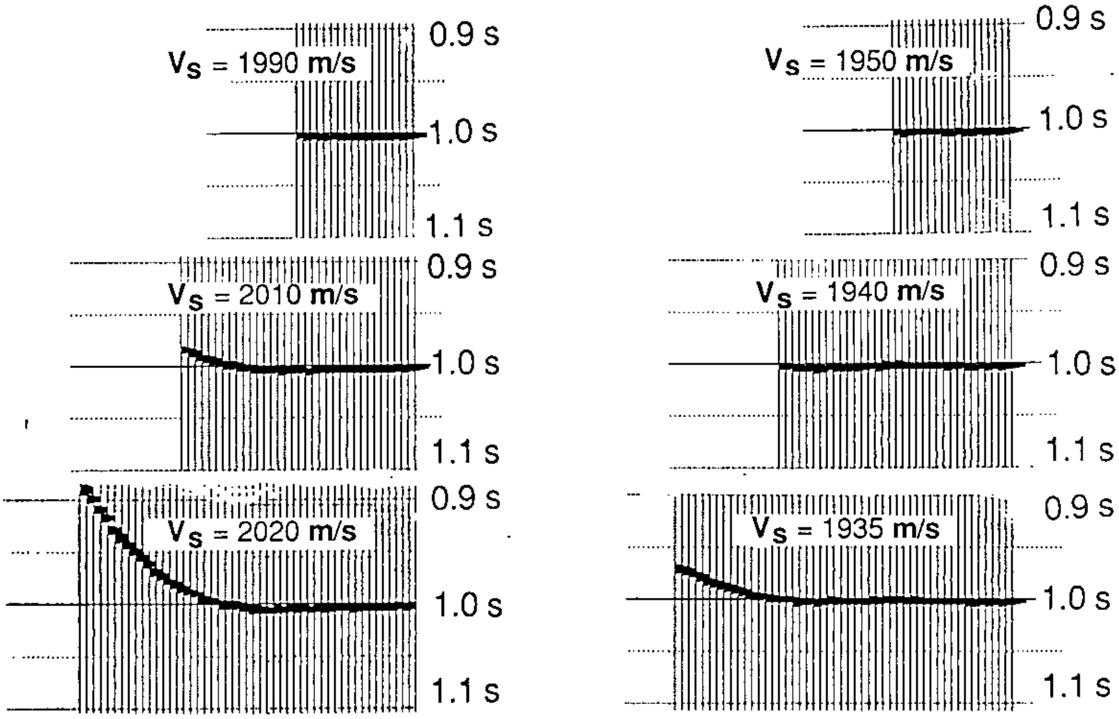


Figure 4. Moveout corrected PS traveltime trajectories for Pekisko event (fourteenth horizon of model in Figure 3). True traveltime was determined by ray tracing. Curve S - standard equation used, Curve C - Castle (1988) equation used, Curve N, new equation used. Model velocities were input into each equation.

DISCUSSION

The main advantage of the new formula is that when far offsets are included, it does much improved NMO- and RMS- velocity estimation - without the extra effort that higher-ordered equations require. Its compactness is also attractive with a view to procedures such as inverse NMO.

It is interesting to note that various time-shifted hyperbolae equations have been presented by other authors including Castle (1988), de Brazelaire (1990), and that they have been shown to better estimate seismic moveout of P-P data. In this paper it has been shown that at least one of these equations (Castle, 1988) also works for converted-wave data (in addition to the new equation), and it is likely that the equation of de Brazelaire (1990), if tested, would also work well for converted-wave moveout estimation. Except for the new equation, however, the other formulae's improvement in accuracy can mainly be attributed to the added degree of freedom they allow into the velocity search - something which requires greater effort in processing.



(A) (B)

Figure 5. Top to bottom: NMO-corrected gathers for Mannville event of model in Figure 3. Velocity analyses used offset-to-depth ranges of one, two, and three respectively and the results from the standard equation are shown in (A), and the results from the new equation are shown in (B). True $V_{rms(ps)}$ is 1952 m/s.

CONCLUSIONS

It has been shown that the standard hyperbolic approximation for seismic traveltime estimation can be used for converted-wave seismic data processing, but that it has important limits in its accuracy, primarily when data with high offset-to-depth ratios are processed. It was shown that adding degrees of freedom to converted-wave velocity analyses gives improved results, although the gain in accuracy is at the expense of additional work. A new equation was presented, and it also was shown to give accurate results for plane and horizontal multiple-layer models. It, however, has the advantage of being a compact equation, readily usable in standard NMO routines, without resorting to the increased work that methods with more degrees of freedom require.

FUTURE WORK

This work would benefit from further testing, in particular using real data. The authors plan to do such testing, as well as testing to see if the new equation can be used for unimodal data as well. Also of use would be to define an error series for the new equation, and also, it is the authors' belief that it should be possible to extend the new equation for dipping layer by adding a cosine term to the equation, as has already been done for the P-wave case.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge support for this work by the sponsors of the Consortium for Research in Elastic Wave Exploration Seismology (CREWES). Malcom Bertram is thanked for his technical assistance.

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APPENDIX

This appendix provides an analytical derivation of the new equation (Equation 4). The derivation is achieved by series manipulation of the exact traveltime series expressed in parametric form.

Tessmer and Behle (1988) showed that for a horizontally layered medium with n layers, converted-wave traveltime and offset can be expressed as a function of the layer thicknesses (where h_k is the thickness of layer k), the P- and S-wave layer velocities (where v_{pk} and v_{sk} are the P- and S-wave velocities of layer k), and the ray parameter p:

$$x_{ps} = \sum_{j=1}^{\infty} q_j \sum_{k=1}^{n} h_k(v_{pk}^{2j-1} + v_{sk}^{2j-1})(p^{2j-1}), \tag{A1}$$

$$t_{ps} = \sum_{j=1}^{\infty} q_j \sum_{k=1}^{n} h_k(v_{pk}^{2j-3} + v_{sk}^{2j-3})(p^{2j-2}), \tag{A2}$$

where

$$q_1 = 1;\ q_k = \frac{1 \times 3 \times \cdots \times (2k\text{-}3)}{2 \times 4 \times \cdots \times (2k\text{-}2)}. \tag{A3}$$

The first step in the derivation is to expand the traveltime series:

$$t_{ps} = \sum_{k=1}^{n} h_k(v_{pk}^{-1} + v_{sk}^{-1}) + \frac{1}{2}\sum_{k=1}^{n} h_k(v_{pk} + v_{sk})p^2 + \cdots. \tag{A3}$$

This in turn can be expressed as:

$$t_{ps} = t_0^{(ps)} + \frac{1}{2} t_0^{(ps)} \widehat{v}_{ps_n}^2 p^2 + \cdots \tag{A4}$$

as

$$t_0^{(ps)} = \sum_{k=1}^{n} h_k(v_{pk}^{-1} + v_{sk}^{-1}), \tag{A5}$$

and

$$\widehat{v}_{ps_n}^2 = \frac{\sum_{k=1}^{n} h_k(v_{pk} + v_{sk})}{t_0^{(ps)}}, \tag{A6}$$

where $\widehat{v}_{ps_n}$ is the converted-wave root-mean-square velocity to layer n. Next t_{ps} is multiplied through (A4). Since p is small, all terms with p^4 or more are eliminated to leave:

$$t_{ps}^2 = t_{ps} t_0^{(ps)} + \frac{1}{2} t_{ps} t_0^{(ps)} \widehat{v}_{ps_n}^2 p^2 . \tag{A7}$$

Expanding t_{ps} in the second term of the right-hand side of (A7) and eliminating terms containing p^4 or more leaves the important intermediate result:

$$t_{ps}^2 = t_{ps} t_0^{(ps)} + \frac{1}{2} t_0^{(ps)2} \widehat{v}_{ps_n}^2 p^2 . \tag{A8}$$

The series for x_{ps} can be expanded, truncated (with terms having p^3 or more being eliminated), and substitutions made in a similar manner to the above to give:

$$x_{ps} = t_0^{(ps)} \widehat{v}_{ps_n}^2 p . \tag{A9}$$

Squaring both sides and rearranging (A9) gives the second important intermediate result

$$\frac{x_{ps}^2}{\tilde{v}_{ps_n}^2} = t_0^{(ps)2} \tilde{v}_{ps_n}^2 \, p^2 . \tag{A10}$$

This can be substituted into (A8) to give the final equation:

$$t_{ps}^2 = t_{ps} t_0^{(ps)} + \frac{1}{2} \frac{x_{ps}^2}{\tilde{v}_{ps_n}^2}. \tag{A11}$$

This analytically derived result is the same as Equation (4), therefore confirming the validity of Equation (5), as well as showing analytically that v of Equation (5) is equivalent to the root-mean-square of the P- and S-wave velocities in a horizontally layered medium.