The relationship between elastic-wave velocities and density in sedimentary rocks: A proposal

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ABSTRACT

When information about density (ρ) is unavailable, it is often estimated from P-wave velocity (Vp) using an empirical relationship such as Gardner's equation. Since access to S-wave velocity is increasing, it is reasonable to consider whether Vs might contribute to improving density estimates. Sandstone and limestone datasets taken from the literature demonstrate correlations between Vp and Vs, Vp and ρ, and Vs and ρ. Both sandstone and limestone show strong correlations between Vp and Vs. The correlations between velocity and density are strong for the limestone data, but are scattered for the sandstone data. The sandstone velocity-density correlations improve significantly when the data are categorized by clay content. When measured densities are compared to densities calculated from Gardner's empirical relation, there is a systematic error in both datasets. The error increases as density decreases. Some of the factors which might affect the uncertainty in the density estimates are porosity, clay content, and lithology. Since Vs is also dependent on these factors, we are seeking empirical relationships among Vp, Vs, and ρ. One goal of this investigation is to estimate density from seismically available measurements.

INTRODUCTION

S-wave velocity information is becoming increasingly available through VSP analysis, full-waveform logging, and multicomponent seismic data. The objective of the proposed work is to determine if this additional information can be combined with P-wave velocity data to improve density estimates.

The elastic impedance of an interface is the product of both velocity and density, and so density is an important parameter in seismic reflection work. Synthetic seismograms are more accurate when density information is incorporated into the reflectivity sequences. Density is a required parameter when seismic data is inverted for velocities. When density data is unavailable, Gardner's relationship (Gardner et al., 1974) is commonly used to estimate density from Vp:

$$\rho = .23\ Vp^{.25} \qquad (1)$$

where: ρ = bulk density in g/cm^3
Vp = P-wave velocity in ft/s

This empirical relationship is based on field and laboratory measurements of saturated sedimentary rocks from a wide variety of basins and depths. As shown in Figure 1, the relationship is essentially an average of the fits for sandstone, shale, and carbonates. Evaporites do not conform to the equation.

Lindseth (1979) proposed an empirical relation between acoustic impedance and density which is used to extract the density from the reflectivity:

$$Vp = 0.308\ \rho Vp + 3460 \qquad (2)$$

where: ρ = bulk density in g/cm^3
Vp = P-wave velocity in ft/s

Lindseth based his equation on the data of Gardner et al., 1974, from which equation (1) is also derived. Both equations (1) and (2) use Vp to estimate density. The two are related by (eg. Sheriff, 1984):

$$Vp = \sqrt{\frac{k + \frac{4}{3}\mu}{\rho}} \qquad (3)$$

where: k = bulk modulus
μ = shear modulus

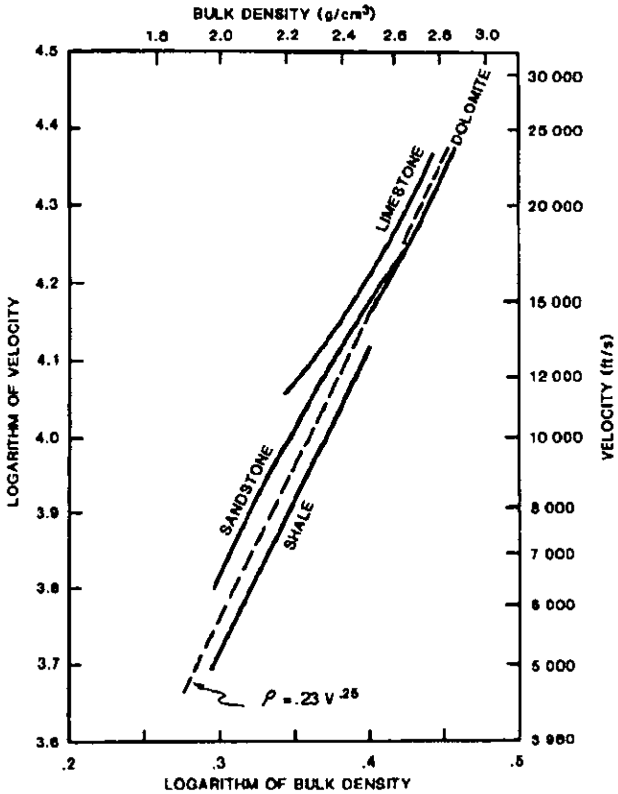

FIG. 1. Velocity-density relationships for a variety of lithologies. Gardner's relation is approximately correct for clastics and carbonates, but not for evaporites. (From Gardner et al., 1974.)

Vs is also related to density as follows (eg. Sheriff, 1984):

$$Vs = \sqrt{\frac{\mu}{\rho}} \qquad (4)$$

Because Vp and Vs are both dependent on density, it is reasonable that a relationship which uses both parameters may provide more accurate density predictions.

Equation (1) describes an empirical exponential relationship between Vp and ρ. A number of workers have observed a empirical linear relationship between Vp and Vs (eg. Pickett, 1963; Domenico, 1984; Castagna et al., 1985). This linearity may lead us to question whether Vs can provide us with any new information about density. However, the relationship between Vp and Vs has been shown to be related to lithology, and is sometimes expressed using the Vp/Vs ratio (eg. Pickett, 1963; Tatham, 1982; Rafavich et al., 1984, Miller and Stewart, 1990). When we recall that Gardner's equation does not account for variations in lithology, it is feasible that the combination of Vp and Vs will improve density predictions by providing the missing lithology factor.

METHODS

The approach we propose is largely an empirical one, such as was used to determine Gardner's equation. Using core and log data, we first determine how closely the data conforms to Gardner's relationship. We then see if the difference between the measured value and the predicted value can be related to some third parameter, such as Vs. If so, the third parameter may be used to perturb the estimate closer to the measured value.

Our objective is to incorporate Vs into a density expression, and so it is useful to consider what form the relationship may take. We may start by accepting that density is related to Vp in the following manner:

$$\rho = b\, Vp^{a} \qquad (5)$$

where: a and b are constants

We are constrained by the fact that when Vp is zero, density is also zero. Conversely, Vs can be zero, as it is in fluids, and density can still have a non-zero value. One possible way to meet these conditions is to arrange the parameters in the following way:

$$\rho = b\, Vp^{a}\,(c + d\, Vs) \qquad (6)$$

where: c and d are constants

Dividing through we obtain a linear equation which relates the ratio of measured to calculated density to Vs, with slope d and intercept c.

$$\frac{\rho}{bVp^a} = c + dVs \tag{7}$$

Plotting these parameters may enable us to determine if this relationship exists, and whether or not it is actually linear.

RESULTS

The two datasets used here, one sandstone and one limestone, are taken from the literature. The sandstone data are from Han et al. (1986). They made extensive measurements on 75 sandstone cores with varying porosities and clay contents. The data used are from saturated samples at 40 MPa. pressure.

Figure 2 shows the logarithm of Vp and Vs plotted against the logarithm of bulk density. The least-squares best-fit lines are also displayed. The equation derived from this dataset for Vp is:

$$\rho = 0.019\ Vp^{.58} \tag{8}$$

Figure 2 indicates that Vs can also be empirically related to density, with the following best fit:

$$\rho = 0.049\ Vs^{.50} \tag{9}$$

(Note that for all the equations derived from data in this paper, velocities are in m/s and density is in g/cm^3, whereas Gardner et al. (1974) and Lindseth (1979) used Imperial units in their equations. To change Gardner's equation to its SI equivalent, substitute 0.31 for 0.23 as the multiplier and use m/s and g/cm^3. Also, density refers to bulk density unless otherwise specified.)

The constants in the expression for Vp differ from those in Gardner's equation. To view the difference graphically, Vp is plotted against density in Figure 3. Equation (8) is used to calculate ρ from the measured velocities and is plotted through the data points. Also shown are the densities which are calculated from the measured velocities using Gardner's equation in SI units. Gardner's equation fits the data as the density approaches the quartz matrix density (2.65 g/cm^3) but deviates at low densities. Density has an inverse correlation with porosity (Figure 4), so that Gardner's equation shows an increasing deviation from the data as porosity increases. Density can be related to porosity by the following equation (eg. Schlumberger, 1989):

$$\rho_b = \rho_f\,\phi + \rho_m\,(1 - \phi) \tag{10}$$

where: ρ_b = bulk density

ρ_f = pore fluid density

ϕ = porosity

ρ_m = matrix density

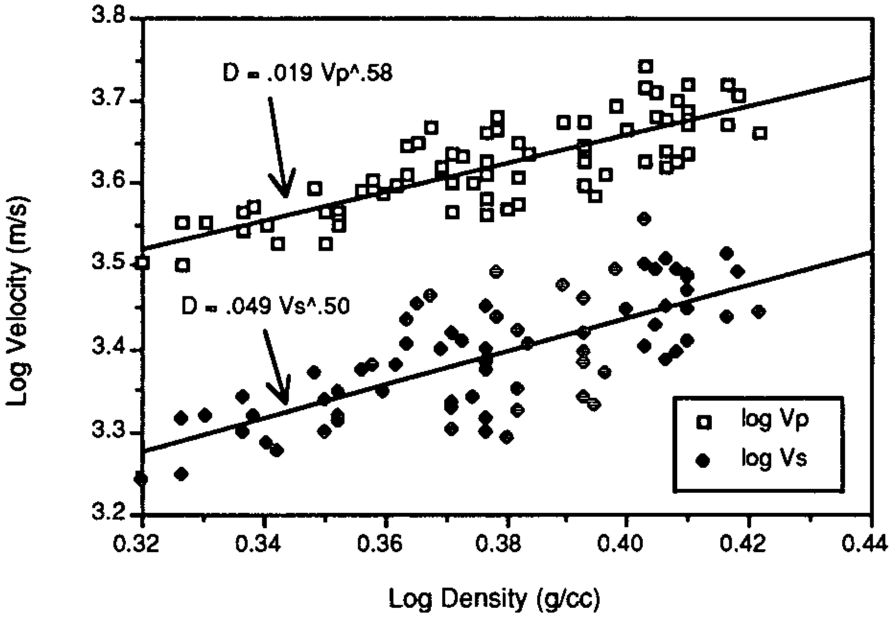

FIG. 2. Logs of Vp and Vs vs. log density for sandstone data from Han et al., 1986. Data are scattered but there are apparent trends for both Vp and Vs. Equations derived from the best-fit lines are shown (D = density).

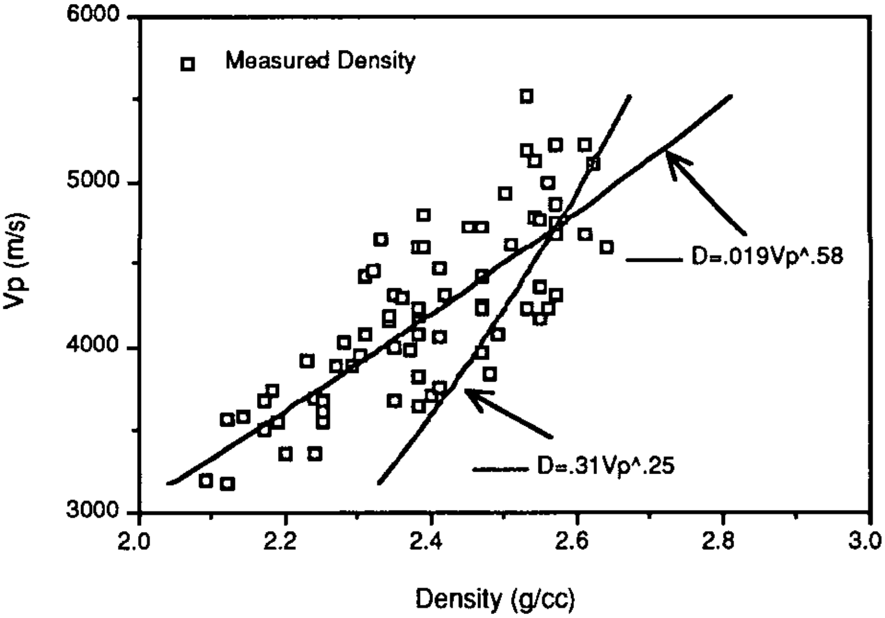

FIG. 3. P-wave velocity vs. density for sandstone data. The overlain lines show the densities that are calculated from the measured velocities, using both Gardner's equation and the equation from Figure 1. Gardner's equation agrees quite well except at low densities.

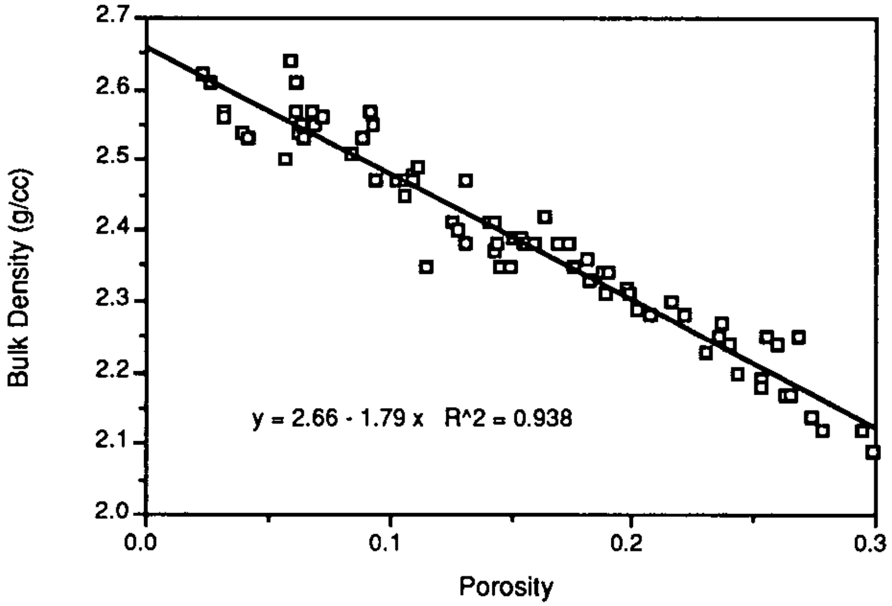

FIG. 4. Density vs. porosity for the sandstone data. Bulk density has quite a high inverse correlation with porosity. This means that Gardner's equation tends to deviate from this dataset at high porosities.

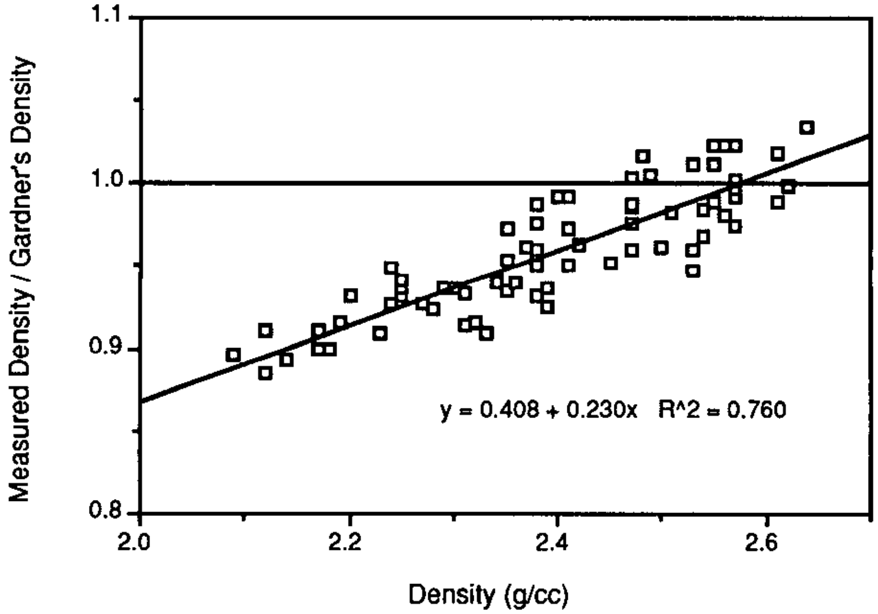

FIG. 5. The ratio of the measured density to the density calculated using Gardner's equation is plotted against the measured density. If the scatter were random, the data would be evenly distributed around the 1.0 line. This plot indicates that the relative deviation is systematic.

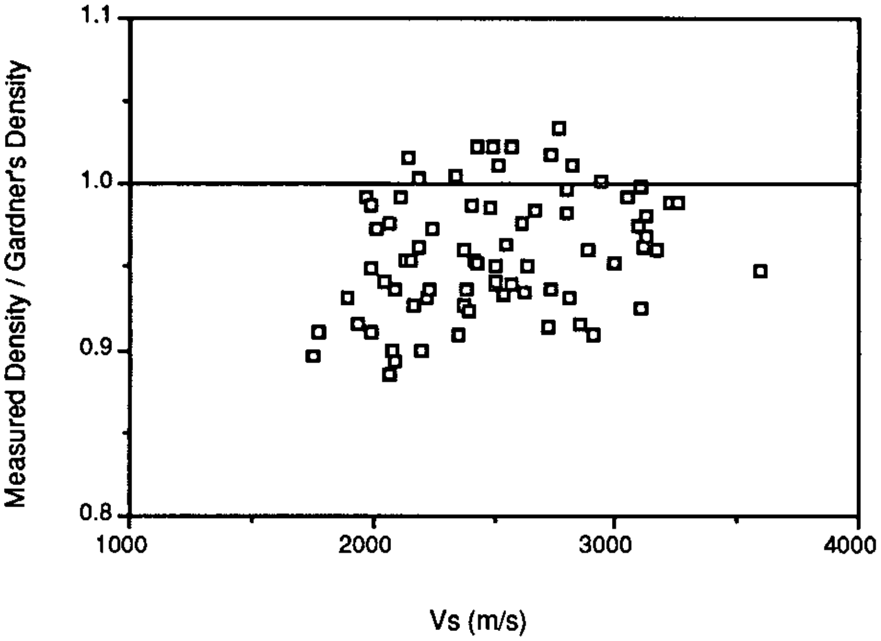

FIG. 6. The relative deviation between Gardner's density and the measured density is plotted against Vs. The deviation does not appear to be related to Vs.

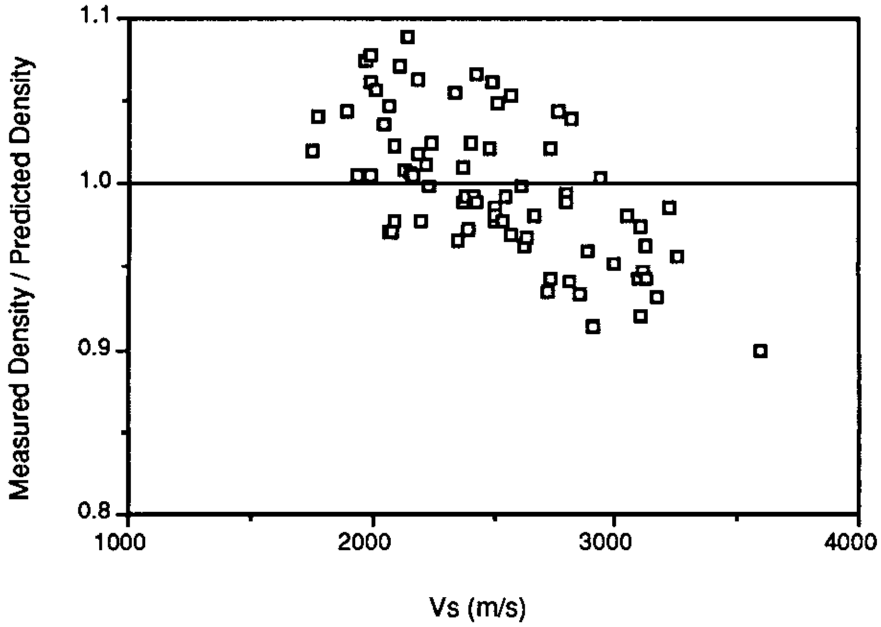

FIG. 7. The ratio of the measured density to the density calculated using equation (8) is also plotted against Vs. The relative deviation in this case does appear to have some dependence on S-wave velocity.

In Figure 5, the ratio of the measured density to the density calculated from Gardner's equation is plotted against the measured density. The deviation from Gardner's relation is systematic, and is greatest at low densities.

Referring back to Figure 1, Gardner et al.'s plot does predict that sandstone densities will be overestimated at low densities. However, Figure 1 predicts an error of 3 - 4 % at 2.2 g/cm^3 whereas Figure 5 shows an error of 9 - 10 %. Both plots show Gardner's estimate converging with the measured density at about 2.6 g/cm^3. This is approaching the density of a pure quartz aggregate with no porosity.

In Figure 6 the data are plotted to see if the relative deviation between the measured and the calculated densities is related to Vs. When the calculated densities determined from Gardner's equation are used, there is no trend. However, if the densities are calculated from expression (8), there does appear to be some dependence on Vs (Figure 7).

When Vp, Vs, and ρ are plotted on three axis and rotated through various angles, it is apparent that the strongest trend in the data is the correlation between Vp and Vs (Figure 8). The linearity of this relationship does not necessarily negate the usefulness of Vs, as the slope and intercept of the Vp-Vs line might potentially be used to identify the samples as sandstones.

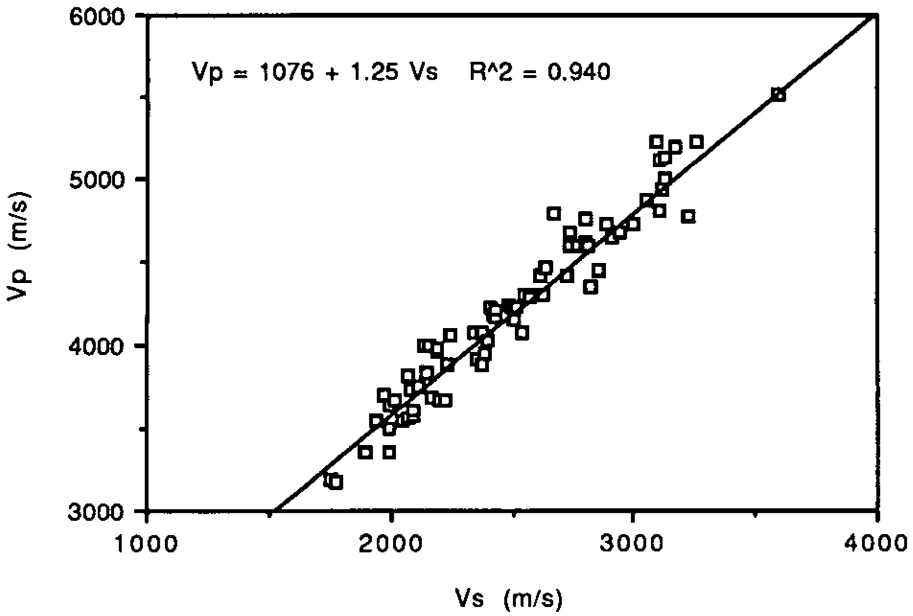

FIG. 8. There is a strong linear correlation between Vp and Vs for the sandstone data. The nature of the Vp-Vs relationship may contain information about lithology.

The relationship between velocity and density as seen in Figure 2 shows considerable scatter for both Vp and Vs. When Han et al. (1986) used this dataset to study the relationship between velocity and porosity, they discovered that the inclusion of a clay term improved the fit to the data significantly. Vernik and Nur (1991) divided sandstones into categories based on clay volume. The divisions they used were:

(1) clean arenite (< 2% clay)

(2) altered arenites and arkoses (2-15% clay)

(3) wackes (15-35% clay)

(4) sandy shales (>35% clay)

The classification system is based on petrophysical characteristics such as texture, minerology, and the structural position of the clay particles. Each category demonstrated a strong correlation between Vp and porosity.

When the sandstone data examined here are divided into those same categories, the correlations between velocity and density improve significantly for both Vp and Vs (Figures 9 & 10). This suggests that the change in rock properties as sandstones grade into shales is not continuous. Rather, clastics with differing clay contents behave as discrete rock types with specific elastic responses. Although Gardner et al. do not give any indication as to the purity of their sandstone samples, it is interesting to note that the sandstone and shale lines in Figure 1 are discrete and fall on either side of the best-fit line.

To observe the effect that clay content has on density predictions, the relative deviation for both Gardner's equation and equation (8) are plotted against clay volume (Figure 11). Both estimates show some dependence on clay.

The second dataset shown here is also taken from the literature (Kuiper et al., 1959). It is a set of 13 limestone cores which were measured dry and at atmospheric pressure. The limestones also exhibit a strong correlation between Vp and Vs (Figure 12). Because the laboratory conditions were so different for the sandstone measurements, we cannot use these particular datasets to compare lithologic differences.

A plot of the logarithms of velocity vs. the logarithm of density indicates strong correlations for both Vp and Vs (Figure 13). The relationship determined from the data for Vp is:

$$\rho = 0.026\ Vp^{.54} \tag{11}$$

and for Vs is:

$$\rho = 0.012\ Vs^{.68} \tag{12}$$

To compare equation (11) to Gardner's equation, the densities calculated from both expressions are shown in Figure 14. The pattern is very similar to the sandstone case. Low density samples are overestimated (by the same percentage as for sandstones), but the prediction improves as densities increase to about 2.6 g/cm^3.

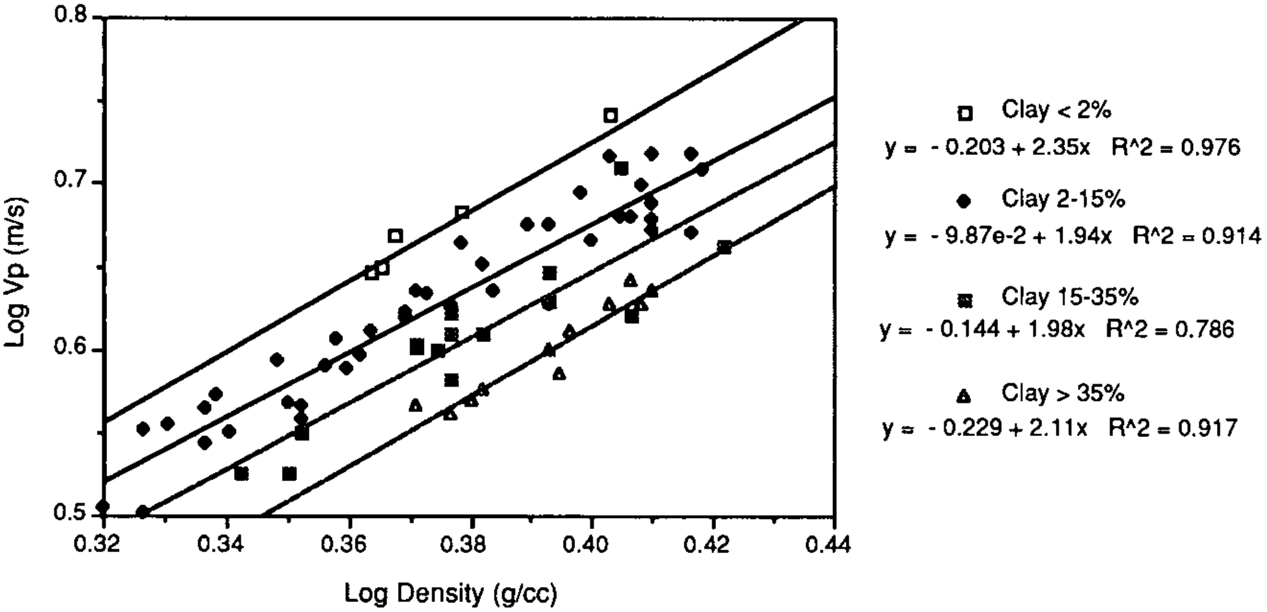

FIG. 9. Log Vp vs. log density for the sandstone data. This time the data has been divided into categories based on the clay content by volume. Correlations are much improved over Figure 2, in which the samples are treated as one rock type.

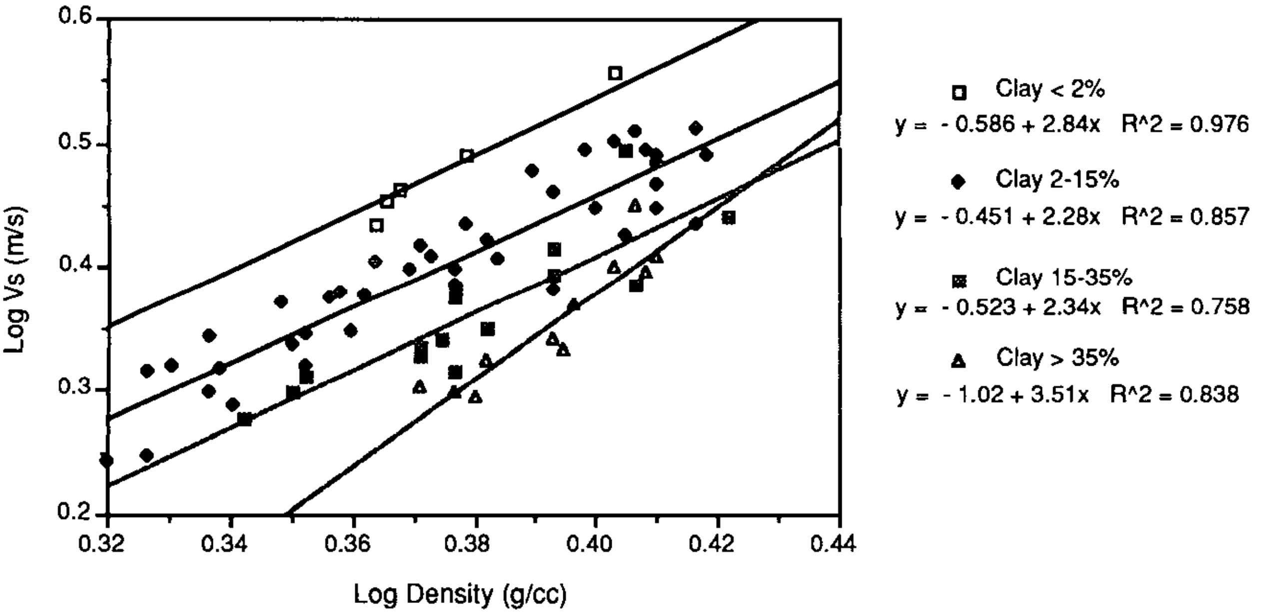

FIG. 10. Log Vs vs. log density for the sandstone data. The data is divided into the same categories as in Figure 9. The correlation between velocity and density again shows a significant improvement over Figure 2, in which all samples are classified as 'sandstones'.

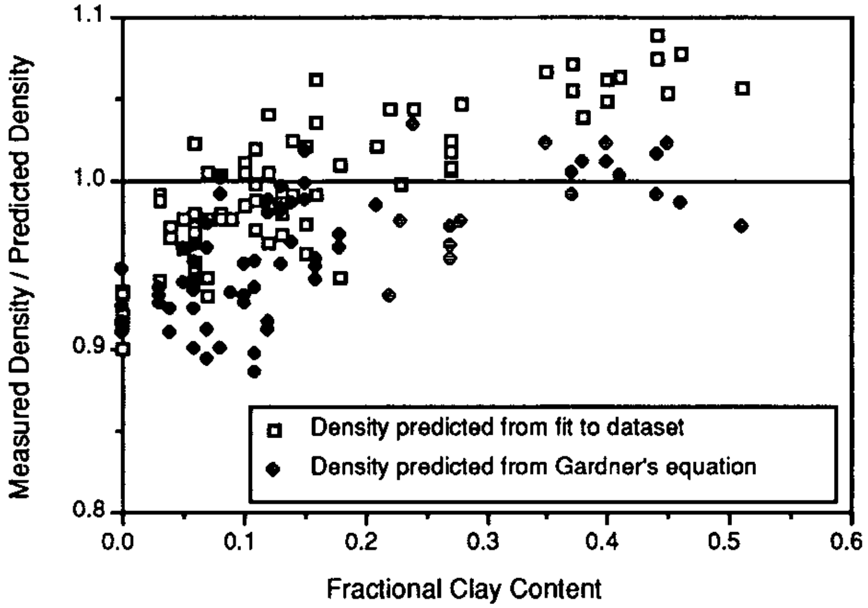

FIG. 11. When density is calculated using equations (1) and (8), the relative deviations between measured and calculated density exhibits some dependence on the clay volume for both cases.

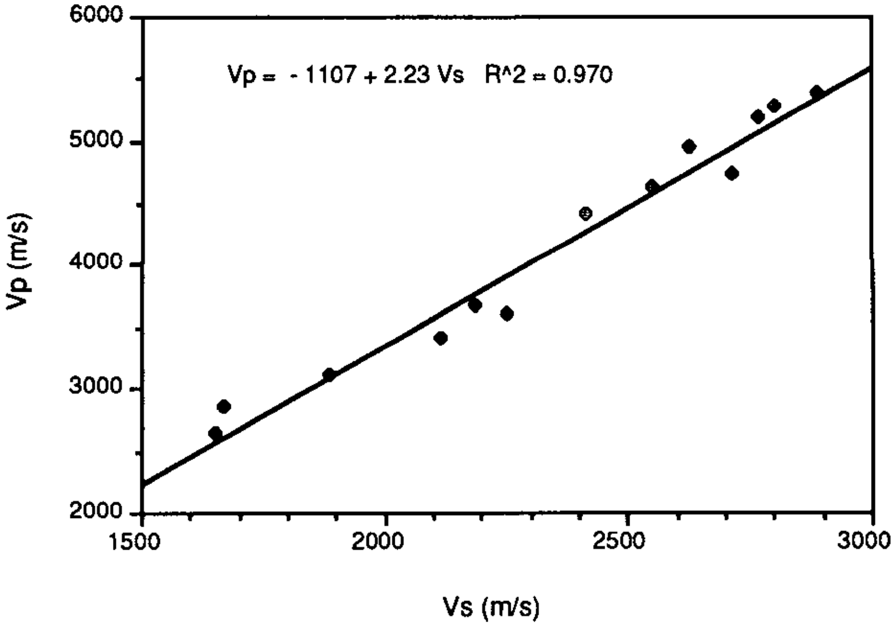

FIG. 12. Vp is plotted against Vs for the limestone data. The correlation is linear, but the negative intercept suggests that the coefficients for this small dataset are not indicative of rock properties.

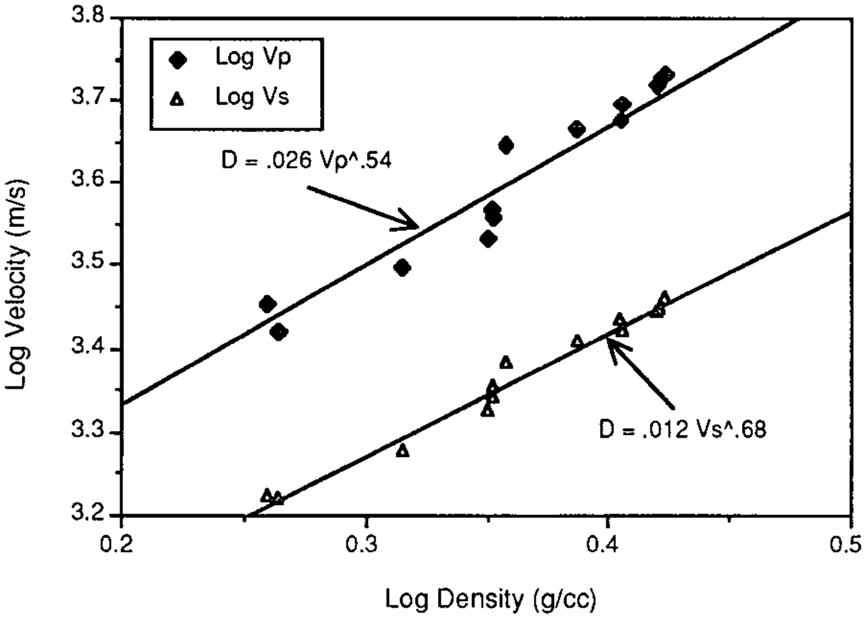

FIG. 13. Logs of Vp and Vs vs. log density for the limestone data. The equations are calculated from the least-squares best-fit lines.

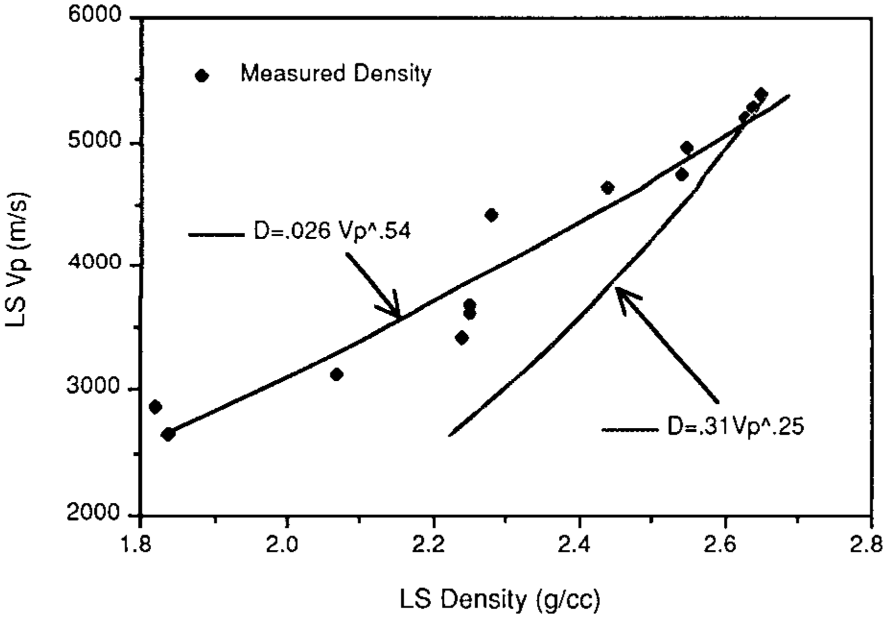

FIG. 14. Vp vs. density for the limestone data: the overlain lines show the densities that are calculated from the measured velocities, using both Gardner's equation and equation (11). Gardner's equation fits the measured data at the higher densities but deviates at lower densities.

The ratio of the measured densities to the densities calculated from Gardner's equation are plotted against Vs in Figure 15. In this case, there is an apparent dependence on S-wave velocity.

It is interesting that all the empirically derived equations from both datasets differ substantially from Gardner's relationship. The constant multiplier is an order of magnitude smaller, and the exponent is at least two times greater than in Gardner's expression.

SUMMARY

Sandstone and limestone datasets taken from the literature demonstrate correlations between Vp and Vs, Vp and ρ, and Vs and ρ. The correlation between velocity and density is scattered for the sandstone data, but improves significantly when the data are categorized by clay content. In the limestone case, the velocity-density correlation is strong for both Vp and Vs. For both sandstone and the limestone, the velocity-density relations derived from the data differ from the Gardner's equation. The relative deviation between the measured densities and the densities predicted by Gardner's relation is systematic, with the error increasing as density decreases. Both datasets show strong correlations between Vp and Vs.

These initial examinations show that the uncertainty in density predictions may be influenced by porosity, clay content, and lithology. Some workers have shown that Vs is more sensitive to porosity and clay content than Vp (eg. Eastwood and Castagna, 1983; Domenico, 1984; Han et al., 1986). Numerous studies indicate that the Vp-Vs relationship is influenced by lithology. Thus, it is possible that S-wave velocity information may improve the density estimates obtained from Vp. These observations encourage us to continue the investigation into elastic velocities and density.

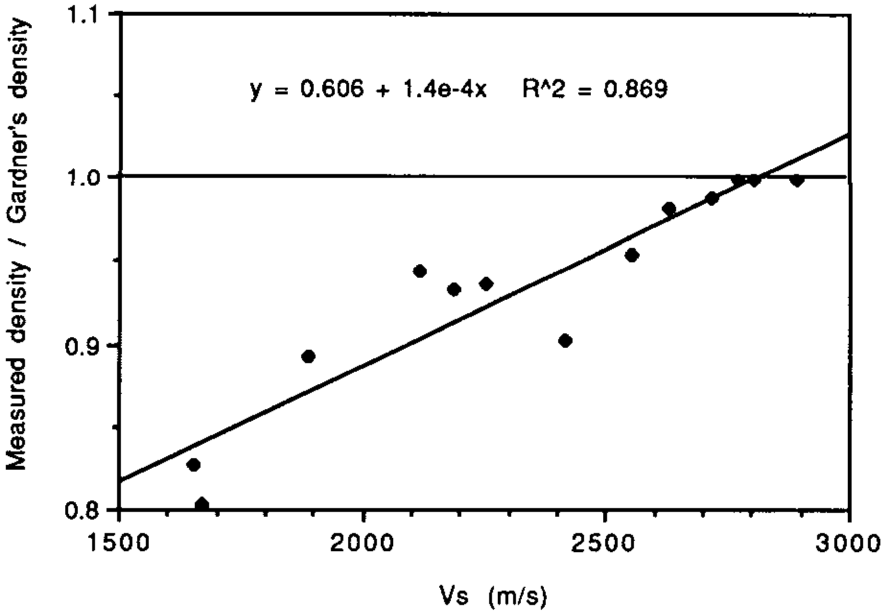

FIG. 15. The relative deviation between the measured density and Gardner's density is plotted against Vs. There is an apparent dependence on S-wave velocity.

FUTURE WORK

We plan to examine more data, using rock core measurements and well-logs. These data will be used to test the relationships shown here, and to examine other possible relationships between various petrophysical parameters. In addition to analyzing the data for empirical trends, we would like to develop theoretical and physical models for relating Vp, Vs, and ρ.

REFERENCES

Castagna, J.P., Batzle, M.L., and Eastwood, R.L., 1985, Relationships between compressional-wave and shear-wave velocities in clastic silicate rocks: Geophysics, 50, 571-581.

Domenico, S.N., 1984, Rock lithology and porosity determination from shear and compressional wave velocity: Geophysics, 49, 1188-1195.

Eastwood, R.L. and Castagna, J.P., 1983, Basis for interpretation of Vp/Vs ratios in complex lithologies: Soc. Prof. Well Log Analysts 24th Annual Logging Symp.

Gardner, G.H.F., Gardner, L.W., and Gregory, A.R., 1974, Formation velocity and density - the diagnostic basics for stratigraphic traps: Geophysics, 39, 770-780.

Han, D.H., Nur, A., and Morgan, D., 1986, Effects of porosity and clay content on wave velocities in sandstones: Geophysics, 51, 2093-2107.

Kuiper, J., Van Ryan, W.M.L., and Koefoed, O., 1959, Laboratory determinations of elastic properties of some limestones: Geophys. Prosp., 7, 38-44.

Lindseth, R.O., 1979, Synthetic sonic logs - a process for stratigraphic interpretation: Geophysics, 44, 3-26.

Miller, S.L.M. and Stewart, R.R., 1990, Effects of lithology, porosity and shaliness on P- and S-wave velocities from sonic logs: Can. J. Expl. Geophys., 26, 94-103.

Pickett, G.R., 1963, Acoustic character logs and their applications in formation evaluation: J. Petr.Tech., June, 659-667.

Rafavich, F., Kendall, C.H.St.C., and Todd, T.P., 1984, The relationship between acoustic properties and the petrographic character of carbonate rocks: Geophysics, 49, 1622-1636.

Schlumberger, 1989, Log Interpretation Principles/Applications: Schlumberger Educational Services.

Tatham, R.H., 1982, Vp/Vs and lithology: Geophysics, 47, 336-344.

Vernik, L. and Nur, A., 1991, Lithology prediction in clastic sedimentary rocks using seismic velocities: presented at the 1991 SEG Summer Research Workshop, St. Louis, Missouri.