Alternate description of the wavefield generated by two vertical vibrators in counterphase

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ABSTRACT

Experimental evidence for the existence of shear waves in the near-vertical direction generated by two vertical vibrators in counterphase was collected by Edelmann, who also made some conjectures as to the cause of these shear waves. Dankbaar put the conjectures of Edelmann on a more theoretical footing, by expressing the wavefield as the far-field radiation of a couple. I propose the possibility of a different dynamic mechanism, namely the radiation from a double-couple system from vibrators with displacements in counterphase.

INTRODUCTION

Edelmann (1981) was able to collect inline shear wave data by using two inline vertical vibrators running in counterphase (180° out of phase) from each other. Figure 1 shows some shot records from this paper. The important feature is the coherent energy on the inside traces, even at a time of over three seconds. This energy, I surmise, must come from a take-off angle quite close to the vertical. Edelmann's paper does not give the exact shooting geometry, so an accurate estimate is not available. Edelmann conjectured that since a disk vibrating vertically on the free surface produces a strong shear-waves around 37° from vertical then the two vibrators running in counterphase would produce strong uncompensated shear-wave between the two vibrators (Figure 2). This is true around 37° from both vibrators, but this is necessarily a near field phenomenon so does not explain the deeper near offset shear-wave energy as observed in Figure 1.

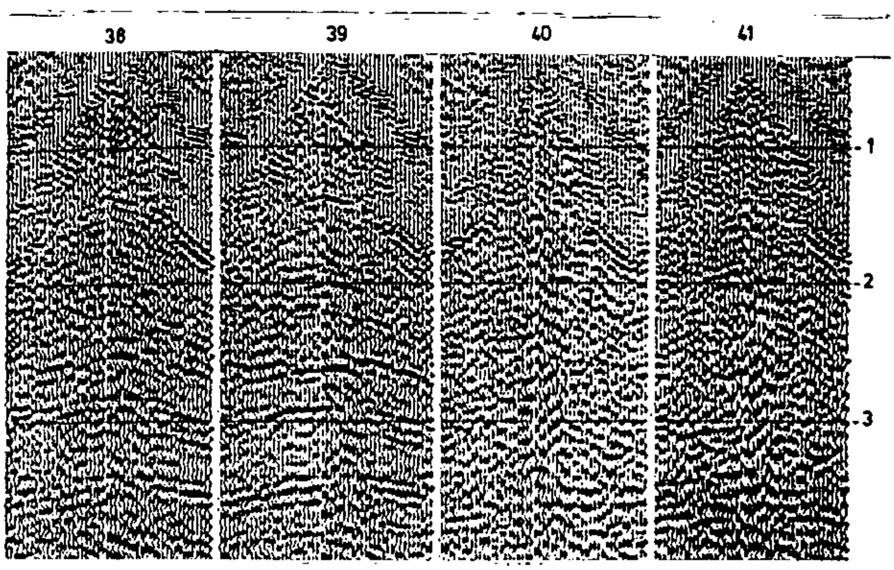

FIG. 1. Shot records as recorded from two in-line vertical vibrators in counterphase. (From Edelmann, 1981, Figure 8)

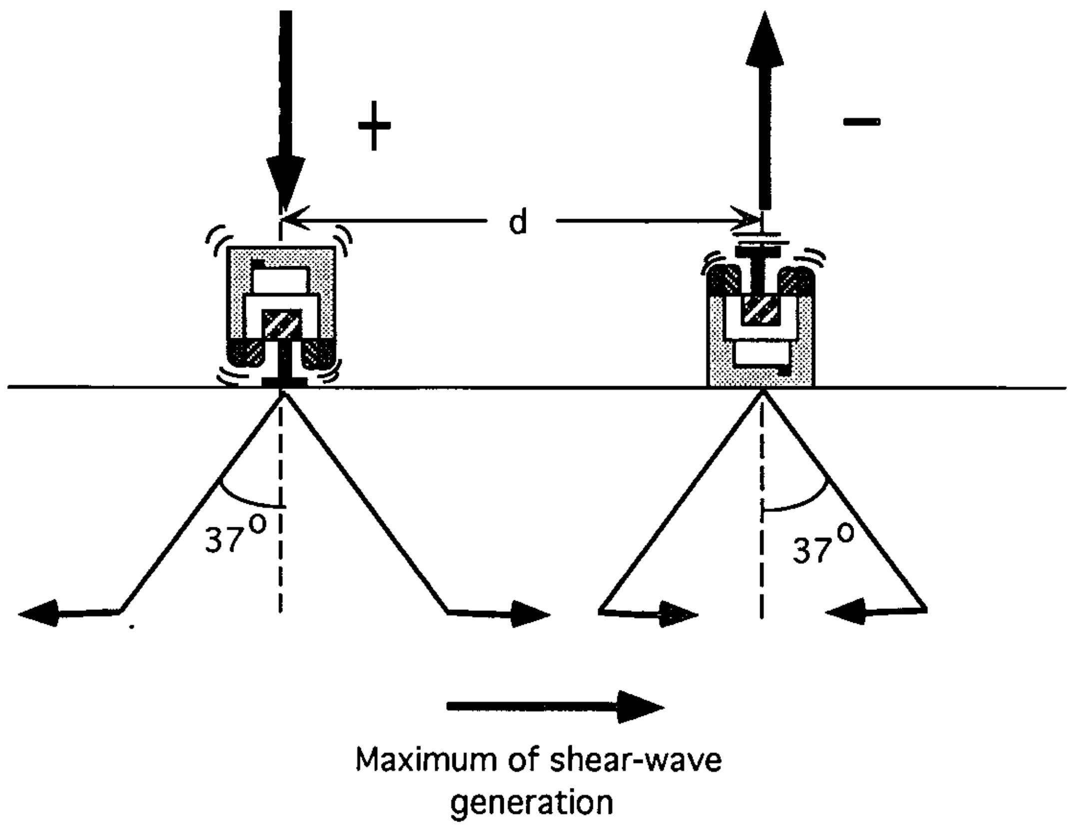

FIG. 2. Horizontal displacement components produced by two vertical vibrators in counterphase (adapted from Edelmann, 1981, Figure 6).

Using the solid elastic modeling facility at The University of Calgary, we were able to show the polarization characteristics of the shear waves, but due to the size of the transducers and the thickness of the elastic medium the results were not clearly in the far-field. The results were, however, supportive of the existence of correctly polarized shear-waves. There exists a need for a good theoretical basis for further progress.

The lack of a good theoretical rational was bridged by Dankbaar (1983), who used the far field displacement of what is essentially the response of a vertical point source on a free surface. Following the convention of Aki and Richards (1980) we will call this response:

$$G_{in}^{\text{free}}(\mathbf{x},t;\xi,\tau), \qquad (1)$$

which is the elastodynamic Green's function, representing the i'th component of displacement at the point $\mathbf{x}$ and time t resulting from a point impulsive source in the n'th direction (which is vertical in the present discussion) located on the free surface at point ξ and detonated at time τ. As mentioned earlier, Dankbaar (1983) did not actually use the exact Green's function but rather the asymptotic far-field representation of the Green's function. However, for our purpose it is enough to keep in mind the difference and proceed. Dankbaar then used a Taylor series to get the response of a source displaced from ξ in the direction of unit vector Δ for a distance D as follows:

$$G_{in}^{\text{free}}(\mathbf{x};\xi + D\Delta) = G_{in}^{\text{free}}(\mathbf{x};\xi) + \left[\frac{\partial G_{in}^{\text{free}}(\mathbf{x};\xi)}{\partial \xi^{i}}\Delta^{i}\right] D + \frac{1}{2}\left[\frac{\partial^2 G_{in}^{\text{free}}(\mathbf{x};\xi)}{\partial \xi^{i}\partial \xi^{j}}\Delta^{i}\Delta^{j}\right] D^2 + \ldots . \tag{2}$$

Einstein summation notation is in effect for repeated indices. Equation (2) allows one to calculate the response of two point sources in counterphase separated by a distance $d = 2D$. This is just the response of a single simple couple, which should have the response:

$$\begin{aligned} G_{mn}^{\text{free}}(\mathbf{x};\xi + D\Delta) - G_{mn}^{\text{free}}(\mathbf{x};\xi - D\Delta) = 2\left[\frac{\partial G_{mn}^{\text{free}}(\mathbf{x};\xi)}{\partial \xi^{i}}\Delta^{i}\right] D + \\ + \frac{1}{3}\left[\frac{\partial^3 G_{mn}^{\text{free}}(\mathbf{x};\xi)}{\partial \xi^{i}\partial \xi^{j}\partial \xi^{k}}\Delta^{i}\Delta^{j}\Delta^{k}\right] D^3 + \cdots \end{aligned} \tag{3}$$

With this description Dankbaar (1983) was able to show that at an angle greater than five to ten degrees shear-wave energy dominates over compressional waves. This can be seen on figure (3a) which shows the far-field radiation characteristics of two vibrators in counterphase. Dankbaar (1983) was also able to show the effect described by Edelmann (1981) as a near-field effect which can also be seen in figure (3b).

(a)

(b)

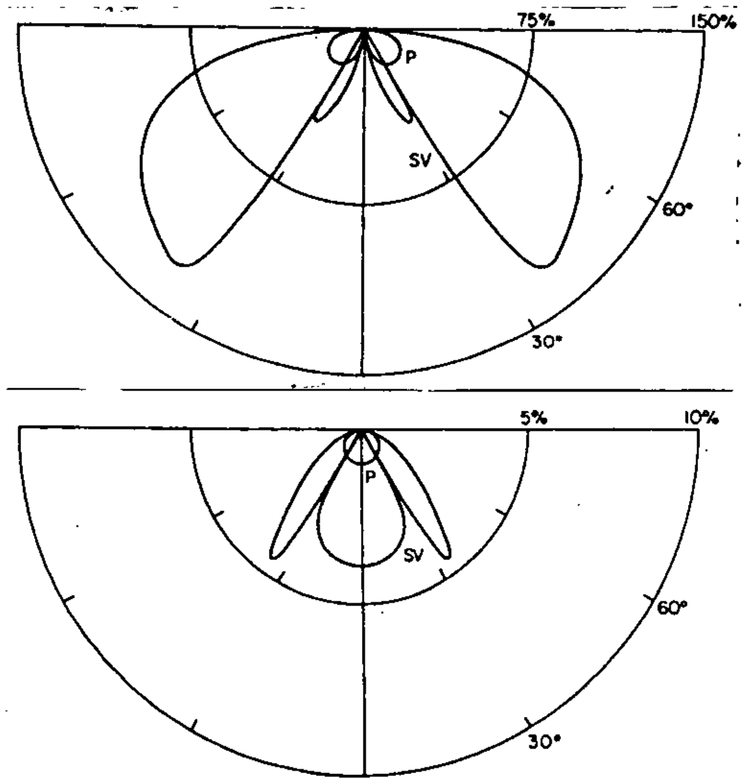

FIG 3. Radiation characteristics of two vibrators in counterphase,
(a) Far field response, (b) Near field response.
(As copied from Dankbaar (1983) figure 5.)

There is much more contained in his paper which is not directly related to the present discussion so will be omitted. Equation (3) represents the response if we were constraining the forcing terms to be in counterphase, but it is also possible to constrain the displacement to be in counterphase; in this case the single couple is no longer the appropriate force description.

THE DOUBLE-COUPLE SOURCE

If we force the displacement to be in counterphase, the possibility arises of describing the source as a double couple. The development here parallels that of Aki and Richards (1981), the difference being that ours was motivated by counterphase surface sources and theirs from earthquake strike-slip sources. The development begins with the representation theorem as given by Aki and Richards (1981, page 29) which is reproduced below:

$$\begin{aligned} u_n(\mathbf{x},t) = & \int_{-\infty}^{\infty} \mathrm{d}\tau \int_V f_m(\xi,\tau)\, G_{nm}(\mathbf{x},t-\tau;\xi,0)\, \mathrm{d}V(\xi) + \\ & \int_{-\infty}^{\infty} \mathrm{d}\tau \int_S T_m(\mathbf{u}(\xi,\tau),\mathbf{n})\, G_{nm}(\mathbf{x},t-\tau;\xi,0)\, \mathrm{d}S(\xi) + \quad , \qquad (4) \\ & \int_{-\infty}^{\infty} \mathrm{d}\tau \int_S u_m(\xi,\tau)\, c_{mjkl}(\xi)\, \mathrm{n_j}\, G_{nk,l}(\mathbf{x},t-\tau;\xi,0)\, \mathrm{d}S(\xi) \end{aligned}$$

where	$\mathbf{u}$	$\equiv$	displacement vector,
	$\mathbf{x}$	$\equiv$	observation point vector,
	ξ	$\equiv$	source point vector,
	$G_{nm}(\mathbf{x},t-\tau;\xi,0)$	$\equiv$	n'th component of the elastodynamic Green's function with unit impulse applied at ξ and time τ,
	$G_{nk,l}$	$\equiv$	partial derivative of the Green's function with respect to source coordinate ξ_l ,
	V	$\equiv$	volume of integration containing source mechanisms,
	S	$\equiv$	closed orientable surface containing V,
	$\mathbf{n}$	$\equiv$	unit outward normal of surface S,
	f_m	$\equiv$	m'th component of body force vector,
	$T_m(\mathbf{u}(\xi,\tau),\mathbf{n})$	$\equiv$	m'th component of traction vector consistent with displacement $\mathbf{u}$ on surface element with unit normal $\mathbf{n}$,
and	$c_{mjkl}(\xi)$	$\equiv$	elastic tensor at source location ξ.

Equation 4 describes the displacement due to the existence of sources within the volume V and on the surface S. I am investigating the dynamic properties of a counterphase displacement source imbedded in a homogeneous isotropic medium. In this case I will assume the Green's function to be resulting from homogeneous boundary conditions. At present I have not brought the source to the free surface but the dynamic properties of the source is clearly shown using equation (4). We will be considering the case of no body forces or surface tractions as sources, which allows us to ignore the first and second integrals in equation (4). The integral then becomes:

$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \int_S u_m(\xi,\tau)\, c_{mjkl}(\xi)\, n_j\, G_{nk,l}(\mathbf{x},t-\tau;\xi,0)\, dS(\xi) \ . \qquad (5)$$

Consider the abstract volume represented in Figure (4a) and the augmented surface represented in Figure (4b) which represents the same volume with an indentation creating new surfaces Σ^+ and Σ^- separated by ε.

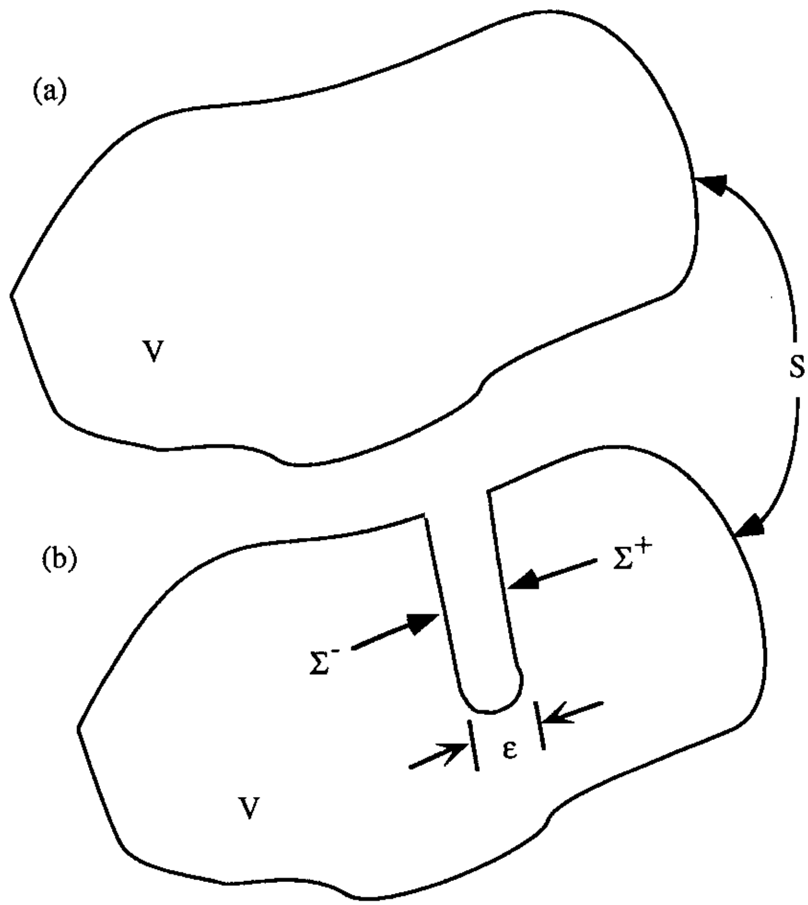

FIG. 4. (a) Volumetric potato representing the volume over which sources are integrated to obtain the total displacement.
(b) A cut is introduced halfway into the potato to create surfaces Σ^+, Σ^- and ε.

We now imbed the displacement discontinuity within the two newly created surfaces and introduce a coordinate system as shown in Figure 5.

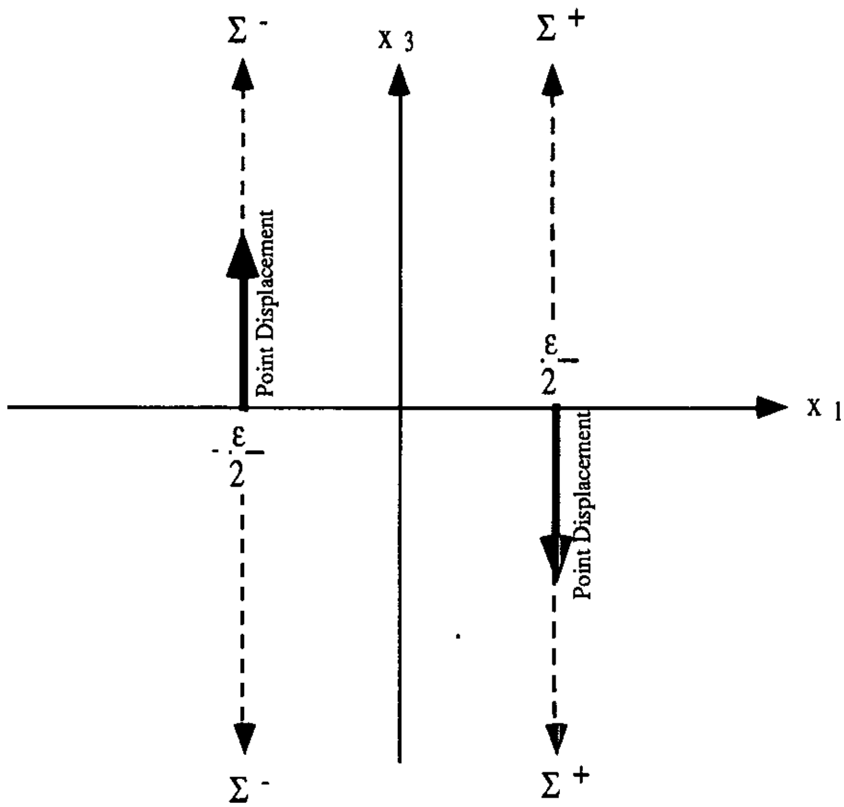

FIG. 5. Graphical representation of point displacements imbedded in surfaces Σ^+ and Σ^-.

Since the only sources are on surfaces Σ^+ and Σ^- we can rewrite equation 5 as:

$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma^+} u_m(\xi,\tau)\ c_{mjkl}(\xi)\ n_j\ G_{nk,l}(\mathbf{x},t-\tau;\xi,0)\ dS(\xi) + \\ + \int_{-\infty}^{\infty} d\tau \int_{\Sigma^-} u_m(\xi,\tau)\ c_{mjkl}(\xi)\ n_j\ G_{nk,l}(\mathbf{x},t-\tau;\xi,0)\ dS(\xi) \qquad (6)$$

The assumption of isotropy allows the elastic tensor to be expressed in the following manner:

$$c_{ijkl} = \lambda\ \delta_{ij}\delta_{kl} + \mu\ (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \qquad (7)$$

where	λ and μ	$\equiv$	Lame constants,
and	δ_{ij}	$\equiv$	Kronecker delta .

On surface Σ^+, $n_i = \delta_{i1}$ and on surface Σ^-, $n_i = -\delta_{i1}$. Substitution of these results and equation (7) into equation (6), noting the displacement is only in the 3 direction allows us to write:

$$u_n(\mathbf{x},t) = -\int_{-\infty}^{\infty} d\tau \int_{\Sigma^+} u_m \left[G_{mn,1} + G_{1n,m}\right] \mu dS + \int_{-\infty}^{\infty} d\tau \int_{\Sigma^-} u_m \left[G_{mn,1} + G_{1n,m}\right] \mu dS \quad . \tag{8}$$

The obvious spatial and temporal arguments have been omitted to simplify the form of the equations. The displacement, as alluded to by figure 5, can be represented as:

$$u_m(\xi,\tau) = \delta_{m3}\, \delta(\tau) \left[\delta\left(\left|\xi + \frac{\varepsilon}{2}\mathbf{x}_1\right|\right) - \delta\left(\left|\xi - \frac{\varepsilon}{2}\mathbf{x}_1\right|\right)\right] \tag{9}$$

where

$\lvert\,.\,\rvert$	$\equiv$	magnitude of the argument,
$\delta(\tau)$	$\equiv$	Dirac delta function (distribution),
and $\mathbf{x}_1$	$\equiv$	unit vector in the positive 1-direction.

Upon substitution of equation (9) into equation (8) we arrive at the conclusion:

$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma^+} \delta(\tau)\, \delta\left(\left|\xi - \frac{\varepsilon}{2}\mathbf{x}_1\right|\right) \left[G_{3n,1} + G_{1n,3}\right] \mu dS + \int_{-\infty}^{\infty} d\tau \int_{\Sigma^-} \delta(\tau) \left[\delta\left(\left|\xi + \frac{\varepsilon}{2}\mathbf{x}_1\right|\right)\right] \left[G_{3n,1} + G_{1n,3}\right] \mu dS \quad . \tag{10}$$

I have not evaluated the Dirac delta functions in equation (10) so that the following steps will hopefully be clearer. If ε is allowed to approach zero then the surfaces Σ^+ and Σ^- will approach a single surface Σ which is located on the 2-3 plane in this instance. This will transform equation (10) into:

$$\mathrm{u_n(x,t)} = 2\int_{-\infty}^{\infty} d\tau \int_{\Sigma} \delta(\tau)\, \delta(|\xi|) \left[G_{3n,1} + G_{1n,3}\right] \mu dS \, . \tag{11}$$

Evaluating equation (11) gives our final result:

$$u_n(\mathbf{x},t) = 2\left[G_{3n,1}(\mathbf{x},t;\xi,0) + G_{1n,3}(\mathbf{x},t;\xi,0)\right]_{(\xi=0)} \, . \tag{12}$$

Equation (12) represents the sum of two single couples. To see that each term within the square brackets represents a single couple, we will write out the definition of the partial derivatives:

$$G_{3n,1}(\mathbf{x},t;\xi,0) = \frac{\partial G_{3n}}{\partial x_1} = \lim_{\varepsilon \to 0}\left(\frac{G_{3n}(\mathbf{x},t;\xi - \varepsilon\xi_3,0) - G_{3n}(\mathbf{x},t;\xi - \varepsilon\xi_3,0)}{2\varepsilon}\right).$$

The term:

$$\frac{G_{3n}(\mathbf{x},t;\xi - \varepsilon\xi_3,0) - G_{3n}(\mathbf{x},t;\xi - \varepsilon\xi_3,0)}{2\varepsilon},$$

is just the response to a couple as shown in figure 6.

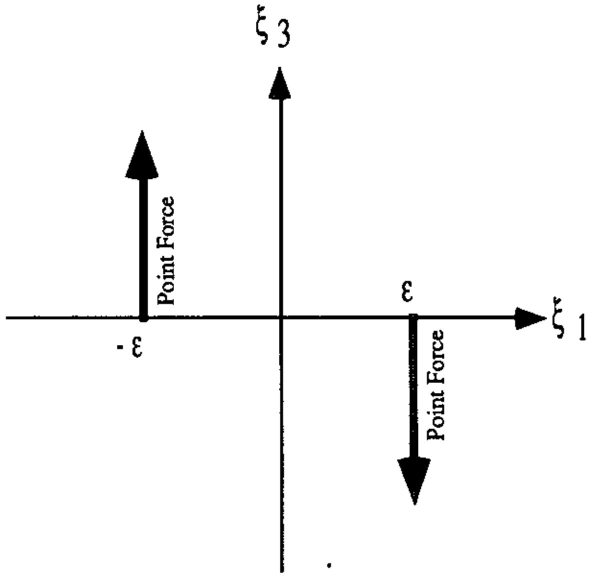

FIG. 6. Single couple corresponding in the limit to response $G_{3n,1}$.

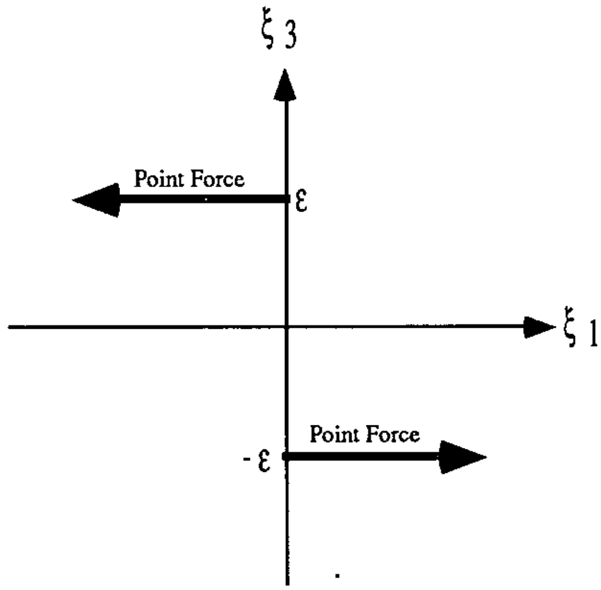

FIG. 7. Single couple corresponding in the limit to response $G_{1n,3}$.

The second term in equation (12) represents the response in the limit of a couple as shown in figure 7. Thus, equations (12) can be seen as the response to a double couple. The radiation characteristics of such a source is given in Aki and Richard's (1980) book. I have duplicated the response in the plane defined by the double couple and is shown in figure 8.

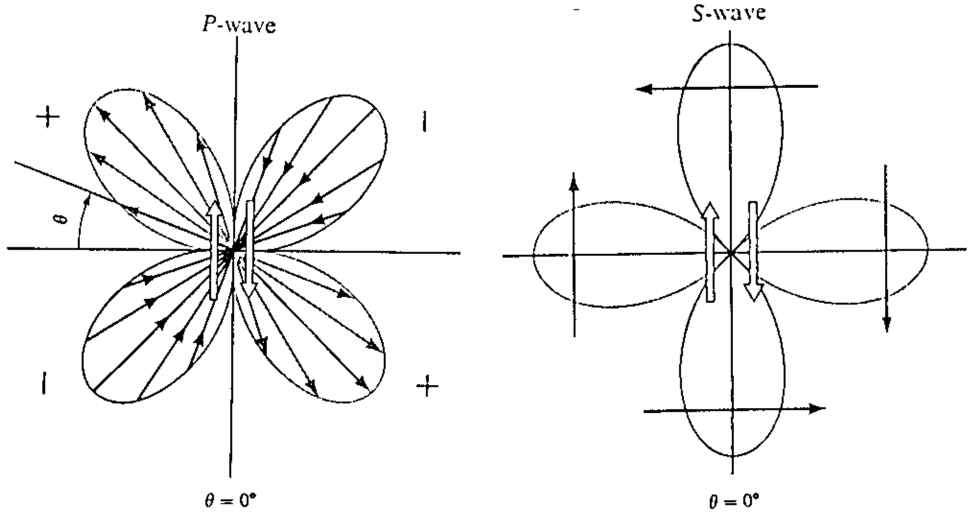

FIG. 8. Radiation pattern for a double couple source in the plane of the double couple (adapted from Aki and Richards, 1980, p. 82-83, Figure 4.5 and 4.6).

The most interesting feature that distinguishes the double-couple from the single-couple response is the shear-wave lobe in the vertical direction. Of course this development is not at the free surface but, even there, differences should be present providing another area for investigation. This may provide a mechanism which will allow us to explain the deep near-offset energy observed on Edelmann's shot records. Another interesting characteristic about double couples is that they have zero moment, while single couples have finite moments. And unless we have countering moments, there will exist an uncompensated torque which will generate free rotation of the material element it acts upon: this would be highly unusual in an elastic medium. This may be another reason why alternate mechanisms, such as the double couple, are appropriate descriptions of counterphase sources.

CONCLUSIONS

The existence of deep near-offset shear-wave energy was pointed out on Edelmann's (1981) shot records. These events must arise from near-zero take-off angles from the counterphase source. Dankbaar (1983) showed that shear-wave energy dominate five to ten degrees from vertical from a counterphase source treated as a single couple. This deviation from vertical is still too great, I feel, to explain the shear-wave energy from the counterphase source records of Edelmann (1981). I have developed a description based on counterphase displacements which results in a double-couple source term. This was done in a homogeneous space where the radiation pattern within an elastic space associated with such a source is shown to be quite different from that of a single couple, especially in terms of the existence of a shear wave maximum in the