Synthetic seismograms for P and S waves using the Goupillaud model

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ABSTRACT

Mendel's state space model using equal time layers has been expanded to include both P and S body waves. The method involves running two simultaneous equal time models, one for each wave type, and injecting the converted wave energy between the models. The speed and flexibility advantages of equal-time or Goupillaud models appears to have survived this adaptation. Evanescent (all interfacial) waves are absent in this method.

INTRODUCTION

The wave equation describes the propagation of plane waves within homogeneous layers as simple time delays. When the wave front impinges on an interface, the boundary condition along with the wave equation specifies a partitioning of the wave field. This naturally gives rise to the definition of reflection and transmission coefficients. Imposing the constraint of having interfaces at discrete equal time intervals results in the Goupillaud model. This model has been studied by many researchers (Hubral et. al., 1980), who use this to study the spectral models of a layered Earth system. Their studies are based on the z-transform. Mendal (1979) proposed a different formalism based on the state-space model. It is this method which has been expanded upon here. The reason for using the state-space approach is the ease by which more sophisticated processes can be implemented, such as absorption (Aminzadeh, 1983), isolation of multiples (Aminzadeh, 1980), and the inclusion of P and S waves within the Goupillaud model. The time domain also appeals more directly to the intuition.

THEORY

Traditional Non-Normal Incident Goupillaud Model

Consider horizontally layered elastic media bounded above and below by a half-space, either or both of which can be vaccums (figure 1). Within the i th layer, we assume the existence of up (u_i) and down (d_i) propagating particle displacements with plane phase fronts. When these disturbances impinge on an interface, the energy is partitioned into up and down going P and S waves (action figure 2). From this partitioning, reflection and transmission coefficients are defined at an interface. Modifying Aminzadeh and Mendel's (1982) notation, it is assumed that at time t the downward propogating disturbance has reached the bottom of layer i ($DP_i(t)$, $DS_i(t)$), and the upward propagating disturbance has reached the top of layer i ($US_i(t)$, $UP_i(t)$). Additionally, the following disturbances are defined:

$UP_i'(t) = UP_i(t+\tau_i) \equiv$ upward going disturbance at the bottom of layer i,

$DP_i'(t) = DP_i(t+\tau_i) \equiv$ downward going disturbance at the top of layer i,

Where τ_i is the traveltime of layer i. These disturbances are assumed to occur at a fixed point in time. Using the transmission and reflection coefficients we can relate these wavefields to each other by:

$$DP_{i+1}'(t) = R_i^{bPP}UP_{i+1}(t) + R_i^{bSP}US_{i+1}(t) + T_i^{tPP}DP_i(t) + T_i^{tSP}DS_i(t)\ , \tag{1a}$$

$$DS_{i+1}'(t) = R_i^{bPS}UP_{i+1}(t) + R_i^{bSS}US_{i+1}(t) + T_i^{tPS}DP_i(t) + T_i^{tSS}DS_i(t)\ , \tag{1b}$$

$$UP_i'(t) = R_i^{tPP}DP_i(t) + R_i^{tSP}DS_i(t) + T_i^{bPP}UP_{i+1}(t) + T_i^{bSP}US_{i+1}(t)\ , \tag{1c}$$

and

$$US_i'(t) = R_i^{tPS}DP_i(t) + R_i^{tSS}DS_i(t) + T_i^{tPS}UP_{i+1}(t) + T_i^{bSS}US_{i+1}(t)\ . \tag{1d}$$

The reflection and transmission coefficients with b in its superscript indicate partitioning of the wavefield as viewed from the bottom of an interface, and those with t in its superscript indicate partitioning of the wavefield as viewed from the top of the interface. The first uppercase superscript indicates that incident wavetype and the second the scattered wavetype.

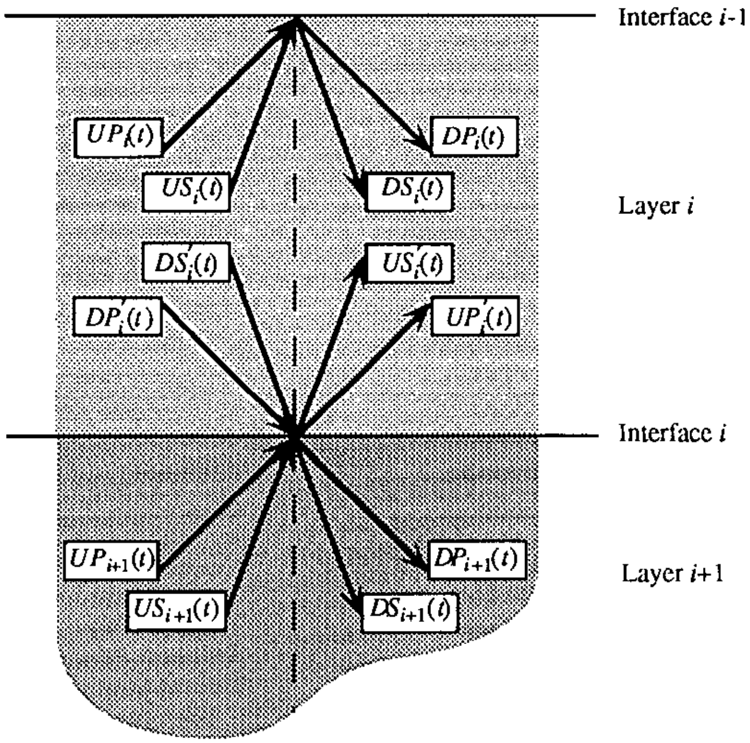

Figure 1: Horizontally layered elastic media

These energy partitioning equations apply to plane waves within arbitrary horizontally layered models. The Goupilllaud models are models in which the travel times in all layers within a model are constant. By the device of interfaces with zero reflection coefficients and unit transmission coefficients, the approximation to a true depth model can be approached to any degree of accuracy by making the travel time in each layer small.

In plane layers with plane waves the wave parameter, $p = \sin(\theta)/V$ is constant by Snell's law and therefore the traveltime for each wavetype is constant within a single layer. But, the P and S wave travel times through a given layer differ from each other. This prohibits the use of a single Goupillaud model for modelling both wave types. It is important to note that the equations thus far deal with the entire wavefield within a single model, while the numerical scheme that is implemented separates the P and S wavefields into two separate Goupillaud models.

Coupled P & S Wave Goupillaud Models

The method used here to accommodate the differing P and S traveltimes is to run separate Goupillaud models for the P and S waves. These models are approximations of the true depth model. The approximation is accomplished by representing the travel time in the i th depth layer as an integral multiple of a small constant time increment, Δt. This approximation constrains the Vp/Vs ratio within each layer of the coupled Goupillaud models to be a rational number. Since rational numbers are dense in the reals, we can achieve any degree of accuracy desired by reducing the time increment Δt. The transformation from the true depth model to the coupled Goupillaud models is shown in figure 2. Note, however, that the coupled Goupillaud models represent an exact depth model which differs slightly from the original.

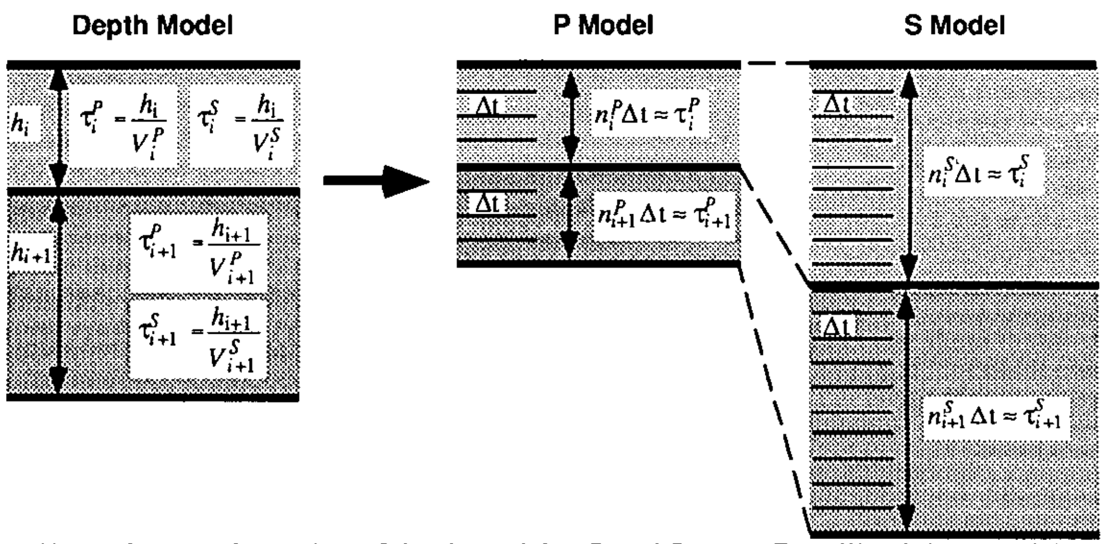

Figure 2: Transformation of depth model to P and S wave Goupillaud time models

The rules to transform the i th depth layer having P velocity V_i^P and S velocity V_i^S and thickness h_i resulting in compression and shear travel times τ_i^P and τ_i^S respectively into the two equal time models are:

$$n_i^P \, \Delta t \approx \tau_i^P \quad , \tag{2}$$

and

$$n_i^S \, \Delta t \approx \tau_i^S \quad , \tag{3}$$

where n_i^P and n_i^S are appropriate integers that minimizes the approximation for a given Δt. This procedure introduces a time error for both P and S wave travel-times in the i *th* layer given by:

$$\delta_i^P = \tau_i^P - n_i^P \,\Delta t \tag{4}$$

and

$$\delta_i^S = \tau_i^S - n_i^S \,\Delta t \ , \tag{5}$$

respectively. The total number of time layers within the two Goupillaud models will be:

$$M_P = \sum_{i=1}^{N} n_i^P, \tag{6}$$

in the P wave Goupillaud model, and

$$M_S = \sum_{i=1}^{N} n_i^S, \tag{7}$$

for the shear wave Goupillaud model.

As stated previously, upward propagating disturbance is assumed to have reached the top of layer i and the downward propagating disturbance is at the bottom of this layer at discrete time indicies. The wavefields designated with a prime are, by our convention, the same as the unprimed wavefields within the same layer advanced in time in the following manner:

$$DP'_{i\,j} = DP_{i\,j+1} \ , \tag{8a}$$

$$DS'_{i\,j} = DS_{i\,j+1} \ , \tag{8b}$$

$$UP'_{i\,j} = UP_{i\,j+1} \ , \tag{8c}$$

and

$$US'_{i\,j} = US_{i\,j+1} \ , \tag{8d}$$

where the index j represents a discrete time variable with unit time step, and therefore $t = j\,\Delta t$. The relationships (8a-d) allows equations (9a-d) to be rewritten in their discrete form and separated into converted and non-converted modes as:

$$U*_{i+1,j+1} = R_i^{t**} D*_{i+1,j} + T_i^{b**} U*_{i,j} \ \ , \tag{9a}$$

$$D*_{i+1,j+1} = R_i^{b**} U*_{i+1,j} + T_i^{t**} D*_{i,j} \ \ , \tag{9b}$$

and for the converted waves:

$$U\Diamond_{i+1,j+1} = R_i^{t*\Diamond} D*_{i,j} + T_i^{b*\Diamond} U*_{i+1,j} \ \ , \tag{9c}$$

and

$$D\Diamond_{i+1,j+1} = R_i^{b*\Diamond} U*_{i+1,j} + T_i^{t*\Diamond} D*_{i,j} \quad . \tag{9d}$$

Equations (9a), (9b), (9a) and (9b) explicitly give the state at time index j+1 in terms of the states at time index j for all layers i ; therefore, these equations provide a means to generate all the states in all the layers from some given initial state at an initial time index. The symbol * represents the state within a single Goupillaud model and the ◊ symbol represents the converted state which must be added to the coupled Goupillaud model at the same time and appropriate layer. The reflection and transmission coefficients are calculated from the Zoeppritz equations (Aki and Richards, 1982). This provides all the necessary theory to generate full P and S body-wave synthetic seismograms within the Goupillaud framework.

Synthetic Seismogram Examples

The following simple model was used to demonstrate the algorithm. The model consists of three layers with the following parameters:

Depth (m)	V_P (m/s)	V_S (m/s)	Density (kg/m^3)
0	4000	2100	2450
1200	4500	2700	2650
3000	5400	3100	2700

Figures 3 contains the synthetic seismic sections for the horizontal and vertical displacements associated with this model. The synthetic was generated with a P-wave source located on the surface and receivers at even depth intervals. Note the reflected up going P wave energy as well as the converted up going S-wave at the first interface. Further down in time the evidence of multiple energy of both wave types is clearly shown. The surface interface was defined as being a perfectly elastic reflector with no mode conversion taking place.

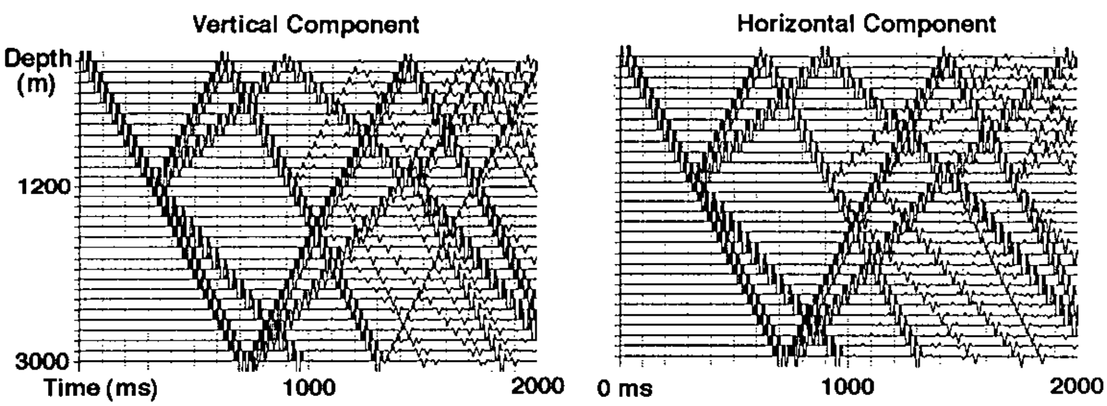

Figure 3: Vertical and horizontal VSP sections

Run Time Considerations

One of the major reasons for the development of this method of modelling is the speed at which the algorithm runs. For a 1200 layer Goupillaud model, a two second synthetic seismogram was generated on a Sun Sparcstation II in 23 seconds. Due to the nature of the algorithm, the code is highly parallelizable and therefore could easily take advantage of modern parallel computers.

Possible Enhancements

The state space framework allows many easy to implement enhancements; some of these are the ability to generate primaries only seismograms as well as seismograms with any particular multiple, this is called Bremmer decomposition (Aminzadeh and Mendel, 1980). Absorption can also be incorporated (Aminzadeh and Mendel, 1983). To more closely match real data the plane wave synthetic seismograms can be combined to form line or point source seismograms (Aminzadeh and Mendel, 1982). The seismograms can also be generated at any offset.

CONCLUSION

A simple adaptation of the Goupillaud model has been implemented to generate full P and S wave synthetic seismograms. The advantages of speed and adaptability of the Goupillaud model has been largely inherited intact. The synthetics generated by this method are shown. Possible enhancements have been suggested and are easily implementable within the statespace framework.

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