Estimating shear-wave velocity logs using P-S seismic inversion.

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ABSTRACT

A method is developed to estimate S-wave velocity logs from pre-stack seismic data. The method requires converted-wave CCP gathers, a P-wave sonic log, and a P-wave macro velocity model. A band-limited S-wave sonic log section and a relative change in S-wave velocity section are produced. The procedure was first tested by computing the S-wave velocity of a single synthetic CCP gather. The synthetic test showed the inversion program to be working correctly. The S-wave velocity inversion was then applied to line EKW-002 of the Lousana multi-component data set. The S-wave sonic section derived from the inversion of the Lousana data set correlated well with the known geology. The effect of frequency band-limiting, and input data scaling were found to have an effect on inversion quality.

INTRODUCTION

Seismic inversion is a process that uses seismic traces recorded at the surface of the earth to infer sub-surface lithologic properties (Russell and Hampson, 1991). Velocity inversion, the focus of this study, converts reflectivities into velocities under various assumptions. These velocities are often used to predict lithology changes across the section (Stewart, 1990).

Many types of seismic inversion are available to explorationists. Band-limited inversion assumes that the seismic trace is a band-limited version of the reflectivity, and performs a simple integration to find impedance (Russell and Hampson, 1991). Sparse spike methods estimate reflection coefficients as an isolated set of delta functions (Oldenburg et al., 1983). Iterative, model-based methods perturb an initial synthetic model until the model best fits a real data set (Russell and Hampson, 1991). The sparse spike and the model-based inversions are solved using model constrained linear programming approaches. The three inversions described above all seek to derive seismic impedance from post-stack, P-wave seismic data. One problem with such post-stack approaches is that amplitude variation with offset (AVO) information is averaged out by the stack (Russell, 1994).

An inversion method that uses P-wave AVO variations was developed by Smith and Gidlow (1987). In their GEOSTACK method, CDP gathers are processed to yield the relative change in P-wave and S-wave velocity. Interval velocity models and correctly processed P-wave reflectivity data are required. The resulting weighted stack section can then be used to compute fluid factors or Poisson's ratio. Again, their method is based on P-wave seismic data only.

Stewart (1990) proposed a method that incorporated mode converted shear wave seismic gathers in a joint P-P and P-S inversion. Because part of the AVO effect involves P to S conversion, use of the P-S reflectivity should result in better estimates of the relative changes in the P- and S-wave velocities. Also the P-S reflectivity is only directly dependent on S-wave velocity changes. Vestrum and Stewart (1993) used synthetic data to show that the joint P-P and P-S inversion was effective in predicting the relative S-wave and P-wave velocities.

This paper is an extension of the pre-stack seismic inversion methods of Smith and Gidlow (1987) and Vestrum and Stewart (1993). A computer program is presented which uses pre-stack P-S seismograms together with P-wave sonic logs to compute an S-wave velocity section. This program has been developed to run as a module in the ProMAX environment (Figure 1).

In the following sections, development of the S-wave velocity estimation method begins with a description of the reflectivity equation for P-S data. The P-S inversion equation will then be derived using a least squares approach. An S-wave velocity inversion processing flow will be described and charted, and the performance of the resulting algorithm on synthetic data will be presented and evaluated. The Lousana 3-component seismic data set will be introduced, the S-wave velocity inversion will be performed on this data set, and the resulting S-wave velocity sections will be discussed.

CONVERTED - WAVE REFLECTIVITY

The S-wave velocity inversion presented here is based on the P-S reflection/transmission coefficient approximation of Aki and Richards (1980). Their equation is an approximation to the P-S seismic energy partition at the welded interface of two differing half spaces (Aki and Richards, 1980). They assume that at the conversion point the differences in rock properties are small, resulting in high transmission coefficients and small reflection coefficients, and that reflection/transmission angles are < 90°. The resulting reflectivity equation is:

$$R^{ps} = 4c\frac{\Delta\rho}{\rho} + d\frac{\Delta\beta}{\beta} . \qquad (1)$$

Assuming a relationship between density and P-wave velocity such as (Gardner et al, 1974):

$$\rho \approx k\alpha^{\frac{1}{4}} .$$

The P-S reflection coefficient becomes:

$$R^{ps} = c\frac{\Delta\alpha}{\alpha} + d\frac{\Delta\beta}{\beta}.$$

Where,

$$c = -\frac{\alpha\tan\phi}{8\beta}\left(1 - \frac{2\beta^2}{\alpha^2}\sin^2\theta + \frac{2\beta}{\alpha}\cos\theta\cos\phi\right).$$

$$d = \frac{\alpha\tan\phi}{2\beta}\left(\frac{4\beta^2}{\alpha^2}\sin^2\theta - \frac{4\beta}{\alpha}\cos\theta\cos\phi\right).$$

at each interface θ and ϕ are the average P-wave and S-wave angles of reflection/transmission across an interface, $\frac{\Delta\alpha}{\alpha}$, $\frac{\Delta\beta}{\beta}$, and $\frac{\Delta\rho}{\rho}$ are the relative changes in P-wave velocity, S-wave velocity, and density, α, β, and ρ are the average P-wave velocity, S-wave velocity and density across an interface (Stewart, 1990).

It then becomes a simple task to cast this equation as a least squares problem and solve for $\frac{\Delta\beta}{\beta}$.

LEAST - SQUARES FORMULATION

Stewart (1990) points out that, with some straight - forward manipulation, and by assuming a relation between density and P-wave velocity, equation (1) can be cast as a least squares error problem. The least squares error formulation for equation (1) is:

$$\varepsilon = \sum (R - R^{ps})^2, \text{ where R is the recorded P-S reflectivity.} \qquad (2)$$

Expansion of the square term and substituting (1) for R^{ps} gives the result:

$$\varepsilon = \sum R^2 - \frac{2\Delta\alpha}{\alpha}\sum Rc - \frac{2\Delta\beta}{\beta}\sum Rd + \frac{\Delta\alpha^2}{\alpha^2}\sum c^2 + \frac{\Delta\beta^2}{\beta^2}\sum d^2 + \frac{2\Delta\alpha\Delta\beta}{\alpha\beta}\sum cd$$

A value of $\frac{\Delta\beta}{\beta}$ is then found to minimize the error function ε. This is accomplished by differentiating (2) with respect to $\frac{\Delta\beta}{\beta}$ and setting the result equal to zero:

$$0 = \frac{\partial\varepsilon}{\partial\frac{\Delta\beta}{\beta}} = -2\sum Rd + \frac{2\Delta\beta}{\beta}\sum d^2 + \frac{2\Delta\alpha}{\alpha}\sum cd \qquad (3)$$

Rearranging (3) gives the inversion equation used in the method presented in this study:

$$\frac{\Delta\beta}{\beta} = \frac{\sum Rd - \frac{\Delta\alpha}{\alpha}\sum cd}{\sum d^2}, \qquad (4)$$

SHEAR - WAVE VELOCITY INVERSION METHOD

An algorithm for the S-wave velocity inversion was written to solve equation (4) as a series of discrete welded layers that satisfy the requirements of small reflectivity, and reflection/transmission angles < 90 degrees. For data handling and further development ease the algorithm was added as a module to the ProMAX processing system. The solution to equation (4) requires P-S reflectivity data, a P-wave sonic log, a P-wave macro velocity model and an average Vp/Vs ratio. The next several paragraphs are summarized in Figure 2.

P-S reflectivity data can be obtained as NMO corrected common conversion point (CCP) seismic trace gathers. CCP gathers can be asymptotically approximated [equations (5) and (6) below]. The data must have as broad a frequency band as is obtainable, and it must be scaled to represent true reflectivity magnitudes (absolute value less than 1.0). This can be done externally using well logs (Russell, 1994) or can be done in the S-wave velocity inversion program with a user-controlled variable.

The surface offsets, which define the CCPs, are calculated using modified versions of the CCP equations of Chung and Corrigan (1985). The modifications involve replacing their stacking velocity term with an average velocity.

$$\zeta = \frac{\chi}{1 + \left(\frac{Vs}{Vp}\right)_{ave}}, \text{ downgoing P-wave source to CCP offset} \qquad (5)$$

$$\xi = \chi - \zeta, \text{ upgoing S-wave receiver-to-CCP offset} \qquad (6)$$

χ is the source-to-receiver offset

The P-wave interval velocity can be obtained from a P-sonic log, correlated to a stack of the S-wave seismic data, and converted to relative P-wave velocity using:

$$\frac{\Delta\alpha}{\alpha}(i) = 2\,\frac{\alpha(i) - \alpha(i-1)}{\alpha(i) + \alpha(i-1)}, \text{ for time sample i.} \qquad (7)$$

The P-wave macro-velocity model is obtained by blocking the correlated P-sonic log. An average Vp/Vs ratio should be estimated over the entire P-velocity model. The ratio will be used to scale the P-wave velocity model to an S-wave velocity model.

Once the data requirements have been satisfied, the S-wave velocity inversion proceeds as follows :

For each CCP gather:

a) Scale all units to meters and seconds.

b) Convert P-sonic to $\frac{\Delta\alpha}{\alpha}$ using (7).

c) Generate S-wave velocity model by copying P-wave velocity model scaled with input average Vp/Vs.

d) For each time sample generate a depth, a downgoing shot-to-CCP offset, and an upgoing CCP to receiver offset using the P-wave and S-wave velocity models and equations (5) and (6).

e) Compute a weighted stack trace using equation (4). Angle's θ and ϕ are calculated using simple straight-ray-to-depth geometry.

TEST DATA

Testing of the S-wave velocity inversion algorithm was performed using a synthetic CCP gather, and synthetic P-sonic and S-sonic logs. The goal of testing was to see how well the S-wave velocity log, generated by the inversion, matched the S-wave velocity log used in the generation of the synthetic CCP gather.

A simple three-velocity P-sonic log (see Figure 3a) was created using a log editing package. An S-wave sonic (see Figure 3b) was then generated by scaling the P-sonic with a Vp/Vs ratio of 2 - except for the zone between 880 and 900 feet where Vp/Vs was set to 2.5.

The synthetic P-wave and S-wave sonic logs were then input to SYNTH, a synthetic trace-gather generator developed by Don Lawton of the CREWES project. A single CCP gather was output which consisted of ten S-wave traces (Figure 4a). The offset range of the traces was 200 m to 2200 m.

The synthetic CCP gather and the P-sonic log were then correlated to each other using STRATA (Figure 4b). Velocity changes on the P-sonic were correlated to the corresponding events on the S-wave seismic data by stretching the P-sonic log. The P-sonic model was then formed by blocking the stretched P-sonic model (Figure 4c), and both logs were converted from velocity-depth, to velocity-time pairs.

The synthetic CCP gather, the P-sonic log, and the P-wave velocity model were then input to the S-wave velocity inversion.

A comparison of the original S-sonic and the one generated by the S-wave velocity inversion algorithm (figure 5a and 5b) shows good correlation. The inversion derived S-wave log returned velocities within 150 m/s of the expected values, and was able to resolve the Vs decrease between 880 and 1200 ms. The inversion derived S-wave log was also found to contain noise in the form of spikes at 750 ms, 880 ms, 1200 ms and 1330 ms. The source of this noise has not been conclusively determined, but is thought to be related to imperfections of the initial synthetic logs.

The results of the above testing indicated that the algorithm was stable enough to attempt an S-wave velocity inversion of real seismic data..

S-WAVE SEISMIC DATA, LOUSANA, ALBERTA

The Lousana seismic data set consists of two orthogonal three-component seismic lines shot by UNOCAL in January 1987 (Miller et al, 1993). The radial component of line EKW-002 was obtained for this study in the form of fully processed, NMO corrected pre-stack gathers, as well as a P-sonic log from an intersecting well.

The Lousana P-S seismic and P-wave sonic log data were first conditioned (Figures 6a and 6b) and then inversion was performed using the flow of figure 2. The inverted sections are given in figures 7a, and 7b. The first section (figure 7a) is $\frac{\Delta\beta}{\beta}$. Peaks (dark colour) indicate an increase in S-wave velocity and troughs (light colour) indicate a decrease in S-wave velocity. The second section (figure 7b) is an S-wave sonic log section, with velocity increasing with darkening colour. Good correlation between the interval velocities calculated from stacking velocities and those predicted by the S-wave velocity inversion are found through three zones (figure 7b). Notice how the S-wave sonic log section oscillates between higher and lower velocities where the S-wave sonic log in the synthetic example is blocky. The difference is because the synthetic data were derived using a broad-band impulse, the trace input of the real data is only a gross approximation to the true reflectivity. The result is that the S-wave sonic log section for the real data looks trace-like.

DISCUSSION

Much was learned during the development of this inversion algorithm. Proper scaling of the input S-wave seismic trace gathers is critical to successful velocity inversion. One method of S-wave seismic data scaling involves finding the rms impedance from a sonic log, then scaling the seismic data to that value (Russell, 1994). This approach was abandoned in favor of a simple qualitative method. A single user controlled scalar was applied to the reflectivity term of equation (4):

$$\frac{\Delta\beta}{\beta} = \frac{\Psi \sum Rd - \frac{\Delta\alpha}{\alpha} \sum cd}{\sum d^2} \text{, where } \Psi \text{ is the user input scale constant} \qquad (8)$$

Then a subset of gathers were sequentially inverted, using different scale values, until a scalar was found that returned an expected velocity from a known time. For example, the expected S-wave velocity of between 1200 and 1300 m/s at the top of the seismic section was found to corresponded to a scalar of 0.03. This scalar was then used in the inversion of the entire data set.

The problem of frequency band-limited input was observed in the S-wave sonic sections. This is apparent when the inversion outputs of the synthetic and real data are compared. The input synthetic data is very broad band and the S-wave sonic log result is correspondingly blocky in appearance. The real input data is band-limited and the resulting S-wave sonic log is trace like. Input data should be carefully processed to restore as much of the frequency band as possible. Preservation or restoration of the low frequencies of the seismic data will result in S-wave sonic logs that are much more representative of true shear wave logs.

CONCLUSIONS

A method of pre-stack P-S seismic velocity inversion has been developed and is available as a module in the ProMAX processing system. The method requires P-S common conversion point seismic trace gathers, and a P-wave sonic log. Output is a band-imited S-wave sonic log section, and a relative change in S-wave velocity section. The S-wave velocity inversion of a single synthetic CCP gather, and the S-wave velocity inversion of line EKW-002 of the Lousana multi-component data set were presented. The S-wave velocity inversion of the Lousana data set showed that frequency band-limiting of the input trace data causes the output S-wave sonic logs to appear trace like. Both data sets showed that the S-wave velocity inversion algorithm is very sensitive to scaling of the input P-S reflectivity.

FUTURE WORK

Future work should include further experimentation with the synthetic P-S gather, and synthetic P-sonic log. This experimentation should first resolve the introduction of noise to the inversion derived S-wave log. Then, comparisons should be made between the best fit scaling approach of this study, and a P-sonic log based rms scaling method.

The program itself should then be opened up and certain improvements made. The first would be to allow the input of laterally varying velocity models. The present program allows only one P-wave velocity model to represent the entire seismic data volume. Then, raytracing should be improved from straight ray to depth, to a bending ray method.

The algorithm could be easily modified to the joint P-P and P-S inversion of Stewart (1990). The added P-P seismic information would enhance the S-wave velocity inversion (Stewart, 1990)

The algorithm uses NMO corrected P-S seismic trace gathers which are not limited to CCP gathers, experimentation with other gather types, for example 3D CCPs or pre-stack migrated shots would require no modification.

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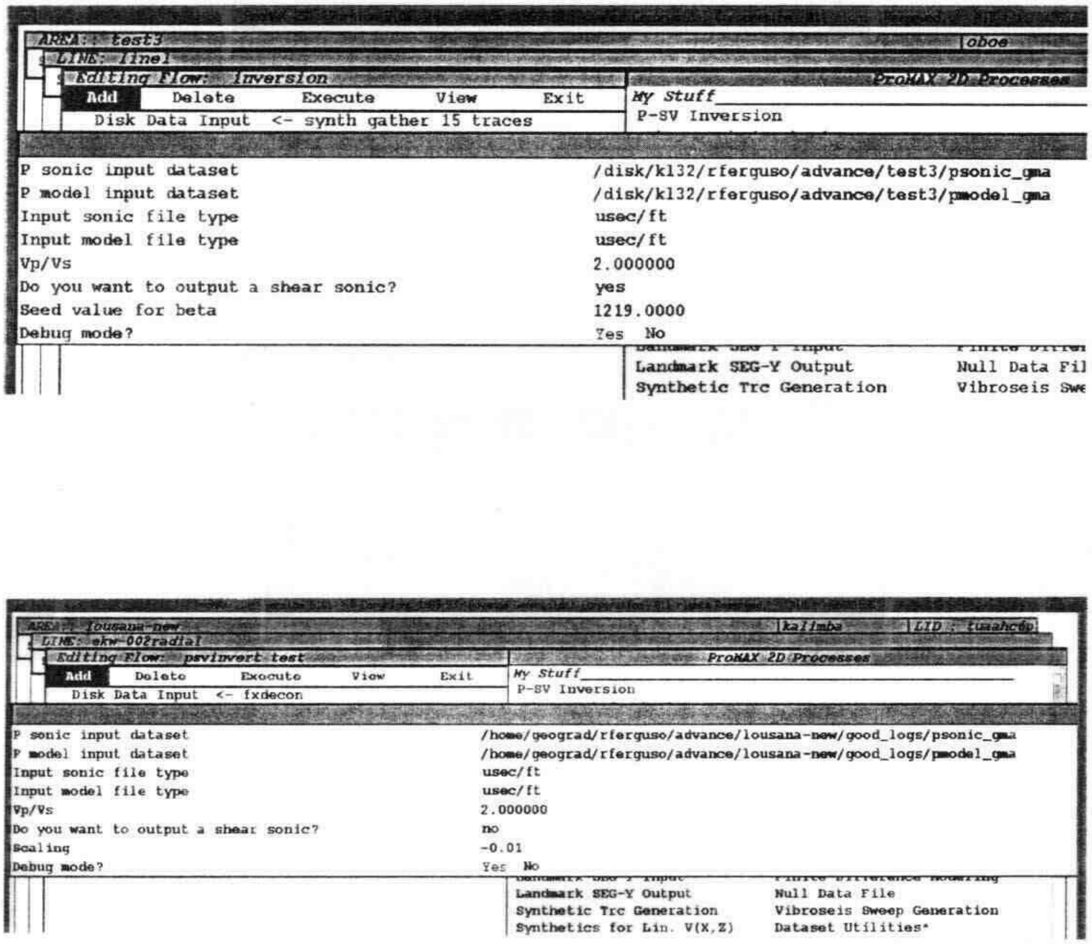

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P model input dataset                    /disk/k132/rferguso/advance/test3/pmodel_gma
Input sonic file type                    usec/ft
Input model file type                    usec/ft
Vp/Vs                                    2.000000
Do you want to output a shear sonic?     yes
Seed value for beta                      1219.0000
Debug mode?                              Yes  No
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P model input dataset                    /home/geograd/rferguso/advance/lousana-new/good_logs/pmodel_gma
Input sonic file type                    usec/ft
Input model file type                    usec/ft
Vp/Vs                                    2.000000
Do you want to output a shear sonic?     no
Scaling                                  -0.01
Debug mode?                              Yes  No
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FIG. 1. ProMAX menus for S-wave velocity inversion.

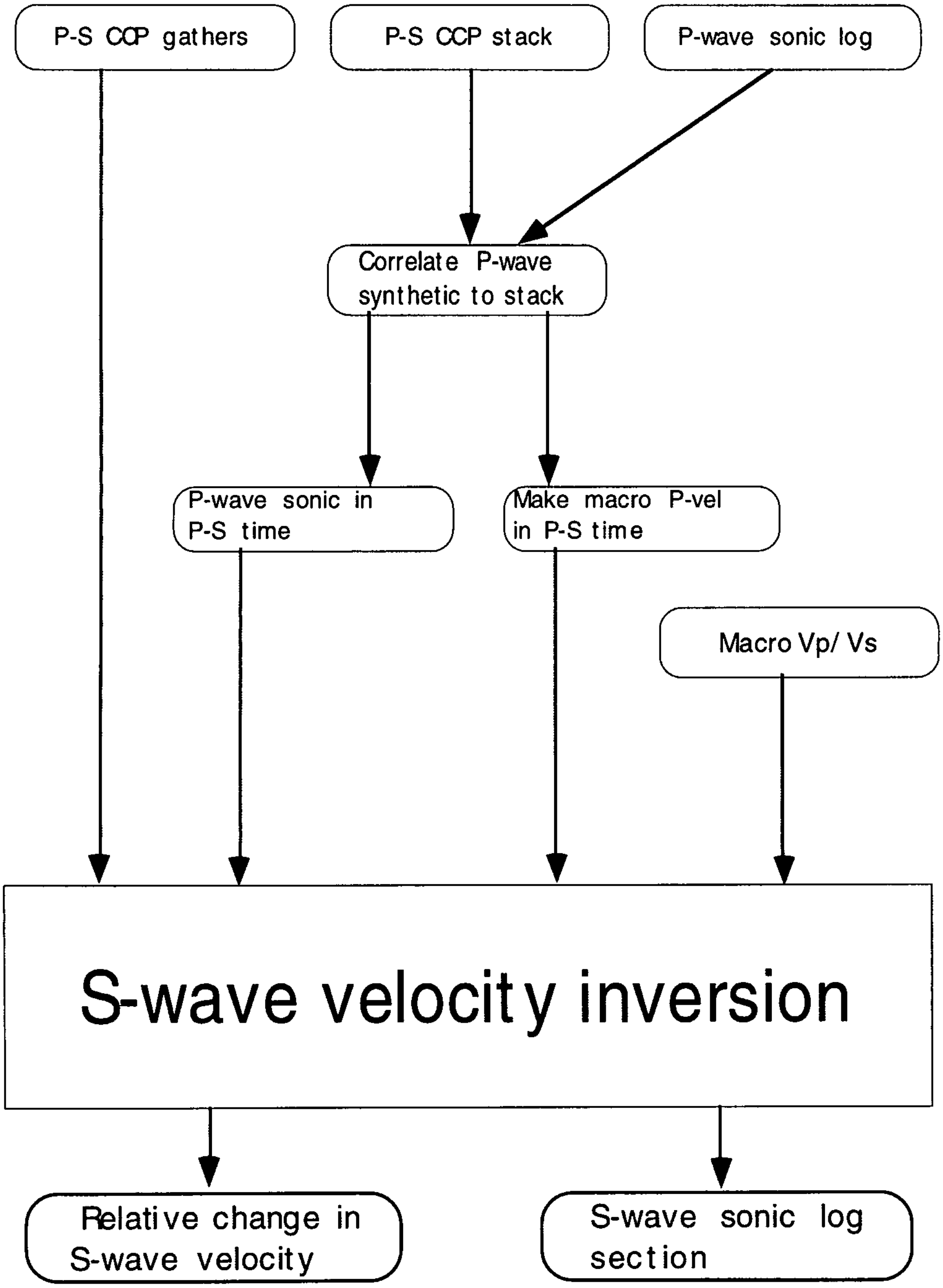
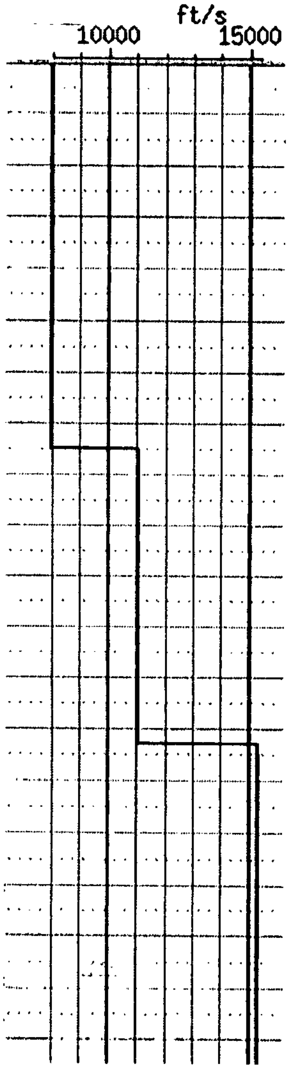


FIG. 2. Flow chart for S-wave velocity inversion.



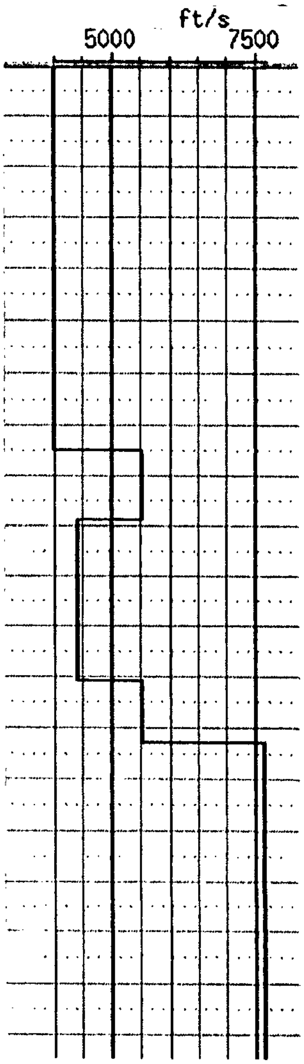


FIG. 3a. Synthetic P-wave sonic log.

FIG. 3b. Synthetic S-wave sonic log.

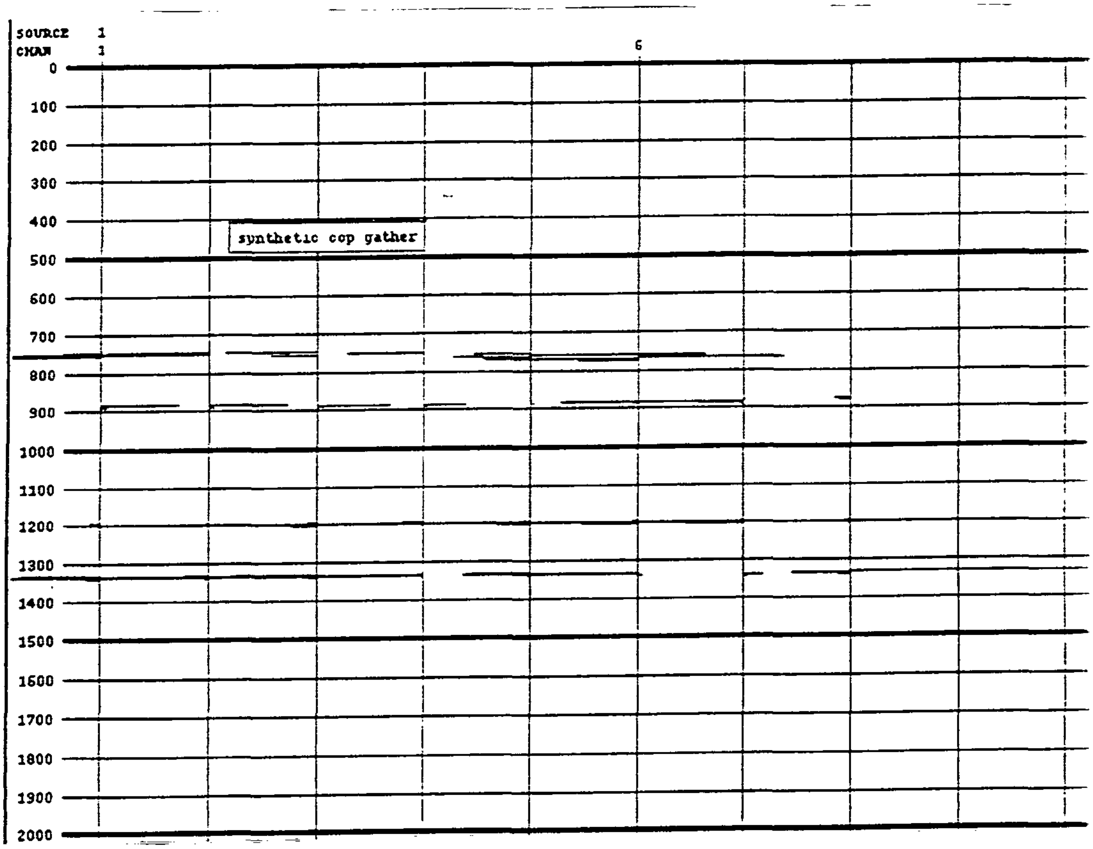
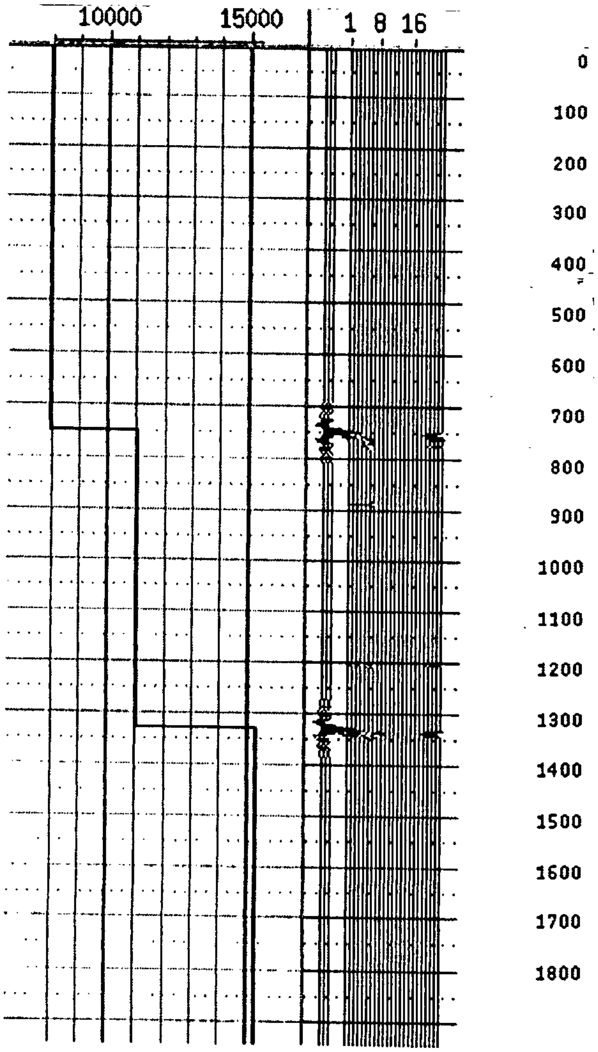


FIG. 4a. Synthetic CCP reflectivity gather.



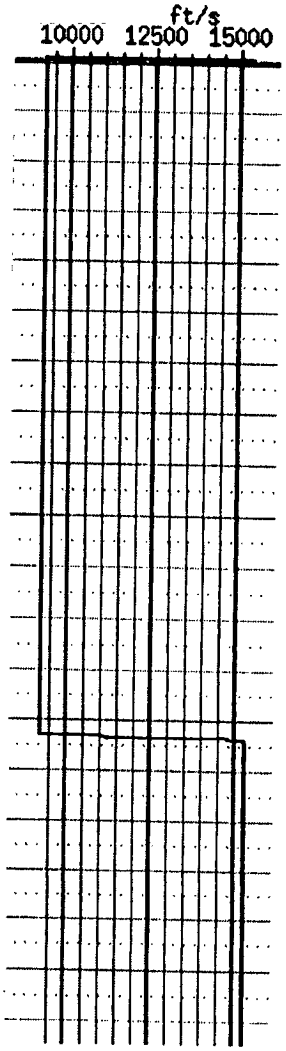


FIG. 4b. Synthetic P-wave sonic log, P-wave synthetic correlated to SYNTH CCP

FIG. 4c. P-wave sonic model.

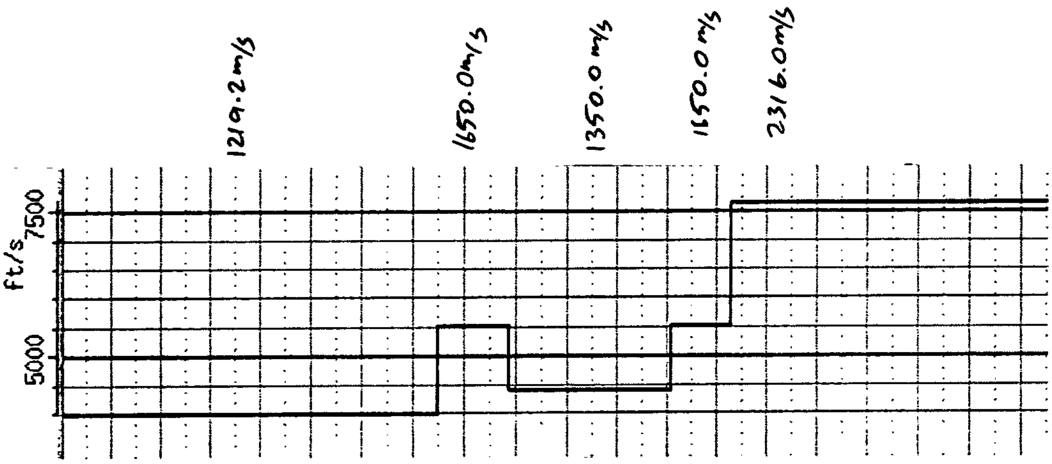


FIG. 5a. Synthetic S-wave sonic log used in computation of synthetic CCP gather

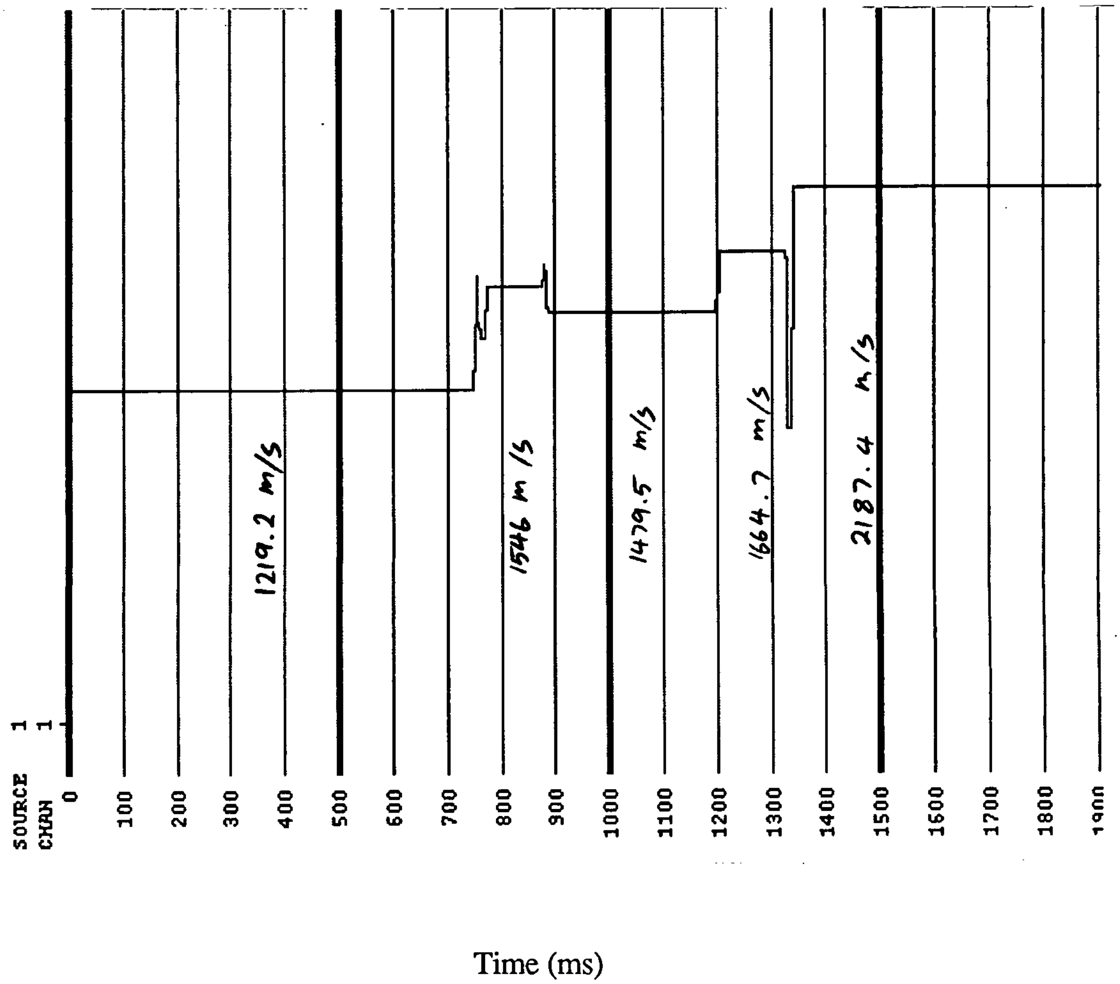


FIG. 5b. Comparison of synthetic S-wave sonic log (top) and inversion derived sonic log (bottom). Velocities correlate within 150 m/s. Vs decrease (800-1200 ms) resolved by inversion. Spikes probably due to imperfections in synthetic logs.

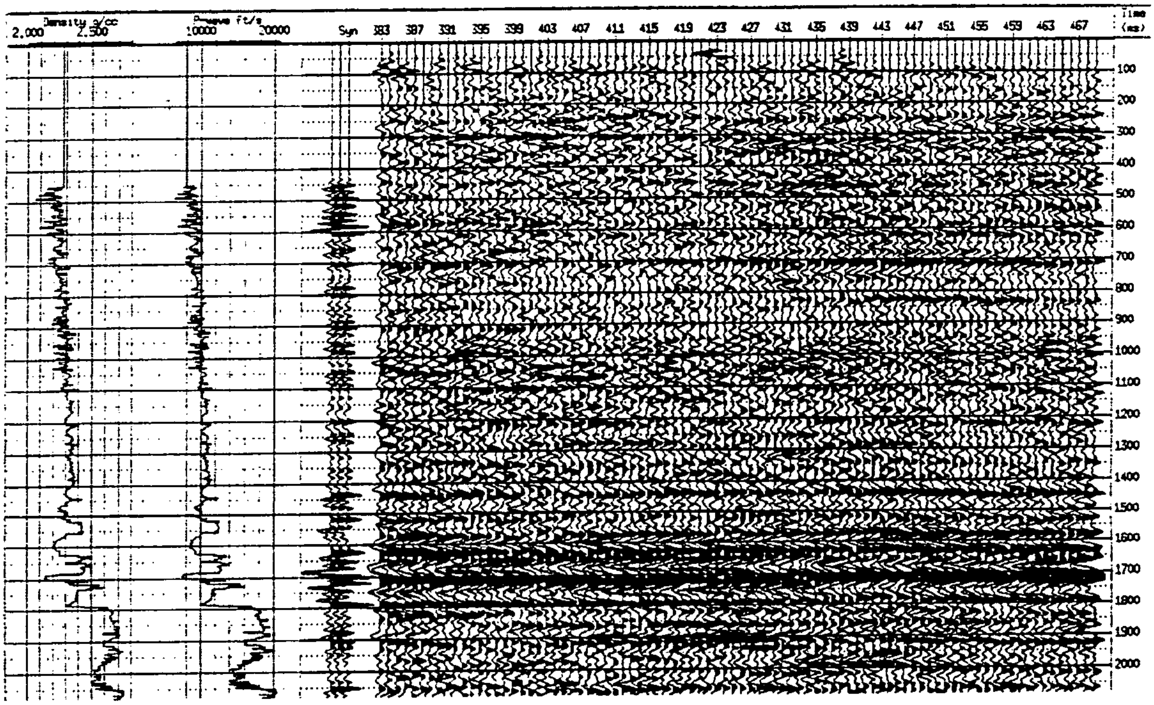


FIG. 6a. P-wave sonic log in time, P-wave synthetic seismoram. Both correlated to P-S section, Lousana data set.

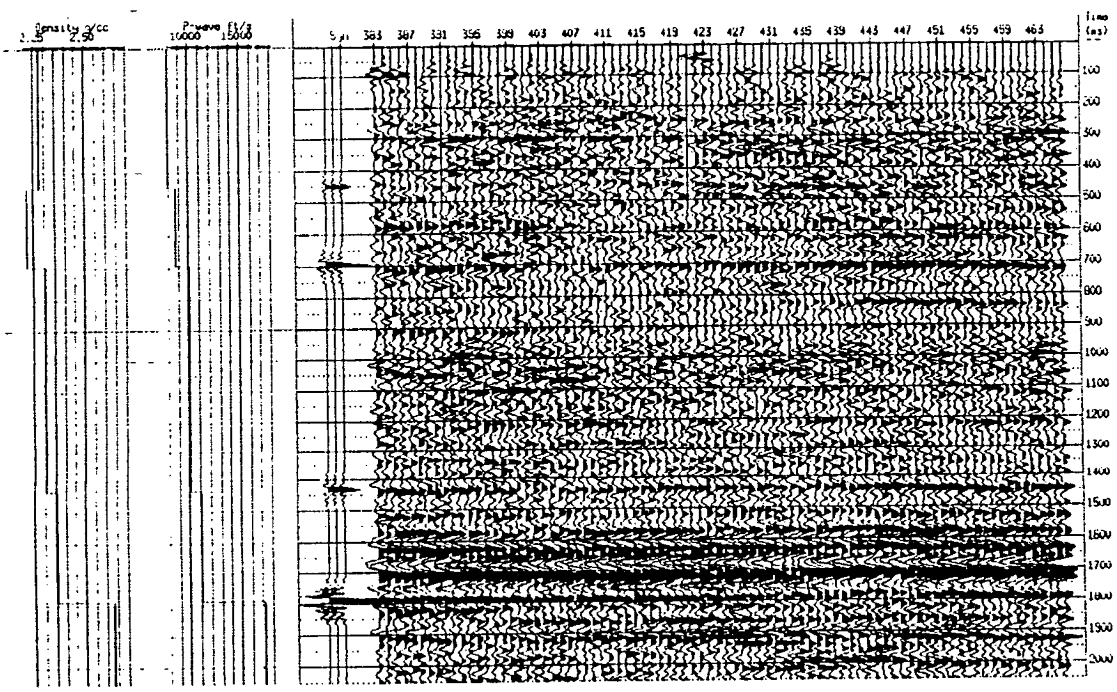


FIG. 6b. P-wave macro velocity model, P-wave synthetic seismogram.

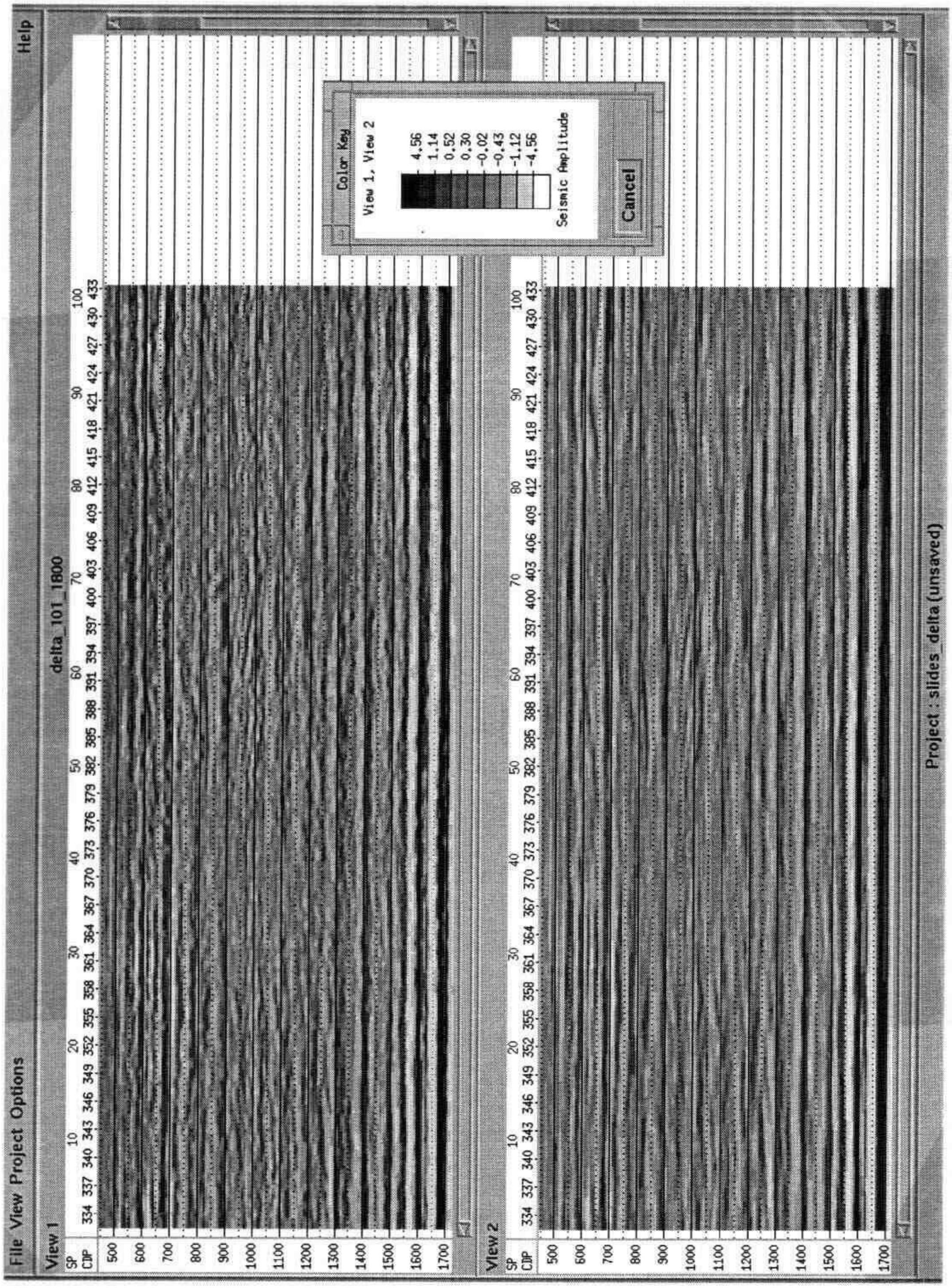


FIG. 7a. Sections showing relative change in S-wave velocity $(\frac{\Delta\beta}{\beta})$. Bottom panel has a 3 trace mix applied to improve trace to trace balance.

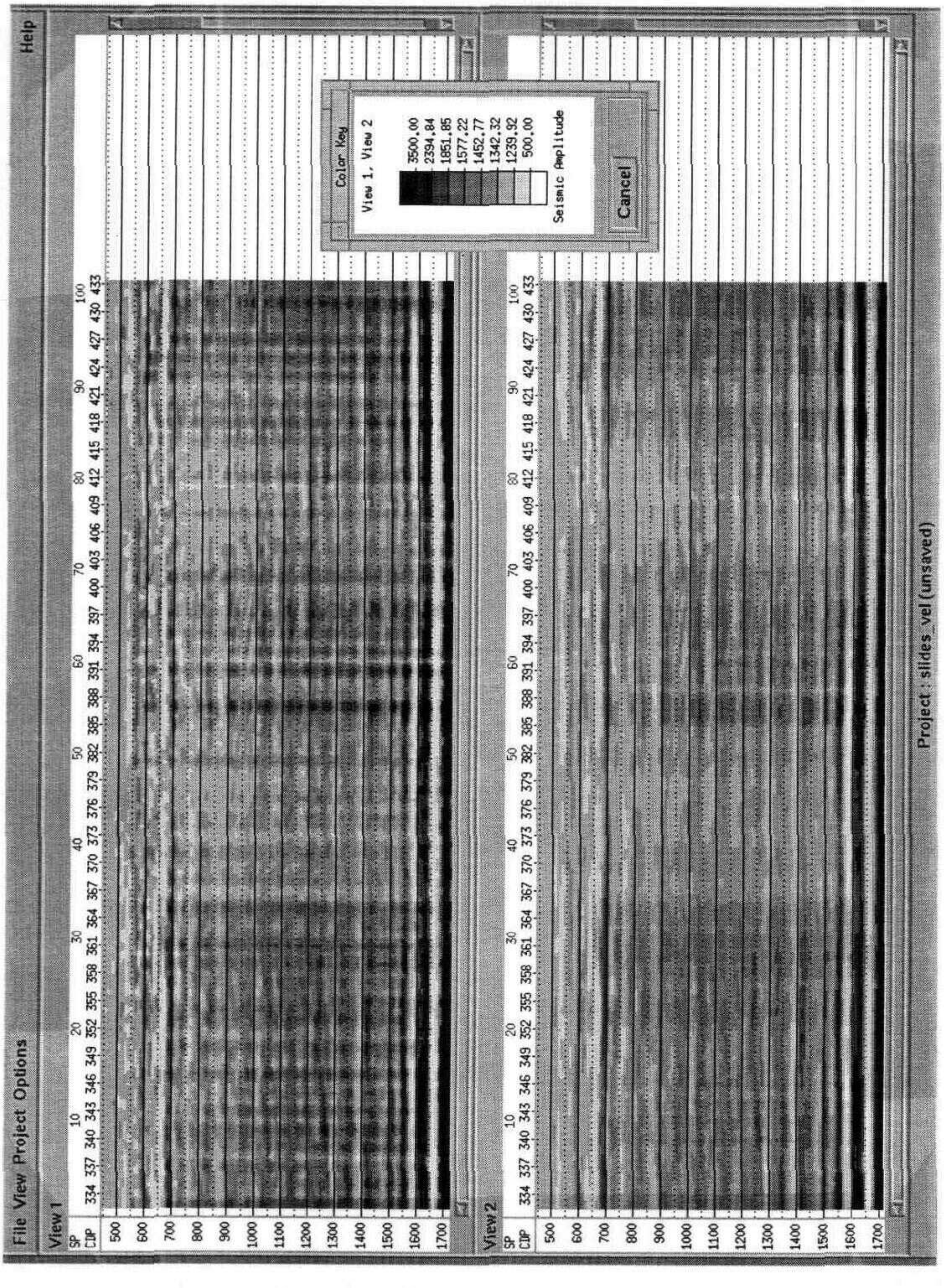


FIG. 7b. S-wave sonic log sections. Bottom panel has a 3 trace mix applied to improve trace to trace balance. S-velocities average 1250 m/s between 500 ms and 670 ms, a velocity kick to 1500 m/s at 700 ms, 1400 m/s between 725 ms and 1500 ms, and increasing through 3000 m/s below 1600 ms. These values correlate well with interval velocities calculated from stacking functions. Band-limited reflectivity makes log velocity values fluctuate.