Aliasing in prestack migration

John C. Bancroft

ABSTRACT

Aliasing is a common problem in seismic processing and is usually associated with steeply dipping events and receiver spacing. However, it also effects horizontal data, and even the simple processing step of normal moveout (NMO). It may appear as noise on a migrated section, or as a coherent non-existing event.

The amount of aliasing in data can be controlled with a number of algorithms such as Kirchhoff migration. Aliasing becomes a compromise between the frequency content of dipping events, the dip angle of the events, the trace spacing of the *input* data, and the (aliasing) noise content. The trace spacing is defined by the source and receiver intervals relating cost to the aliasing compromise.

This paper will review the basic principles behind aliasing, address the techniques used to eliminate or reduce aliasing, and describe the natural antialiasing effects of Equivalent Offset migration.

ALIASING CONCEPTS

Aliasing criteria for 1-D data

The term frequency is used to define the number of cycles in a unit measure such as time where the symbol F is used to represent cycles per second, or distance where the symbol K is used to represent cycles per meter. Data sampled at a uniform sampling rate F_s, is limited to a maximum frequency that is referred to as the Nyquist frequency F_{nyq}. The Nyquist frequency is half the sampling frequency. Frequencies higher than F_{nyq} will have a different apparent frequency and are referred to as being aliased. It is difficult or usually impossible to recover the original sinusoid once aliasing has occurred. An other way of looking at the aliasing criteria is there must be at least two samples per sinusoidal period.

Aliasing criteria for 2-D data

The same Nyquist principle applies to two dimensional data, with sample rates defined for each axis. Each axis has a Nyquist frequency that should limit the frequency content of the data. An advantage with two dimensional data is that aliased information can sometimes be recovered and/or removed easily where data and aliased energy can be separated.

Aliasing criteria of linear events in 2-D data is controlled by the slope of the event, the frequency content of the event, and the sampling rates. For a typical seismic time section with trace spacing Δx, seismic dip θ, and velocity of interest V, the maximum frequency on the time axis before aliasing is given by

$$f_{al} = \frac{v}{4\Delta x \tan(\theta)} . \tag{1}$$

Frequencies higher than f_{al} will be spatially aliased, and will appear as noise to many processing algorithms. This aliased noise may often be separated from the desired signals and removed by anti aliasing filters (AAF's).

Two dimensional aliasing is represented in Figure 1, which contains two dipping events. The dip from the upper left is composed of a low frequent wavelet, while the dip from the upper right contains higher frequencies. Each vertical 2-D trace is finely sampled with the maximum signal content well below the Nyquist frequency. Aliasing does not occur in these time traces

The data could be plotted with horizontal traces as illustrated with the horizontal line that passes through the peaks of both wavelets. Points on the line identify intersection with the low and high frequency wavelets. The displacement of these points would normally be plotted in a vertical direction to represent a horizontal trace as indicated below at the bottom of the figure. Seven samples are required to

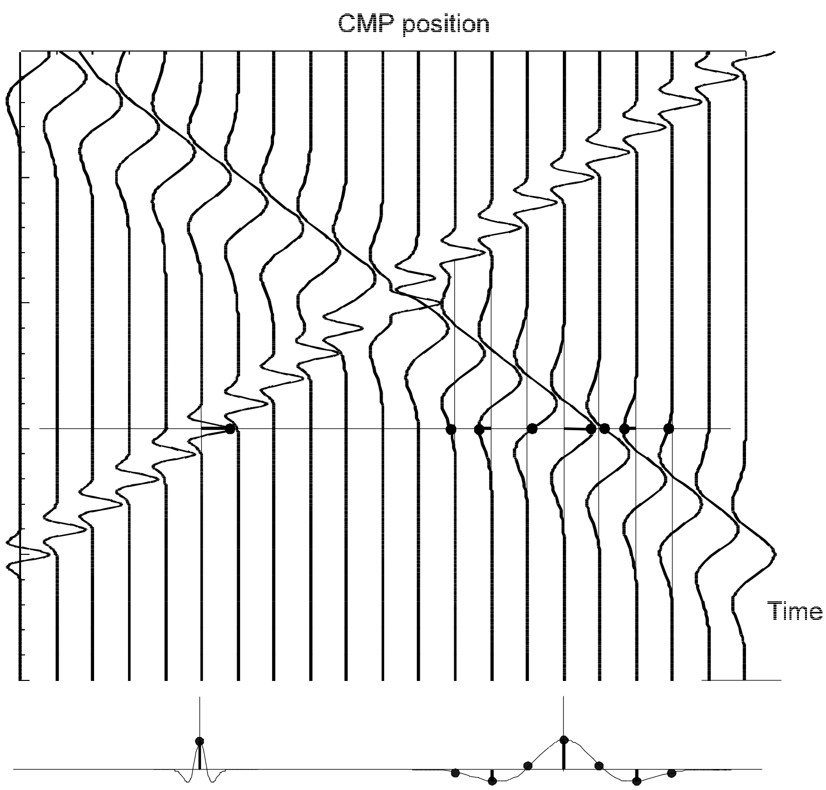

Figure 1. Two dipping events with different bandwidths. The higher bandwidth event is aliased.

represent the low frequency wavelet, while only one is used for the high frequency wavelet. The low frequency wavelet is adequately sampled, while the high frequency wavelet is aliased.

Aliasing in the horizontal 1-D direction results in aliasing of the 2-D data. The aliased dipping event is visually identifiable, and appears to present no problem. However, problems will occur when processing either the 1-D horizontal traces, or the 2-D image. The aliasing of the event may be controlled by inserting other traces and interpolating the data, or by high cut filtering.

Another way of looking at the same data is in the 2-D Fourier transform domain. This transform of seismic data is referred to as the *FK* domain where *F* represents the frequency transformed from the time (vertical) axis, and *K* represents the wave number from the distance (horizontal) axis. In this FK domain, all events at the same dip become collinear with the dip angle defined from the *F* axis to the *K* axis as illustrated in Figure 2. The figure is plotted with origin at the top right and with the spatial Nyquist *Kn* at the vertical center. The Nyquist frequency *Fn* is at the bottom of the display. This cartoon view of the FK domain permits the evaluation of energy at the spatial Nyquist location. The FK domain contains a representation of the data in figure 1. Energy of dip $\theta 2$ stops before crossing the Nyquist axis and is not aliased. Energy along dip $\theta 1$ crosses the Nyquist axis with the aliased portion shown in gray.. The extent of the dips may be evaluated by their frequency content, f1 for θ1 and f2 for θ2. As mentioned above, a high cut filter at Fc would prevent aliasing.

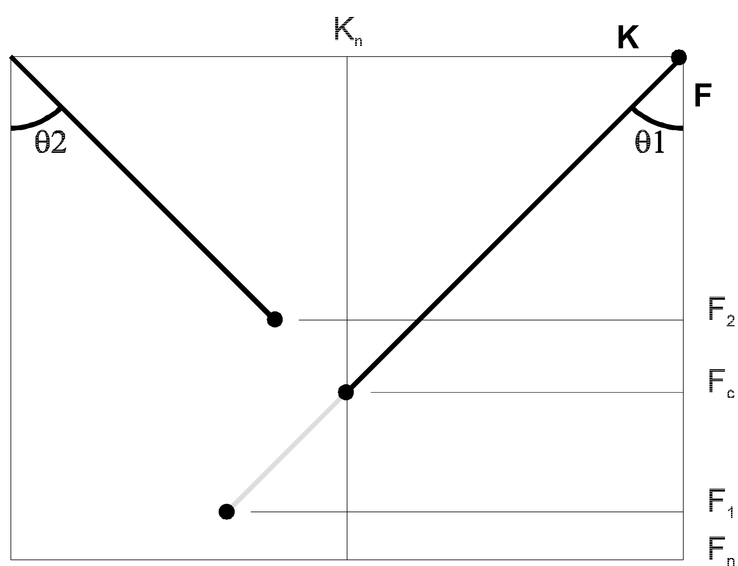

Figure 2. FK diagram showing two dipping events, with the gray portion aliased.

The high frequency aliased dip in Figure 1 appears to be quite acceptable for visual interpretation, but would create significant noise in processing algorithms such as migration.

ALIASING OF GEOPHYSICAL DATA AND PROCESSING

Aliasing of seismic data

The station interval of the receivers defines the subsurface trace spacing (half the station interval). This spacing should be designed for the maximum dip expected in the seismic data, and must include the expected dips of the diffractions. This isnot the general case. For example, a 30 meter trace spacing, designed for all dips (45° before migration), and with a velocity of 3,000m/s, will have an aliasing frequency of 25 hz. Another way of looking at the same problem is specifying a maximum frequency of 60 hz and limiting dip to 22°. To maintain 60 Hz for all dips, the trace spacing needs to be 12.5 meters.

The above criterion defines the guaranteed limits to prevent aliasing and must be adhered to for steeply dipping events. However, dips shallower than the maximum dips do not require the same frequency restrictions, and may contain higher frequencies without aliasing.

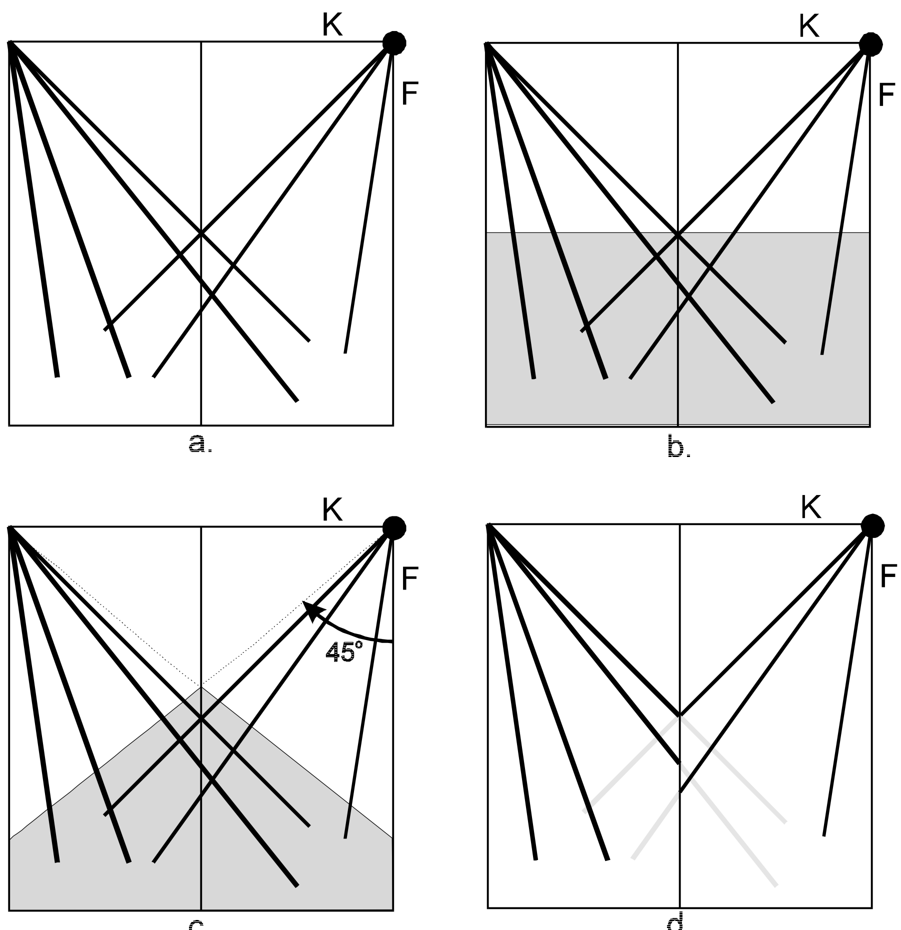

Figure 3. Examples of designing anti aliasing filters, a) the aliased input data, b) low cut filtering to the lowest aliased frequency, c) FK filter designed to eliminate aliased data at 45 degrees, and d) filter internal to algorithm such as Kirchhoff migration.

Once data has be acquired, the extent of aliasing may be controlled by filtering. This control is best viewed in FK domain as illustrated in Figure 3 that shows four FK diagrams and methods of aliasing control. Figure 3a shows the input data with aliasing occurring on the steeper dips. The maximum dip is less than 45 degrees. The three remaining figures contain shaded areas that represent the cut bands to reduce or remove aliasing. Figure 3b represents high cut filter trace filtering with the cut off frequency set to the lowest aliased frequency. The filter, while easy and quick will also high cut filter unaliased data. Figure 3c shows an FK filter designed to cut the aliased frequencies on all dips below 45 degrees. Note that this is an improvement over method (b), but still attenuates some low dip frequencies. The aliased data in Figure 3d is removed internal to a processing algorithm such a Kirchhoff migration.

Lowering the dip range of the Kirchhoff migration will also reduce the amount of aliasing that will occur. This is equivalent to limiting the extent of the summation hyperbola to a dip related to the maximum desired geological dip (including diffractions). The dip on the hyperbola α is related to the geological dip β by $\tan(\alpha) = \sin(\beta)$. Figure 4 illustrates the effect on the FK domain when single traces are interpolated between the original data traces where the dimension in the K direction has doubled, moving the data away from the aliasing condition. Figure 4a shows a simple and quick method of interpolation that begins by inserting null traces, duplicating the original FK image into one image as shown. Most of the unwanted noise at the new Nyquist wave number *Kn2* can be removed by an FK filter that filters unaliased signal above 45 degrees as represented by the shaded area. Some of the original aliased signal remains and will appear as noise on the x-t section. More aliased noise could be removed by reducing the dip of the FK filter. It should be noted that a full dip FK or Kirchhoff migration will achieve the same filtering as the FK filter. These migrations take the dips below 45 degrees and stretch them to 90 degrees.

When a perfect (non-linear) interpolation is used, the Fk diagram will appear as Figure 4b. This figure shows the separation of the dipping signals with no aliasing. Note that non-linear processes are based on assumptions that will vary between data sets. Some of the assumptions used for interpolation are high signal to noise ratios, and identifiable events.

Coherent noise such as ground roll, or air blast, may also appear on seismic data with dips that exceed 45° and contain high frequencies that are aliased. These aliased signals often occupy the same position as data in the FK transform and require special filters to attenuate some of the energy of this aliased signal.

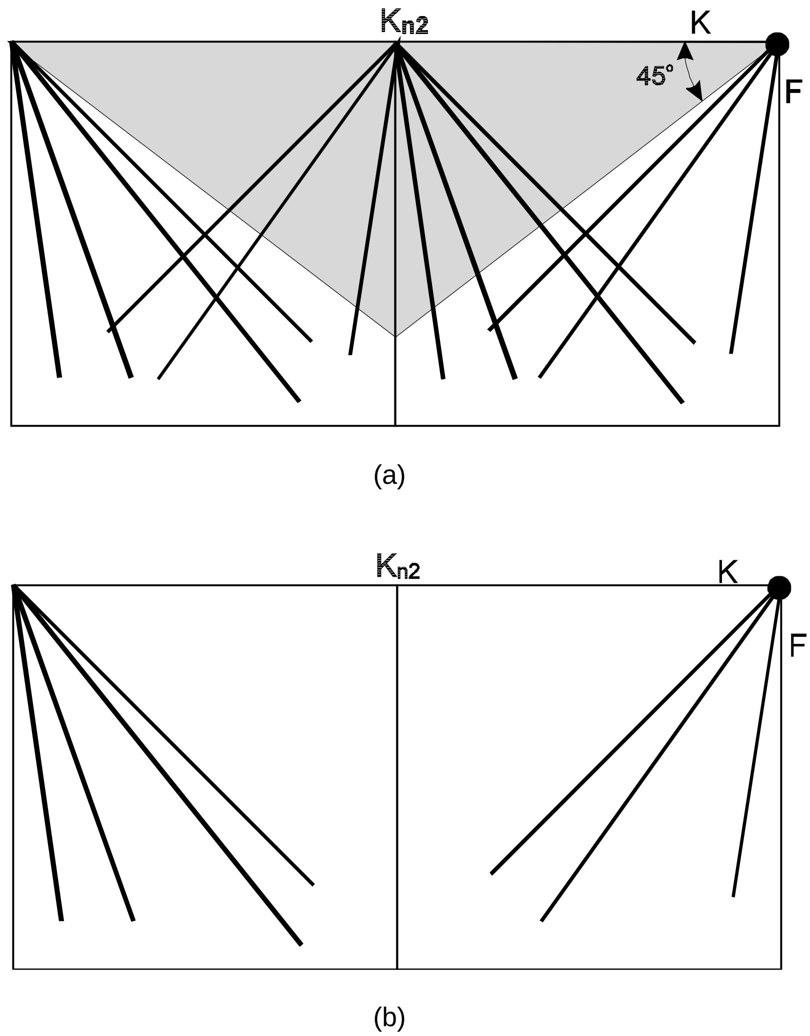

Figure 4. FK plot of the data in Figure 3a when a) a single null trace is inserted between each original trace and FK filtered (shaded area) to reduce aliasing, and b) the resulting FK plot where inserted traces are perfectly interpolated between each existing traces.

Aliasing of migrated data

After migration, seismic data is also subject to aliasing, but the consequences are usually not sever. This could occur in a Kirchhoff migration where the migrated traces have a spacing greater than the input data. However, if the input data is not aliased, and the migration process does not alias, then the migration will not be aliased. Migration does increase the dips of dipping events, while lowering the frequencies of the new dips. This lowing of frequencies is a desired feature that prevents aliasing of the steeper dipping data.

Migration operator aliasing

Migration algorithms have a wide variety in their ability to handle noise and control aliasing. Algorithms that use finite difference approximation are usually limited to a dip range, and disperse the remaining dips as noise. This dispersed noise combines with the noise that existed before migration. Algorithms based on the FK transform are usually able to remove large areas of noise that are identifiable in the FK domain, such as dips over 45 degrees. These Fk based migrations are subject to an aliasing that creates wrap around in the x-t domain. Algorithms based on Kirchhoff migration, also have a large control of noise, and may be applied to migrate all aliased signal, or to optimally prevent aliasing at all dips. Because of this special capability, Kirchhoff migration will be discussed in more detail.

It should be noted that removal of noise from a section may result in a wormy appearance. Some interpreters find this appearance to be objectionable, so options for adding back noise, or permitting a small amount of aliasing, is often included. The wormy appearance of migration should not be confused with that of mixing.

ALIASING IN KIRCHHOFF MIGRATION

Kirchhoff migration is rapidly becoming the migration of choice. This is due to its ability to create an excellent image, handle rugged topography, limit the range of migrated dips, and migrate select portions of the output area while maintaining use of all the input data. The use of anti-aliasing filters (AAF) is a major design consideration as they increase the run time. Designs implemented for speed, usually do not include an AAF, and may introduce considerable aliasing noise.

Time migration defines the diffraction shape to be hyperbolic. The FK transform of the hyperbola contains a continuum of signal at dips below 45 degrees. In order to visualize the dips of the diffraction, Figure 5a shows a piece wise linear approximation to the hyperbola. Input data that lies under the hyperbola is scaled, summed, and placed at the apex of the diffraction. Data under the diffraction that lies between time samples will require interpolation. Figure 5b show the FK domain of (a), and the resulting dips. The steeper dips alias and will continue to wrap around until the Nyquist frequency F_m is reached. Migrations performed with this operator will migrate a large portion of the data twice, once relative to positive dips, and once for negative dips as illustrated by the shaded areas in Figure 5c and d. All positive dips will be fully migrated in Figure 5c, and all negative dips that fall within this range will become noise. The wrapped around portion of the operator will also contribute noise from the shallower dips with the same sign. Note that the operator does extend to the zero dip portions of the data and that some noise may appear as spurious horizontal events.

To prevent aliasing from occurring, the Kirchhoff operator must be high cut filtered, where the high cut frequency is defined by equation (1). This is illustrated in Figures 5e where the thickening of the hyperbola is used to represent the width of a convolutional type filter that is applied to the input data before summing. These filters limit the input data as illustrated in Figure 5f. One location on the input data will be filtered many times with different filters that depend on the position of the migrated sample and the corresponding position of the input sample on the hyperbola.

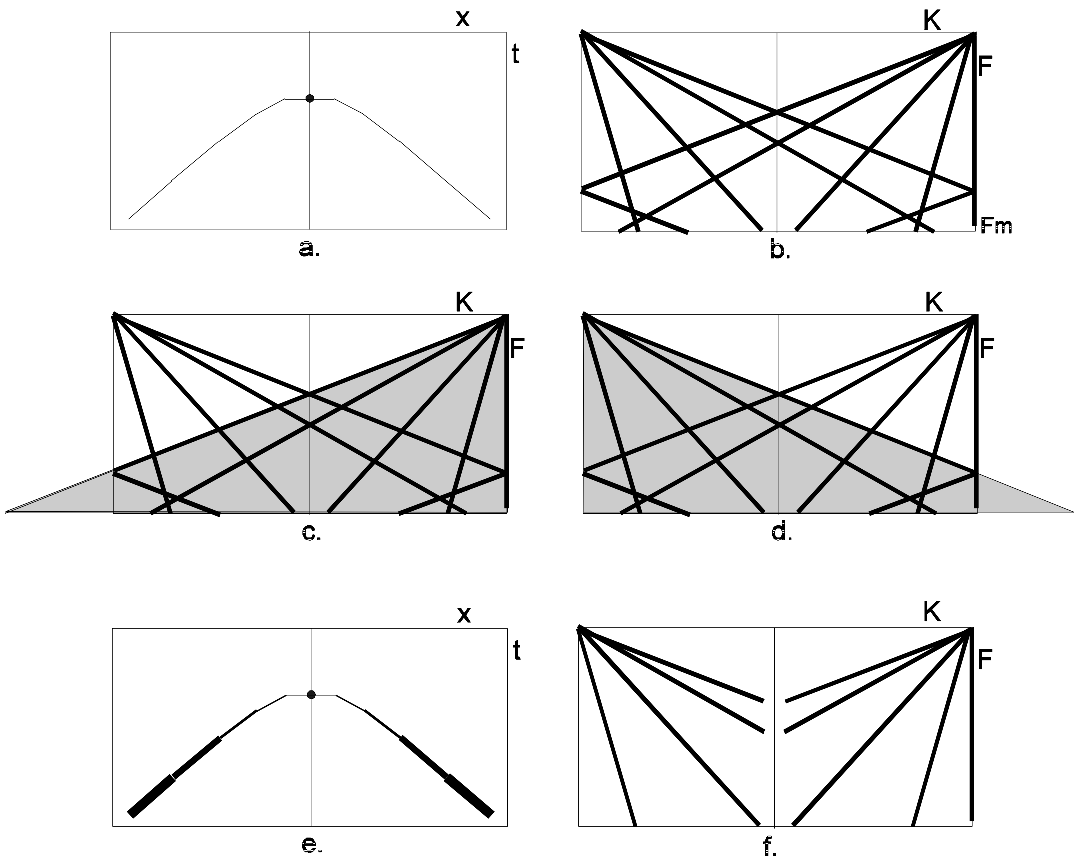

Figure 5. a) Piece wise linear diffraction with b) the FK domain, c) the aliased migration window for positive dips and d) for negative dips. Figure e) illustrates the band limited diffraction to prevent aliasing, and f) the corresponding FK domain.

ANTI ALIASING FILTERS IN KIRCHHOFF MIGRATION

To prevent aliasing in Kirchhoff migrations, the input data under the diffraction must be filtered with a high cut filter to remove the high frequencies at steep dips. Each input sample will contribute to many different positions on many different diffractions, each of which will require a different high cut filter. As only one output sample is required, a convolution form of the high cut filter is used. The size in samples of these filters is proportional to the period of the high cut frequency. Many filter options are available and may range from a simple box car shape, triangular shape, or sin(x)/x type filter that has sharp transitions in the frequency domain.

The choice of filter depends on the processing time and acceptable results. In addition, the spectral shape of these filters must be considered in the way they truncate the end up the dip. The desired sharp cut off frequency of the sin(x)/x filter will preserve all the energy in the pass band and attenuate all the aliased energy. It may however, require an order of magnitude greater processing time than the box filter. The triangular filter will attenuate more energy in the pass band and aliased energy than the box car filter, but it will also attenuate more of the desired signal than the box car. The box-car filter is a good compromise as it usually attenuates a sufficient amount of the aliased energy, and preserves a reasonable amount of the desired signal. The width of the box car filter T_{box} is often defined as one wave length of the cutoff frequency defined by equation (1), i.e.,

$$T_{box} = \frac{4\Delta x \tan(\theta)}{V}. \tag{2}$$

This filter usually produces a wormy appearance to the data, and a box car filter with width less than the full period T_{box} may be used.

Rather than sum all the points in a box car filter, the trace may be integrated by a simple accumulation of the samples. The energy in the box car may be found by a difference of two integrated samples, one at each end of the box car. An example of a similarly efficient triangle filter is given by Lumley (1994).

In addition to providing the filtering of the input data, the convolutional filters may also be used to interpolate between time samples. This is accomplished in the box car filter by interpolating the samples at the ends of the box car.

Another fast method of achieving an anti-aliasing filter action has been to high cut the input data into a number of (say three) sections that cover the range of filtering required to prevent aliasing. Data from each filtered section will contribute to different portions of the diffraction by direct insertion, or by interpolating between the appropriate filtered sections, (Gray 1992).

Kirchhoff migration allows the output trace (and samples for that matter) to be arbitrarily located. Often the input data may be at a given trace spacing, and the migration desired at half the original trace spacing. This is a perfectly acceptable process and the migration is used as an interpolator. The consequences of such an operation are susceptible to aliasing noise, but can increase the frequency content of the dipping events. For a noise free migration, the aliasing filter should use the input trace space and not the output trace spacing. Using the output trace spacing for the anti-aliasing filter will allow the steeper dips to contain higher frequencies, but will also contain more aliasing noise.

PRESTACK CONSIDERATIONS OF ALIASING

It is traditionally assumed that the aliasing criterion is based on trace spacing Δx. A source record for example, may be migrated with ideal AAF's designed on half the receiver spacing. No aliasing occurs and the data is assumed to be a sub set of the final prestack migration. Simply adding the source records (it is often assumed) will produce the desired migrated section. What possible aliasing could still occur? Lots!

Aliasing due to acquisition geometry

Significant aliasing occurs in most acquisition geometrys. The only time when aliasing does not occur is when sources are placed at each half station intervals. If one assumes horizontal data, the source spacing may be increased to the station interval, but then the sources must be placed midway between the receivers. These principles are discussed in Vermeer 1990.

Consider a typical line with the sources spaced at every fourth receiver station. Traces in a CMP gather will separated by four surface station intervals. A similar problem of aliasing also occurs in the constant offset sections where the trace spacing is also four surface stations. NMO and stacking could be considered Kirchhoff migration process (but rarely recognized as such) and is extremely aliased with this shooting configuration. Prestack migrations and DMO that are performed on the constant offset sections require special attention to the aliasing condition and usually involve some form of interpolation or offset mixing. Conventional NMO processing ignores the aliasing by muting the data to small offsets when the NMO stretch becomes too large.

The consequence of this sparse shooting is an increase in noise. This noise could be reduced by using Kirchhoff migration for the NMO and stacking, taking care to use the “correct” input trace spacing for the anti-aliasing filter. The results would be a section with less noise and a slight lowering of the frequency content. As above, the results are a compromise between noise, bandwidth, and dip. Conventional NMO and stacking are equivalent to a Kirchhoff migration with no aliasing control, and without the root differential filter.

An ideal solution is to shoot each half station and use Kirchhoff migration principles to perform the NMO and stacking. The dreaded NMO stretch is actually desired as it controls aliasing. Note that the effect of NMO spreading is reduced by the root differential filter that is part of 2-D Kirchhoff migration.

Aliasing in prestack processes

Data acquired with aliasing will affect all prestack processes, including all prestack migrations. This has long been recognized in the s-g prestack migration in which data is alternately sorted into source (s) and receiver (g for geophone) gathers and downward continued one time increment, or tau step. The process of resorting is repeated for all tau steps until the migration is complete. Data in the source gathers have a trace spacing equal to the receiver interval. However, when the data is sorted in to the receiver gathers, the trace spacing is equal to the source interval. When the sources are every fourth station, the receiver gathers are aliased.

Prestack migration of source record is also a compromise between the aliasing noise, bandwidth, and dip. After the source record is migrated, much of the steeply dipping energy is moved beyond the range of the original shot geometry. This steeply dipping information is often muted away and the range of the original source record used. This lowers the dip range of the migration and the accompanying noise of the aliased steeper dips. If the migrated source records are preserved to full migrated offset, then a migrated output trace will be the sum of the receiver gather. This gather will typical have traces at every fourth station, and will be aliased.

Full prestack Kirchhoff migration that uses the double square root equation to evaluate traveltimes is also susceptible to aliasing. The anti-aliasing filter is usually based on the output trace interval that may be arbitrarily set, and is typically half the receiver interval to match conventional post stack process. The aliasing noise is tolerated for an increase in the frequency content of dipping events. The anti-aliasing filter should use the source interval instead of the output trace interval for an aliasing noise free section.

ALIASING CONSIDERATION FOR EQUIVALENT OFFSET MIGRATION

Equivalent offset migration is a relatively new process (Bancroft 1994 a and b) in which all input traces that will contribute energy to a migrated trace are gathered into one common scatter point (CSP) gather. The input traces are efficiently gathered before any time shifting of the data is performed. The offsets in these gathers are typically three times greater than the maximum source receiver offset (for 2-D data), and may be binned at an arbitrary spacing. An important question is what is the optimum bin spacing. The actual number of bins, or bin spacing has little effect on the time required for CSP gathering and should not be a serious factor in defining the bin size. A fewer number of bins requires a smaller memory to save the gathered data and may be a consideration for maximizing the bin size.

A fine bin spacing was suggested to minimize the amount of data smear that accompanies bin sorting. Due to the extremely high fold in the equivalent offset bins (up to many thousands for 3-D), the spread of offsets within a bin are linear. This linear distribution will act as a box-car filter and attenuate the higher frequencies within the bins. The time spread in a bin T_{bin} is proportional to the slope α of the NMO hyperbolas, and is found from

$$T_{bin} = \frac{2\Delta x \tan(\alpha)}{V} \qquad (3)$$

where Δx is the bin spacing, and V the velocity. This potential lowering of the frequency content is not a detriment but an asset. The filtering effect is a natural anti-aliasing filter, as is evident by comparing with equations (3) with (2). A bin spacing equal to the final trace spacing will contain a boxcar period equal to one half the ideal required for the antialiasing filter. Varying the size of the bins allows control over the amount of aliasing noise and the frequency content of the data. The CSP gathers may still be formed with a finer bin spacing and a sin(x)/x filter used to control aliasing if desired. The use of an expensive sin(x)/x antialiasing filter is still economical as it is applied to the bin after the CSP gathers are formed, and not at each input trace relative to the CSP location. An example of the bin smear is shown in Figure 6 where one scatter point is modelled with a 3-D geometry. Reflections from the scatter point are defined as a spike on the source records. The figure contains the CSP gather located directly above the scatter point, and contains the spikes from all the input traces. Note that the spread of the spikes are reasonably distributed across the bins, and that the energy in each bin steps down to the energy in the next bin.

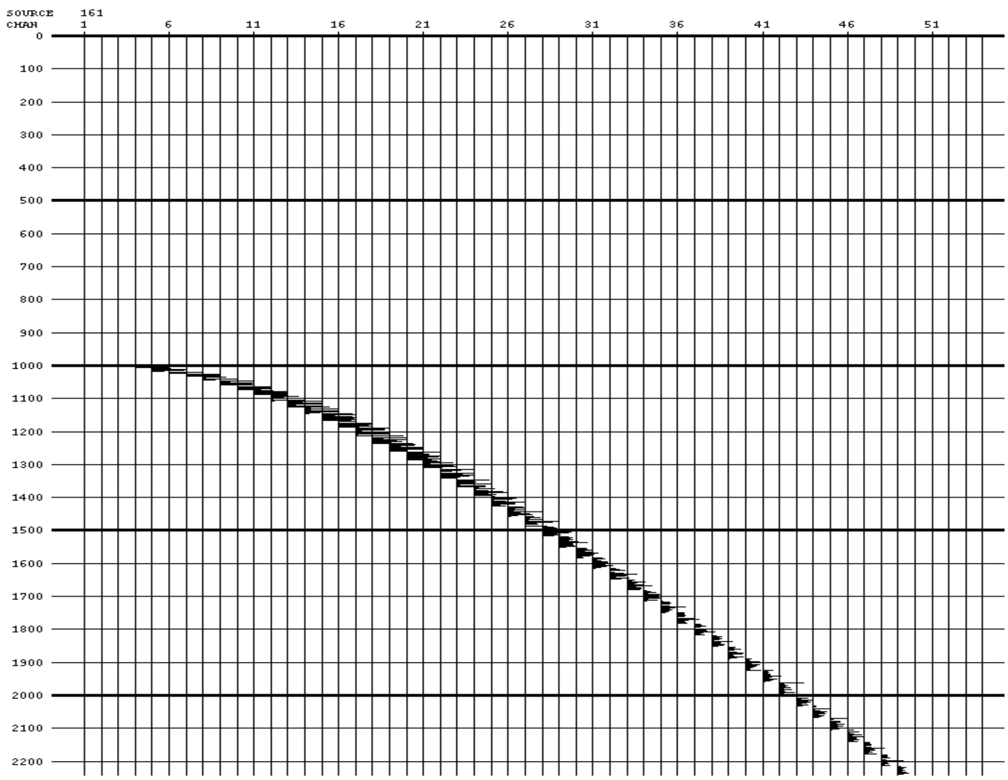

Fig. 6 One scatterpoint modelled from a 3-D volume to a CSP gather. The wavelet is represented by a spike to illustrate bin smearing, and the natural anti-aliasing effect.

ALIASING CONSIDERATIONS FOR 3-D ACQUISITION

3-D projects as in 2-D projects should be shot with the same distance between the sources and receivers to maximize frequency content with minimal aliasing noise. With this criteria, the anti-aliasing filters will be able to use the same bin width criteria for all input traces. Projects that have different source and receiver spacing, will have anti-aliasing filter bin sizes that vary with offset, or require a compromise to be established for the appropriate bin size of the anti- aliasing filter. An anti-aliasing filter that removes all aliasing noise requires the cutoff frequency to be established by the larger value of either the source or receiver spacing.

The size of the trace spacing for an anti-aliasing filter can be visualized by a two pass post stack Kirchhoff migration. The two pass method will first migrate the in line direction, then the cross line direction. The anti aliasing filter will be applied for the appropriate trace spacing that would be either half the receiver spacing, or half the source spacing. If an ideal high cut filter is used, only the filter with the lowest high cut frequency, or highest period, would be required. The period *P* of the AAF is used in the following displays to compare the effects of unequal source or receiver spacing.

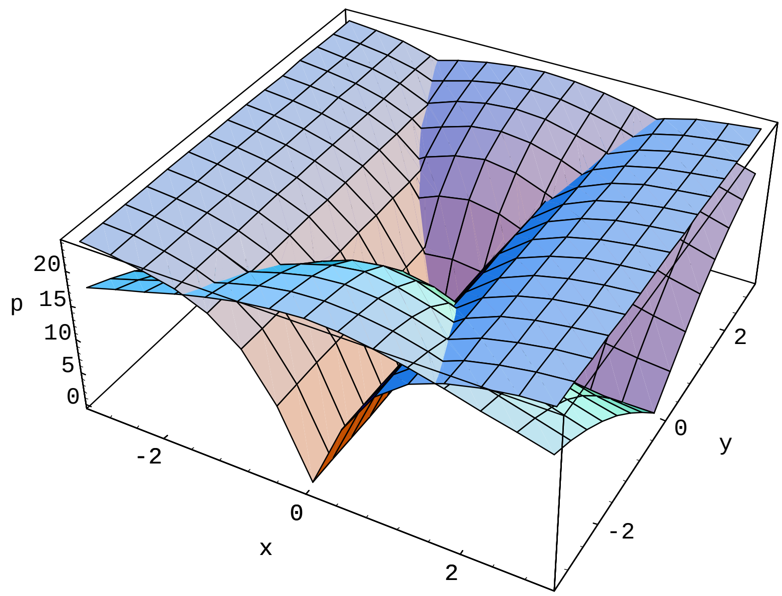

Figure 7. Two surfaces showing the period of the AAF for two pass migrations when source and receiver spacings are equal. Only the upper surface is required.

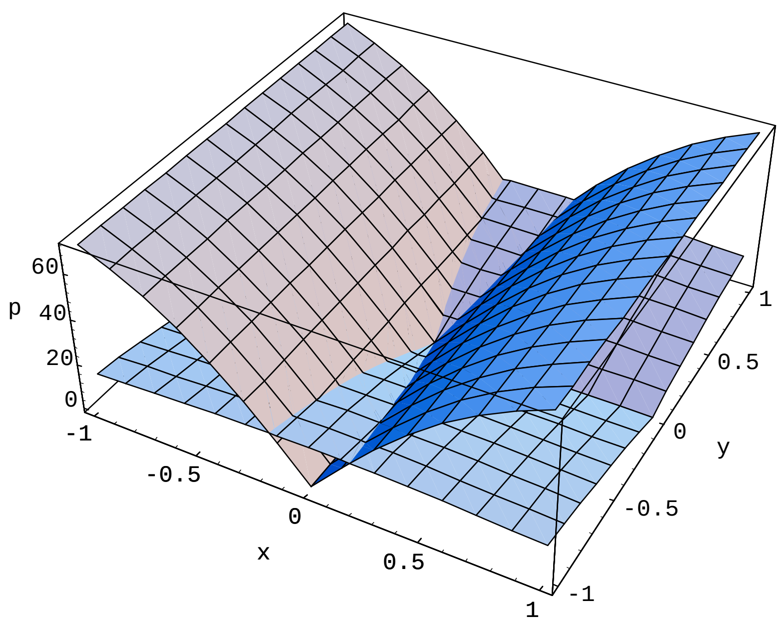

Figure 8. Two pass AAF periods when the source spacing is four times the receiver spacing.

A map of these periods is displayed in Figure 7, for one sample of one migrated trace located at the center of a 3-D. The two surfaces represent the period of the AAF's at the input location for the respective two pass migration directions. The source and receiver spacing are equal in Figure 7. Figure 8 shows the source spacing (x) to be four times the receiver spacing (y). Note the AAF period is now dominated by the source spacing. Data in the receiver direction does not have the full effect of the source spacing. A direct one pass method of migration should use an AAF designed for the appropriate azimuth.

As in 2-D design, 3-D design desiring a maximum band width, maximum dip range, and minimal aliasing noise, requires the source spacing to be equal to the receiver spacing. Relaxing this constraint results in a compromise between aliasing noise and the bandwidth of the dipping events.

EXAMPLES

The first example in Figure 9 is a Kirchoff migration of a synthetic model. The algorithm was written to allow aliasing on the left side of the section, while the right side included an antialiasing filter. Note the large amplitude of the aliasing noise, and its characteristic appearance of alternating polarity between traces.

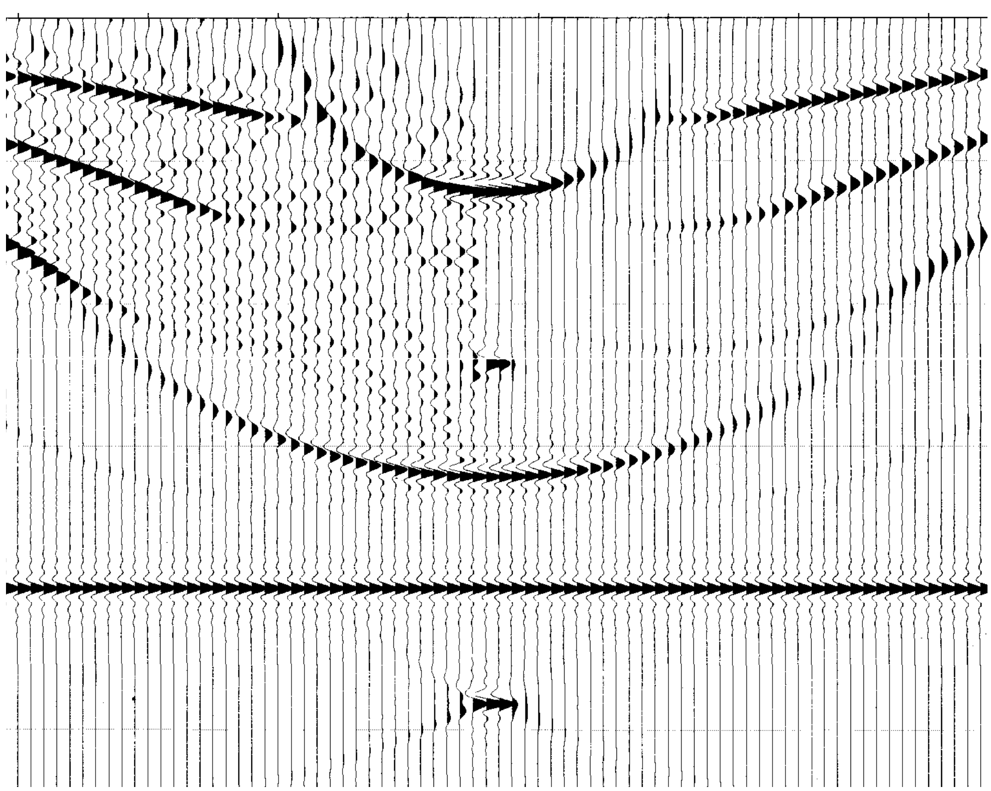

Fig. 9. A Kirchhoff migration on a synthetic model with the an anti-aliasing filter applied to right side, while the left side id aliased.

The second example in Figure 10 shows the effects of applying AAF's to the equivalent offset method of prestack migration (Bancroft 1994). A simple prestack model was created with the source spacing four times the receiver spacing. The processing was based on an equal grid size. Figure 10a shows aliasing noise above the reflector. Figure 10b shows the effect of using the receiver interval for the AAF's, while 10c shows the results with the AAF designed for the source spacing. Note the reduction in aliasing noise with little reduction in the reflector's bandwidth.

The spacing of the source and receiver lines also contribute an aliasing effect on the data and appears as geometry imprinting. Rather than use AAF's designed on extremely large offsets, their effects are usually reduced by some form of trace scaling and binning.

CONCLUSIONS

A review of aliasing was presented to aid in the design of anti aliasing filters for 3-D data. The AAF's should use the source and receiver offset in their design, and not be based on the processing grid size. As the 3-D field design becomes more sparse, the loss of information due to larger source and receiver spacing will result in a compromise with acceptable aliasing noise levels and reduced bandwidth.

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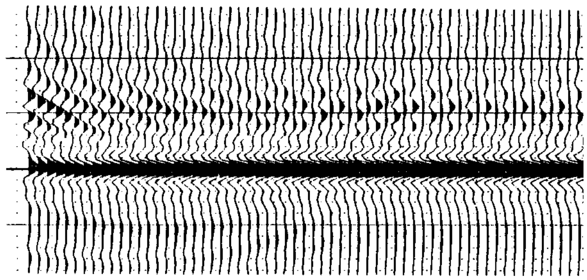
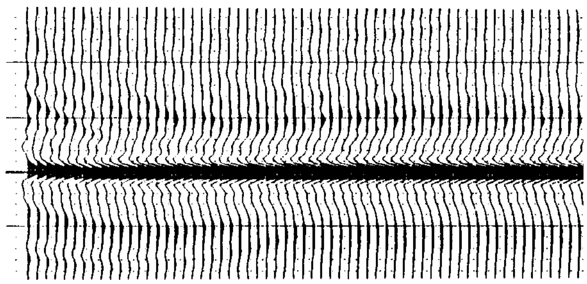
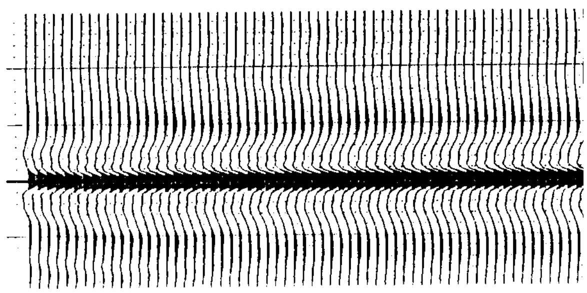


(a)



(b)



(c)

Fig. 10. Effects of AAF design, a) the aliased data, b) AAF designed for receiver spacing, and c) for the source spacing.