Predicting density using V_s and Gardner's relationship

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ABSTRACT

We use Gardner's relationship, $\rho = aV^{1/4}$, with both P-wave and S-wave velocities to predict density. The data are from two wells in the Blackfoot region of Alberta. A linear least-squares polynomial is fit to the data. The S-wave data gives 0.2357 and 0.2928 as approximations to the Gardner exponent, while the P-wave velocities give estimates of the exponent to be 0.1297 and 0.1248. The densities predicted by the S-wave velocities show similar scatter to the P-wave predictions.

INTRODUCTION

Gardner et al. (1974) conducted a series of empirical studies and determined the following relationship between velocity and density:

$$\rho = aV^{\frac{1}{4}}, \tag{1}$$

where ρ is in g/cm^3, a is 0.31 when V is in m/s and is 0.23 when V is in ft/s. Their results are summarized in the graph shown in figure 1.

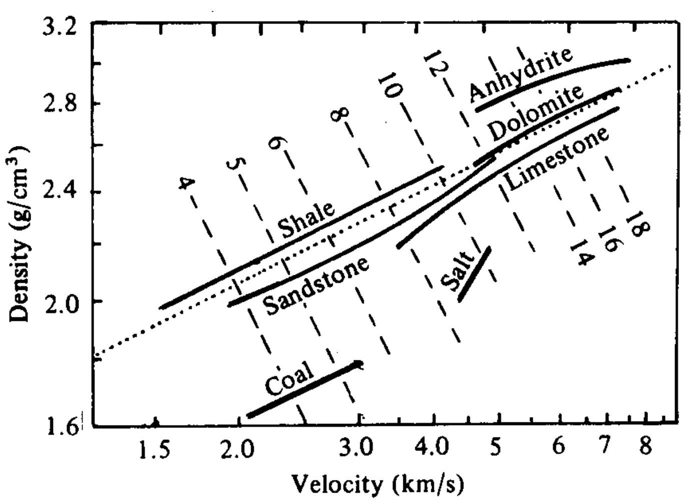

Fig. 1: P-wave velocity's relationship to density for different lithologies (log-log scale). The dotted line represents the predictions from equation 1 and the dashed lines show constant acoustic impedance (from Sheriff and Geldart, 1995).

Major sedimentary rocks define a rather narrow corridor across the graph and the Gardner predictions are approximately in the middle of this corridor. The major deviations from this trend are anhydrites, coals, and salts. Further progress was made when Rafavich et al. (1984) preformed detailed laboratory experiments to study velocity relationships with petrographic character in carbonates. Their work indicated that V_p and V_s are primarily influenced by rock porosity and rock density.

Lithology, itself, appeared to have only a small influence as did pore shape and interstitial fluid.

GEOLOGY OF THE REGION

The log data studied in this report are taken from two wells within the Blackfoot field located near Strathmore, Alberta, in Township 23, Range 23, west of the 4th meridian. Miller et al. (1995) discuss the initial interpretation of this area's geology. Figure 2 shows the stratigraphy near the zone of interest.

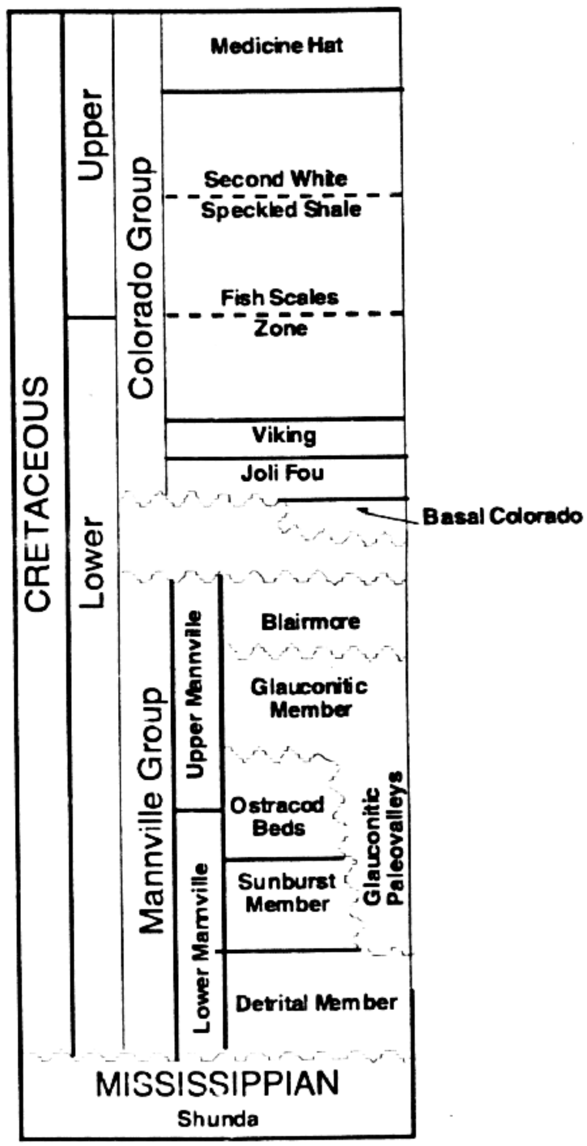

Fig. 2: Stratigraphic sequence near the zone of interest (from Miller et al., 1995).

Table 1: Major geological formation units near zone of interest (from Potter et al., 1996).

ABBREVIATION	FORMATION NAME
2WS	Second White Speckled Shale
BFS	Base of Fish Scales Zone
VIKING	Viking
MANN	Blairmore-Upper Mannville
COAL 1	1st Coal Layer
COAL 2	2nd Coal Layer
COAL 3	3rd Coal Layer
GLCTOP	Glauconitic Channel Top
LITHCH	Lithic Channel
GLCSS	Glauconitic Channel Porous Sandstone
OST	Ostracod
SUNB	Sunburst
DET	Detrital
MISS	Shunda-Mississippian

RESULTS

Tables 2 and 3, modified from Potter et al. (1996), show parameters that are taken from the well log data. This study uses the 12-16 and 9-17 wells from the study area because they have dipole sonics over the interest zone, as well as the complete array of conventional logs. These logs are blocked across the lithological units in Table 1 using MATLAB's LOGEDIT algorithm with the mean value between the formations used for the blocking. Tables 2 and 3 show a depth value that corresponds to the top of the formation and its interval width.

Table 2: Log data for various horizons from the 12-16 well (modified from Potter et al., 1996).

UNIT	DEPTH(m)	V_p(m/s)	V_s(m/s)	ρ (kg/m^3)
Above	1229	3562	1837	2512
2WS	1241	3557	1819	2485
BFS	1328	3303	1520	2390
VIKING	1353	3865	2008	2516
MANN	1445	3987	2018	2525
COAL 1	1519	3485	1683	2256
COAL 2	1525	4121	2120	2472
COAL 3	1538	4071	2103	2516
GLCTOP	1566	3999	2113	2558
GLCSS	1586	4062	2297	2493
DET	1595	4458	2339	2571
MISS	1611	5897	3063	2690

Table 3: Log data for various horizons from the 9-17 well (modified from Potter et al., 1996).

UNIT	DEPTH(m)	V_p(m/s)	V_s(m/s)	ρ (kg/m3)
Above	1440	3597	1865	2547
MANN	1450	3862	2011	2528
COAL 1	1526	3416	1659	2305
COAL 2	1532	3971	1996	2508
COAL 3	1545	3890	2031	2561
SUNB	1593	4072	2122	2550
DET	1606	4170	2184	2538
MISS	1637	5061	2421	2669

Below are V_p versus ρ plots and V_s versus ρ plots for two well logs from the Blackfoot region of Alberta. The data being cross-plotted are the densities, P-wave velocities, and S-wave velocities from the 12-16 and 9-17 well logs. If the data is plotted in log-log style, then the slope of a best-fit line to this data is an approximation to the Gardner exponent. This is shown by the following equation:

$$\log_e(\rho) = \frac{1}{4}\log_e(V) + \log_e(a). \tag{2}$$

Equation 2 also tells us that the y-intercept of the line, $\log_e(a)$, is the natural logarithm of an estimate to the Gardner coefficient. The line shown on these plots is the best-fit (in a least-squares sense) linear polynomial to the given data.

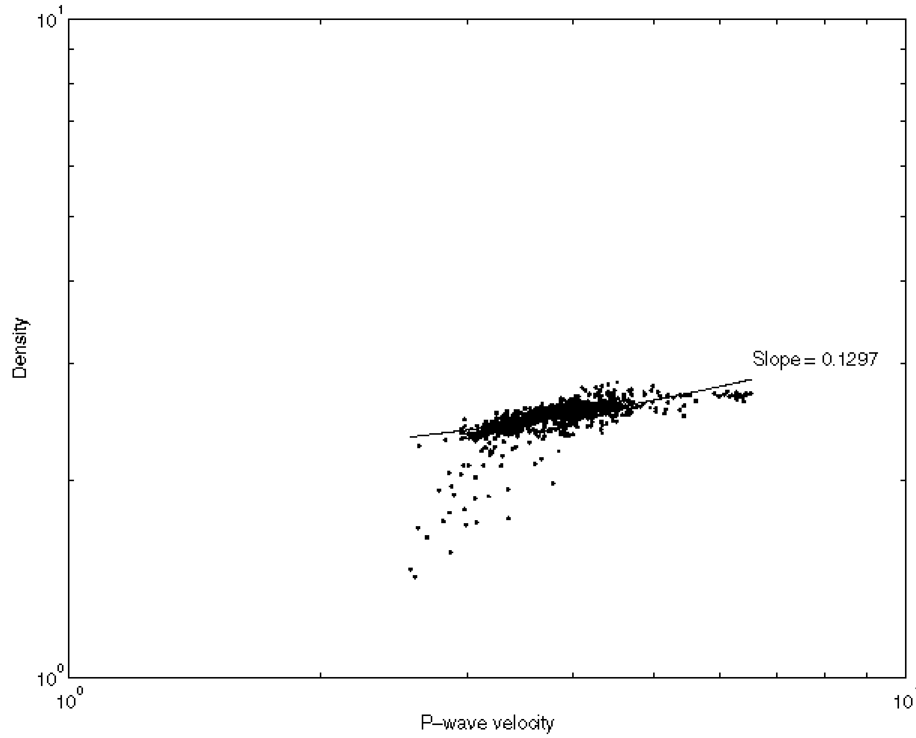

Fig. 3: V_p versus ρ log-log plot with linear curve fitting for the 12-16 well.

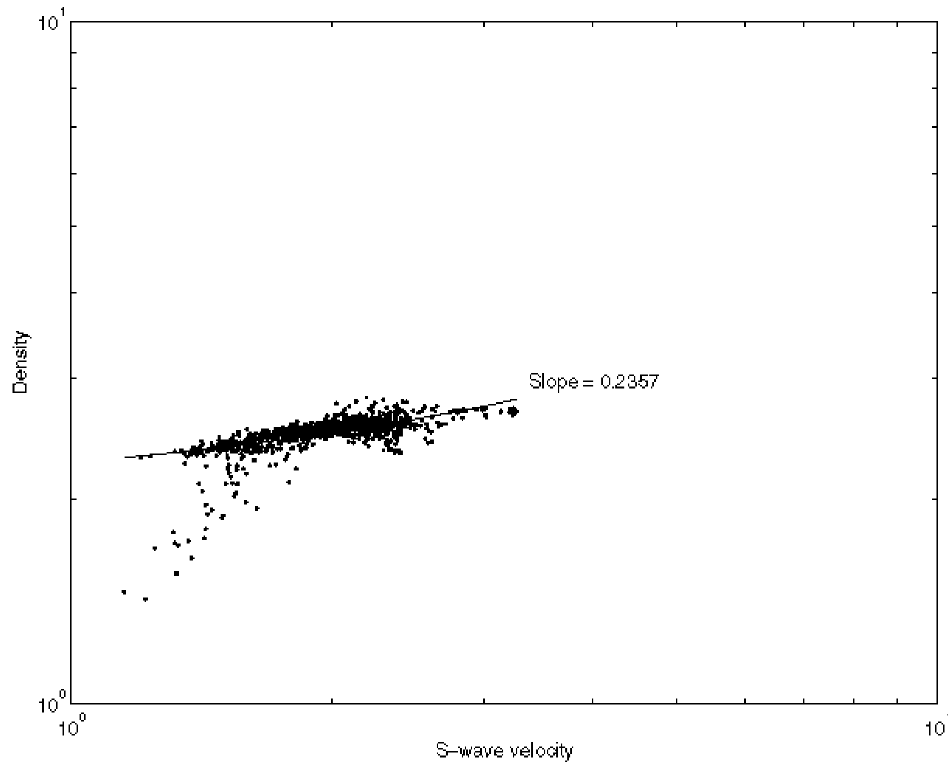

Fig. 4: V_s versus ρ log-log plot with linear curve fitting for the 12-16 well.

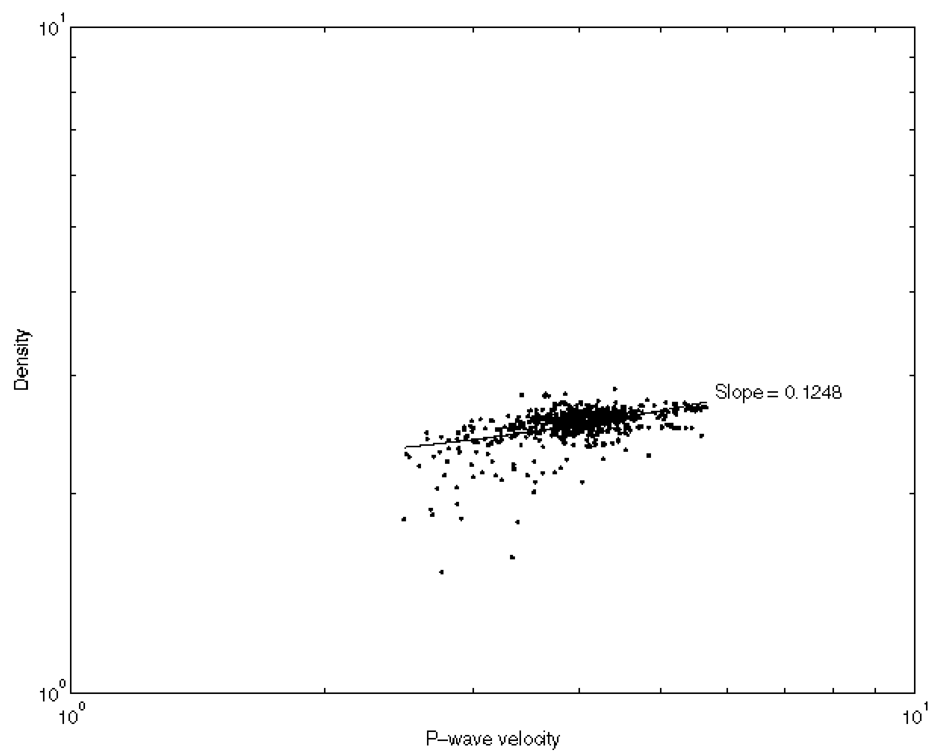

Fig. 5: V_p versus ρ log-log plot with linear curve fitting for the 9-17 well.

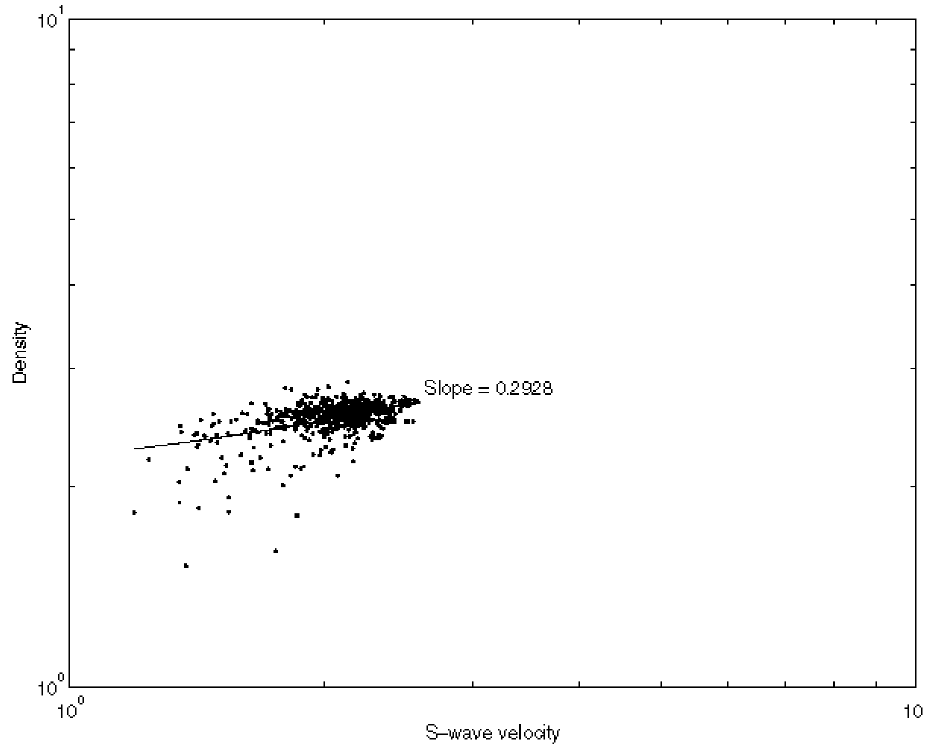

Fig. 6: V_s versus ρ log-log plot with linear curve fitting for the 9-17 well.

What we see from all of these four plots is that the conventional method, i.e. using V_p, of treating Gardner's relationship seems to give a poorer estimation of the Gardner exponent than if we use shear wave velocity. For the 12-16 well, the P-wave velocity approximation to the Gardner exponent has an absolute deviation from the expected value, 0.24, of 0.1103. In the meantime, the S-wave velocity approximation has only a 0.0043 absolute deviation from the expected value. When considering the 9-17 well, it is seen that the P-wave estimate deviates from the norm by 0.1152 and the S-wave estimate only fluctuates by 0.0528. While it is unconventional to use V_s in Gardner's relationship, these preliminary results seem to encourage its use. As a means to understand the goodness of this fit to the data, we use the MATLAB functions *std* and *cov*. These functions give us values for the standard deviation, σ, the variance, σ^2, and the correlation coefficient, cc, respectively. With these values, we can evaluate the scatter of the data. One should note that the *std* function normalizes by (n-1), where n is the number of samples in the sequence, and this makes *cov* the best estimate of the variance in the data. Table 4 summarizes these statistical results.

Table 4: Statistical analysis values for density models.

	V_p Density Model	V_s Density Model
12-16 Well:	$\sigma = 0.0869$	$\sigma = 0.0907$
	$\sigma^2 = 0.0076$	$\sigma^2 = 0.0082$
9-17 Well:	$\sigma = 0.0674$	$\sigma = 0.0653$
	$\sigma^2 = 0.0045$	$\sigma^2 = 0.0043$

These statistical results seem to strengthen the argument for using shear wave velocities to predict density. We see that for the 12-16 well, the standard deviation and variance for the V_s model are quite similar to the values for the V_p model. The 9-17 well results seem to offer better statistics in that both the standard deviation and the variance are lower for the V_s model.

CONCLUSIONS AND FURTHER DIRECTIONS

A preliminary conclusion that we draw from this investigation is that using S-wave velocities to predict density with Gardner's relationship seems to give a better approximation to the Gardner exponent. In addition, the statistical analysis seems to suggest that the shear wave model has standard deviations and variances that are similar to, or possibly better than, those of the P-wave model. While these results are very encouraging, there are still some significant issues to address. First, we must test the use of V_s in Gardner's relationship with well log data from other areas. Namely, we wish to test this hypothesis with data from regions that have a lithology that varies from that which is seen in the Blackfoot region. In addition to this, we must also extrapolate these best fit lines back to the y-axis. This will give us a value, when converted under logarithmic inversion, that estimates the Gardner coefficient. Only when this component can also be shown to have a better approximation with V_s can we recommend the use of V_s as routine practice when considering density predictions with Gardner's relationship. As a final path of investigation, we hope to look at Lindseth's relationship, $\rho\beta = a\beta + k$, and see how its estimates of density compare to approximations given by Gardner's relation.

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