Investigating the randomness assumption in wavelet estimation

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ABSTRACT

In the February 1991 issue of *GEOPHYSICS*, Anton Ziolkowski gives a scathing criticism of statistical wavelet estimation methods. Among other points, Ziolkowski questions the validity of the randomness assumption. That is, when trying to estimate a wavelet by statistical means, it is common place to assume that the seismic reflectivity sequence is a random signal so that the wavelet autocorrelation can be obtained from the trace autocorrelation. In examining the randomness assumption, we shall present some very preliminary results concerning these statistical methods and their assumption of randomness. We will discuss the methods used and analyze their results. Finally, we shall indicate some directions that will be taken in the future.

INTRODUCTION

Statistical wavelet estimation methods usually assume that the reflectivity is a random uncorrelated signal (that is, the reflectivity has an autocorrelation which approximates a delta function) and this effectively means that the seismic trace autocorrelation is approximately equal to the seismic wavelet autocorrelation, $X(z)X^*(z) \cong W(z)W^*(z)$. Consider the following figure.

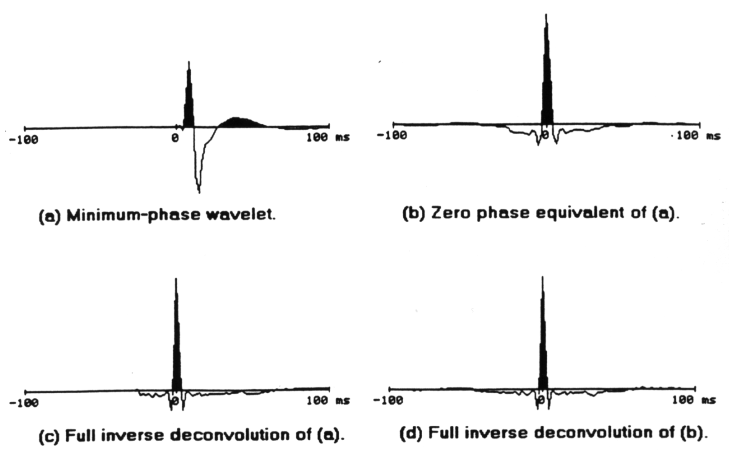

Fig. 1: The effect of deconvolution on known minimum-phase and zero-phase wavelets (from B. Russell, SEG inversion course notes).

Figure 1 shows that if the wavelet is known, then deconvolution will give consistently good results since the application of a Wiener deconvolution filter produces a good approximation to a bandpassed delta function. The deconvolution of data is only effective if the input wavelet is a reasonable approximation to the true wavelet. To establish the best, and most effective, means of wavelet extraction is the key to creating a more interpretable, high resolution, seismic section. This is precisely why we must investigate the assumptions upon which our commonly used statistical methods are based. The random reflectivity assumption allows one to estimate the wavelet's autocorrelation, and consequently the amplitude spectrum, from the trace autocorrelation. Hence, given an input seismic trace, it is possible to estimate the wavelet needed for deconvolution. For the synthetic data that is used in this study, a comparison of the model wavelet and wavelet estimates obtained by methods that use the randomness assumption will be made.

METHODOLOGY

Two synthetic seismic sections are generated for use in this investigation. We first look at the validity of the randomness assumption with a primaries only section derived from a well in central Alberta and then with a section that also has multiples, also derived from the same well. The use of synthetic data is done because we know what the wavelet used to generate the section is and, therefore, can make meaningful comparisons with the statistical estimates. For this experiment, we consider the wavelet estimates that are created via the Hilbert transform method and the Wiener-Levinson double inverse method. Both of these methods use the assumption of reflectivity randomness to estimate minimum phase wavelets. Detailed descriptions of the methods are given by White and O'Brien (1974), Claerbout (1976), and Lines and Ulrych (1977).

The Wiener-Levinson double inverse method can use the wavelet autocorrelation rather than the wavelet itself. If the desired output is set to a spike at zero delay (which will be the optimum only if the wavelet is minimum phase), then the inverse filter for a minimum phase wavelet is obtained. To obtain the wavelet estimate, another Wiener filter is applied to invert the inverse filter and thereby estimating the minimum phase wavelet from its autocorrelation.

The Hilbert transform method (sometimes called the Kolmogorov method) also uses the autocorrelation and the amplitude spectrum which can be derived from this autocorrelation. If a wavelet is minimum phase, then its phase spectrum can be uniquely derived from its amplitude spectrum by taking the Hilbert transform of the log amplitude spectrum (Robinson, 1967).

These estimates are computed by FORTRAN codes and their autocorrelations are compared to the trace autocorrelation. In addition, we will consider how these wavelet estimates compare to the actual wavelet that is used to generate these sections. The input traces are both 10 traces long with 261 samples per trace. The actual wavelet is a single trace with 44 samples. We use the conventional wisdom of choosing a filter length that is equal to the wavelet length. This results in estimated wavelets that are also a single trace with 44 samples. To prevent computational blowup, a prewhitening value of 1.01 is used. After the generation of these estimates, autocorrelations are made of the input trace and wavelet estimates. Since our estimates are just a single trace with 44 samples, we use a single trace from our input that is 44 samples long when we do these autocorrelations. Again, FORTRAN is used to create these sequences. To understand the goodness of these estimation methods, visual considerations are used. That is to say, we will qualitatively evaluate our estimates based on a visual comparison with the known wavelet.

RESULTS AND DISCUSSIONS

Some very preliminary results will now be presented and discussed. Figure 2, shown below, displays the synthetic trace that consists of reflections from primaries only. It is generated by the convolution of a reflectivity sequence and the wavelet that we are trying to estimate.

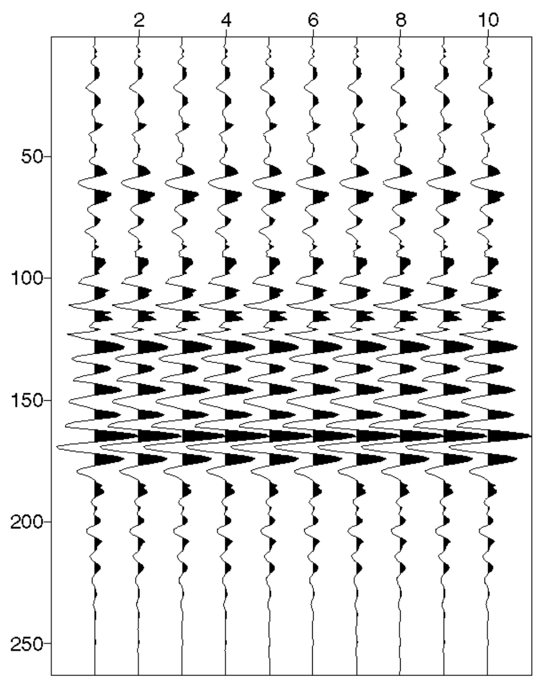

Fig. 2: A synthetic seismogram with only primary reflections.

The trace autocorrelation, in figure 3, shows a progressive decrease in amplitude, while figures 4 and 5 show the actual wavelet and its autocorrelation. Note the close similarities between figure 3 and figure 5. It is this wavelet and this autocorrelation that we wish to reasonably estimate so that an accurate reflectivity sequence can be recovered.

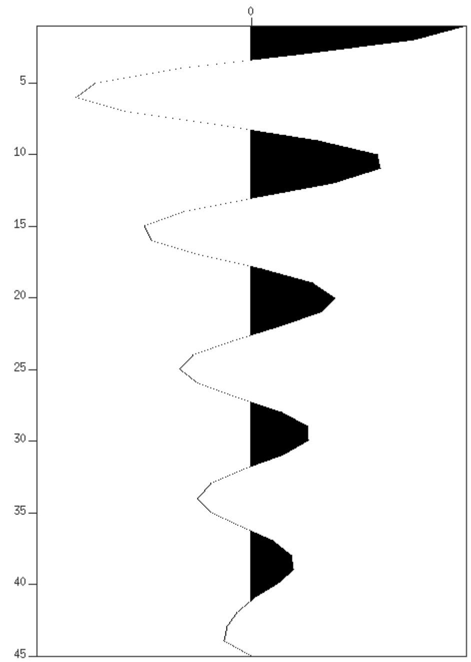

Fig. 3: The autocorrelation of a single trace from the input data.

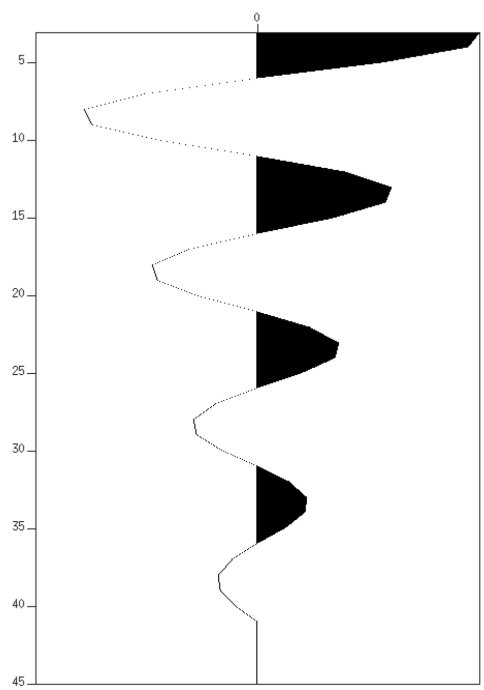

Fig. 4: The actual wavelet used to generate the synthetic sections.

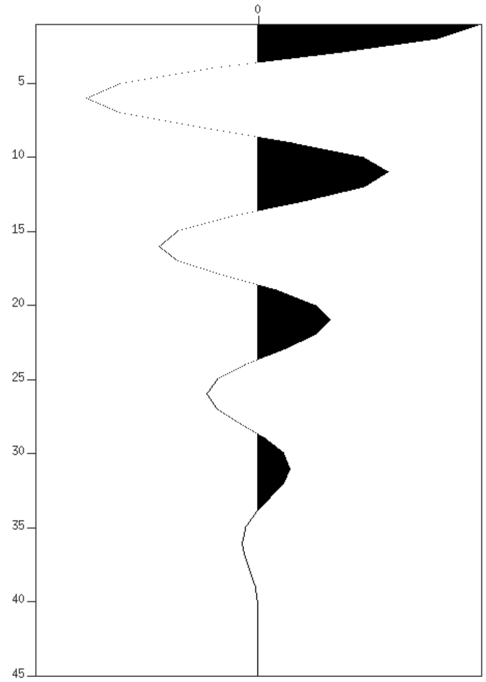

Fig. 5: The autocorrelation of the actual wavelet.

According to the theory, we expect that the wavelet estimates will be fairly good. This is because there is no inherent periodicity since there are no multiples in the data. Shown in figures 6 and 7 are the autocorrelations of the estimated wavelets.

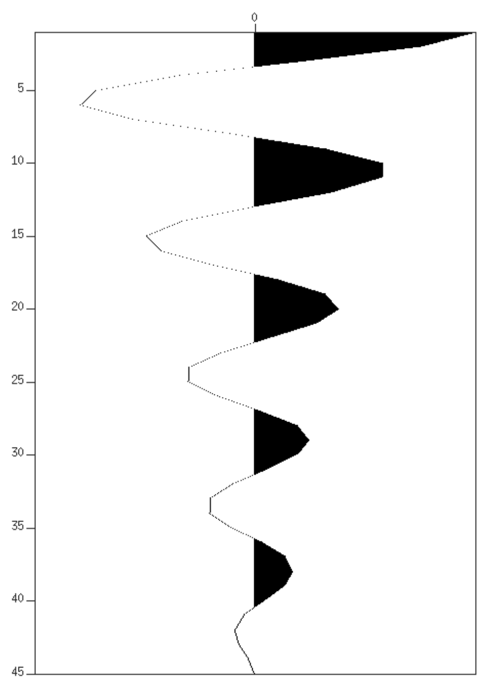

Fig. 6: The autocorrelation of the Hilbert transform wavelet.

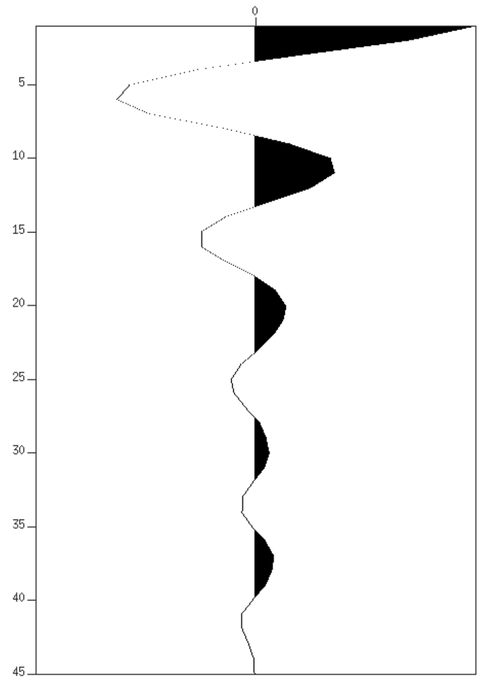

Fig. 7: The autocorrelation of the Wiener-Levinson double inverse wavelet.

From a qualitative standpoint, these images show a remarkable similarity to the plot in figure 3. Closer inspection reveals that the Hilbert transform estimate has a better correlation with input than the Wiener-Levinson estimate. We see that the Hilbert transform wavelet estimate has almost the same amount of energy as the trace. With such a close correlation, it can be said that there is random reflectivity in the input synthetic seismic trace since the approximation $X(z)X^*(z) \cong W(z)W^*(z)$ holds true. The Wiener-Levinson double inverse wavelet estimate also seems to be a close approximation to the trace autocorrelation and the same can be said in this case as well. The analysis shows that there is a good similarity between the autocorrelation of the trace and the autocorrelations of the estimates. As shown in the introduction, we need these wavelet estimates to be good approximations to the actual wavelet because then the deconvolution of the data to obtain a reflectivity sequence will be valid. In that vain, consider the following two plots.

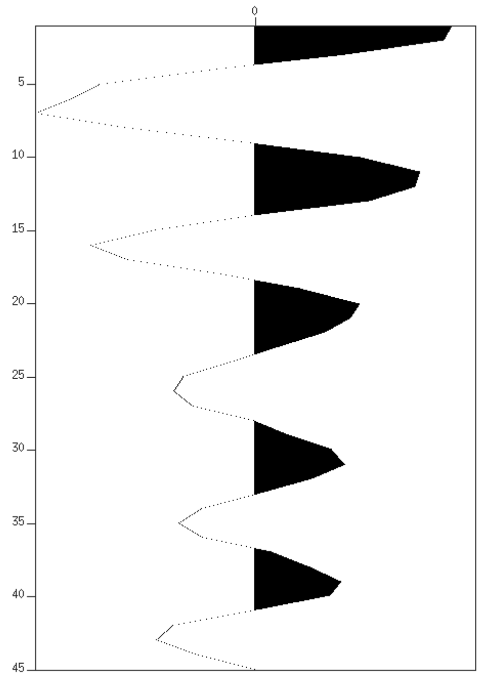

Fig. 9: Hilbert transform wavelet estimate for the primaries only data.

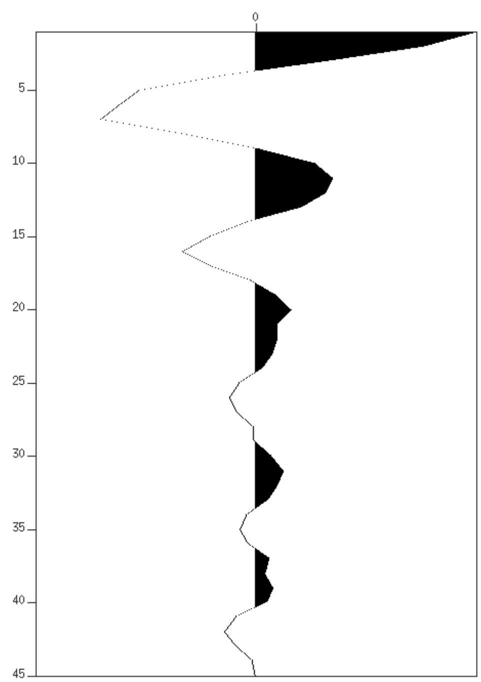

Fig. 10: Wiener-Levinson double inverse wavelet estimate for the primaries only data.

We can see that these estimates are fairly good reproductions of the actual wavelet that is displayed in figure 4.

From here, we investigate the next logical scenario of a synthetic seismic trace that includes primaries and multiples. The plot of such a seismic trace is displayed in figure 11.

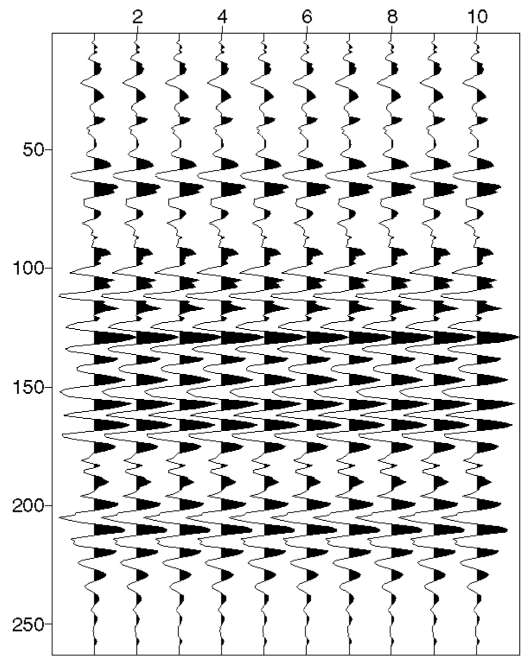

Fig. 11: A synthetic seismic trace with primaries and multiples.

Again, we consider just a single trace and compute its autocorrelation. We see this trace in figure 12 and note that it too has decreasing amplitudes as sample number increases. Also note that at the end of this trace, there seems to the onset of a doublet.

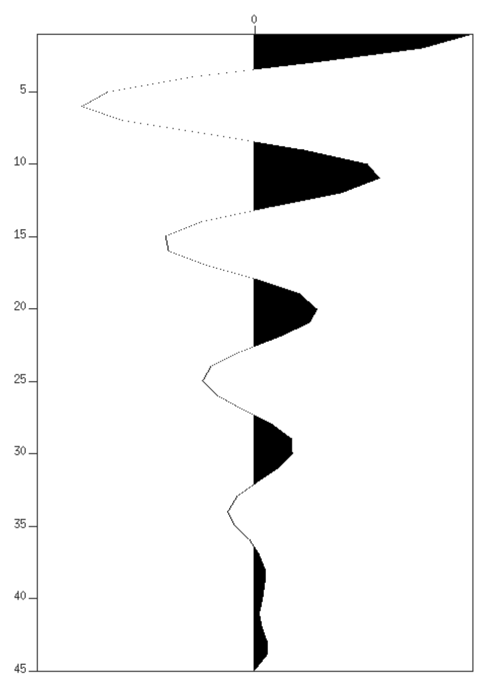

Fig. 12: The autocorrelation of a single trace from input data.

In the same manner as above, we consider the plots in figures 13 and 14. The Hilbert transform data shows a good correlation but there are some noticeable differences.

Meanwhile, there seems to be quite a noticeable difference between the input trace autocorrelation and the Weiner-Levinson double inverse autocorrelation.

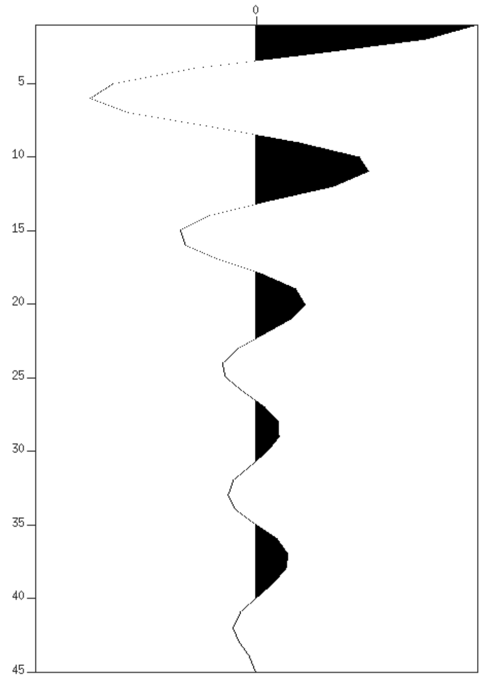

Fig. 13: The autocorrelation of the Hilbert transform wavelet.

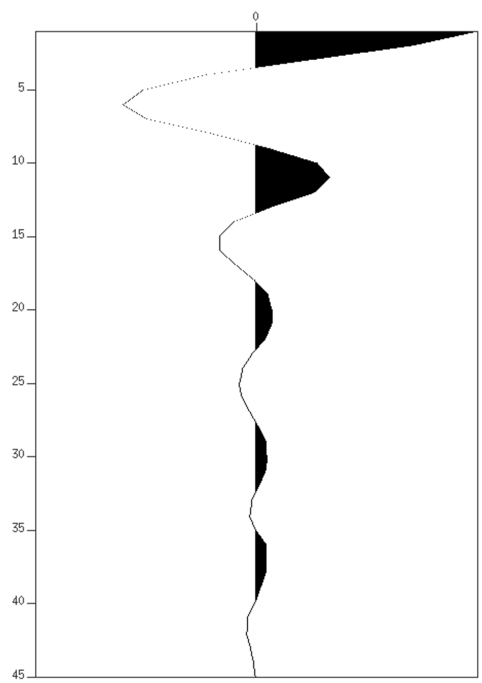

Fig. 14: The autocorrelation of the Wiener-Levinson double inverse wavelet.

As before, the Hilbert estimate is better than the Wiener-Levinson one. Again, we compare the wavelet estimates for this data to the actual wavelet that is shown in figure 4.

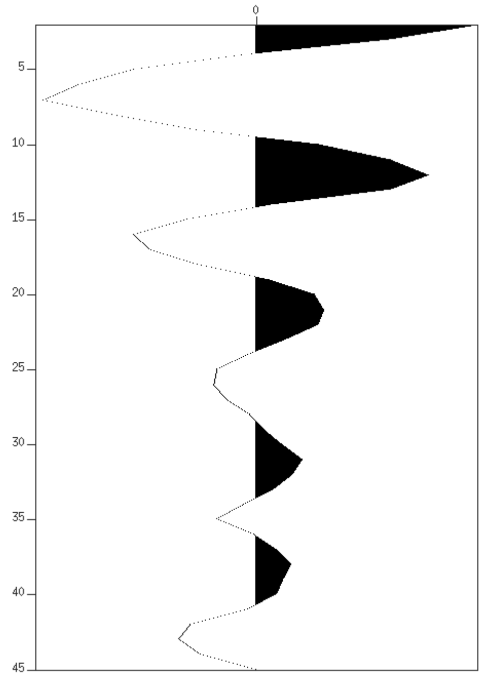

Fig. 15: Hilbert transform wavelet estimate for data with multiples.

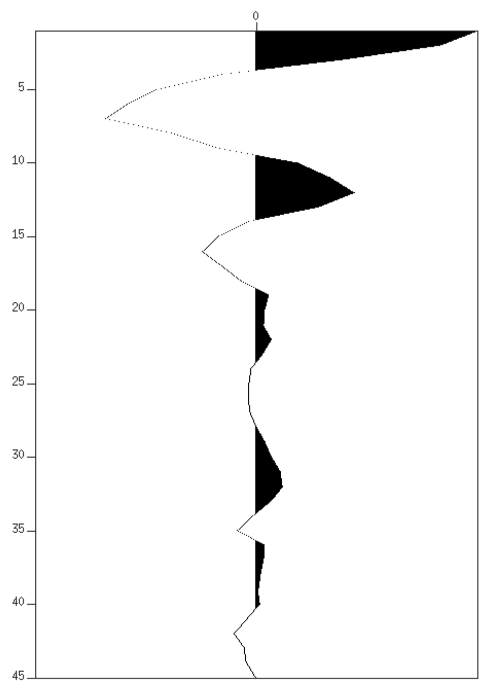

Fig. 16: Wiener-Levinson double inverse wavelet estimate for data with multiples.

Note that these estimates are not quite as good as those before. This is to be expected because the data being considered now is not a totally random signal since it contains multiples and these have an inherent periodicity to them.

CONCLUSIONS

The preceding results give some very mixed impressions regarding the randomness of the reflectivity sequence. As expected, both of these methods give better results for the primaries only trace since there are no periodic multiple events involved. Certainly, further and a more detailed investigation will be pursued. Also to be investigated will be how these statistical methods work on real data. To that end, the validity of the assumption can be tested by using sonic and density logs to compute a reflectivity series for a geological area and then using this information to measure the goodness of the estimates. Through all of the investigations, it is expected that the randomness assumption for a reflectivity sequence will be closely tied to the lithology of an area. That is to say, if an exploration area has periodic properties, then its reflectivity will not have randomness.

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