Migration velocity analysis by perturbation for converted waves

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ABSTRACT

Prestack depth migration can handle dipping reflectors and lateral velocity variations, but its success depends on a more accurate velocity model than may be obtained from simple velocity analysis methods, i.e. normal moveout. For converted waves, imaging is more complex due to prestack depth migration requiring different propagating velocities, P-wave (downgoing) and S-wave (upgoing) velocity fields.

This paper shows an approach to P-S prestack depth migration for converted waves, based on an extension of the method proposed by Liu (1993). This method works on common-image gathers (CIG) and migration velocity is obtained through residual moveout analysis. The migration velocities are updated by a perturbation approach. When the correct migration velocity is obtained, reflectors in CIG gathers, after depth migration, will be nearly horizontal.

BACKGROUND

The converted-wave processing sequence includes common conversion point (CCP) gathering, velocity analysis, stacking and migration, processes that assume horizontal layering and limited offset range. When complex structures and lateral velocity variations are present, CCP binning, velocity analysis and migration processes can fail in locating the reflector in the right position in depth or time. For dipping layers DMO is required, but even for a constant velocity situation, P-S DMO may be expensive.

With complex structures or lateral velocity variations, prestack depth migration may be required. Prestack depth migration can handle dipping reflectors and lateral velocity variations, however, it needs to have a more accurate velocity model than may be obtained from simple velocity-analysis methods, such as normal moveout (Liu, 1993). Fortunately, prestack depth migration also provides a powerful tool for doing velocity analysis in complex media. Stewart and Lawton (1996) indicate that P-S imaging may produce credible images in structural environments when poor data recording is present.

Two approaches of prestack depth migration for converted waves can be found in the literature, which provide migration velocity analysis (MVA) for improving the imaging process.

In the first, Zuurbier et al.(1987) migrated shot gathers, generated with a P-wave source and inline components of the geophones, by applying P- and S-wave velocities together during wavefield extrapolation of converted waves. Prestack depth migration is achieved by employing finite difference solutions in the frequency-space domain, where downgoing wavefield is extrapolated with P-wave velocity while upcoming wavefield extrapolation uses S-wave velocity. Both wavefields are extrapolated recursively along the depth axis. At each depth level, both wavefields are correlated with each other to produce the migrated result at that level. After resorting to receiver gathers, the correctly migrated P-S reflections should show up as aligned events at a constant depth. This prestack depth migration type implies a lot of sorting to check MVA, increasing the computational costs.

As a second option, Chan and Stewart (1994) developed a MVA in time. It is assumed that before applying MVA, a P-wave RMS velocity function is known from P-P processing. The RMS V_p/V_s value, γ_{rms}, is introduced during MVA. This time-and-space variant γ_{rms} function is sought rather than the RMS velocity function for S-wave itself during this velocity analysis. Then, MVA must address two issues to succeed: (1) how to establish a criterion for knowing if a migration velocity is acceptable, and (2) how to update the velocity, if it unacceptable. A new method for converted wave MVA is presented based on perturbation theory, such as used by Liu (1993; 1995) for P-wave data. This method derives a quantitative relationship between residual moveout and velocity error, which is valid for any offset, dip and velocity distribution. Additionally, the assymetrical raypaths assumption is not important and there are not offset or dip limitations.

METHODOLOGY

This migration velocity analysis by perturbation is based on a layer-stripping procedure for both P-P and P-S waves. The layer-stripping procedure for MVA, from pre-stack depth migration, can be stated as follows:

(1) Sort input data into common-offset gathers

(2) Kirchhoff depth migration with an initial velocity guess

(3) Sort migrated data into common-image gathers

(4) Measure imaged depths through RMO (Residual Moveout analysis)

(5) Evaluate perturbation formula through a derivative term

(6) Update the layer migration velocity by using the evaluated perturbation formula

(7) Image interfaces by using corrected velocities

(8) Check imaged depth in common image gathers

(9) Repeat steps from 1 to 8 for next layer

KIRCHHOFF MIGRATION

Conventional techniques, such as the downward continuation of sources and geophones by finite-difference (S-G finite-difference migration), are relatively slow and dip-limited. Compared to S-G finite-difference migration, the Kirchhoff integral implements prestack depth migration relatively efficiently, handles lateral velocity variations and has no dip limitation (Liu, 1993; 1995).

The Kirchhoff integral method uses a finite-difference algorithm to calculate traveltimes and WKBJ approximation to obtain WKBJ amplitudes (Liu, 1993). This method treats amplitude in migration so that the output is the reflectivity function (Bleistein et al., 1987).

Traveltimes satisfy the eikonal equations, and amplitude terms satisfy linear partial differential equations that depend on traveltime derivatives. For solving these equations to obtain the traveltimes and traveltime derivatives an explicit finite-difference scheme is introduced to solve for the WKBJ amplitudes. The Crank-Nicolson scheme is accurate to second-order and absolutely stable so that computation cost is relatively small for variable velocity by choosing large step sizes (Liu, 1993).

In this approach, paraxial raytracing is used to obtain the traveltime and cosine of emergence angles tables from the velocity model. These tables will be used in the prestack depth migration. When converted waves are considered, we will need to generate two traveltimes times. One of them, associated to P-wave downgoing waves and, the second one, related to the S-wave upgoing waves.

RESIDUAL MOVEOUT

When an incorrect velocity is used to migrate multichannel data, the imaged depths in a common image gather (CIG) will differ from each other. In this situation, residual moveout (RMO) is observed. Residual moveout is defined as a small amount of moveout which remains because of incomplete traveltime removal. Like normal moveout, residual moveout contains information from which we can estimate the medium velocity, then RMO has been used into migration velocity analysis (Liu, 1993; 1995).

The P-S recorded traveltime can be approximated by a time-shifted hyperbola (Slotboom et al., 1990) as

$$t_{ps} = \frac{t_0}{2} + \sqrt{\frac{t_0^2}{4} + \frac{x^2}{2V_{ps}^2}} \tag{1}$$

and the imaging P-S time equation is given by

$$t_{ps} = \frac{\left(h_e^2 + z^2\right)^{1/2}}{V_{p,m}} + \frac{\left(h_e^2 + z^2\right)^{1/2}}{V_{s,m}} \tag{2}$$

where t_0 = zero-offset traveltime,

x = offset source-receiver,

$V_{p,m}$ = migration P-wave velocity,

$V_{s,m}$ = migration S-wave velocity,

$V_{ps,m} = 2V_{p,m}V_{s,m} / (V_{p,m} + V_{s,m})$ = Migration P-S wave velocity (Bancroft and Wang, 1995),

V_{ps} = Stacking P-S velocity, and

$h_e = X - X_{ccp}$ (Figure 1).

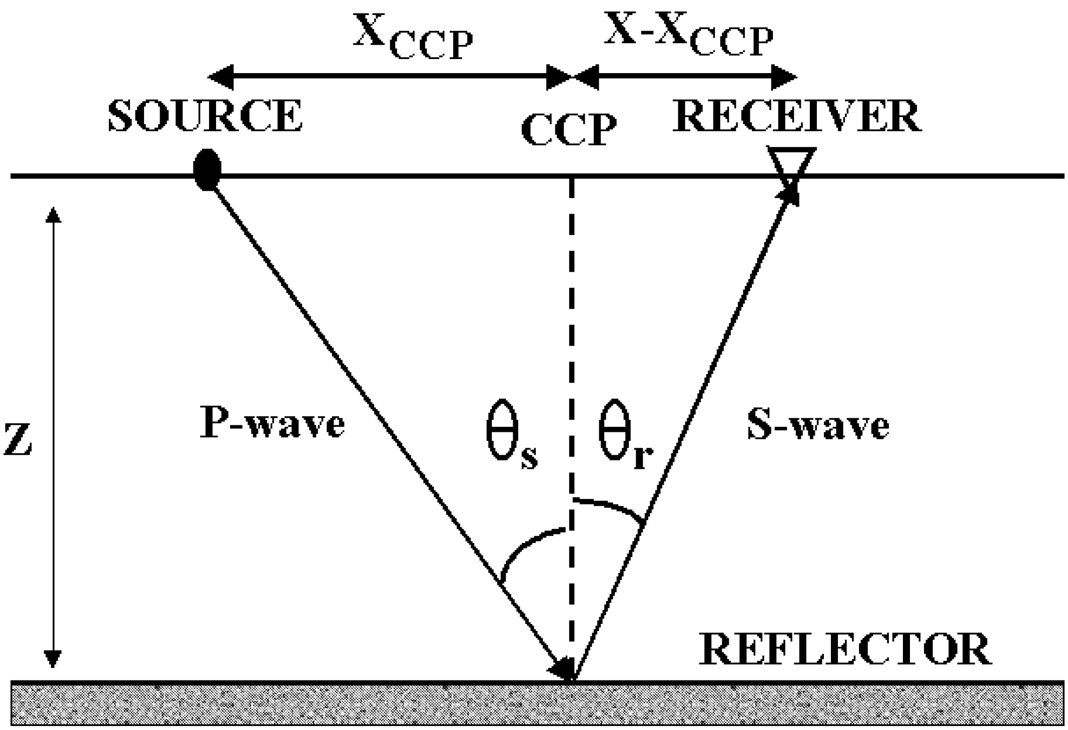

Figure 1. Converted-wave propagation geometry

From equations (1) and (2) can be obtained the image depth, z, as follows

$$z^2 \cong \frac{z_0^2}{4} + \left(\frac{1}{4} \frac{V_{ps,m}^2}{V_{ps}^2} - 1 \right) X^2 + 2XX_{cpp} - X_{cpp}^2 \qquad \left(if\ \sqrt{2}X / t_O V_{ps} << 1 \right) \tag{3}$$

If $X_{ccp} = \xi X$ (where $0 \leq \xi \leq 1$), then

$$z^2 \cong \frac{z_0^2}{4} + \left(\frac{1}{4} \frac{V_{ps,m}^2}{V_{ps}^2} - 1 + 2\xi - \xi^2 \right) X^2 \qquad \left(if\ \sqrt{2}X / t_O V_{ps} << 1 \right) \tag{4}$$

From eqs. (3) and (4), we conclude that P-S residual moveout equation (RMO) depends on common conversion point (CCP) location (X_{ccp}), total offset (X), P-S migration velocity ($V_{ps,m}$) and P-S stacking velocity (V_{ps}). If $\xi = 0.5$ (CMP location), equation (4) is reduced to

$$Z^2 = (2z)^2 \cong z_0^2 + \left(\frac{V_{ps,m}^2}{V_{ps}^2} - 1 \right) X^2 \quad \left(if \sqrt{2} X / t_O V_{ps} << 1 \right) \tag{5}$$

which resembles the residual moveout equation for P-wave given by Liu (1995).

For both P-P and P-S waves, parallel to NMO velocity analysis, semblance and velocity scans can be used to do RMO velocity analysis. RMO velocity analysis is similar to the NMO velocity analysis, except in the residual term. The RMO formula (eq. 4) provides a criterion to estimate migration P-S velocity, $V_{ps,m}$, from stacking P-S velocity, V_{ps}, through the analysis of the migrated depth, z, for a given reflector in a common-image gathers.

After prestack depth migration of common-offset gathers, using a constant velocity, common-image gathers are constructed in depth. When the migration velocity is not the same of the true velocity (assumed V_{ps},) of the media, events in depth are not flat with offset. In other words, at each CIG, the imaged depth is a function of offset (eq. 4) When the migration velocity is correct, $z(x) = z_O$ for all offsets. Otherwise, for a wrong velocity, one should expect that $z(x) \neq z_O$.

VELOCITY PERTURBATION ANALYSIS

When velocity has lateral variations, the residual moveout (RMO) cannot be approximated by a hyperbola and the RMO velocity may be quite different from the RMS velocity. Therefore, the velocity estimate cannot be simply done by using residual moveout correction. In this situation, iterative approaches are required to update velocity. Iterative formulas used in conventional approaches are derived under assumptions such as small offset, small dip, and lateral velocity homogeneity (Liu, 1993; 1995). Although iteration generally is helpful in obtaining a more accurate velocity , too coarse an approximate formula for updating velocity not only increases the number of iteration steps but may result in divergence. Thus, applications of these approaches to velocity analysis are limited when there are complex structures.

Liu (1995) proposed to use perturbation theory for performing migration velocity analysis. The update of the migration velocity can be done from residual moveout by computing a derivative function of imaged depths, with respect to velocity. Significantly here, this formula has no limitations on offset, reflector dip, or velocity distribution if the velocity perturbation is sufficiently small. This formula gives a general description of the relationship between residual moveout and residual velocity. In addition, this formula provides both sensitive and error estimation for migration-based velocity analysis, which is helpful in explaining the reliability of the estimated velocity.

Suppose that S-wave velocity V_s is characterized by a parameter or a family of parameters, λ

$$v = v(x;\lambda) \tag{6}$$

For example, when $v_s(x:\lambda) = v_o + ax + bz, \lambda$ is any set of one to three parameters chosen from v_0 , a, and b. Thus, the problem of velocity estimation becomes a parameter estimation (Liu, 1995). To simplify the derivation, λ is just a single parameter at first. Denoting source and receiver traveltimes by τ_x and τ_r, respectively, Liu (1995) showed that

$$\left[\frac{\partial \tau_s}{\partial_z} + \frac{\partial \tau_r}{\partial_z}\right]\frac{dz}{d\lambda} = \left[\frac{\partial \tau_s}{\partial \lambda} + \frac{\partial \tau_r}{\partial \lambda}\right] \tag{7}$$

and
$$\frac{\partial \tau_s}{\partial z} = \frac{\cos\theta_s}{V_p(x;\lambda)} \qquad \frac{\partial \tau_r}{\partial z} = \frac{\cos\theta_r}{V_s(x;\lambda)} \tag{8}$$

where V_p is P-wave velocity between source and conversion point, x, V_s is S-wave velocity between conversion point and receiver, and θ_s and θ_r are the angle between the raypath from the source and the receiver, and the vertical at x (Figure 1).

Equations (6) and (7) can be combined for obtaining the derivative of imaged depth with respect to λ as follows (Liu, 1995)

$$\frac{dz}{d\lambda} = g_s(x,h) \tag{9}$$

where

$$g_s(x,h) = \frac{\partial \tau_r}{\partial \lambda}\left(\frac{\cos\theta_s}{V_p(x;\lambda)} + \frac{\cos\theta_r}{V_s(x;\lambda)}\right)^{-1} \tag{10}$$

Here, $\partial \tau_r / \partial \lambda = 0$ because P-wave velocity is assumed not show dependence on parameter λ.

P-S MIGRATION VELOCITY ESTIMATION

The function g_s (eq. 9) characterizes the relationship between the imaged depth and the migration velocity in a general medium context, for converted waves. The computation of this function will result in a new migration velocity analysis method, compared to conventional ones based on hyperbolic residual moveout. Suppose that the true parameter λ^* and the true reflection depth is z^*. If there is a small perturbation $\delta\lambda = \lambda^* - \lambda$ between the true parameter and the parameter used in migration, then the imaged depth will have a corresponding perturbation

$$\delta z(x,h) \equiv z^* - z(x,h) \approx \frac{dz}{d\lambda}\delta\lambda \qquad \delta z = g(x,h)\delta\lambda \tag{11}$$

or

$$\delta z = g(x,h)\delta\lambda \tag{12}$$

If residual moveout (RMO) is represented by δz (eq. 11), which is a function of migration velocity, v_m, then the true velocity of the medium, v is the one for which

$$\delta z(v_m) = 0 \tag{13}$$

In a practical sense, Liu (1995) proposed to use Kirchhoff integral for estimating function g_s (eq. 10) in a complex medium. In this approach g_s is solved through the calculation of two migration outputs which have the same phase but different amplitudes. The first output uses the original amplitude and the second output uses the original amplitude multiplied by the quantity g_s. Thus, the ratio of the amplitudes of these two outputs will evaluate g_s at the specular source-receiver position according to the stationary-phase principle, without requiring knowledge of the specular source-receiver pair (Bleistein et al., 1987; Liu, 1995). This technique will work better if the dominant seismic wavenumber is larger than length scale of the velocity variation; then, it is required apply a smoothing velocity operator in Kirchhoff migration before calculating P-S migration velocity by perturbation (Liu, 1993; 1995).

CONCLUSIONS

Velocity analysis in areas of significant structure may be complicated for P-S imaging. We propose a velocity analysis method that uses Kirchhoff depth migration and residual moveout in common image gathers to update the velocity field. The depth residuals are related to velocity perturbations to provide a method to correct the velocities.

REFERENCES

Bleistein, N., Cohen, J., and Hagin, F., 1987, Two and one-half dimensional Born inversion with an arbitrary reference: Geophysics, 52, 26-36.

Chan, W., and Stewart, R. R., 1994, Migration velocity analysis for P-S data: CREWES Research Report, 6, 6-1 - 6-20.

Helbig, K., and Mesdag, C. S., 1982, The potential of shear-wave observations: Geophys. Prosp., 30, 413-431.

Meissner, R., and Hegazy, M. A., 1981, The ratio of the PP- to the SS-reflection coefficient as a possible future method to estimate oil and gas reservoirs: Geophys. Prosp., 29, 533-540.

Liu, Z., 1993, A Kirchhoff approach to seismic modeling and prestack depth migration: 62th Annual International Meeting, SEG Expanded Abstracts, 888-891.

Liu, Z., 1995, Migration Velocity Analysis: Ph. D. thesis, Center for Wave Phenomena, Colorado School of Mines.

Stewart, R. R., and Lawton, D. C., 1996, P-S seismic exploration: A mid-term overview: CREWES Research Report, 8, 1-1 - 1-34.

Zuurbier, N. P., Krajewski, P., Schneider, J., and Fertig, J., 1987, Prestack Migration of Converted Waves: 57th Annual International Meeting, SEG Expanded Abstracts, 750-752.

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