Elevation statics, wave equation datuming, and EOM

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ABSTRACT

Most migration algorithms assume that zero time on a recorded seismic trace corresponds to the onset of energy that is applied at the surface. Large elevation differences across a seismic line may violate this assumption and require wave equation datuming to move the input time traces to a satisfactory datum. For similar reasons, seismic acquisitions that use large elevation changes with vertical receiver arrays may also require wave equation datumming.

Equivalent offset migration (EOM) is a prestack migration that has been applied to data with rugged surface elevations and to data recorded with vertical receiver arrays. Data that are correctly processed with EOM do not require wave equation datumming.

INTRODUCTION

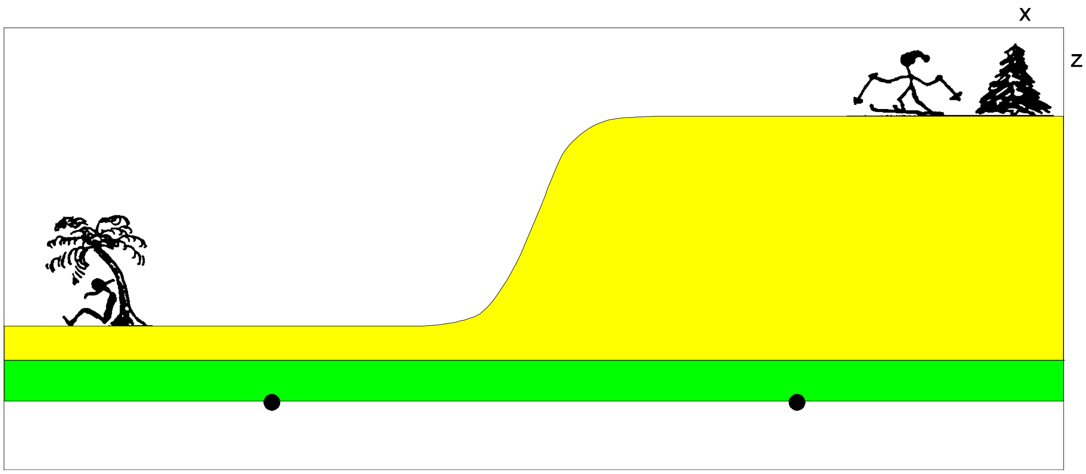
Surface elevation problems

The stacking of seismic data require reflected energy that comes from the same reflection point to be aligned at the same time (or depth). Physical obstacles that prevent this alignment may be found in the difference between the elevations of the sources and receivers, or in the varying velocities on the near surface layers. The time differences between traces are usually corrected by

- choosing a datum
- selecting a velocity to represent the traveltime between the actual elevation and the datum
- computing the vertical traveltime in this replacement layer, and
- applying the time shift to the corresponding input trace.

The datum may be located at a fixed elevation, or it may be allowed to follow a smoothed value of the surface elevation. The corresponding elevation corrections are simple to apply and prove effective for focussing the data, provided that the elevation differences between the surface and datum are small enough; typically a distance that is less than a few trace intervals.

When the elevation differences are large, the vertical traveltime assumption fails. This problem is illustrated by the geological cross-section in Figure 1a, which shows a large elevation change above a number of horizontal reflecting layers and two scatterpoints that are at the same elevation. The zero-offset section is illustrated in the time section (b) that shows the horizontal reflectors to be curved and the diffractions to have different shapes. Migrating this section will destroy the horizontal reflections.



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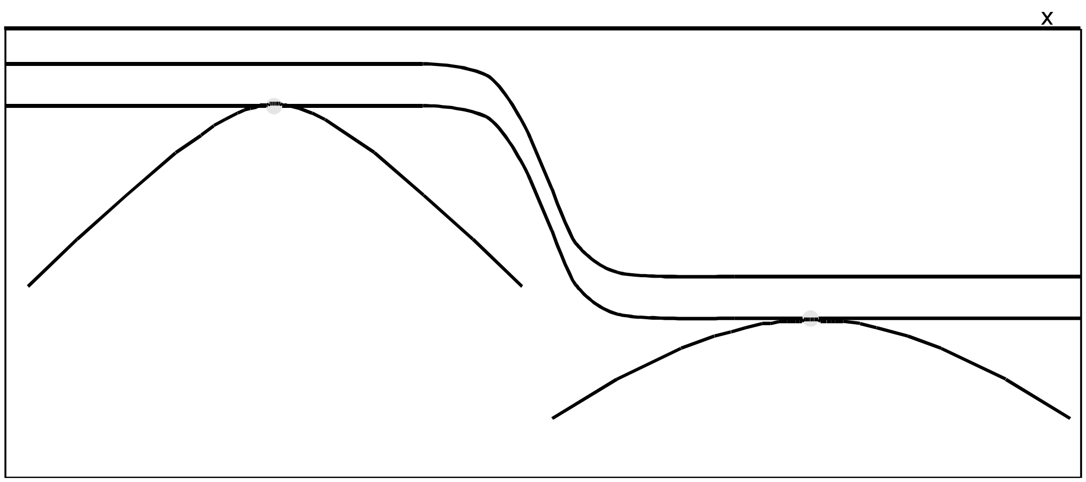

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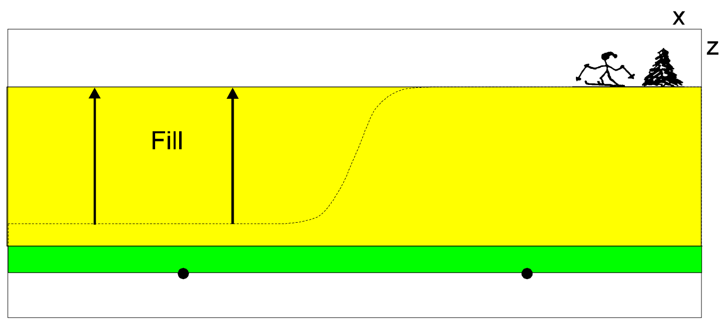
Figure 1. A geological structure (a) with a large elevation change; and (b) the resulting zero-offset time section. Time migration of this section will create large errors in the flat reflections.

In the above example, the diffractions were recorded in an area where the surface elevation was constant. The hyperbolic shape of each scatterpoint is unique as each one has asymptotes that intersect at the surface. This very important property, that the asymptotes intersect at the surface, is required by most poststack migration algorithms. In addition, this example also illustrates how migration should be performed on time data where time zero corresponds to energy insertion at the surface.

Elevation statics

The first correction to the varying surface elevation problem is to apply elevation statics that correct the traveltimes from the surface to a horizontal datum. The following illustrations will use a datum that is chosen at the maximum elevation, however similar datums could have been chosen such as the minimum or average elevation. The effect of elevation statics is illustrated in Figure 2a, in which the lower elevations have been conceptually filled to the maximum elevation with material that matches the replacement velocity. Part (b) of the figure displays the resulting time section. The reflections are now horizontal, but that the left diffraction has been shifted vertically down. The asymptotes of the left diffraction will now intersect deep in the time section and will not be migrated correctly.



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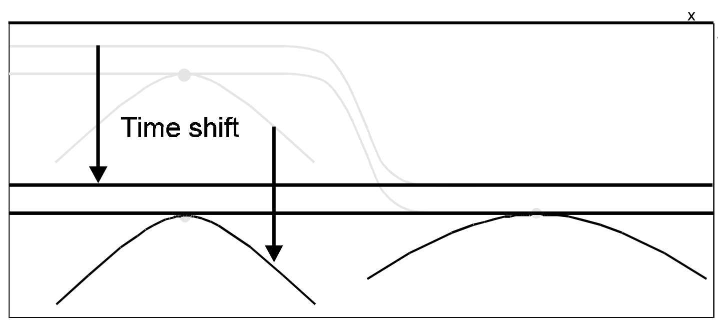


b)

Figure 2. The maximum elevation datum showing: (a) the filled valley; and (b) the time section.

Wave equation datumming

The main assumption of the elevation correction in Figure 2 is that the raypath to the surface is vertical. The actual raypaths could have been at any angle, as illustrated in Figure 3a. The traveltimes from the new surface to the old surface-point maps out a hyperbolic shape that is illustrated in part (b). This hyperbolic shape becomes the modelling operator that moves the energy on the left side to its new location in part (c). The shape of this operator is defined by the thickness of the replacement layer, and will rapidly become smaller at the center of the model until it reaches zero at the maximum elevation. Note that the left diffraction has been modelled to a new shape that is similar to that of the right side diffraction.

This process is referred to as wave equation datumming and is required when a section with large elevation changes is to be converted to one that has a flat datum. The poststack migration will now correct both the horizontal reflectors and the diffraction energy.

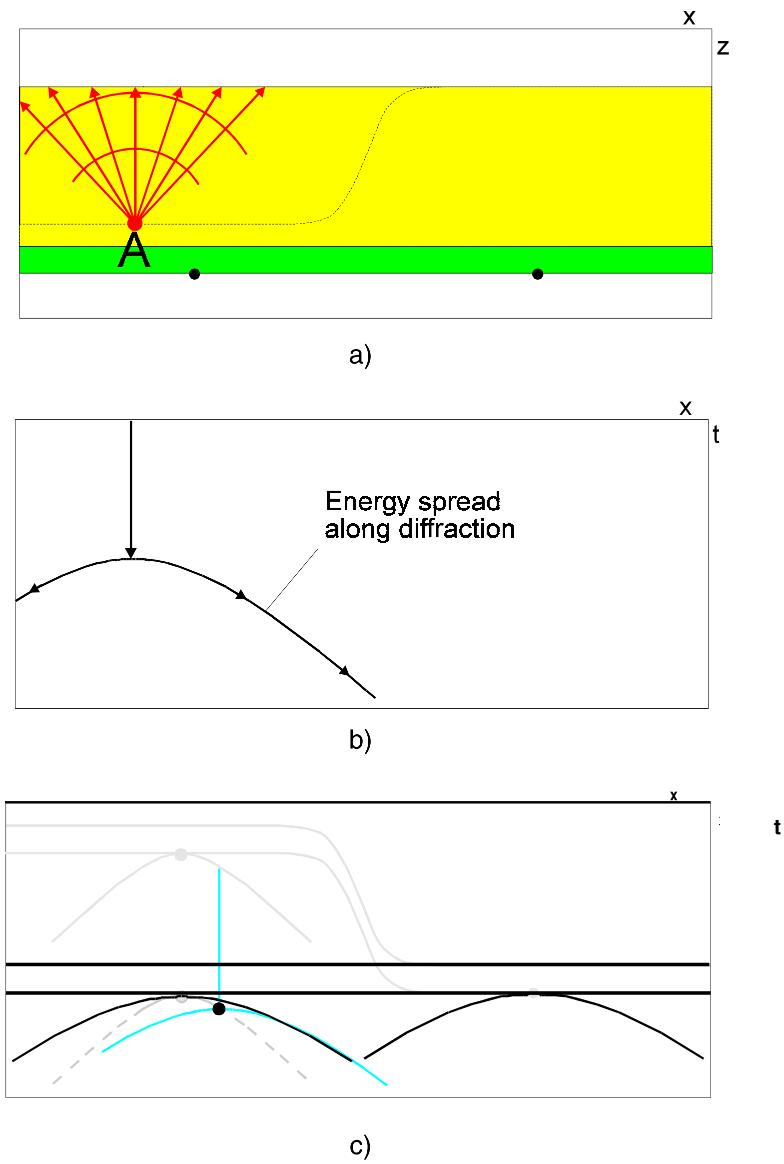


Figure 3. Wave equation datum modelling with: (a) the geological structure; (b) the shape of the dispersion diffraction; and (c) the time response at the new datum.

Other forms of datuming

There are other processes that accomplish the same task as wave equation datumming. Moving traveltimes on traces to the minimum elevation is a migration process that could use a Kirchhoff operator with a shape that is defined by the thickness between the surface and datum.

Downward continuation depth migration also accomplishes the same task by starting at the maximum elevation and by only migrating data that are directly below the intersection of the depth layer with the actual geology. All other data with surface elevations below the migration depth level are left untouched.

Downward continuation *time* migrations can also accomplish wave equation datumming by starting at the maximum elevation, and using a velocity model that incorporates the elevation information. Elevations above the surface elevation are defined to have zero velocity, in order to indicate to the migration algorithm to copy the data from one depth level to the next (i.e. perform on calculations). It is important that vertical elevation statics must have been applied before the start of the migration.

Kirchhoff migrations enable the flexibility of computing traveltimes from the location of any source or receiver. Consequently, if the velocity model is correct, traveltimes can be computed directly from the source or receiver at any elevation, be it the actual surface or any floating datum.

The elevations of prestack traces may also be addressed using the above procedures. For example, a source record may be wave equation datumed to move the elevations of the receivers to those of the source. This process is quite simple for 2-D data, as it assumes that all raypaths are confined to the same vertical plane. However, for 3-D land data the process becomes much more complex as the downward propagating energy moves out of the vertical plane of the 2-D receiver lines. In these situations, prestack Kirchhoff migration from the surface, directly to the scatterpoint, may be a better solution

THE EFFECT OF ELEVATION CHANGE ON EOM

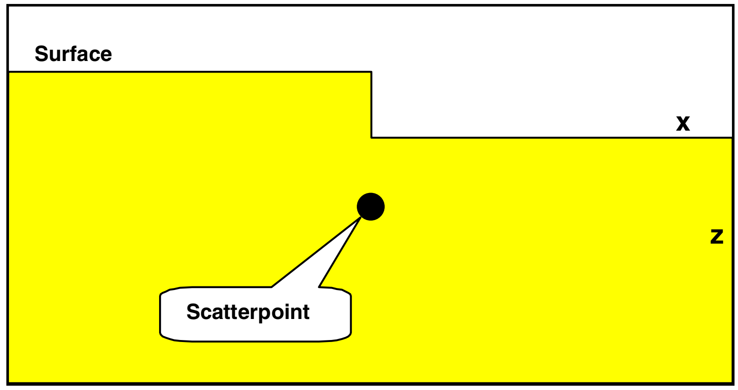
Introduction

The principles of equivalent offset migration (EOM) have been extended to include data areas with rugged topographies and to include acquisition geometries with vertical receiver arrays. Part of the EOM solution may require the application of a time shift to the recorded trace. This time shift represents the vertical traveltime from the receiver to the surface. Is this vertical shift similar to the elevation static that requires some form of wave equation datumming?

Consider a simple geometry that shows a scatterpoint below a step change in surface elevation, as illustrated in Figure 4a. The corresponding zero-offset time section is shown in part (b). The diffraction from the scatterpoint is in two parts, the left side having been acquired from the higher elevation and the right side from the lower elevation. The mirror images of the diffractions are also included as dashed curves to provide a relative shape that emphasizes the difference in diffraction shapes. Since elevation static correction alone will not prepare the data for migration, wave equation datuming is required.

A more general description of scattered energy includes all offsets, as illustrated by the prestack surface in Figure 5. The two views of the prestack surface illustrates that the scattered energy surface is composed of three sections, which correspond to

1. raypaths that go to the higher elevation, (source and receiver to the left of the step)
2. raypaths that straddle the step, and
3. raypaths that only go to the lower elevation.



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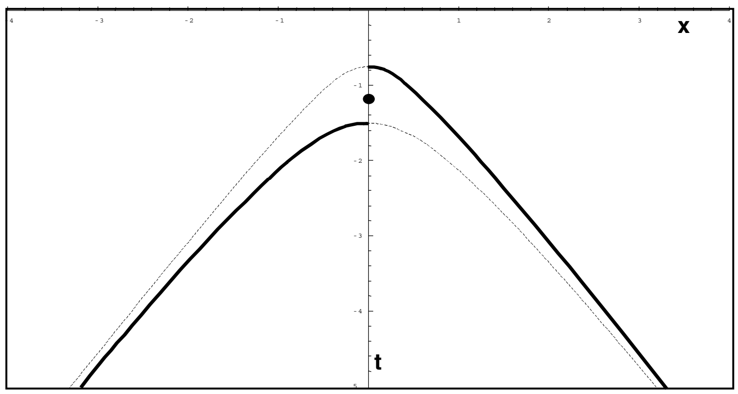
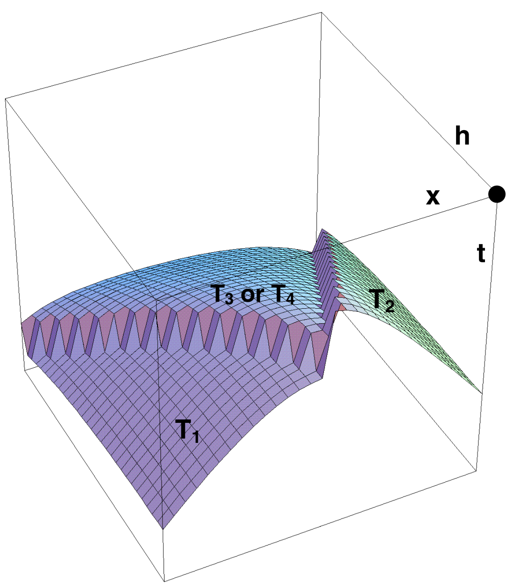
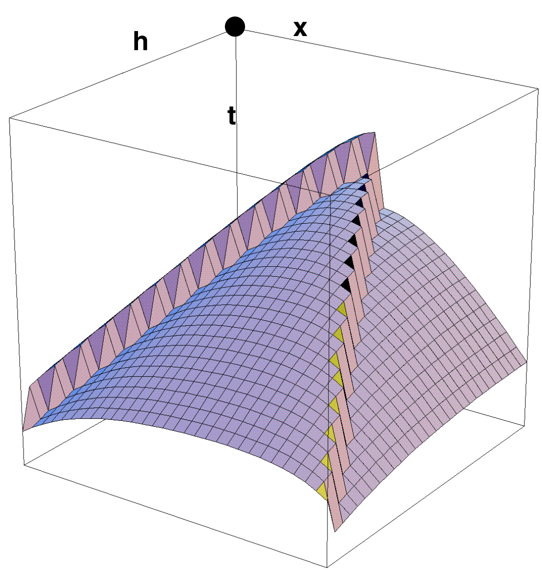


b)

Figure 4. Scatterpoint below a step change in elevation with: a) showing the geological cross-section and; b) the resulting zero-offset time section.



a)



b)

Figure 5. Front a) and rear b) perspective views of the prestack surface where a scatterpoint lies below a step change in surface elevation.

The outside surfaces of Figure 5 are portions of Cheops pyramids that are defined from the depth of the scatterpoint relative to the surface. Although the center portion of the surface is more complex, it retains a similarity to a Cheops pyramid. The objective of prestack Kirchhoff migration is to sum up the energy on this complex surface, then place the summed energy at the scatterpoint location.

Equivalent offset migration (EOM) can also accomplish the task of migrating data in which the sources and receivers are at different elevations. EOM accomplishes this task with an intermediate step that moves one sample of the input energy to a single moveout hyperbola on a CSP gather that is located at the scatterpoint. This energy on the hyperbola is then NMO corrected and stacked at the scatterpoint location.

EOM

A datum for a common scatterpoint gather is chosen for convenience (such as the source elevation for a vertical array of receivers). In the example of Figure 5, a datum is chosen at the maximum elevation. All data, with a source and receiver at the *maximum* surface elevation, will map directly to the CSP gather at the equivalent offset at time T_1, as defined by the double-square root (DSR) equation and equivalent offset hyperbola in equation (1). In this equation, T_1 is the two-way (total) traveltime, T_0 is the vertical two-way traveltime from the scatterpoint to the maximum elevation, x is the spatial coordinate with $x = 0$ defined at the scatterpoint, h is the half source - receiver offset, h_e is the equivalent offset, and V is the velocity.

$$T_1 = \left[\frac{T_0^2}{4} + \frac{(x+h)^2}{V^2}\right]^{1/2} + \left[\frac{T_0^2}{4} + \frac{(x-h)^2}{V^2}\right]^{1/2} = \left[T_0^2 + \frac{4h_e^2}{V^2}\right]^{1/2} . \tag{1}$$

All traces with the *minimum* source and receiver elevation have times T_2 defined by the DSR portion of equation (2). The vertical traveltimes are reduced by t_s, which is the one-way vertical traveltime through the elevation change. The loss of these times is partially recovered by adding $2t_s$ to the DSR equation giving T_2, which is then equated to the equivalent offset hyperbola.

$$T_2 = \left[\frac{(T_0 - t_s)^2}{4} + \frac{(x+h)^2}{V^2}\right]^{1/2} + \left[\frac{(T_0 - t_s)^2}{4} + \frac{(x-h)^2}{V^2}\right]^{1/2} + 2t_s = \left[T_0^2 + \frac{4h_e^2}{V^2}\right]^{1/2} . \tag{2}$$

Equations (3) and (4) define the central portion of the reflecting surface, where the source and receiver are at different elevations. Traces with the source elevation higher than the receiver elevation define the travel time T_3 with equation (3), and those traces with the source elevation lower than the receiver elevation define the travel time T_4 with equation (4). In both cases, the time t_s is subtracted from the appropriate vertical traveltime and then added to the total traveltime.

$$T_3 = \left[\frac{T_0^2}{4} + \frac{(x+h)^2}{V^2}\right]^{1/2} + \left[\frac{(T_0 - t_s)^2}{4} + \frac{(x-h)^2}{V^2}\right]^{1/2} + t_s = \left[T_0^2 + \frac{4h_e^2}{V^2}\right]^{1/2} , \tag{3}$$

$$T_4 = \left[\frac{(T_0 - t_s)^2}{4} + \frac{(x+h)^2}{V^2}\right]^{1/2} + \left[\frac{T_0^2}{4} + \frac{(x-h)^2}{V^2}\right]^{1/2} + t_s = \left[T_0^2 + \frac{4h_e^2}{V^2}\right]^{1/2} . \qquad (4)$$

The addition of the time shift $2t_s$, or t_s, in equations (2), (3), and (4), ensures that the total traveltime T remains larger than T_0, forcing h_e to always be a real value, i.e.

$$h_e = \frac{V}{2}\sqrt{T^2 - T_0^2} . \qquad (5)$$

In addition, adding the time t_s has the same effect as adding an elevation static or time shift to the entire trace. This enables efficient copying of the input trace to the bins of the CSP gather by the use of a fixed sample offset n (or index) for the input trace. The value of t_s can be increased to t_{sn}, which is the next integer multiple of the time sample rate δt, i.e. $t_{sn} = n\delta t$, where $t_{sn-1} < t_s < t_{sn}$. Substituting t_{sn} for t_s in the last term in the central part of equations (2) through (4) enables the equivalent offset to ensure accurate positioning of the input sample as given by

$$T_2 = \left[\frac{(T_0 - t_s)^2}{4} + \frac{(x+h)^2}{V^2}\right]^{1/2} + \left[\frac{(T_0 - t_s)^2}{4} + \frac{(x-h)^2}{V^2}\right]^{1/2} + 2t_{sn} = \left[T_0^2 + \frac{4h_e^2}{V^2}\right]^{1/2} \qquad (6)$$

$$T_3 = \left[\frac{T_0^2}{4} + \frac{(x+h)^2}{V^2}\right]^{1/2} + \left[\frac{(T_0 - t_s)^2}{4} + \frac{(x-h)^2}{V^2}\right]^{1/2} + t_{sn} = \left[T_0^2 + \frac{4h_e^2}{V^2}\right]^{1/2} \qquad (7)$$

$$T_4 = \left[\frac{(T_0 - t_s)^2}{4} + \frac{(x+h)^2}{V^2}\right]^{1/2} + \left[\frac{T_0^2}{4} + \frac{(x-h)^2}{V^2}\right]^{1/2} + t_{sn} = \left[T_0^2 + \frac{4h_e^2}{V^2}\right]^{1/2} . \qquad (8)$$

Referring to the time shift t_{sn} as an elevation static does not imply that wave equation datuming is required. The equivalent offset approach (as with the Kirchhoff approach) maps all the input samples directly to the migrated position via the single hyperbola on the SP gather.

FLOATING DATUM COMMENTS

Areas with large elevation changes may use a floating datum that is found by spatial filtering the surface elevations. The floating datum is closer to the surface elevation and the vertical elevation statics are now smaller than those using a horizontal datum.

The floating datums for NMO and stacking may use a boxcar filter with a width of three to five acquisition spread lengths. This type of datum assumes a linear elevation slope across the traces in a CMP gather and also that the normal moveout (NMO) remains hyperbolic. This datum should only be used for NMO, DMO, and stacking, but not used for migration.

Migration should be performed from a horizontal datum, or a floating datum in which the elevations are smoothed with a boxcar filter that is three to five migration apertures in width. This larger spatial filter is required to minimize distortion of the diffraction shape.

CONCLUSIONS

The elevation static defined in EOM of rugged surface elevations or data recorded with vertical receiver arrays does not require wave equation datuming. A quantized form of the static shift can be applied efficiently and accurately.

REFERENCES

Bancroft, J. C., Geiger, H. D., and Margrave, G. F., 1998, The equivalent offset method of prestack time migration: Geophysics, Vol.63, No. 6, P. 2042-2053.

Bancroft, J. C., and Xu, Y., 1998, Equivalent offset migration for vertical receiver arrays: CREWES Report, Chapter 11.

APPENDIX

The above discussion used a simple model in which the scatterpoint is located below the elevation step. The same principle applies when the scatterpoint is laterally displaced from the elevation step, as illustrated below in Figure A1.

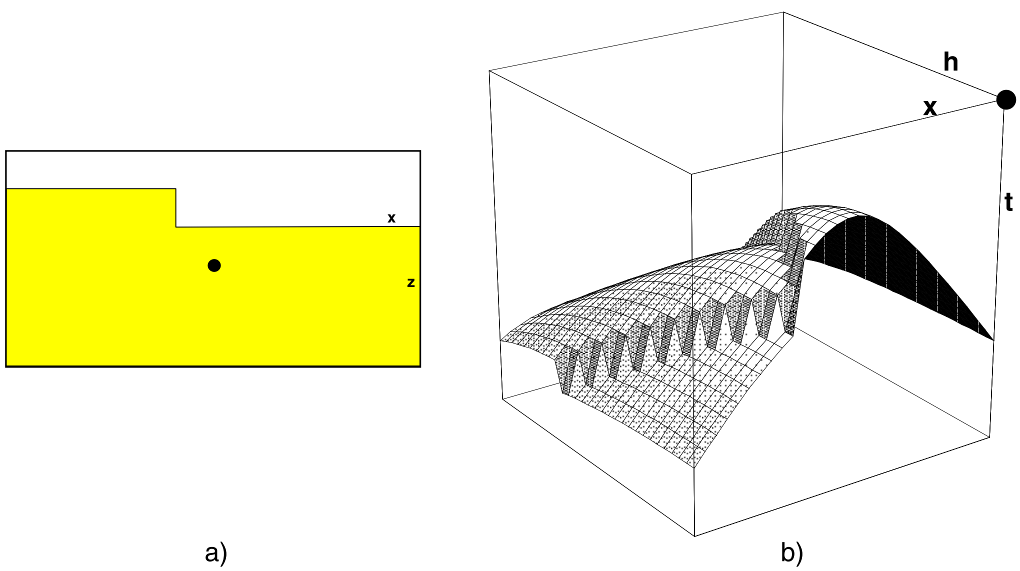


Figure A1. Example: a) of stepped elevation change when the elevation step is moved to the left and; b) the corresponding traveltimes in the prestack volume.