Introduction to matched filters

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ABSTRACT

Matched filters are a basic tool in electrical engineering for extracting known wavelets from a signal that has been contaminated by noise. This is accomplished by cross-correlating the signal with the wavelet. The cross-correlation of the vibroseis sweep (a wavelet) with a recorded seismic signal is one geophysical application. Another geophysical application is found in Kirchhoff migrations, where the summing of energy in diffraction patterns is equivalent to a two-dimensional cross-correlation.

The basic concepts of matched filters are presented with figures illustrating the applications in one and two dimensions.

INTRODUCTION

1D model for matched filtering

Matched filtering is a process for detecting a known piece of signal or wavelet that is embedded in noise. The filter will maximize the signal to noise ratio (SNR) of the signal being detected with respect to the noise.

Consider the model in Figure 1 where the input signal is *s(t)* and the noise, *n(t)*. The objective is to design a filter, *h(t)*, that maximizes the SNR of the output, *y(t)*.

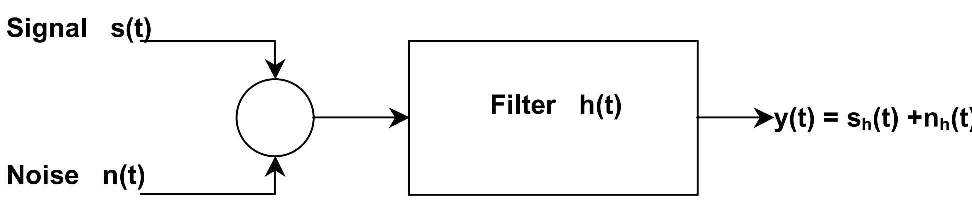

FIG. 1: Basic filter

If the input signal, *s(t)*, is a wavelet, *w(t)*, and *n(t)* is white noise, then matched filter theory states the maximum SNR at the output will occur when the filter has an impulse response that is the time-reverse of the input wavelet. Note that the convolution of the time-reversed wavelet is identical to cross-correlation of the wavelet with the wavelet (autocorrelation) in the input signal. When the wavelet is of length, *T*, then the matched filter is defined by:

$$h(t) = w(T - t).$$

This result is derived in many signal processing texts such as Ziemer and Tranter (1988) and Lathi (1968), and will not be derived in this paper. Essentially, the least-squares principle is used to maximize the output signal energy with respect to the output noise energy. The matched filter improves the SNR by reducing the noise's spectral bandwidth to that of the wavelet, and in addition, reduces the noise within the wavelet's bandwidth by the shape of the wavelet's spectrum.

The duration of the wavelet can be small, as used in radar or sonar, and it can be much larger when used with the vibroseis source with the acquisition of seismic data. Other applications may involve the detection of weak signals from satellite transmissions, or the detection of military equipment from visual images. In addition, Kirchhoff migration is a form of 2D or 3D matched filtering that estimates the location, size, and shape of scattered or diffraction energy.

EXAMPLES

Example with a short wavelet

The first example is a short wavelet added to noisy traces. In this situation, the signal-to-noise ratio (SNR) is defined by taking the peak amplitude of the wavelet and dividing it by the root-mean-squared (RMS) measure of the noise (standard deviation).

Various amplitudes of the wavelet in Figure 2 are added to ten noisy traces and are illustrated in Figure 3. The SNR of these traces varies from 2.0 at the top to 0.2 at the bottom as indicated on the left side of the figure.

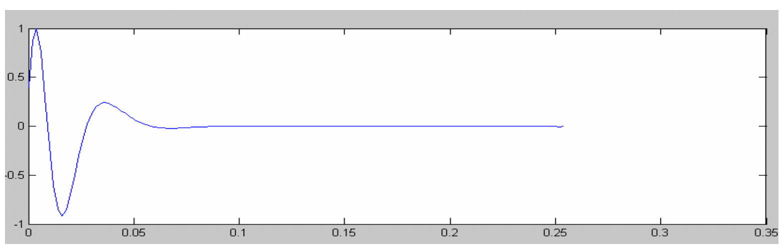

FIG. 2: Wavelet, only first 100 ms (50 points) used.

The cross-correlation of the known wavelet in Figure 2 with the traces in Figure 3 produces the corresponding matched-filter traces in Figure 4. Note the improved SNR of the traces, and that even the bottom trace with an initial SNR of 0.2 demonstrates a higher probability of detection.

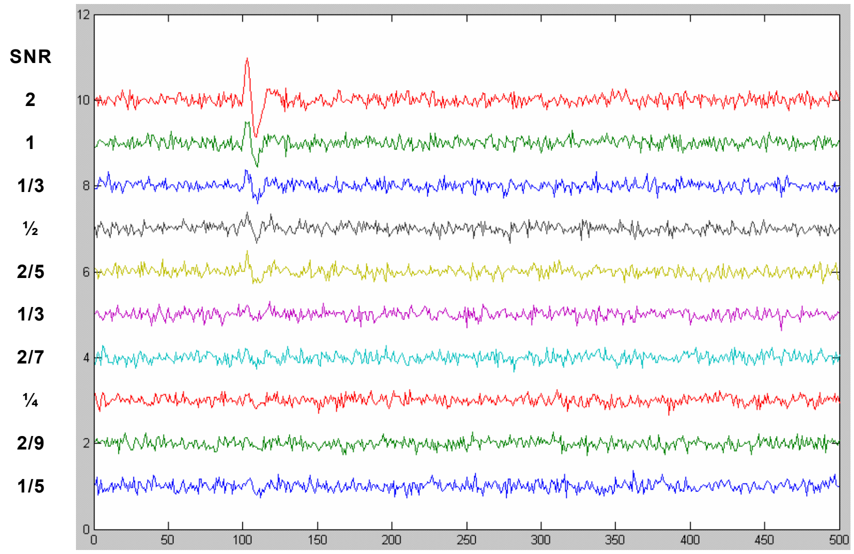

FIG. 3: Ten noisy traces and wavelets with a SNR that varies from 2.0 to 0.2.

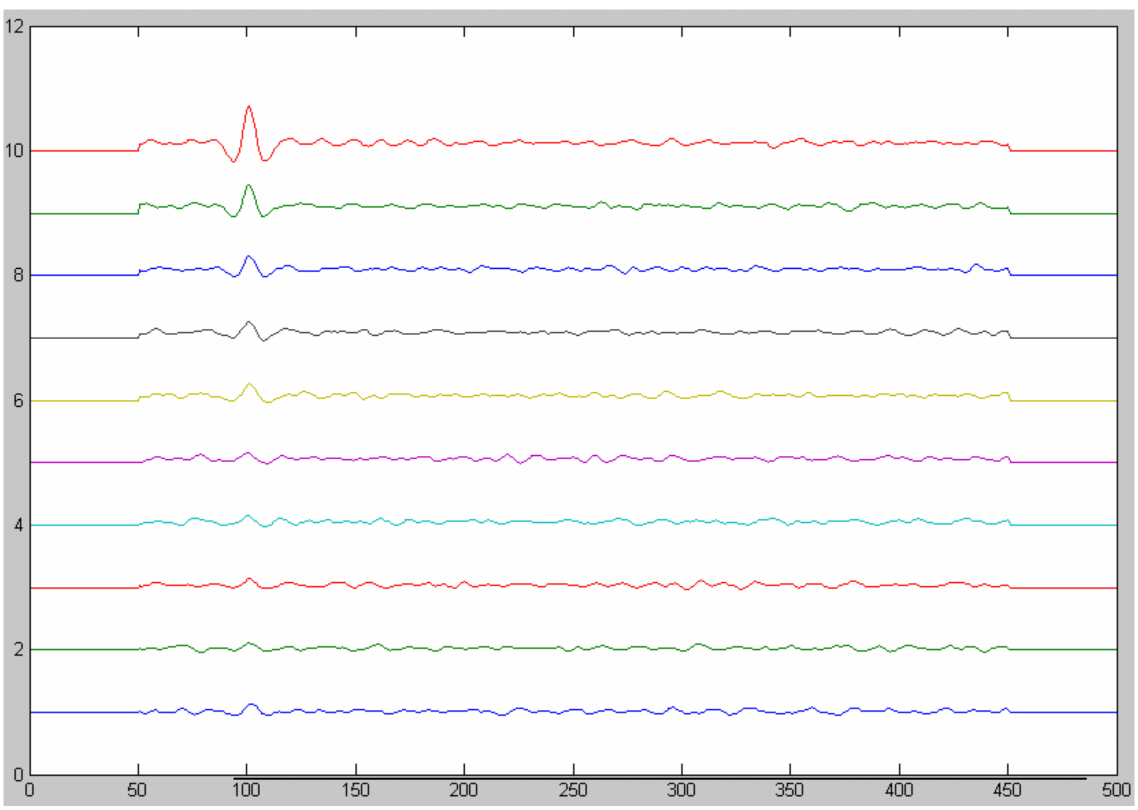

FIG. 4: Results of a matched filter from cross-correlating the wavelet in Figure 2 with the noisy signals in Figure 3,

Figure 4 illustrated the attenuation of the noise and the preservation of the wavelet energy. The shape of the wavelet, however, has been modified to zero-phase, corresponding to the cross-correlation of the wavelets being an auto-correlation. The peak of the zero-phase wavelet identifies the beginning of the initial wavelet.

Example with a longer chirp wavelet

The above experiment is repeated with a chirp wavelet shown in Figure 5 and the noisy signals in Figure 6. This wavelet is longer and contains higher frequencies than the short wavelet. These higher frequencies also define the shape of the matched filter and will allow higher frequencies of the noise to pass as observed in the matched-filter result in Figure 7.

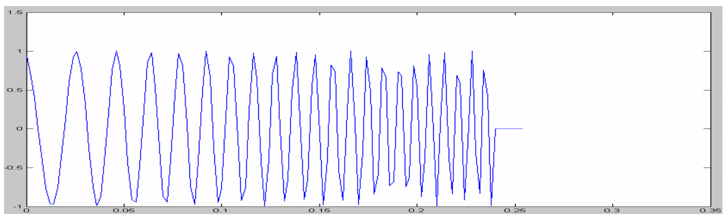

FIG. 5: Chirp wavelet.

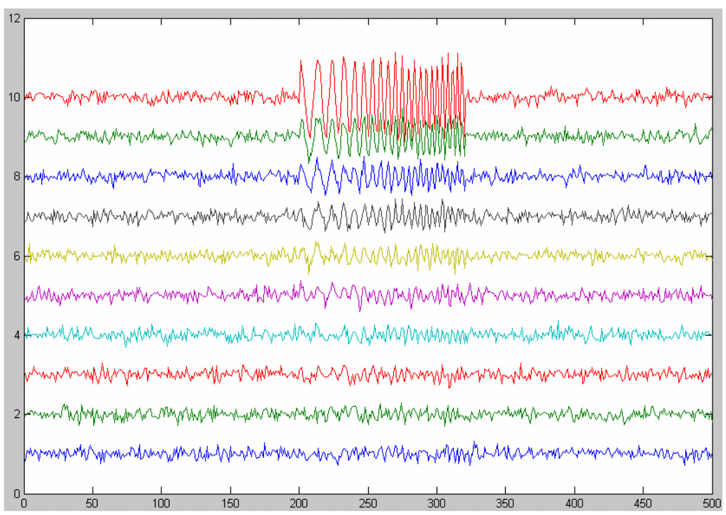

FIG. 6: Noisy traces and chirp wavelet.

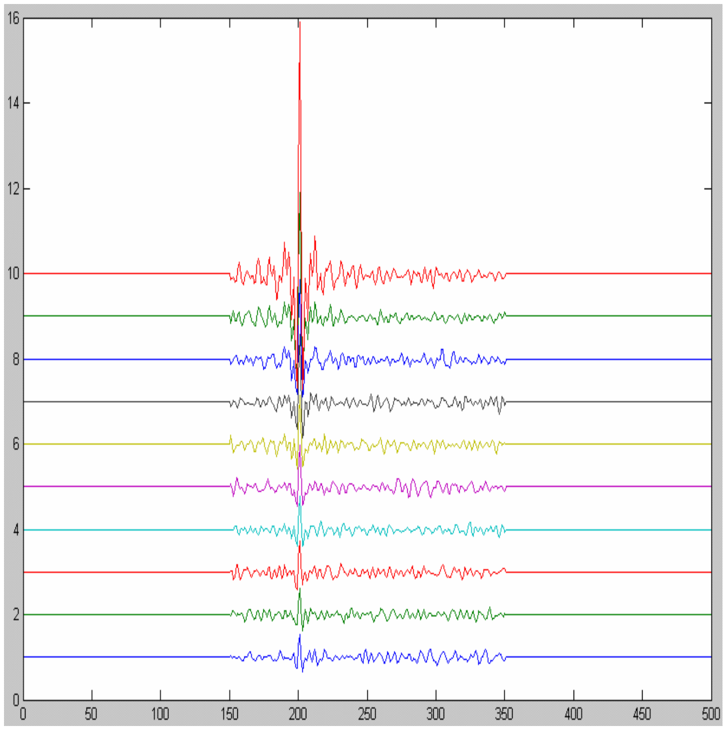

FIG. 7: Match-filtered result of the noisy chirps.

With a larger energy wavelet, there is more energy in the cross-correlation, and better detection. Note that the input SNRs was the same as in the shorter wavelet example.

2D data

The same principles apply to detecting a 2D wavelet in a 2D signal. Such applications are used to detect potential military equipment in video or optical images. In geophysics, it can be used to detect seismic diffractions in a process we know as seismic migration. Note that the matched filter concept tells us that we need to cross-correlate with the same size and amplitude of the "object" being detected.

Consider the noisy section, the 2D wavelet, and the matched filter result in Figure 8. The noisy section in (a) has 100 by 100 samples, and contains one barely detectable wavelet. The wavelet in (b) has 21 by 21 samples and is much narrower when inserted into the grid of the noisy section. The cross-correlation in (c) shows a significant improvement in the matched filter result.

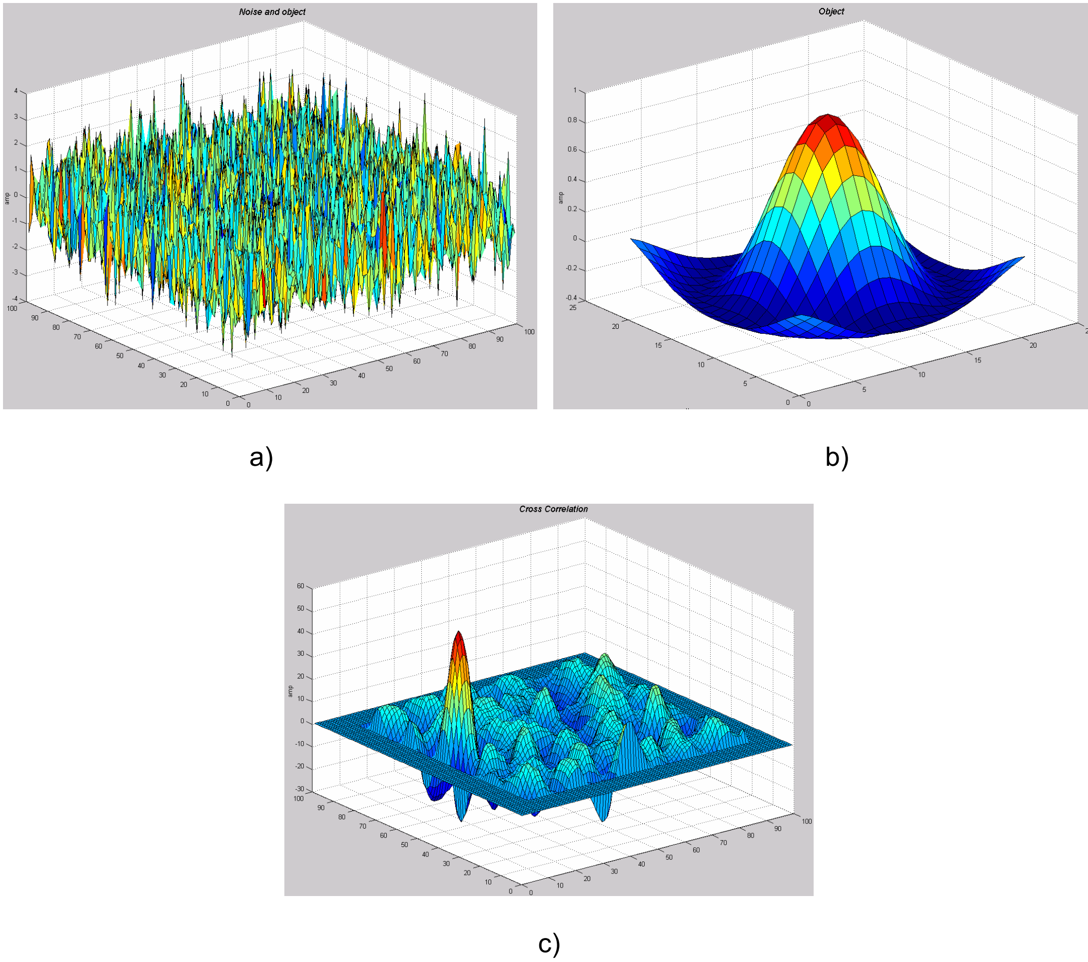

FIG. 8: Example of 2D matched filtering with a) the noisy section containing the wavelet and b) matched filter.

COMMENTS

In the cross-correlation process, the amplitudes of the input wavelets or events are squared and will represent some form of energy.

Seismic migration is similar to the detection process. Both processes try to collapse the energy in a diffraction to the smallest possible point with the highest energy.

In migration, the diffraction stack algorithm cross-correlates a time and spatially varying diffraction with a seismic section. For maximum detection, the Kirchhoff operator should match the shape and amplitude of the diffractions in the section. Note that the amplitudes on the migrated section would now represent the energy, and in the exploding reflector model, the energy is proportional to the reflectivity of the interfaces. This heuristic or *ad hoc* approach to migration coincides with the Kirchhoff integral solution to the wave equation.

The reflectivity coefficients vary with the angles of incidence, reflection and transmission. The inclusion of these amplitude factors in the weighting of the summation diffraction can improve the signal-to-noise ratio of specific targets in the migrated section. This has been demonstrated in the migration of P-P Class 2 AVO data and in the migration of converted-wave data (Beckett and Bancroft, 2002).

In Class 2 AVO, the reflection coefficient varies from a positive to a negative value, and summing (or stacking) these amplitudes tends to give a very small amplitude, relative to the surrounding reflectors. Including these amplitude weightings in the prestack migration operator (the diffraction shape) will boost the amplitude of the Class 2 event, and attenuate the amplitudes of the data with a constant amplitude with offset. This filtering should then aid in the detection of these events.

Converted-wave reflection amplitudes tend to zero at zero-offset and increase in magnitude with offset. In addition, the noise toward zero-offset may be large and contribute significant noise to the migrated section. Using the matched filter concept to bias the migration operator with the same weighting will tend to attenuate the near zero-offset noise to produce a migrated section with less noise.

Authors such as Claerbout (1992), point out that the inclusion of these AVO weightings in prestack migrations will prevent further evaluation of subsequent AVO parameters in any following prestack measurements.

CONCLUSION

Matched filters are designed to extract the maximum SNR of a signal that is buried in noise. Some applications in geophysics are in the cross-correlation of vibroseis data, and in Kirchhoff migration.

ACKNOWLEDGEMENTS

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