Improved Radon transforms for filtering of coherent noise

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ABSTRACT

Radon transforms rely on the ability to predict the moveout of coherent events. Most algorithms assume parabolic or hyperbolic moveout, a characteristic that many reflections do not adhere to. Standard parabolic and hyperbolic Radon transforms typically involve smearing of reflections across Radon space, which reduces the effectiveness of coherent-noise suppression. We present a method developed to specifically remove reflections having nonhyperbolic moveout. The shifted-hyperbolic and anisotropic Radon transforms employ a curve-fitting technique to allow for flexibility in predicting the true moveout of specific reflections. In addition, approximations to the damping factors used in the low- and high-resolution Radon algorithms are presented. These alternate parameters are feasibly employed and improve the efficiency of the algorithms.

INTRODUCTION

The Radon transform is a processing tool utilized to exploit differences in the moveout of seismic events. Variants of the algorithm are commonly employed in discriminating between primary reflections and other types of coherent noise. The linear function typically is used in the suppression of ground roll and other linear noise events (Trad et al., 2002; Kelamis et al., 1990). Parabolic and hyperbolic Radon-domain processes are commonly geared towards noise attenuation or data interpolation. The time-invariant hyperbolic transform is used in suppressing multiples, but is accurate only at depths approximately corresponding to offset (Foster and Mosher, 1992).

Although the Radon space is frequently utilized for multiple suppression, much work has been completed involving the transform for other innovative applications. Radon transforms are commonly employed in attenuating sampling artifacts, gap-filling of missing data, and various migration and inversion techniques (Trad et al., 2002; Ottolini and Claerbout, 1984; Miller et al., 1987; Rüter, 1987; Hubral, 1991; Diebold and Stoffa, 1981; Thorson and Claerbout, 1985). Additionally, Radon algorithms are also effective for plane-wave decomposition, wavefield separation, and filtering (Zhou and Greenhalgh, 1994).

The Radon transform is advantageous because it requires no inherent knowledge of the coherent-noise-generating mechanism and works relatively well with nonuniform geometries (though it may require extensive computing time). Although the Radon transform is practical for implementation in a wide variety of cases, there are limitations on the effectiveness of Radon methods due to the inherent assumptions made.

The most popular multiple-removal technique involves Hampson's (1986) method of applying NMO to CMP gathers and isolating events in the data that have residual moveout. This method is relatively inexpensive to implement and works well in laterally homogeneous media. However, the algorithm breaks down in cases of strong lateral velocity variations when NMO assumptions are not met. Nonhyperbolic moveout, such as that found for converted waves, anisotropic media, or long-offset data, also will not be completely suppressed with this method.

Thorson and Claerbout (1985) described some implicit assumptions made in using the Radon transform on regular data. A reflector on a CMP gather should have uniform amplitude and vary smoothly in moveout from trace to trace for the Radon transform to be able to effectively focus the event. Specifically, the traces on the gather must be free of static shifts and be balanced in amplitude.

In order to avoid problems due to violation of these assumptions, the data should be preconditioned prior to Radon analysis to remove dip and static problems (Thorson and Claerbout, 1985). However, it seems counterproductive to remove AVO effects for improving multiple suppression when the identification of hydrocarbon reservoirs through AVO is dependent upon the preservation of true amplitudes. Better focusing of reflections in the Radon panel can lead to better handling of nonuniform reflection amplitudes, thereby diminishing some of the problems associated with smearing of amplitude with offset.

The high-resolution Radon technique proposed by Sacchi and Ulrych (1995) minimizes smearing problems in the Radon panel due to limited-aperture problems; however, it is rather expensive to implement and the vertical smearing of events on the τ-p gather is not removed. It follows that a Fourier-domain method for performing fast inversions with sparse constraints in both the τ- and p-directions may well be the next key endeavor for efforts in this area. Alternative methods for improving τ-p focusing, such as employing alternate summation paths and damping factors, will prove important for effective removal of noise until a fast transform that is computationally sparse is established.

The first part of this paper catalogues equations for improved approximations of reflection moveout, reducing the error and smear involved in the transform space. The second part of this paper introduces an alternative damping factor that may be used in the low-resolution transform. Approximations for the high-resolution damping-factor parameters are also presented to allow for reduced cost and ease in applying this computer-intensive technique. The implementation of these enhanced equations attacks the limitations of existing Radon transforms in order to attain a better coherent-noise suppression technique.

METHODS

The simplified formula for two-dimensional generalized Radon transforms is given as:

$$m(\tau, p) = \iint d(t, h)\,\delta[t - t'(\tau, p, h)]\,\mathrm{d}t\,\mathrm{d}h \qquad (1)$$

where the function d denotes the CMP input signal in t-h or *data* space and m denotes the output function in *model* space. Thorson and Claerbout (1985) also refer to the domains of d and m as offset and velocity space, respectively. The Dirac delta function (δ) in equation (1) identifies rectilinear paths that are parallel to a specified set of projections, given by t'. The function t' is defined for the particular type of τ-p transform. Slant stacks involve summation along lines where $t' = \tau + ph$, where the transform is given as:

$$m(\tau, p) = \iint d(t,h)\delta(t - \tau - ph)\mathrm{d}t\mathrm{d}h. \tag{2}$$

Parabolic transforms involve summation along curves such that $t' = \tau + qh^2$, and poststack migrations involve summation along $t' = \left[\tau^2 + 4(q-h)^2 / v^2\right]^{1/2}$ curves (Cary, 1998). In these two equations, the curvature variable, q, replaces p, representing the moveout for the parabolic and other specified transforms. The parabolic transform is given as:

$$m(\tau, q) = \iint d(t,h)\delta\left(t - \tau - qh^2\right)\mathrm{d}t\mathrm{d}h. \tag{3}$$

The parameter τ is zero-offset time for a particular summation curve.

Hyperbolic summation paths can be integrated directly in an expensive time-variant manner where $t' = (\tau^2 + q^2h^2)^{1/2}$. Specialized summation curves may also be developed for efficient targeting of particular reflections. Foster and Mosher (1992) proposed a summation curve that optimized the focusing of hyperbolic multiple reflections around a specific focusing depth; $t' = \tau + q\left(\sqrt{h_k^2 + z^2} - z\right)$ is their summation path, and z is the chosen fixed focusing depth for the multiple reflection. Their hyperbolic multiple transform is written as:

$$m(\tau, q) = \iint d(t,h)\delta\left[t - \tau - q\left(\sqrt{h_k^2 + z^2} - z\right)\right]\mathrm{d}t\mathrm{d}h. \tag{4}$$

Traditional Radon transforms assume that events maintain a parabolic or hyperbolic shape. This imperfection of the Radon transform makes the process ill-suited for event isolation. The inaccurate approximation of the shapes of reflections leads to smearing and defocusing of the events in the Radon panel. The smearing of multiples into primary events in the Radon panel can counteract coherent-noise muting techniques.

The compelling question is how do we obtain less smearing and higher resolution in the Radon panel? Disregarding the aperture and discrete sampling problems, a better approximation of the shape of the reflections would, in theory, decrease the smearing and limit the overlap of events in the Radon panel. Castle (1994) showed that the Dix (1955) NMO curve is a small-offset approximation for moveout of reflections for a horizontally layered Earth. He recommended using the shifted-hyperbola equation as a solution to the inaccuracies of reflection moveout at long offsets. Castle gave the sixth-order shifted-hyperbola equation as:

$$t^2 = c_1 + c_2h^2 + c_3h^4 + c_4h^6, \tag{5}$$

where the coefficients are defined as:

$$c_1 = t_0^2, \tag{6}$$

$$c_2 = \frac{1}{\mu_2}, \tag{7}$$

$$c_3 = \frac{1}{4}\frac{\mu_2^2 - \mu_4}{t_0^2 \mu_2^4}, \tag{8}$$

$$c_4 = \frac{2\mu_4^2 - \mu_2\mu_6 - \mu_2^2\mu_4}{t_0^4 \mu_2^7}, \tag{9}$$

and

$$\mu_j = \sum_{k=1}^{N} \Delta t_k V_k^j \Big/ \sum_{k=1}^{N} \Delta t_k \ . \tag{10}$$

The parameter t_0 is the two-way vertical traveltime from source to receiver; $\Delta\tau_k$ is the vertical traveltime in the *k*th layer, and V_k is the interval velocity of the *k*th layer.

In a more simplified approach, we propose the fourth-order nonhyperbolic Radon transform that employs the first three terms of equation (5) for the summation of reflections for long-offset data. The focusing parameters, t_0 and μ_4, are extrapolated from the data as input parameters in each transform for a particular primary, multiple, or converted-wave reflection. The fourth-order summation curve is represented as $t' = \tau + qf(h_k)$ and the elements of the function $f(h_k)$ are given as:

$$f(h_k) = h^2 + \frac{\left(1 - \mu_4 q^2\right) q h^4}{4t_0^2}. \tag{11}$$

The fourth-order equation operates on the data in the t^2-*h* domain, a process that considerably increases the size of the data to be transformed and the computation time. An alternative shifted-hyperbola equation in the *t*-*h* domain is also given by Castle (1994), where:

$$t' = \tau - \frac{t_0}{S} + \sqrt{\left(\frac{t_0}{S}\right)^2 + \left(\frac{hq}{\sqrt{S}}\right)^2} \tag{12}$$

The summation curve used in the linear operator requires input tuning parameters, *z* and *S*, for the function

$$f(h_k) = \sqrt{\frac{4z^2}{S^2} + \frac{h^2}{S}} - \frac{2z}{S}. \tag{13}$$

Foster and Mosher's (1992) hyperbolic summation curve can be represented as an approximation of the shifted hyperbola in equation (13) (Oppert, 2002). The shifted-hyperbolic Radon transform can be implemented through direct data estimation of the focusing parameters, t_0 and μ_4, for equation (11). Alternatively, curve-fitting techniques such as that described by Elapavuluri and Bancroft (2002) can be employed to estimate the shift parameter as a function of velocity for implementation in equation (12).

Anisotropic Radon:

Reflections involving anisotropic media typically have a nonhyperbolic shape, requiring unconventional equations to describe their moveout. Reflection traveltimes for the case of transverse isotropy with a vertical symmetry axis (VTI) can be summarized using the equation derived by Alkhalifah and Tsvankin (1995), that is:

$$t^2 \cong t_0^2 \left[1 + \left(\frac{x}{t_0 V_{PNMO}} \right)^2 - 2\eta \frac{\left(x / t_0 V_{PNMO} \right)^4}{1 + (1+2\eta)(x / t_0 V_{PNMO})^2} \right] \tag{14}$$

where the anisotropic parameter η is defined as:

$$\eta = \frac{(\varepsilon - \delta)}{(1 + 2\delta)} \tag{15}$$

and the parameters ε and δ are dimensionless measures of anisotropy defined by Thomsen (1986). The Radon transform for anisotropic reflections in equation (14) requires input tuning parameters ε, δ and t_0 defined to focus specific reflections. The anisotropic linear operator function is defined using:

$$f(h_k) = h^2 - \frac{2\eta h^2}{(1+2\eta) + t_0^2 / (qh^2)}. \tag{16}$$

The parameters involved in the anisotropic Radon transform should be estimated from the data. Accurate knowledge of the anisotropic parameters leads to improved focusing for reflections in the anisotropic model space.

The least-squares approximation

The time-domain transform given in equation (1) is sufficient for use with continuous and infinite data. In practice, however, field data are finite and discretely sampled functions. Thorson and Claerbout (1985) used the idea of minimum entropy to formulate an expression to calculate the model space, *m*, for a finite number of *q* and τ. This formulation later was called the discrete Radon transform (DRT). The proposed least-squares formulation of the DRT employs sparsity constraints along the *q* and τ axes to reduce amplitude smearing in the *t-h* domain. The computer-intensive time-domain DRT can be written as:

$$m(q,\tau) = \int_{h_{\min}}^{h_{\max}} d(h,t')\,\mathrm{d}h. \tag{17}$$

In order to examine the problem in a least-squares sense, it is easier to express (17) as a summation, where

$$m(q,\tau) = \sum_{j=1}^{M} \sum_{k=1}^{N} d(h_k, t'_j), \{j = 1,\ldots,M; k = 1,\ldots,N\}. \tag{18}$$

The summation can be written in matrix form as:

$$\mathbf{m'} = \mathbf{L}^{\mathbf{T}}\mathbf{d}, \tag{19}$$

where $\mathbf{L}^{\mathbf{T}}$ is an $N \times M$ linear operator defined by the transformation curves of t'. This formulation of the problem involves applying the linear operator, $\mathbf{L}^{\mathbf{T}}$, to the data to obtain a low-resolution version of the transformed function in model space, denoted by $\mathbf{m'}$. The linear operator, $\mathbf{L}^{\mathbf{T}}$**,** applies a moveout stretch to the data, where the moveout is dependent on the type of *τ-p* (or *τ-q*) transform being calculated. The transpose of $\mathbf{L}^{\mathbf{T}}$, $\mathbf{L}$, is also an adjoint of $\mathbf{L}^{\mathbf{T}}$, provided the elements of $\mathbf{L}^{\mathbf{T}}$ are real numbers. The associated inverse transform of equation (19) is given as:

$$\mathbf{d} = \mathbf{L}\mathbf{m} \tag{20}$$

(Thorson and Claerbout, 1985).

Equation (20) provides the inverse transform back to data space by application of the linear operator directly to the Radon panel. The adjoint, $\mathbf{L}$**,** applies an inverse-moveout compression on each trace to obtain the data space (Thorson and Claerbout, 1985). Because $\mathbf{L}$ is an inexact inverse of $\mathbf{L}^{\mathbf{T}}$, a least-squares (or stochastic) approach to the inversion can provide higher resolution in the forward transform domain (Kabir and Verschuur, 1994).

The stochastic inversion approach derived by Thorson and Claerbout (1985) asserts that the transformation of the linear operator $\mathbf{L}$ on some function $\mathbf{m_0}$ results in the combination of the CMP gather and a noise term, $\mathbf{n}$, such that

$$\mathbf{d} = \mathbf{L}\mathbf{m_0} + \mathbf{n}. \tag{21}$$

The solution to equation (21) is obtained by taking a least-squares approach to minimizing the noise term, $\mathbf{n}$, which represents the difference between the actual data and the modelled data. The cumulative squared noise term, $\mathbf{S} = \mathbf{n}^{\mathbf{T}}\mathbf{n} = (\mathbf{d} - \mathbf{L}\mathbf{m_0})^{\mathbf{T}}(\mathbf{d} - \mathbf{L}\mathbf{m_0})$, is minimized with respect to $\mathbf{m_0}$, to yield the desired least-squares solution:

$$\mathbf{m} = \left[\mathbf{L}^{\mathbf{T}}\mathbf{L}\right]^{-1}\mathbf{L}^{\mathbf{T}}\mathbf{d} \tag{22}$$

(Lines and Treitel, 1984). The generalized inverse of $\mathbf{L}$ is thus computed to be $\left[\mathbf{L}^{\mathbf{T}}\mathbf{L}\right]^{-1}\mathbf{L}^{\mathbf{T}}$.

The calculation of the inverse of $\mathbf{L}^{\mathbf{T}}\mathbf{L}$ is required to solve equation (22) directly, a process that typically is impractical due to the large nature of the matrix and the instability of the inversion. Furthermore, the operator $\mathbf{L}^{\mathbf{T}}\mathbf{L}$ is diagonally dominant; however, if the side lobes of the matrix are significant, then smearing occurs along the *q*-axis. Prewhitening the operator $\mathbf{L}^{\mathbf{T}}\mathbf{L}$ suppresses the side lobes and stabilizes the inversion.

A stable solution for equation (19) is computed by perturbing the matrix $\mathbf{L}^{\mathbf{T}}\mathbf{L}$ with a damping factor. The resultant stochastic inversion formula is given as:

$$\mathbf{m} = \left[\mathbf{L}^{\mathrm{T}}\mathbf{L} + \mu\mathbf{I}\right]^{-1}\mathbf{L}^{\mathrm{T}}\mathbf{d}, \tag{23}$$

where the constant μ is the damping factor incorporated to add white noise along the main diagonal of the inversion matrix, and $\mathbf{I}$ is the identity matrix (Thorson and Claerbout, 1985).

The offset-weighted damping factor

If the variances for the solution $\mathbf{m}$ are undetermined, the sparsity constraint, μ, can either be bootstrapped from the data or iteratively refined. In practice, μ is defined as 1% of Λ, the maximum of the main diagonal of the matrix $\left(\mathbf{L}^{\mathrm{T}}\mathbf{L}\right)^{\mathrm{T}}$ (Yilmaz, 1989). An offset-weighted damping factor, $\mathbf{\Gamma}$, may be alternatively used in place of $\mu\mathbf{I}$, such that

$$\mathbf{\Gamma} = \boldsymbol{\gamma}\mathbf{I}, \tag{24}$$

and $\boldsymbol{\gamma}$ is the vector with elements $h_{\max}/(1+h_k)$.

The alternative damping factor was designed to represent the error of the least-squares algorithm as a function of offset. This methodology assumes the summation curves represent near-offset data better than far-offset data and compounds errors based on offset. The offset weighting typically acts to reduce smear in the low-resolution Radon panel, an effect that makes it preferable to the standard damping factor. A longer computing time may be required when using the offset-weighted matrix due to the nonconstant diagonal in the regularization matrix.

The task of performing time-domain Radon transformations on field data is computer-intensive and very costly due to calculations involving very large matrices. Hampson (1986) overcame this problem by performing integration for independent frequencies in the Fourier domain. This methodology relies on the similarity of the integration over curved lines in the time domain to the integration over phase shifts in the Fourier domain. A forward Fourier transform is applied to the data and the transform equivalent to formula (17) for a given summation curve, $t' = \tau + qf(h)$, is written as:

$$\hat{m}(q,\omega) = \int_{h_{\min}}^{h_{\max}} \hat{d}(h,\omega) e^{i\omega q f(h)} \mathrm{d}h. \tag{25}$$

where $\hat{m}$ and $\hat{d}$ represent the Fourier-transformed model and data sets, respectively. The function $f(h)$ is dependent upon the type of transform being computed and is usually given as x or x^2. Equation (25) expressed in summation form is

$$\hat{m}(q_j,\omega) = \sum_{k=1}^{N} \hat{d}(h_k,\omega) e^{i\omega q_j f(h_k)}, \{j = 1,\ldots,M; k = 1,\ldots,N\}. \tag{26}$$

The summation can be written in an equivalent matrix form as:

$$\hat{\mathbf{m}}' = \hat{\mathbf{L}}^{\mathbf{A}} \hat{\mathbf{d}}, \tag{27}$$

where $\hat{\mathbf{m}}'$ is a low-resolution version of the transformed function in Fourier-domain model space. Equation (27) is the Fourier-domain representation of equation (19). The elements for the Fourier-domain linear operator, $\hat{\mathbf{L}}$, are now defined as:

$$\hat{L}_{k,j} = e^{-i\omega q_j f(h_k)}. \tag{28}$$

The adjoint of $\hat{\mathbf{L}}$ involves complex numbers and is denoted as $\hat{\mathbf{L}}^{\mathbf{A}}$. The constrained least-squares formulation of the inversion in the Fourier domain is given as:

$$\hat{\mathbf{m}}(q_j) = \left[\hat{\mathbf{L}}^{\mathbf{A}} \hat{\mathbf{L}} + \mu\mathbf{I}\right]^{-1} \hat{\mathbf{L}}^{\mathbf{A}} \hat{\mathbf{d}}_{\omega}\left[f(x_k)\right]. \tag{29}$$

The focusing power of the regularization parameter, μ, provides an inexact approximation of events outside of the finite aperture range of the data, subsequently smearing energy along the q-axis. A variable regularization term is required to constrain smearing of the transform in a data-dependent manner. Sacchi and Ulrych (1995) proposed a *high-resolution* technique that involves an iterative method of employing the data within the sparsity constraint to allow for a better reconstruction.

The high-resolution method requires an initial computation of the model, $\mathbf{m_i}$, using equation (29):

$$\hat{\mathbf{m}}_{\mathbf{i}} = \left[\hat{\mathbf{L}}^{\mathbf{A}} \hat{\mathbf{L}} + \mu\mathbf{I}\right]^{-1} \hat{\mathbf{L}}^{\mathbf{A}} \hat{\mathbf{d}}. \tag{30}$$

The approximations for the high-resolution damping-factor parameters

After computation of the initial model, the resultant matrix is then used to determine the regularization parameter in an iterative method designed to minimize the smearing problems along the q-axis. The nonconstant diagonal regularization matrix, **D,** replaces $\mu\mathbf{I}$ in equation (30), and is defined for each iteration as:

$$\mathbf{D}_{\mathbf{i}} = \frac{\lambda}{b + \left|\hat{\mathbf{m}}_{\mathbf{i}}^{\mathbf{2}}\right|}. \tag{31}$$

The constant regularization parameters, λ and b, are optimized for a CMP gather prior to application to the entire data set. The parameter b is included in the damping factor to provide for white noise. The parameter b may be alternatively estimated as 1% of the maximum of $\left|\hat{\mathbf{m}}_{\mathbf{i}}^{\mathbf{2}}\right|$, while the parameter μ may be substituted for λ. This alternative estimation of the regularization parameters saves time in otherwise testing parameters to optimize the solution.

The elements of the matrix **D** are computed during each iteration of the high-resolution transform, where

$$\hat{\mathbf{m}}_{\mathbf{i}+1} = \left[\hat{\mathbf{L}}^{\mathbf{A}} \hat{\mathbf{L}} + \mathbf{D}_{\mathbf{i}} \right]^{-1} \hat{\mathbf{L}}^{\mathbf{A}} \hat{\mathbf{d}}, \{\mathbf{i} = 0,1,....,K\}. \quad (32)$$

Three iterations typically are necessary to provide an optimally constrained solution.

CONCLUSIONS

The shifted-hyperbola method acts to create a curve-fitting technique to approximate the actual moveout of targeted noise for improved suppression. Proper employment of the shifted-hyperbolic transform should result in well focused transform domains, less overlap of primary P-events and multiples in Radon space, and an overall improvement in multiple suppression in comparison with the parabolic and hyperbolic techniques.

The shifted hyperbola creates a more accurately focused transform space when applied with correct parameters. The input shift parameter may be extrapolated from the data or, in cases where coherent events only slightly deviate from hyperbolae, they may be visually estimated. The flexibility of choosing the shift parameter allows the operator to design a focused transform for nonhyperbolic events.

In summary, this work highlights several new insights dealing with Radon transforms. The offset-weighted damping factor [equation (24)] provides an alternative, empirical method for computing the least-squares damping factor. The approximations for the high-resolution damping-factor parameters [equation (31)] expedite the process of choosing adequate parameters for various data sets. Ultimately, the alternative shifted-hyperbolic and anisotropic Radon equations [equations (11), (13), and (16)] should provide an enhanced algorithm for better focusing in Radon space. These methods should prove beneficial in the separation of coherent noise from reflections with variable amplitudes and phases.

ACKNOWLEDGEMENTS

We would like to express our appreciation to Dr. Gary Margrave and Dr. Mauricio Sacchi for their analysis of this work. We would also like to thank the CREWES sponsors for their support of this research.

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