Short note: Normal incidence synthetics in viscoelastic media

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ABSTRACT

One of the major tasks when introducing viscoelasticity (anelasticity) into synthetic seismogram computations requires selecting a method for accomplishing this and at the same time not introducing non-physical (causality) artefacts into the synthetics. This must be done within a mathematical framework, which can pass at least moderate scrutiny, without invoking questions as to, among other things, its accuracy, applicability and theoretical correctness. If the SEG reprint series (1981), which contains a number of reprints on viscoelastic theory applied to seismic problems, is consulted several (often contentious) points of view may be encountered. After a significant amount of numerical testing and consultation with other texts and papers related to this matter and with several academic and industry researchers, the theory presented by Futterman (1962) was deemed to be the most useful and accurate when used together with the high frequency geometrical optics solution method of computing synthetic traces. An assumption used in his discussion of seismic wave propagation in a viscoelastic medium is that $Q > 30$. (More realistically, the minimum value of Q should be such that $Q > 10^1$.)

Although the problem considered here is fairly simplistic, an earlier version of this algorithm is part of a software package, which as of the date of writing, is still in use in an industry processing package.

BASIC THEORY

The equations of particle motion for a viscoelastic medium derived by Boltzman are the basis for the theory on which the software used here has been written. The theoretical development of this problem will not be pursued here as this is meant to be only a basic introduction to the topic that is probably better served by presenting some results rather than a sequence of fairly mathematically intense derivations. To further keep matters as simple as possible only normal incidence will be considered so that the saddle point at all frequencies, which are required to cover the spectrum of the source pulse in the frequency domain, is zero. In the more general non-zero offset case, the saddle point must be computed at each required frequency point for each ray comprising the synthetic trace and is a generally complex quantity, not laying on the real axis in the $\mathbf{p}$ (slowness) plane, but rather in the first or third quadrant depending on the time dependence ($e^{\pm i\omega t}$) used.

The viscoelastic equivalents of the elastic coefficients are assumed to be time dependent and as a consequence, due to the complexity of even a simple problem type, a time transformation to the frequency domain is made and most computations are carried out in that domain. This allows for significant latitude in specifying the physical mechanisms that result in a medium being viscoelastic. The attenuating mechanism discussed in Futterman (1962) assumes that at some reference circular frequency, $\omega_R = 2\pi f_R$ a reference attenuation factor, $Q_R = Q(\omega_R)$, and reference $P-$wave velocity,

$V(\omega_R)$, are known. These quantities are real valued. At some other frequency, ω, the values of $Q(\omega)$ and $V(\omega)$ are given by the approximations to those of Futterman's as

$$Q(\omega) = Q(\omega_R)\left[1.0 - \frac{1.0}{\pi Q(\omega_R)} \ln\left(\frac{\omega}{\omega_R}\right)\right] \tag{1}$$

and

$$V(\omega) = V(\omega_R)\frac{Q(\omega_R)}{Q(\omega)}. \tag{2}$$

The two values $Q(\omega)$ and $V(\omega)$ obtained above are also real. Viscoelasticity or attenuation is introduced into a medium through a complex velocity obtained by an analysis of the attenuating mechanism, which as previously stated may be found in Futterman's (1962) paper. His high frequency expression for this complex velocity in terms of the real parameters $Q(\omega)$ and $V(\omega)$ is

$$\frac{1}{C(\omega)} = \frac{1}{V(\omega)}\left[1 + \frac{i}{2Q(\omega)}\right]. \tag{3}$$

It should be noted that density is taken to be a real quantity throughout.

The velocity defined by equation (3) is that which is used in the computation of the complex "traveltimes", geometrical spreading and reflection and transmission coefficients.

In the case of normal incidence for plane parallel layers the complex traveltime of a primary *P* ray propagating through *N* layers from a source and receiver located at the surface is given by

$$\tau = \sum_{n=1}^{N} \frac{2h_n}{C_n(\omega)} = \tau_{\mathrm{Re}} \pm i\tau_{\mathrm{Im}} \tag{4}$$

where τ_{Re} and τ_{Im} are real and positive and the "$\pm$" choice is indicated by the time dependence used, $e^{\mp i\omega t}$, as it is required that the solution be proportional to $e^{-\tau_{\mathrm{Im}}}$ satisfying physical radiation conditions.

If reflection and transmission coefficients are initially taken as modifications of those for a solid/solid interface as in Aki and Richards (1980), they degenerate to the normal incidence acoustic case (Brekhovskikh, 1980) for the problem considered here where at the *j-th* interface are functions of the complex velocity impedances, $I_n(\omega) = \rho_n C_n(\omega)$ $(n = j, j+1)$. The geometrical spreading is similar to the elastic case with the exception that complex quantities are used rather than the real values which result in the elastic case.

NUMERICAL RESULTS

The model chosen for computing synthetic traces is composed of 8 layers over a halfspace. Actually, it is a four layered model which is repeated twice. This is a common practice in testing certain software packages to ascertain the accuracy of the results computed as there should be an observable relationship between the first and second occurrences of a layer with the same viscoelastic parameters. Only one layer in each of the two sequences is viscoelastic. This has the effect of the viscoelasticity of the single layer having its greatest affects on the reflection coefficient from its top interface and the interface between the viscoelastic layer and the underlying layer. However, all arrivals from deeper layers are affected in some manner.

The normal incidence trace is computed for reference values of $Q(\omega_R)$ of 1000, 100, 50, 30 and 10. The value of $Q(\omega_R) = 1000$ is a progression towards a nearly elastic layer. The value of $Q(\omega_R) = 10$, which falls below the upper bound of 30 for which the theory is assumed valid, has been included to show that progressively smaller values of $Q(\omega_R)$ behave in a predictable manner even if the theoretical limit of applicability has been passed. The use of $Q = 30$ as a lower bound of applicability may also be questioned. The fact that the high frequency approximation appears to produce reasonable results outside of its region of applicability is not an uncommon occurrence.

The reference frequency is chosen to be the same as the predominant frequency of the source wavelet; in this case, 30 *Hz*. A Gabor wavelet is used and defined as

$$f(t) = \sin\left[\omega_R (t - t_h)\right] \exp\left[-\left(\frac{\omega_R (t - t_h)}{\gamma}\right)^2\right] \tag{5}$$

where $\omega_R = 2\pi f_R$ with $f_R = 30Hz$ and the dimensionless damping factor controlling the side lobes chosen to be $\gamma = 4.0$. The half length in the time domain of this pulse, t_h, is approximately given by $t_h \approx \gamma/(2 f_R)$. The time dependence of the wavelet and the frequency spectrum are shown in Figure 1 while the velocity, density – depth structure is given in Figure 2.

The normal incidence synthetic traces for the 5 different values of *Q* are presented in Figures 3 and 4. As no amplitude scaling of any kind is used, Figure 4 is just Figure 3 with the first arrival removed to allow for better viewing. In Figure 4 the grid lines perpendicular to the time axis have been added to emphasize that no time shift in the arrivals occur at different values of *Q*. Only *P*-wave primaries are included in the traces. There are provisions for the introduction of multiples, which has not been implemented for this preliminary presentation. As $Q = 1000$ corresponds to the layer approaching the elastic case, conclusions as to the effect of introducing viscoelasticity are left to the reader's observations of variations between the traces with different values of $Q(\omega)$ shown in Figures 3 and 4.

REFERENCES

Aki, K. and Richards. P.G., 1980, Quantitative Seismology, vol. 1, W.H. Freeman and Company, San Francisco.

Brekhovskikh, L.M., 1980, Waves in Layered Media, Academic Press, New York.

______ (1981) Seismic Wave Attenuation, Geophysics Reprint Series, Society of Exploration Geophysicists, N.M. Toksöz and D.M. Johnston: Eds.

Futterman, W.I., (1962), Dispersive body waves, Journal of Geophysical Research, **67**, 5279-5291.

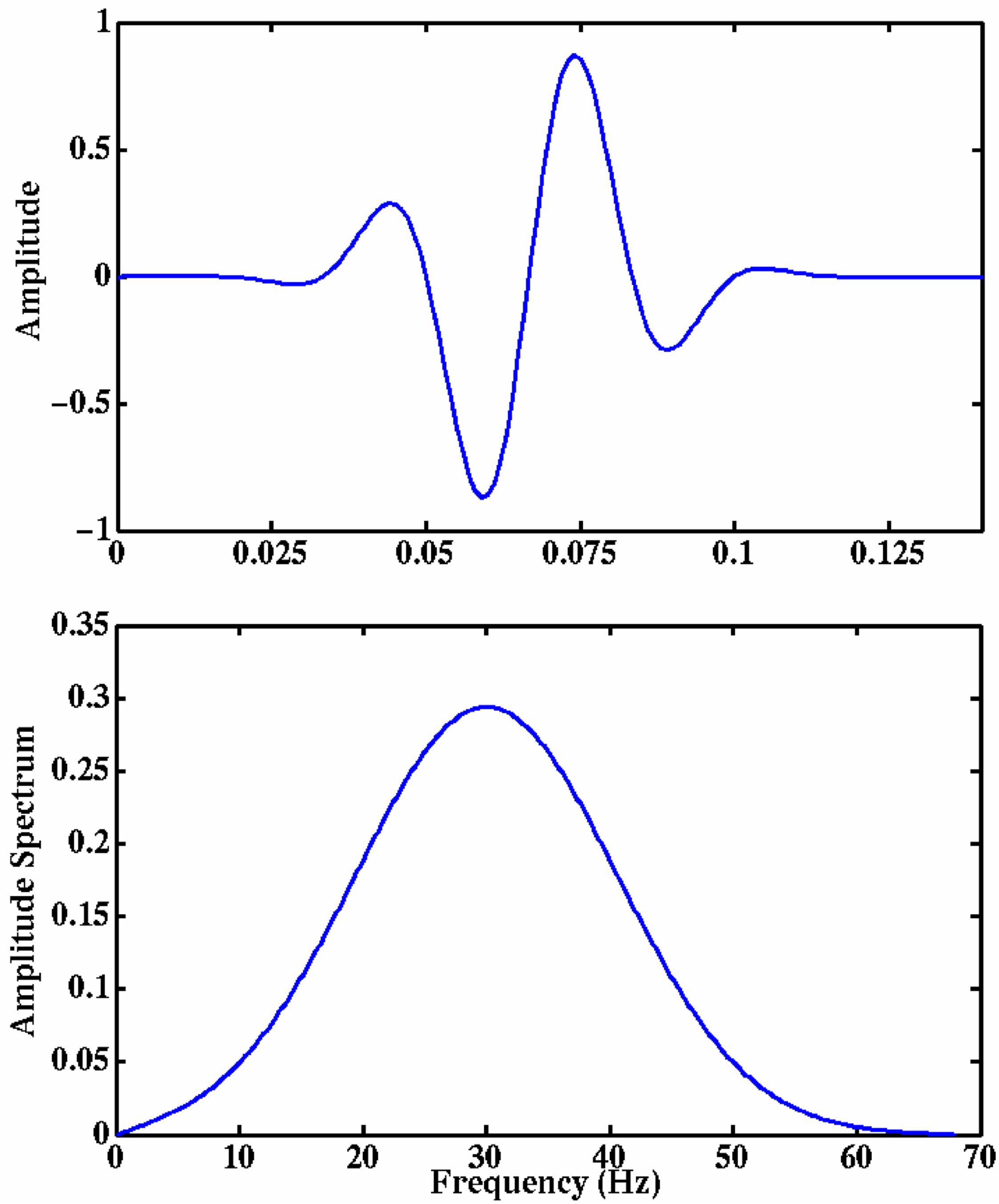

FIG. 1 The time and frequency dependence of the Gabor wavelet with a predominant frequency of 30 Hz and a damping factor of 4.

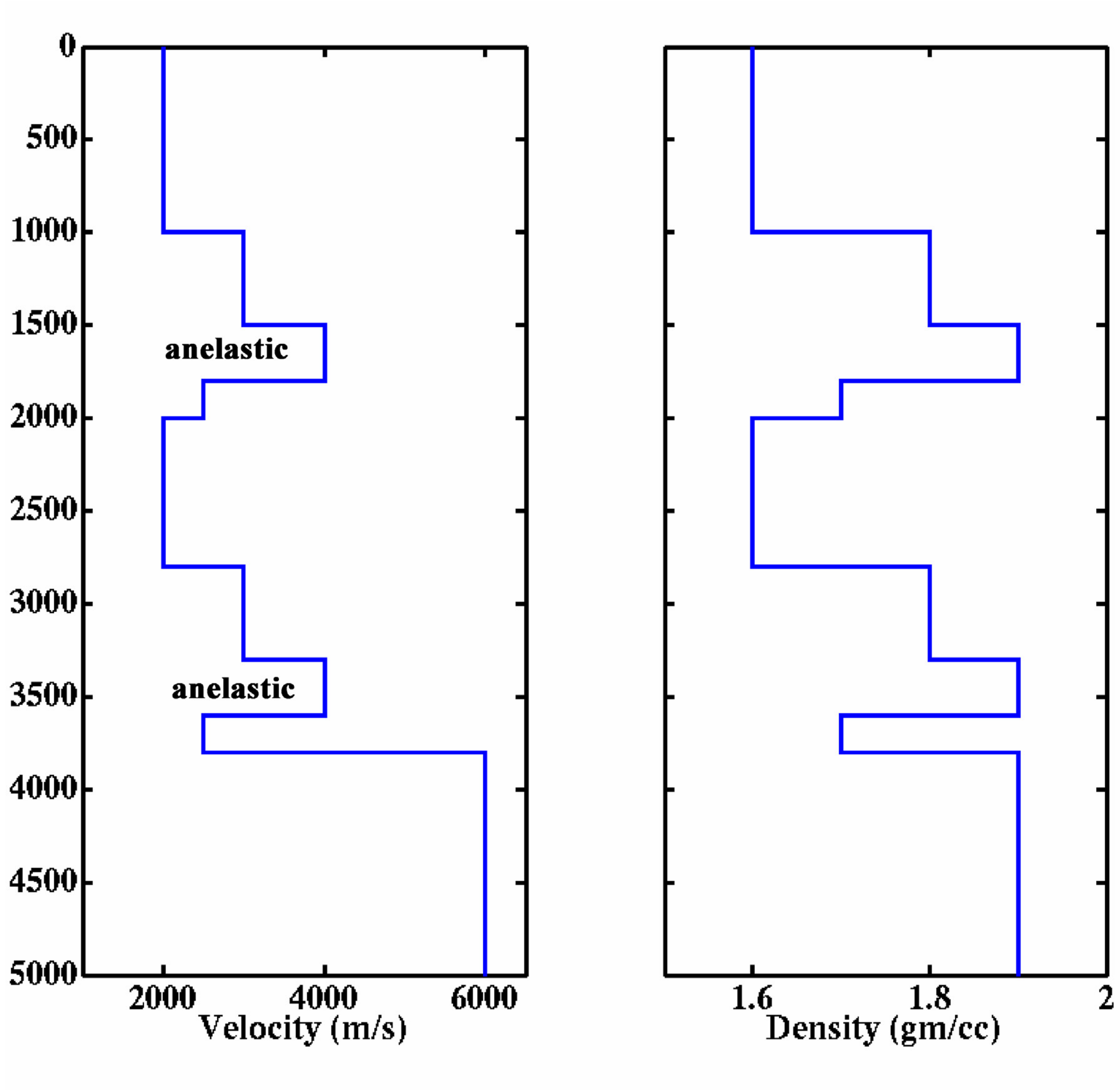

FIG. 2. Velocity and density versus depth profiles for the model considered in this report. The velocities are *reference* velocities supposedly measured at some reference frequency. In actuality they reflect the velocities that would be measured if the media were elastic. The two anelastic layers are indicated on the velocity depth plot.

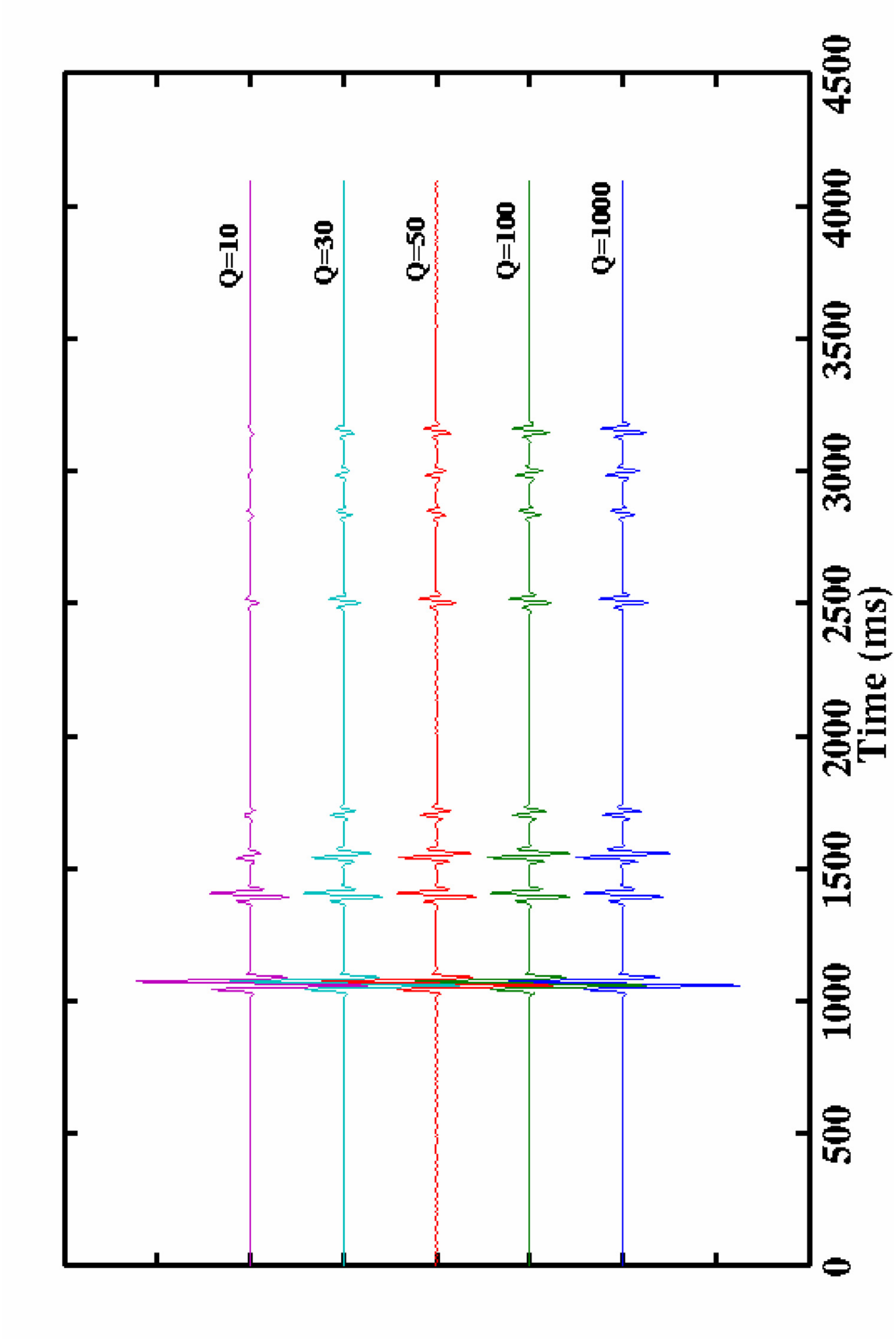

FIG. 3. Normal incidence plots of traces using the model given in Figure 2. Five different reference values of *Q* are shown. The viscoelastic layers are 3 and 7 and the other layers are constrained to be elastic. This aids in discerning what the effects are of an individual viscoelastic on a synthetic trace.

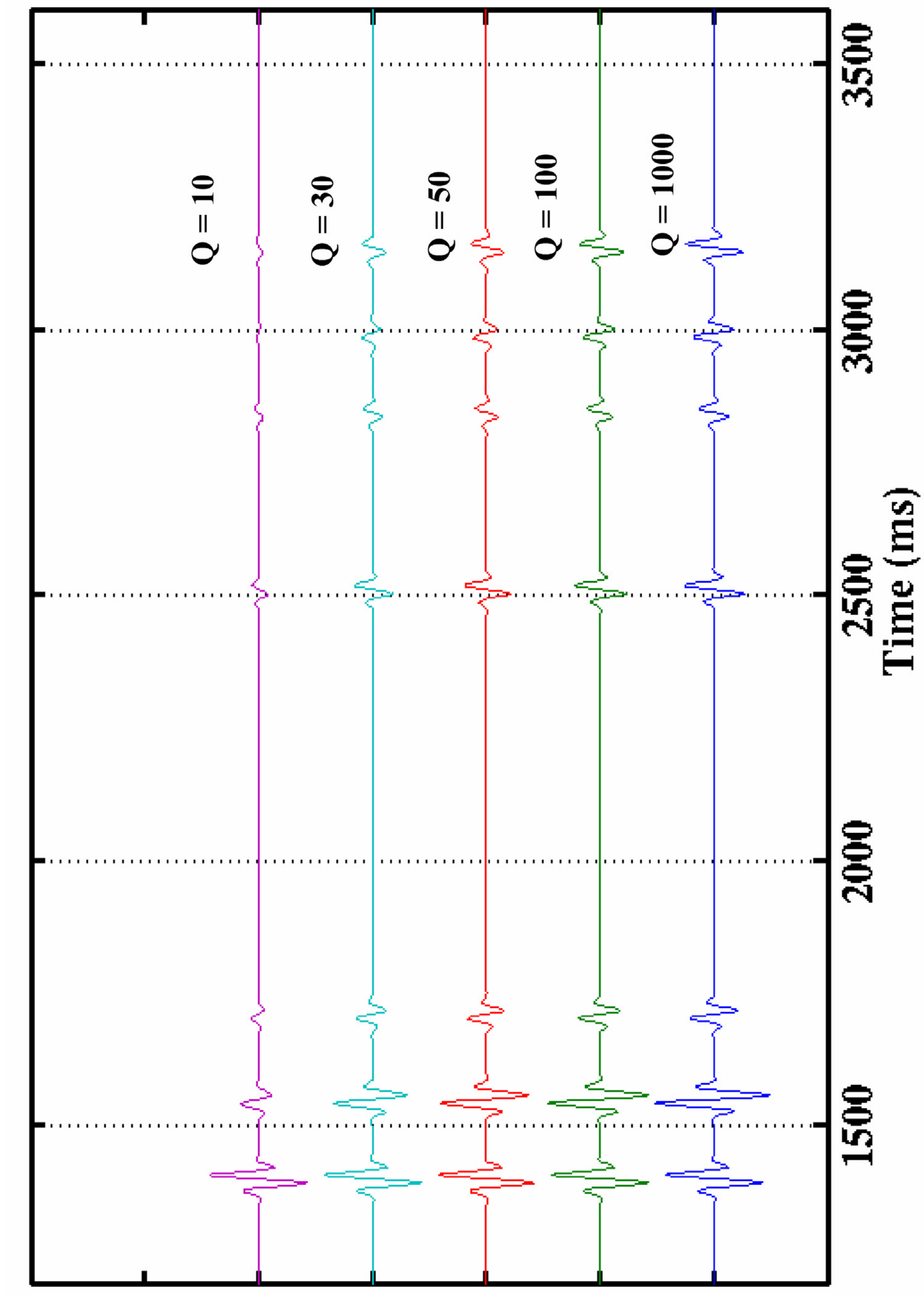

Fig. 4. The same as Figure 3 with the large first primary arrival removed to give a better view of what occurs when anelasticity is introduced into synthetic trace computation. The time grid lines are to indicate that there is no introduction of acausal effects.