Testing pseudo-linear Zoeppritz approximations: P-wave AVO inversion

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ABSTRACT

The Zoeppritz coefficients describe seismic reflection and transmission properties of idealized solid-solid interfaces. The Aki-Richards approximations linearize these coefficients in terms of elastic property contrasts across the interface. Such expressions, especially for the P-P and P-S reflections, are the starting point for most AVO (amplitude variation with offset) analysis of seismic data. We have previously introduced the idea of an Optimal Zoeppritz Approximation, which is designed to be both accurate and simple. We have shown that it is naturally expressed in a form that is pseudo-linear, analogous to the form of the Aki-Richards approximation. We have proposed that these forms would be of potential value in AVO studies.

In this report we review our earlier results and present two extensions. In the first we have extended the range of validity of the pseudo-linear approximation for the P-P reflection coefficient. In the previous form it was accurate only up to the critical point, which is often adequate for seismic applications. However, attempts to derive density contrasts from AVO require large-offset surveys, and this increases the likelihood of dealing with post-critical data. Accordingly we have developed an approximation which is somewhat more complex, but which is reasonably accurate for much of the post-critical regime. In the second extension we demonstrate that the exact Zoeppritz expression itself can be expressed in a pseudo-linear form, which may be useful in theoretical and/or practical studies. Finally, we use numerical tests to assess the potential practicality and accuracy of pseudo-linear approximations in AVO applications.

INTRODUCTION

The two best-known reflectivity expressions in seismic exploration are the Zoeppritz coefficients and the Aki-Richards approximation, the latter being obtained from the former by linearization in contrasts across the interface. The Aki-Richards approximation is convenient for use in AVO, but is not as accurate as the exact Zoeppritz coefficients. We aim to bridge the gap between them by creating methods which are accurate but still amenable to AVO procedures.

We have previously presented the idea of an Optimal Zoeppritz Approximation (Ursenbach, 2002a,b). This involved two key ideas. First we showed that contrast variables should not all be treated equally in creating approximations. In particular the dependence on P-wave velocity contrast ($\Delta\alpha/\alpha$) must be treated exactly, while the dependence on S-wave velocity and density contrasts ($\Delta\beta/\beta$ and $\Delta\rho/\rho$) may be represented using a Taylor expansion. Second we showed that approximations more complex than linear expansions may still be represented in a linear form, which makes their structure more intuitive and convenient for use in AVO methods.

In the previous study (Ursenbach, 2002a) we presented an expression for R_{PP} (and for R_{PS}) which was exact in $\Delta\alpha/\alpha$ and linear in $\Delta\beta/\beta$ and $\Delta\rho/\rho$. In the present study we build on our previous work in two directions. First, we develop more accurate expressions than that given previously. This is motivated by a desire to extend accuracy in the post-critical regime, which may be helpful in dealing with large-offset data required in order to extract $\Delta\rho/\rho$ accurately. Second, we carry out inversion tests to begin evaluating the performance of various approximations in actual AVO methods.

An earlier version of this study has been previously presented (Ursenbach, 2003).

SECOND-ORDER APPROXIMATION AND POST-CRITICAL REGIME

Using symbolic mathematics software it is straightforward to obtain various approximations to the Zoeppritz expressions and to plot them in comparison to an exact expression. Using this approach we ascertained that an approximation which treated the dependence of $\Delta\alpha/\alpha$ exactly and which was quadratic in $\Delta\beta/\beta$ and $\Delta\rho/\rho$ would be much more accurate in the first half of the post-critical region than the corresponding pseudo-linear expression (which is also exact in $\Delta\alpha/\alpha$ but only linear in $\Delta\beta/\beta$ and $\Delta\rho/\rho$). (See Appendix for supporting figures.) Once such a fact is determined, however, some effort is still required to reduce the formal output from the software down to a compact and useful expression. We present the result of this effort below, and refer to it as the **pseudo-quadratic approximation**, in that it has the form of an approximation which is quadratic in $\Delta\beta/\beta$ and $\Delta\rho/\rho$, but in which the coefficients are in fact dependent upon $\Delta\alpha/\alpha$.

$$
\begin{aligned}
R_{PP}^{PQ} = \frac{4\cos\theta_1\cos\theta_2}{Q^2}\Bigg\{ & \frac{\Delta\alpha/\alpha}{2\cos\theta_1\cos\theta_2} - 2\sin\theta_1\sin\theta_2\frac{\beta^2}{\alpha^2}\left(\frac{\Delta\mu}{\mu}\right)_{Lin}\Bigg[1+\Bigg(-\frac{\beta}{\alpha}\frac{\cos\varphi\cos\theta_2}{2+\Delta\alpha/\alpha} \\
& +4\frac{\beta^2}{\alpha^2}\frac{\sin^2\theta_1}{[1-\Delta\alpha/(2\alpha)]^2Q^2}\frac{\Delta\alpha}{\alpha}+\frac{\beta^3}{\alpha^3}\frac{\sin^3\theta_1\sin\theta_2}{2[1-\Delta\alpha/(2\alpha)]^3\cos\varphi\cos\theta_2}\Bigg)\left(\frac{\Delta\mu}{\mu}\right)_{Lin} \\
& -\left(\frac{2}{Q^2}\frac{\Delta\alpha}{\alpha}+\frac{\beta}{\alpha}\frac{\sin\theta_1\sin\theta_2}{2[1-\Delta\alpha/(2\alpha)]\cos\varphi\cos\theta_2}\right)\frac{\Delta\rho}{\rho}\Bigg] \\
& +\frac{1}{2}\frac{\Delta\rho}{\rho}\left(1-\left[\frac{\Delta\alpha}{2\alpha}\right]^2\right)\left(1-\frac{1}{Q^2}\frac{\Delta\alpha}{\alpha}\frac{\Delta\rho}{\rho}-\frac{\beta}{\alpha}\frac{\sin\theta_1\sin\theta_2}{2[1-\Delta\alpha/(2\alpha)]\cos\varphi\cos\theta_2}\frac{\Delta\rho}{\rho}\right)\Bigg\}.
\end{aligned}
$$

In this expression, θ_1 and θ_2 are the P-wave reflection and transmission angles, and φ_1 and φ_2 are the same for S-waves. We also employ the following definitions:

$$
Q = \left(1+\frac{\Delta\alpha}{2\alpha}\right)\cos\theta_1 + \left(1-\frac{\Delta\alpha}{2\alpha}\right)\cos\theta_2
$$

$$
\cos\varphi = \sqrt{1-\frac{\beta^2}{\alpha^2}\frac{\sin^2\theta_1}{[1-\Delta\alpha/(2\alpha)]^2}}, \qquad \left(\frac{\Delta\mu}{\mu}\right)_{Lin} = 2\frac{\Delta\beta}{\beta}+\frac{\Delta\rho}{\rho}.
$$

The approximation that we have just termed pseudo-quadratic might also reasonably be termed pseudo-linear, in the sense that we may view it as being linear in $\Delta\beta/\beta$ and $\Delta\rho/\rho$, but with coefficients that depend not only on $\Delta\alpha/\alpha$, but also linearly on $\Delta\beta/\beta$ and $\Delta\rho/\rho$.

EXACT PSEUDO-LINEAR EXPRESSIONS

In the same sense that the pseudo-quadratic expression can be written in a pseudo-linear form, we consider that it should be possible even to write the exact Zoeppritz expression in a pseudo-linear form, with coefficients depending on all three variable contrasts. To this end, we have first manipulated the exact P-P coefficient, normally written in terms of ρ_1, ρ_2, α_1, α_2, β_1, and β_2, into the useful form below.

$$R_{PP}^{Exact} = \frac{D_-}{D_+} \tag{1}$$

$$\begin{aligned}
D_\pm = \frac{1}{4}&\left[\left(1+\frac{\Delta\beta}{2\beta}\right)\left(1+\frac{\Delta\rho}{2\rho}\right)\cos\varphi_1 + \left(1-\frac{\Delta\beta}{2\beta}\right)\left(1-\frac{\Delta\rho}{2\rho}\right)\cos\varphi_2\right]\times \\
&\left[\left(1+\frac{\Delta\alpha}{2\alpha}\right)\left(1+\frac{\Delta\rho}{2\rho}\right)\cos\theta_1 \pm \left(1-\frac{\Delta\alpha}{2\alpha}\right)\left(1-\frac{\Delta\rho}{2\rho}\right)\cos\theta_2\right] - \frac{\beta^2}{\alpha^2}\frac{\sin\theta_1\sin\theta_2}{(1-[\Delta\alpha/2\alpha]^2)}\frac{\Delta\mu}{\mu}\times \\
&\left[\left(1+\frac{\Delta\alpha}{2\alpha}\right)\left(1+\frac{\Delta\beta}{2\beta}\right)\left(1+\frac{\Delta\rho}{2\rho}\right)\cos\theta_1\cos\varphi_1 \mp \left(1-\frac{\Delta\alpha}{2\alpha}\right)\left(1-\frac{\Delta\beta}{2\beta}\right)\left(1-\frac{\Delta\rho}{2\rho}\right)\cos\theta_2\cos\varphi_2\right] \\
&+\frac{\beta^3}{\alpha^3}\frac{\sin\theta_1\sin\theta_2\cos(\theta_1\mp\varphi_1)\cos(\theta_2-\varphi_2)}{(1-[\Delta\alpha/2\alpha]^2)}\left(\frac{\Delta\mu}{\mu}\right)^2 \\
&\mp\frac{\beta}{\alpha}\sin\theta_1\sin\theta_2\left[\sin\varphi_1\sin\varphi_2\frac{\Delta\mu}{\mu} - \frac{1}{4}\left(1-\left[\frac{\Delta\beta}{2\beta}\right]^2\right)\frac{\Delta\rho}{\rho}\right]\frac{\Delta\rho}{\rho}
\end{aligned}$$

Here we must use the cubic expression $\Delta\mu/\mu = 2\,\Delta\beta/\beta + \Delta\rho/\rho + (1/4)(\Delta\beta/\beta)^2\Delta\rho/\rho$. This expression corrects some errors made in an earlier presentation (Ursenbach, 2003).

We note the overall structure of this expression, namely,

$$D_\pm = C_0^{\pm} + C_1^{\pm}\frac{\Delta\mu}{\mu} + C_2^{\pm}\left(\frac{\Delta\mu}{\mu}\right)^2 \pm C_3\frac{\Delta\mu}{\mu}\frac{\Delta\rho}{\rho} \pm C_4\left(\frac{\Delta\rho}{\rho}\right)^2.$$

Linearizing C_0^- yields terms linear in $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$, while linearizing C_0^+ yields a constant term $[\cos(\theta_1)\cos(\varphi_1)]$ as well as linear terms, so D_+ and D_- are each pseudo-quadratic. Let us define a quantity $D_+^{(0)}$ to contain the constant portion of C_0^+ but not the linear terms. Then if we define $D_+^{(1)} \equiv D_+ - D_+^{(0)}$ we can rewrite Equation (1) as

$$R_{PP} = (D_- - D_+^{(1)} R_{PP}) / D_+^{(0)} \quad \text{or} \quad R_{PP} D_+^{(0)} = D_- - R_{PP} D_+^{(1)}$$

Why is an expression such as this useful? We have just shown that D_- and $D_+^{(1)}$ are both pseudo-quadratic with no constant terms, while $D_+^{(0)}$ has a constant term and thus is non-zero even when all contrasts are zero. Thus, if we don't mind having R_{PP} appear in our operator, or perhaps have some functions mixed in with our data vector, we can write the exact Zoeppritz coefficients implicitly in pseudo-quadratic expressions. To do this explicitly we first note that

$$\cos\theta_1 - \cos\theta_2 = -\sin\theta_1 \tan\theta_1 \frac{\Delta\alpha}{\alpha} + O\left(\left[\frac{\Delta\alpha}{\alpha}\right]^2\right).$$

We write the final result as

$$\begin{aligned}
&R_{PP}\left\{\frac{1}{4}(\cos\theta_1+\cos\theta_2)\left(1+\frac{1}{4}\frac{\Delta\alpha}{\alpha}\frac{\Delta\rho}{\rho}\right)\left[\left(1+\frac{1}{2}\frac{\Delta\beta}{\beta}\right)\left(1+\frac{1}{2}\frac{\Delta\rho}{\rho}\right)\cos\varphi_1+\left(1-\frac{1}{2}\frac{\Delta\beta}{\beta}\right)\left(1-\frac{1}{2}\frac{\Delta\rho}{\rho}\right)\cos\varphi_2\right]\right\} \\
&=\frac{1}{4}\left[\left(1+\frac{1}{2}\frac{\Delta\beta}{\beta}\right)\left(1+\frac{1}{2}\frac{\Delta\rho}{\rho}\right)\cos\varphi_1+\left(1-\frac{1}{2}\frac{\Delta\beta}{\beta}\right)\left(1-\frac{1}{2}\frac{\Delta\rho}{\rho}\right)\cos\varphi_2\right]\times \\
&\left[\left(1+\frac{1}{4}\frac{\Delta\alpha}{\alpha}\frac{\Delta\rho}{\rho}\right)\frac{\cos\theta_1-\cos\theta_2}{\Delta\alpha/\alpha}\frac{\Delta\alpha}{\alpha}+\frac{1}{2}\left(\frac{\Delta\alpha}{\alpha}+\frac{\Delta\rho}{\rho}\right)([1-R_{PP}]\cos\theta_1+[1+R_{PP}]\cos\theta_2)\right] \\
&-\frac{\beta^2}{\alpha^2}\frac{\sin\theta_1\sin\theta_2}{[1-(\Delta\alpha/2\alpha)^2]}\left[(1-R_{PP})\left(1+\frac{1}{2}\frac{\Delta\alpha}{\alpha}\right)\left(1+\frac{1}{2}\frac{\Delta\beta}{\beta}\right)\left(1+\frac{1}{2}\frac{\Delta\rho}{\rho}\right)\cos\theta_1\cos\varphi_1\right. \\
&\qquad\left.+(1+R_{PP})\left(1-\frac{1}{2}\frac{\Delta\alpha}{\alpha}\right)\left(1-\frac{1}{2}\frac{\Delta\beta}{\beta}\right)\left(1-\frac{1}{2}\frac{\Delta\rho}{\rho}\right)\cos\theta_2\cos\varphi_2\right]\frac{\Delta\mu}{\mu} \\
&+\frac{\beta^3}{\alpha^3}\frac{\sin^2\theta_1}{[1-(\Delta\alpha/2\alpha)]^2}\left[(1-R_{PP})\cos\theta_1\cos\varphi_1-(1+R_{PP})\sin\theta_1\sin\varphi_1\right]\cos(\theta_2-\varphi_2)\left(\frac{\Delta\mu}{\mu}\right)^2 \\
&+(1+R_{PP})\frac{\beta}{\alpha}\sin\theta_1\sin\theta_2\left[\sin\varphi_1\sin\varphi_2\frac{\Delta\mu}{\mu}-\frac{1}{4}\left(1-\left[\frac{\Delta\beta}{2\beta}\right]^2\right)\frac{\Delta\rho}{\rho}\right]\frac{\Delta\rho}{\rho}.
\end{aligned}$$

Note that this corrects an error which appears elsewhere (Ursenbach, 2003).

APPLICATION: ITERATIVE SOLUTION OF PSEUDO-LINEAR EXPRESSIONS

The expressions derived above are typically of the form

$$R_{PP}(\theta_i) = c_\alpha(\theta_i)\frac{\Delta\alpha}{\alpha} + c_\beta(\theta_i)\frac{\Delta\beta}{\beta} + c_\rho(\theta_i)\frac{\Delta\rho}{\rho} \tag{2}$$

where θ_i takes on a range of values. This results in a matrix equation

$$\underline{R} = \underline{\underline{C}} \begin{bmatrix} \Delta\alpha/\alpha \\ \Delta\beta/\beta \\ \Delta\rho/\rho \end{bmatrix} \equiv \underline{\underline{C}}\,\underline{x}.$$

As θ_i normally takes on more than three values, this equation can be solved by the least-squares solution,

$$\underline{x} = (\underline{\underline{C}}^T \underline{\underline{C}})^{-1} \underline{\underline{C}}^T \underline{R}.$$

However, the coefficient matrix, C, also depends to some extent upon the contrasts as well. In the Aki-Richards approximation there is an implicit dependence on $\Delta\alpha/\alpha$ via the average angle $\theta = (\theta_1+\theta_2) / 2$. In the pseudo-linear approximation there is an explicit dependence on $\Delta\alpha/\alpha$. In the pseudo-quadratic and exact expressions there is an explicit dependence on all three contrasts.

There are at least two approaches one may take in carrying out an inversion with a pseudo-linear expression. In one approach, we assume that approximate values of the contrasts are known (e.g., $\Delta\alpha/\alpha$ from a velocity model) and these are employed in the matrix C, which is thus determined prior to inversion. Inversion can then be accomplished by a single least-squares solution of the matrix equation. In the other approach, we may initially set values of the contrast in C to zero, which again yields true linear equations. These are solved, and the contrast values obtained are then substituted into the coefficients for a second round. Continuing on in this way we may obtain a self-consistent solution through iteration, if a convergent solution exists. In this study we will employ the latter approach, considering first convergence properties, and then accuracy of convergent solutions.

Earth models

We employ the velocity and density data given in Table 1 of Castagna and Smith (1994) for 25 sets of shale, brine sand, and gas sand. For each set we consider the shale-over-brine, brine-over-shale, shale-over-gas, gas-over-shale, and gas-over-brine interfaces. This yields a set of 125 interfaces. For each interface we calculate three contrasts and β/α. Figure 1a contains the six possible crossplots of these four quantities, and we observe no strong correlations. A weak Gardner correlation (slope = 1/4) is present between $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$, and a weak 1:1 correlation may be present between $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$, but in both cases there is considerable scatter. This data then represents a broad sampling of earth interface models.

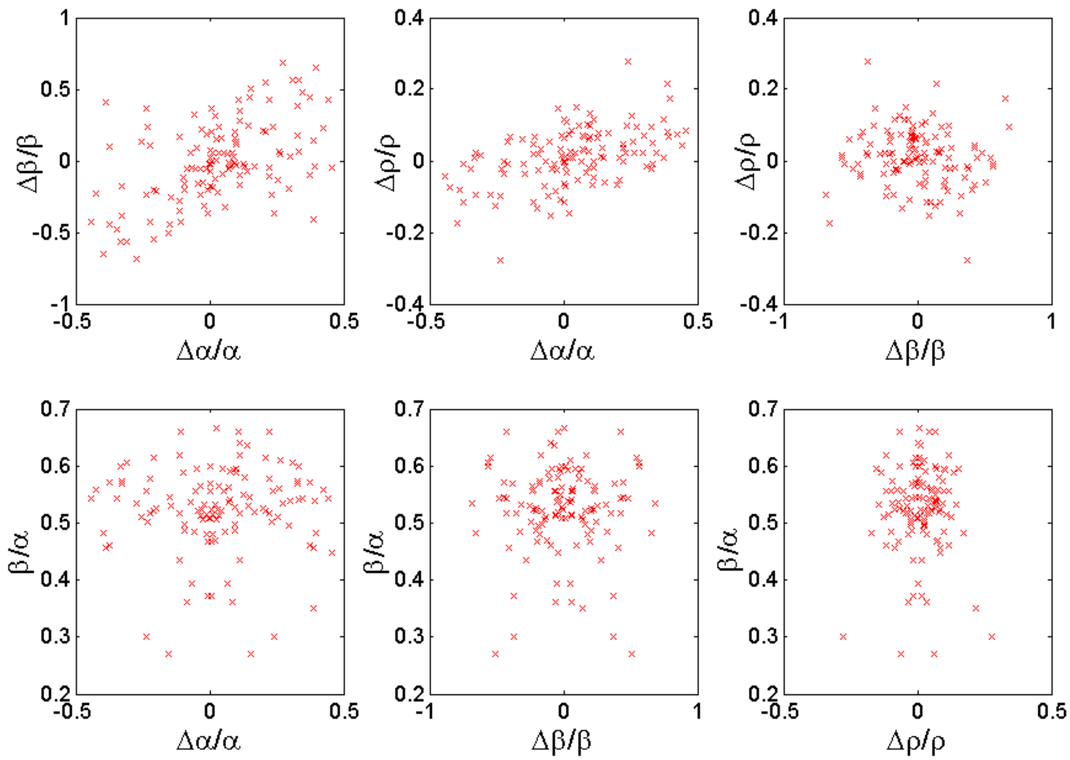

FIG. 1a. Crossplots of earth-parameter ratios used in this study. Based on data from Table 1 of Castagna and Smith (1994).

In Figure 1b we show this data in a different format, namely, by cross-plotting the AVO gradient and intercept for each of the 125 interfaces. This shows that there is representation of all common AVO classes in the dataset, again pointing out its usefulness.

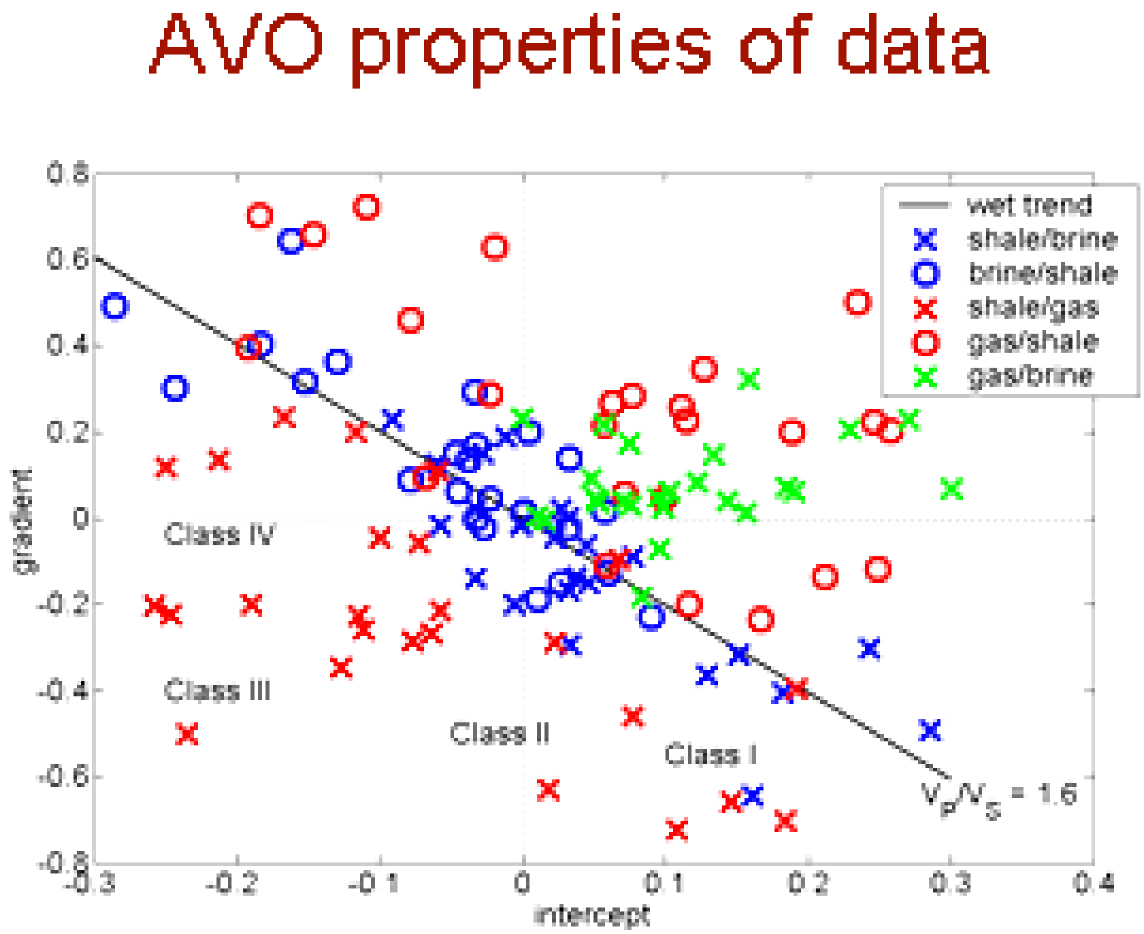

FIG. 1b. Crossplots of AVO gradient and intercept derived from earth-parameter ratios used in this study. Based on data from Table 1 of Castagna and Smith (1994).

Convergence

To solve Equation (2) for a particular interface we generate $R_{PP}(\theta_i)$ from the exact Zoeppritz expression for θ_i = 0°, 1°, 2°, … 30°. We begin iteration by setting all contrasts to zero in the 31×3 matrix, C. We obtain new values of contrasts by inversion, and then substitute these into C, and iterate this process until convergence is achieved, or until divergence becomes apparent.

In Figure 2a we display the $\Delta\alpha/\alpha$ result from inversion for all 125 interfaces. The values are plotted against the exact $\Delta\alpha/\alpha$. Unconverged results are shown in red. All methods shown suffer from poor convergence above $\Delta\alpha/\alpha \sim 0.2$. This corresponds to critical points below ~ 55°. We have not shown results from using the exact expression (as when it converges it yields exactly the correct value), but its convergence properties are similar to those of the pseudo-linear and pseudo-quadratic methods. We have also applied a Newton-Raphson method and obtained similar convergence behaviour. The Aki-Richards method differs from the others in that it has convergence difficulties at large negative values of $\Delta\alpha/\alpha$ as well. This is reasonable, since the other methods all treat the $\Delta\alpha/\alpha$ dependence exactly. Their problem is not with the magnitude of $\Delta\alpha/\alpha$, but only with proximity to the critical point.

We have also performed similar plots but against $\Delta\beta/\beta_{exact}$. In these the uncoverged values are scattered quite randomly throughout the range. Inspection of velocities also shows that all critical points are determined entirely by $\Delta\alpha/\alpha$ (i.e., all shear velocities are less than all compressional velocities).

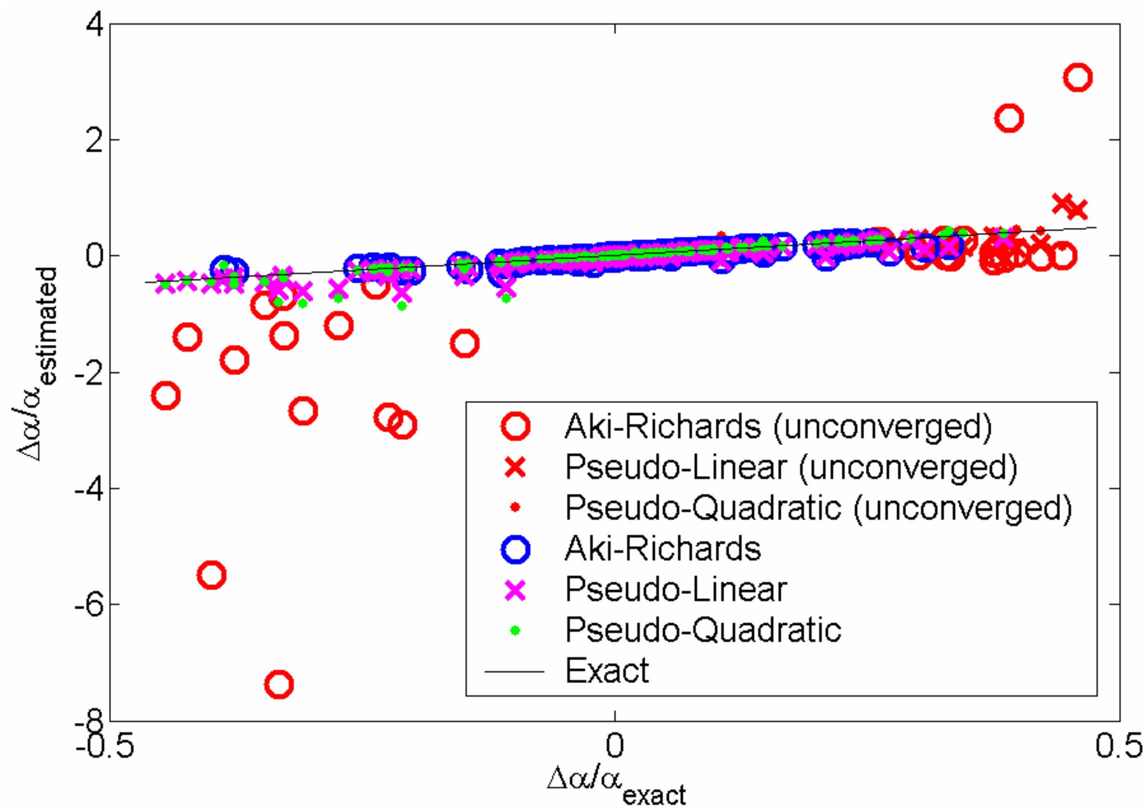

FIG. 2a. Plot of estimated $\Delta\alpha/\alpha$ values. Note the poor convergence of all methods at large $\Delta\alpha/\alpha$ (for low critical points) while only the Aki-Richards method converges poorly for negative $\Delta\alpha/\alpha$.

Also of interest is that the condition number is correlated with $\Delta\alpha/\alpha_{exact}$ as well, as shown below in Figure 2b. The convergence issues described above are not affected by the condition number; rather, this relates to each individual least-squares procedure. In this case none of the condition numbers are large enough to inhibit solution. Such would be more likely when noise is added to the data.

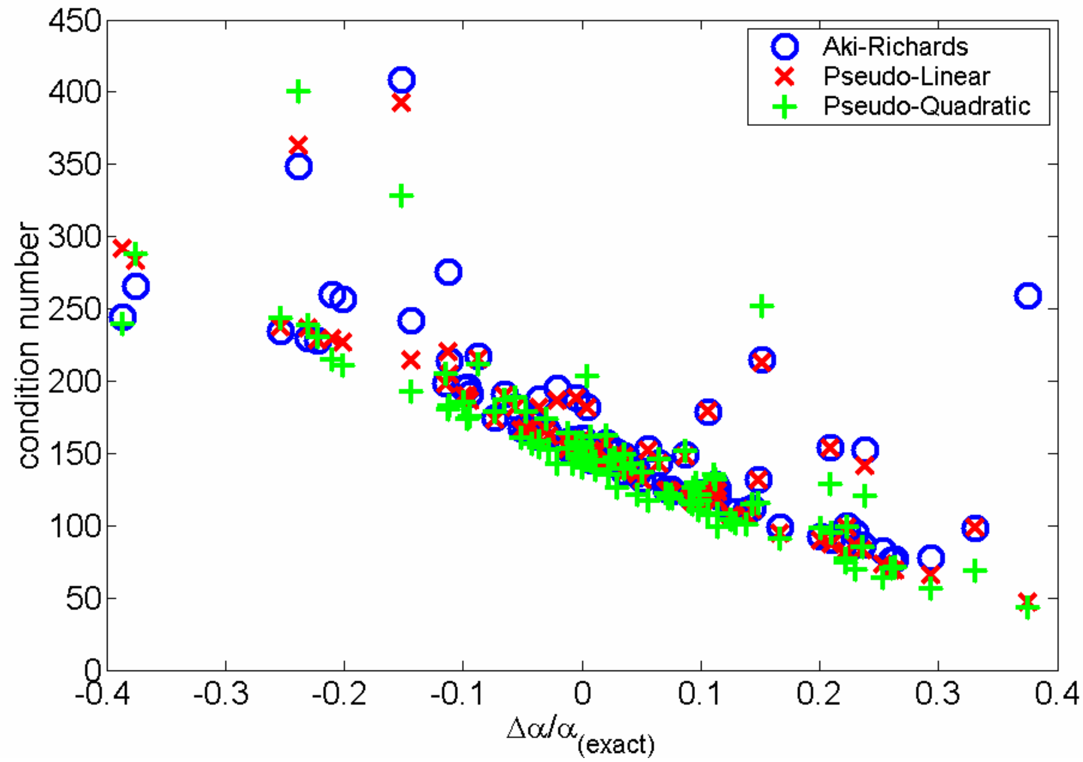

FIG. 2b. Plot of condition numbers and their variation with $\Delta\alpha/\alpha$. Note that this is unrelated to convergence, as the smallest condition numbers occur for large $\Delta\alpha/\alpha$.

Accuracy

We have found that an instructive way to consider error is to plot it against $\Delta\beta/\beta_{exact}$ as shown in Figure 3. Here we display the error in prediction of all three contrast variables for the pseudo-linear and pseudo-quadratic methods. A plot of the Aki-Richards results is similar in appearance to the pseudo-linear plot. (Only converged values have been employed.) A clear systematic trend is apparent for all variables and methods, combined with a random scatter. In contrast, when the same data is plotted against $\Delta\alpha/\alpha_{exact}$ or $\Delta\rho/\rho_{exact}$, the errors follow no trend at all, and are only random.

It is reasonable that the errors in the linear theory are quadratic, and in the quadratic theory the errors are cubic. We can perform a rough fit of this data to obtain the trend as an explicit function of $\Delta\beta/\beta_{exact}$. More useful though would be to obtain them as a function of $\Delta\beta/\beta_{estimated}$, as this could be used to correct for systematic errors in actual applications. Before seeking such relations, we observe in Figure 3 that the errors of the different contrast variables appear roughly proportional to each other. Such a relationship should be incorporated into our functions. In Figure 4 we plot the ratios of the errors. Despite significant scatter in the values there is a definite clustering about -1 for the ratio of $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$ errors for all methods. In the pseudo-quadratic method, the error in $\Delta\beta/\beta$ is in a ratio of ± 1 with both $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$ errors, while in the Aki-Richards and pseudo-linear methods the ratios are closer to $\pm$ ½. We can now incorporate these ratios into our functions. After analyzing our data, we obtain the following relations:

$$\Delta\alpha/\alpha^{PQ}_{corr} = \Delta\alpha/\alpha^{PQ}_{est} - 0.35(\Delta\beta/\beta^{PQ}_{est})^3, \qquad \Delta\beta/\beta^{PQ}_{corr} = \Delta\beta/\beta^{PQ}_{est} - 0.35(\Delta\beta/\beta^{PQ}_{est})^3$$
$$\Delta\rho/\rho^{PQ}_{corr} = \Delta\rho/\rho^{PQ}_{est} + 0.35(\Delta\beta/\beta^{PQ}_{est})^3$$
$$\Delta\alpha/\alpha^{PL}_{corr} = \Delta\alpha/\alpha^{PL}_{est} + 0.5(\Delta\beta/\beta^{PL}_{est})^2 + 0.35(\Delta\beta/\beta^{PL}_{est})^3$$
$$\Delta\beta/\beta^{PL}_{corr} = \Delta\beta/\beta^{PL}_{est} + (\Delta\beta/\beta^{PL}_{est})^2 + 0.7(\Delta\beta/\beta^{PL}_{est})^3$$
$$\Delta\rho/\rho^{PL}_{corr} = \Delta\rho/\rho^{PL}_{est} - 0.5(\Delta\beta/\beta^{PL}_{est})^2 - 0.35(\Delta\beta/\beta^{PL}_{est})^3.$$

The equations for $\Delta\alpha/\alpha^{AR}{}_{corr}$, etc., use the same coefficients as in the pseudo-linear case. These expressions are promising as a means of removing systematic error (but not random error) from predicted results. However further study will be required to properly assess their validity with real seismic data.

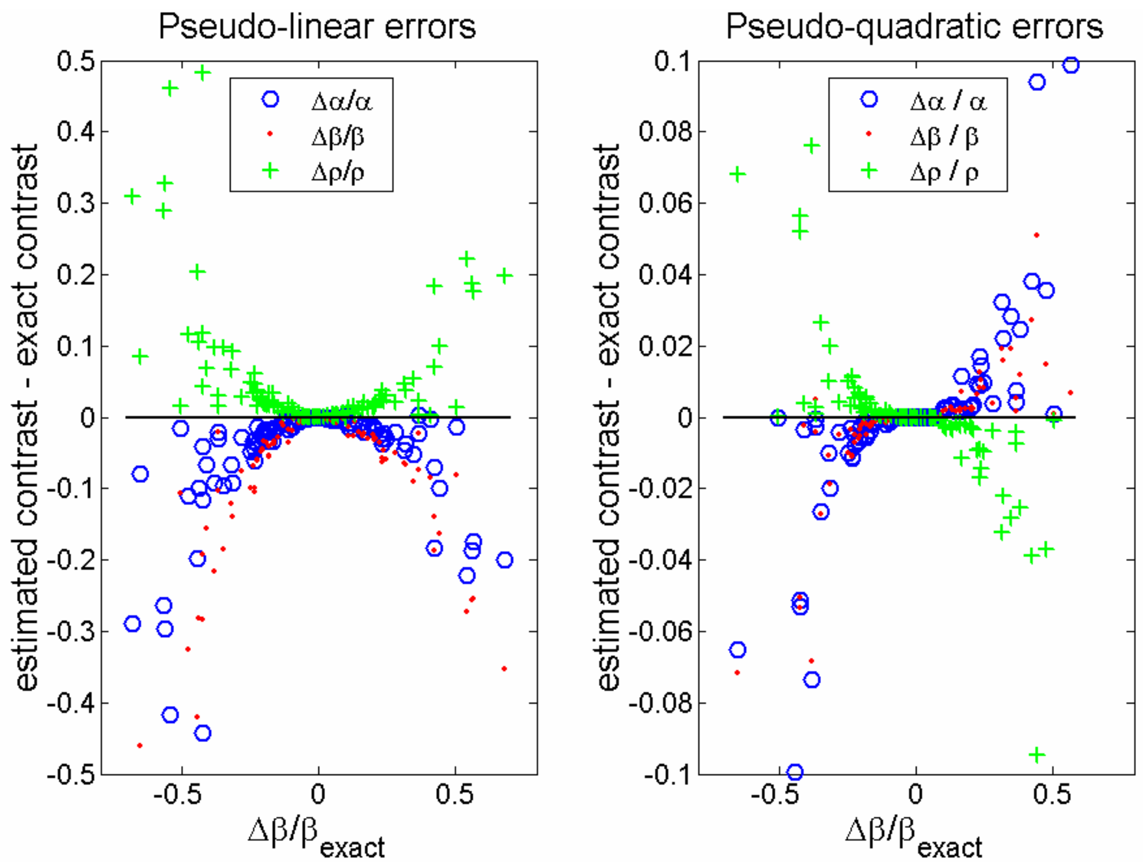

FIG. 3. Errors in the estimate of earth parameter contrasts for pseudo-linear and pseudo-quadratic methods. (The Aki-Richards predictions appear similar to those of the pseudo-linear method.) Note that the errors are strongly correlated with the value of $\Delta\beta/\beta$.

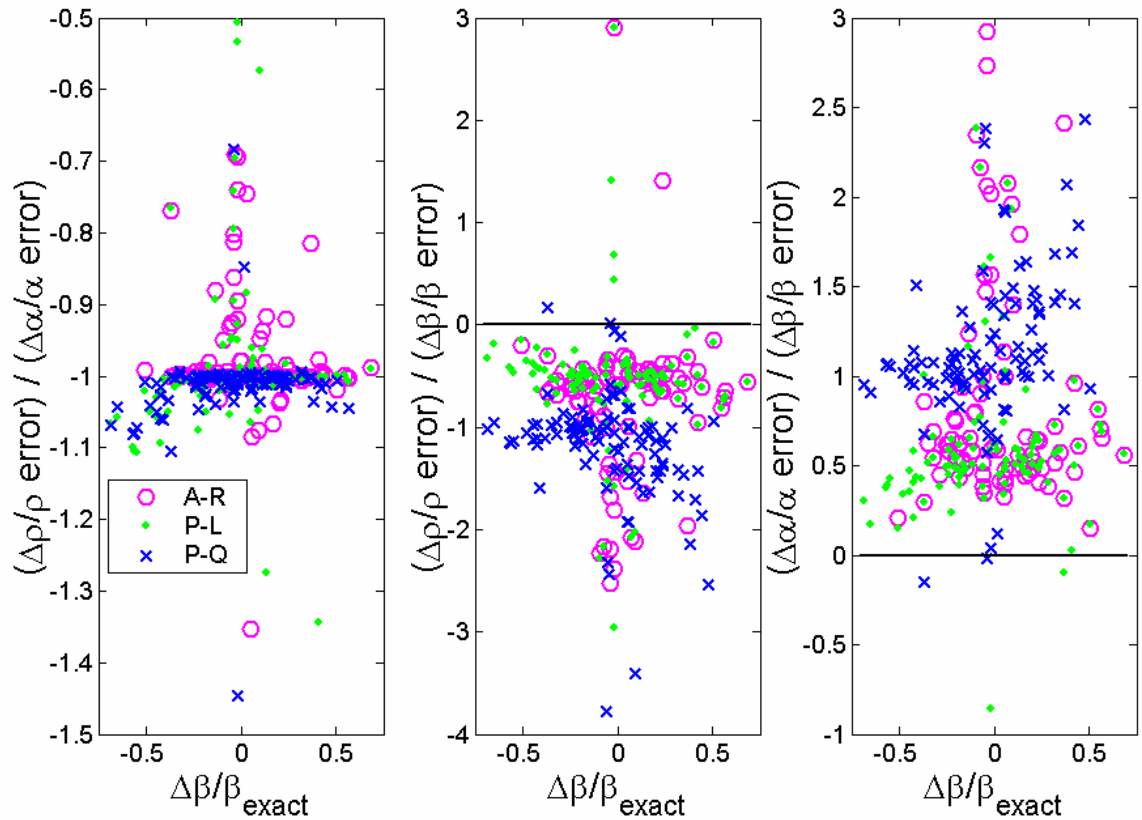

FIG. 4. Ratios of estimation errors for different variable contrasts. Note the clustering about ±1 and ±½.

We conclude this section by considering the relative magnitude of errors from the three methods being considered as displayed in Figure 5. In one case we have plotted against $\Delta\alpha/\alpha_{exact}$, and in the other against $\Delta\beta/\beta_{exact}$. In the first plot we see that the Aki-Richards and pseudo-linear methods are roughly equivalent for positive $\Delta\alpha/\alpha$, although the pseudo-linear method is sporadically superior. For negative $\Delta\alpha/\alpha$ however, the pseudo-linear method becomes rapidly superior, except for a few isolated points. In the second plot we see that for $|\Delta\beta/\beta_{exact}| < 0.2$, the pseudo-quadratic method is typically an order of magnitude superior to the linear methods. This difference diminishes outside that region.

CONCLUSIONS

We are able to draw a number of conclusions from the results presented here. We have first of all demonstrated that quadratic approximations and even the exact Zoeppritz equation can be easily employed in an AVO inversion using an iterative application of the pseudo-linear forms described here. However this convergence becomes more difficult near the critical point. The Aki-Richards method also has convergence problems for large negative $\Delta\alpha/\alpha$, but other methods discussed here do not.

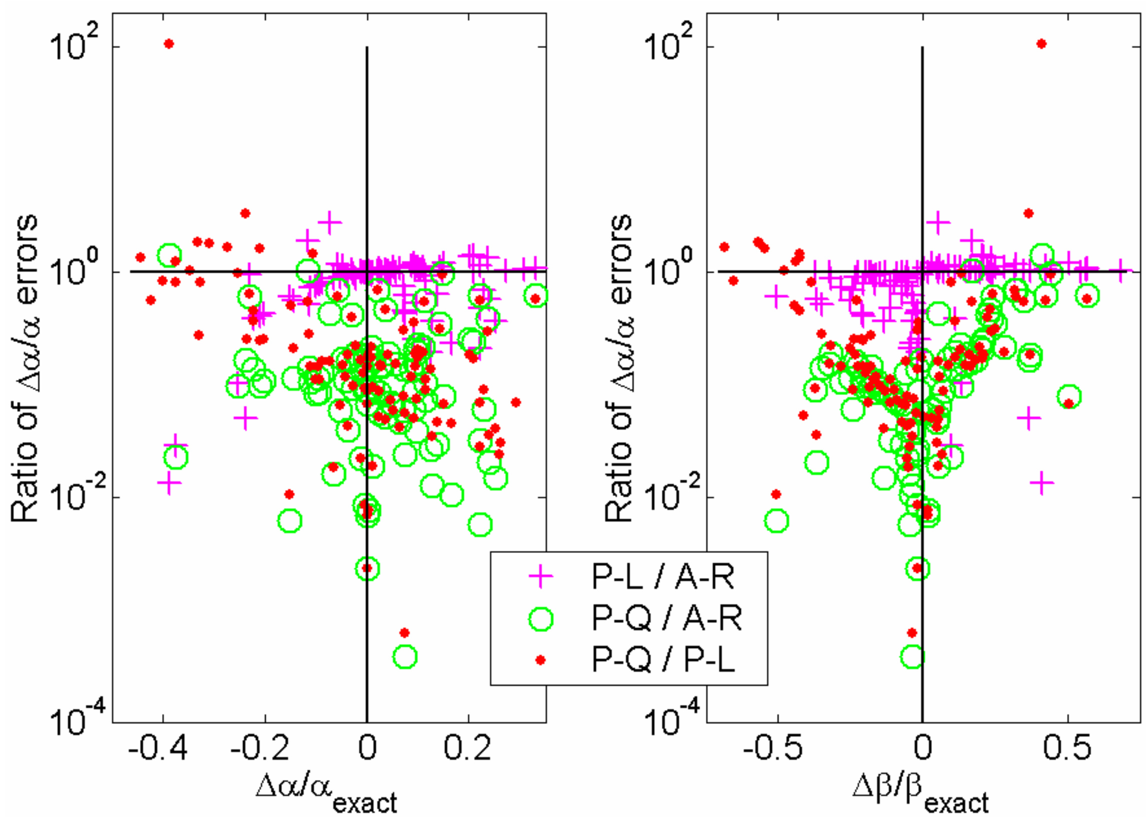

FIG. 5. A comparison of the accuracy of three theoretical expressions for AVO inversion.

For convergent solutions we have seen that errors in predicted contrasts are markedly correlated with $\Delta\beta/\beta$, so much so that it is reasonable to create empirical relations for correcting the systematic (but not random) error for this synthetic data. Finally we have seen that the pseudo-linear and Aki-Richards methods are of similar quality for positive $\Delta\alpha/\alpha$, but that the former is superior for negative $\Delta\alpha/\alpha$, and that the pseudo-quadratic method is strongly superior for moderate values of $\Delta\beta/\beta$.

ACKNOWLEDGEMENTS

The author wishes to thank Jon Downton for several helpful discussions on AVO topics.

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APPENDIX

In Figure A-1 below we demonstrate that the magnitudes of the pseudo-quadratic approximation are accurate in the initial part of the critical region. In the critical region one must also consider phase, and we show in Figure A-2 that both the pseudo-linear and pseudo-quadratic functions yield quite accurate phases, particularly in comparison with the Aki-Richards approximation.

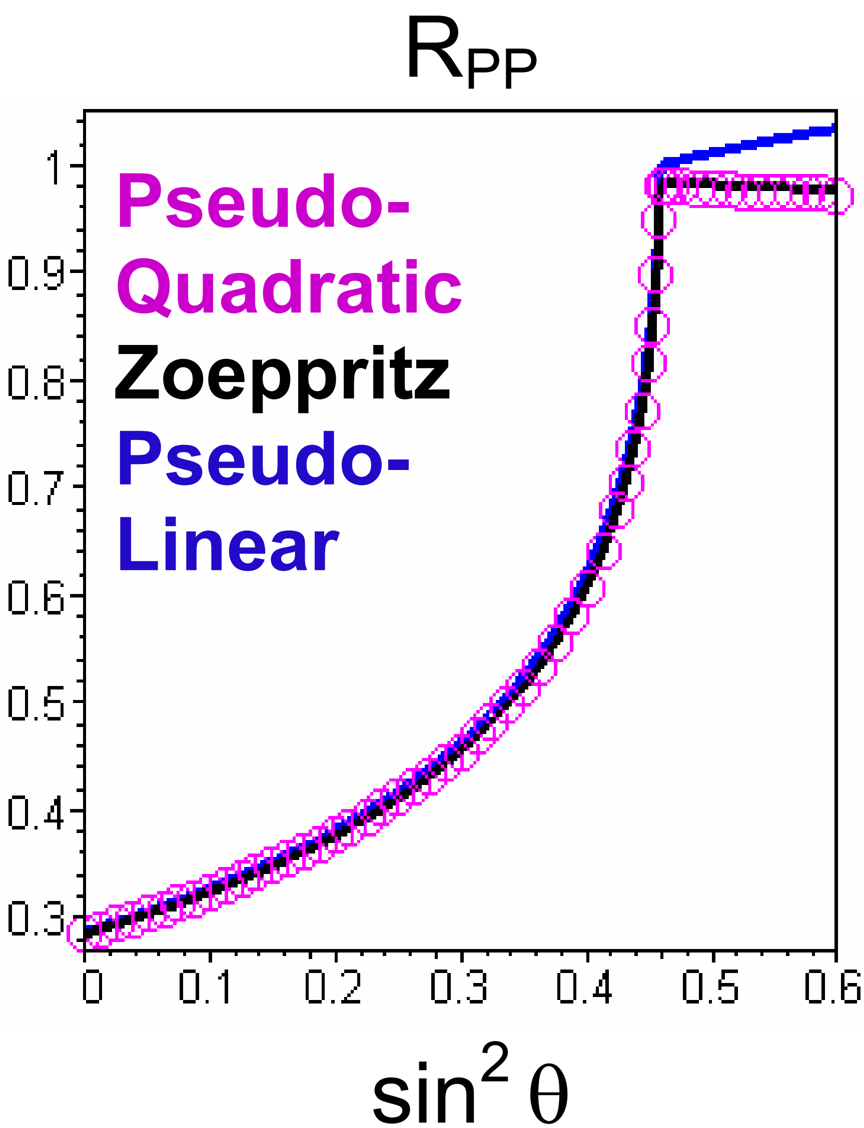

FIG. A-1. Both the pseudo-linear and pseudo-quadratic approximations appear reasonably accurate below the critical point, although a more careful analysis would should that the quadratic level is still considerably more accurate. Beyond the critical point, even visual inspection reveals that only the pseudo-quadratic theory is faithful to the exact magnitude.

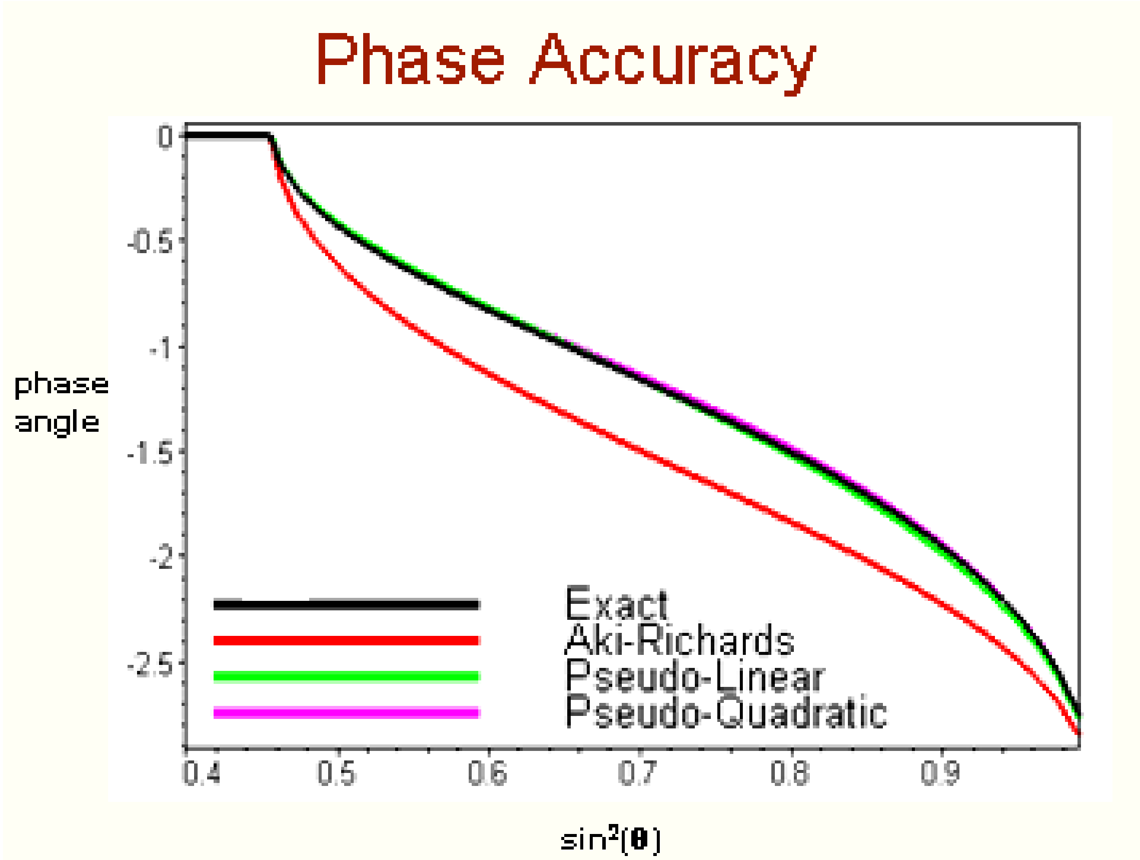

FIG. A-2. The phase angles (in units of pi) are plotted from just before the critical angle to nearly 90°. It is apparent that, while the Aki-Richards approach yields a reasonable approximation to the phase, the pseudo-linear and pseudo-quadratic methods provide a more quantitative representation.