Testing pseudo-linear Zoeppritz approximations: Multicomponent and joint AVO inversion

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ABSTRACT

Formulas for AVO (Amplitude Variation with Offset) inversion are briefly reviewed and two new expressions are presented. AVO inversion of P-wave, converted-wave, and joint data is carried out using experimental parameters and three theoretical techniques. It is shown that inversion of converted-wave data is more accurate than other sources. The accuracy of multicomponent inversion can be further improved using the pseudo-linear approximation presented here.

INTRODUCTION

AVO inversion can be used to extract elastic data in a variety of forms. One common objective is the extraction of impedance contrasts. These are given as $\Delta I_P/I_P = \Delta\alpha/\alpha + \Delta\rho/\rho$ and $\Delta I_S/I_S = \Delta\beta/\beta + \Delta\rho/\rho$, where I_P and I_S are P-wave and S-wave impedances, α and β are P-wave and S-wave velocities, and ρ is the density. Contrasts of the form $\Delta A/A$ are defined as the difference in A across the interface, divided by its average across the interface. Fatti et al. (1994) demonstrated that one could reasonably extract I_P and I_S contrasts and discard ρ information. Goodway et al. (1997) showed that I_P and I_S could be profitably converted to the Lamé parameters, λ and μ. These references highlight some of the current interest in impedances.

In this study we consider the extraction of impedance contrast from conventional and converted-wave data using three theoretical methods (some of these presented here for the first time). In this study we employ a large sample of experimental interface parameters.

We have previously put forward a "pseudo-linear" (Ursenbach, 2002) and "pseudo-quadratic" (Ursenbach, 2003a) expression for the P-wave reflectivity, R_{PP}, for use in AVO inversion. In these approximations to the Zoeppritz coefficients, R_{PP} is expanded to linear or quadratic order in $\Delta\rho/\rho$ and $\Delta\beta/\beta$, but retains exact dependence on $\Delta\alpha/\alpha$. The latter was found to be important for accuracy near the critical point. It was also shown that relatively simple expressions can still be preserved. For instance, the pseudo-linear expression has the same linear structure as the Aki-Richards approximation, but the coefficients are explicitly dependent upon $\Delta\alpha/\alpha$ – hence the "pseudo". An attempt was also made to obtain comparable expressions for converted-wave reflectivity, R_{PS}, (Ursenbach, 2002), but these were somewhat more cumbersome. We here present new pseudo-linear (P-L) and pseudo-quadratic (P-Q) expressions for R_{PS} that are quite compact, and we test them against the traditional Aki-Richards (A-R) method, paying particular attention to their value in estimating impedances.

THEORY

Using symbolic mathematics software it is straightforward to obtain a Taylor expansion of R_{PS}, but some effort is still required to reduce the formal result down to a compact and useful expression. We present the result of this effort below, and refer to it as the pseudo-quadratic approximation, in that it has the form of an approximation which is quadratic in $\Delta\beta/\beta$ and $\Delta\rho/\rho$, but the coefficients are in fact nontrivially dependent upon $\Delta\alpha/\alpha$.

$$
\begin{aligned}
R_{PS}^{PQ} = \frac{-\sin\theta_1\cos\theta_1}{Q\cos\varphi} & \left\{ \left(1+\frac{\Delta\alpha}{2\alpha}\right)\left(-2\sin^2\varphi\frac{\Delta\mu}{\mu}+\frac{\Delta\rho}{\rho}\right) \right. \\
& \times\left[1+\frac{1}{2}\frac{\Delta\beta}{\beta}-\frac{\cos\theta_2}{Q}\left(1-\frac{\Delta\alpha}{2\alpha}\right)\left(4\sin^2\varphi\frac{\Delta\mu}{\mu}-\frac{\Delta\rho}{\rho}\right)\right] \\
& +2\frac{\beta}{\alpha}\cos\theta_2\cos\varphi\frac{\Delta\mu}{\mu} \\
& \left. \times\left[1-\frac{1}{2}\tan^2\varphi\frac{\Delta\beta}{\beta}+\frac{\cos\theta_1}{Q}\left(1+\frac{\Delta\alpha}{2\alpha}\right)\left(4\sin^2\varphi\frac{\Delta\mu}{\mu}-\frac{\Delta\rho}{\rho}\right)\right]\right\}.
\end{aligned}
\tag{1}
$$

(Note that elsewhere this expression was incorrectly labelled as R_{PP} instead of R_{PS} (Ursenbach, 2003b).) In this expression θ_1 and θ_2 are the P-wave reflection and transmission angles, and φ_1 and φ_2 are the same for S-waves. We also employ the following definitions:

$$
Q=\left(1+\frac{\Delta\alpha}{2\alpha}\right)\cos\theta_1+\left(1-\frac{\Delta\alpha}{2\alpha}\right)\cos\theta_2
$$

$$
\sin\varphi=\frac{\sin\varphi_1+\sin\varphi_2}{2}=\frac{\beta}{\alpha}\left(1-\frac{\Delta\alpha}{2\alpha}\right)^{-1}\sin\theta_1
$$

$$
\frac{\Delta\mu}{\mu}=2\frac{\Delta\beta}{\beta}+\frac{\Delta\rho}{\rho}.
$$

To reduce Equation (1) to the pseudo-linear expression we simply replace by unity each of the two bracketed quantities of the form [1 ± …].

The approximation that we have just termed pseudo-quadratic might in practical terms also reasonably be termed pseudo-linear, in the sense that we may view it as being linear in $\Delta\beta/\beta$ and $\Delta\rho/\rho$, but with coefficients that depend, not only on $\Delta\alpha/\alpha$, but also linearly on $\Delta\beta/\beta$ and $\Delta\rho/\rho$.

For completeness we also include the exact Zoeppritz given in a pseudo-linear form, analogous to that given for R_{PP} in Ursenbach (2003a):

$$1/4\,R_{PS}\left(\left(1+1/2\,(\Delta\beta/\beta)\right)\left(1+\frac{\Delta\rho}{2\rho}\right)\cos\phi_1+\left(1-\frac{\Delta\beta}{2\beta}\right)\left(1-1/2\,(\Delta\rho/\rho)\right)\cos\phi_2\right)$$

$$\times\left(\left(1+1/4\,(\Delta\alpha/\alpha)\,(\Delta\rho/\rho)\right)\left(\cos\theta_1+\cos\theta_2\right)+1/2\left((\Delta\alpha/\alpha)+(\Delta\rho/\rho)\right)\left(\cos\theta_1-\cos\theta_2\right)\right)$$

$$=\left(-\left(\frac{\beta}{\alpha}\right)\left(\cos\theta_2\cos\phi_2+\frac{\sin^2\theta_1\left(\frac{\beta}{\alpha}\right)\left(1+1/2\,(\Delta\alpha/\alpha)\right)\left(1+1/2\,(\Delta\beta/\beta)\right)}{\left(1-\frac{\Delta\alpha}{2\alpha}\right)^2}\right)\right.$$

$$\times\left(\cos\theta_1\left(1-\frac{\Delta\rho}{2\rho}\right)+\left(2\cos\theta_1+\left(\cos\theta_1\cos\phi_1+\frac{\sin^2\theta_1\left(\frac{\beta}{\alpha}\right)\left(1-\frac{\Delta\beta}{2\beta}\right)}{1-\frac{\Delta\alpha}{2\alpha}}\right)R_{PS}\right)\left(\frac{\beta}{\alpha}\right)^2\sin^2\theta_1\,(\Delta\mu/\mu)\left(1-\frac{\Delta\alpha}{2\alpha}\right)^{-2}\right)$$

$$\left.+\frac{\left(\frac{\beta}{\alpha}\right)^2\sin^2\theta_1\left(\left(1+\frac{\Delta\rho}{2\rho}\right)\left(1+\frac{\Delta\alpha}{2\alpha}\right)\left(1+\frac{\Delta\beta}{2\beta}\right)\cos\theta_1\left(2+R_{PS}\cos\phi_1\right)-R_{PS}\left(1-\frac{\Delta\rho}{2\rho}\right)\left(1-\frac{\Delta\beta}{2\beta}\right)\left(1-\frac{\Delta\alpha}{2\alpha}\right)\cos\theta_2\cos(\phi_2)\right)}{\left(1-\frac{\Delta\alpha}{2\alpha}\right)^2}\right)\frac{\Delta\mu}{\mu}$$

$$-1/2\left(1+\frac{\Delta\alpha}{2\alpha}\right)\left(1+\frac{\Delta\beta}{2\beta}\right)\left(\cos\theta_1\left(1+\frac{\Delta\rho}{2\rho}\right)+\frac{R_{PS}\sin^2\theta_1\left(\frac{\beta}{\alpha}\right)\left(1-\frac{\Delta\beta}{2\beta}\right)\frac{\Delta\rho}{\rho}}{2\left(1-\frac{\Delta\alpha}{2\alpha}\right)}-2\frac{R_{PS}\sin^4\theta_1\left(\frac{\beta}{\alpha}\right)^3\left(1-\frac{\Delta\beta}{2\beta}\right)\frac{\Delta\mu}{\mu}}{\left(1-\frac{\Delta\alpha}{2\alpha}\right)^3}\right)\frac{\Delta\rho}{\rho}$$

CALCULATIONS

We employ the velocity and density data given in Table 1 of Castagna and Smith for 25 sets of shale, brine sand, and gas sand. For each set we consider the shale-over-brine, brine-over-shale, shale-over-gas, gas-over-shale, and gas-over-brine interfaces. This yields a set of 125 interfaces. For each interface we calculate three contrasts and β/α. Figure 1 contains a snapshot of these four quantities, and we observe no strong correlations. A weak Gardner correlation (slope = 1/4) is present between $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$, and a weak linear correlation appears to be present between $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$, but in both cases there is considerable scatter. This valuable data set then represents a broad sampling of earth interface models.

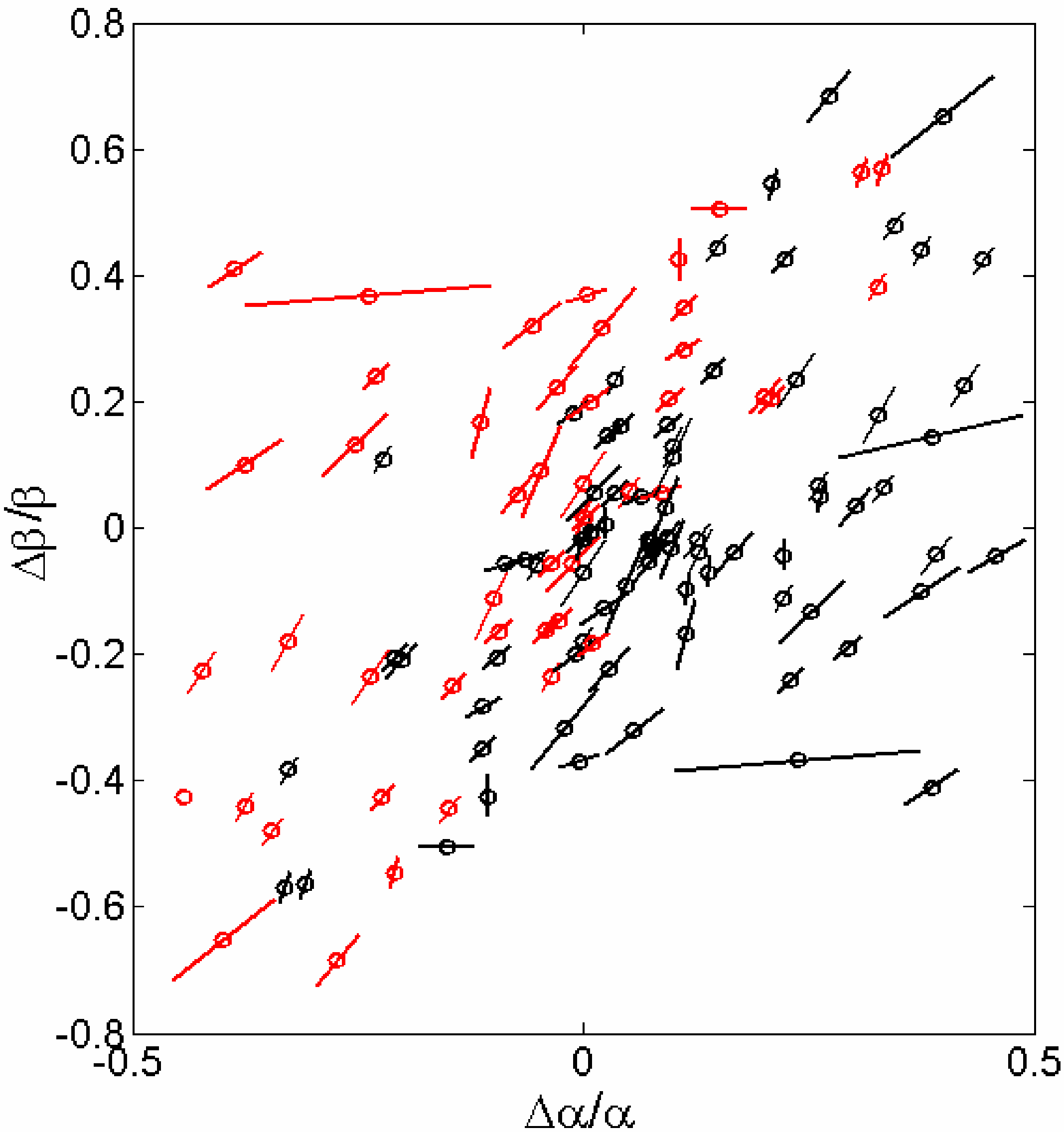

FIG. 1: A snapshot of data for 125 interfaces generated from Table 1 of Castagna and Smith. For each interface we calculate Δα/α, Δβ/β, Δρ/ρ, and β/α. Data for each interface is plotted above, with Δα/α and Δβ/β forming the axis coordinates, and Δρ/ρ being proportional to the length of each line (red negative, black positive). β/α is related to the angle of each line, with β/α = 0.45 given by horizontal lines, β/α = 0.60 given by vertical lines, and intermediate values (which form the bulk of the data) distributed linearly between with positive slopes. The trend from red to black with increasing Δα/α is evidence of Gardner's relation.

To carry out inversion, synthetic amplitudes were generated from the exact Zoeppritz expressions for R_{PP} and R_{PS} at $\theta = 0°, 1°, 2°, \ldots, 30°$. The various theories (A-R, P-L, P-Q) are then each used to construct a matrix relating the contrasts to the synthetic amplitudes. For A-R and P-L methods we require background β/α and Δα/α values. The P-Q method requires these and also Δβ/β and Δρ/ρ. We begin by using the exact β/α and setting all contrasts initially to zero. This system is inverted using a simple least squares procedure to obtain the estimated contrasts. The contrasts thus obtained are substituted back into the matrix of coefficients, which are again inverted. We iterate in this manner until convergence is reached. In some cases convergence is not achieved. We have found that this happens more frequently when inverting R_{PP} and joint data than when inverting R_{PS} data alone. Convergence problems are also more frequent for the Aki-Richards method than for the new methods presented here. There is little difference though whether one is inverting for velocities or directly for impedances. For this study we will simply discard the unconverged values. For the majority of parameters, convergence is obtained, yielding an ensemble of estimated contrasts that can then be analyzed.

In Figure 2 we display the error in prediction of $\Delta\rho/\rho$, $\Delta\alpha/\alpha$, $\Delta I_P/I_P$, $\Delta\beta/\beta$, and $\Delta I_S/I_S$ for the familiar Aki-Richards inversion of R_{PP} data. At least two interesting features appear in these plots. First, a systematic trend combined with scatter is apparent for all variables, especially for the impedance. This is because errors have been plotted against $\Delta I_S/I_{S,exact}$ which appears to be the dominant factor in determining the error. In contrast, when the same data is plotted against $\Delta I_P/I_{P,exact}$ or $\Delta\rho/\rho_{exact}$, the errors appear not to follow a significant trend.

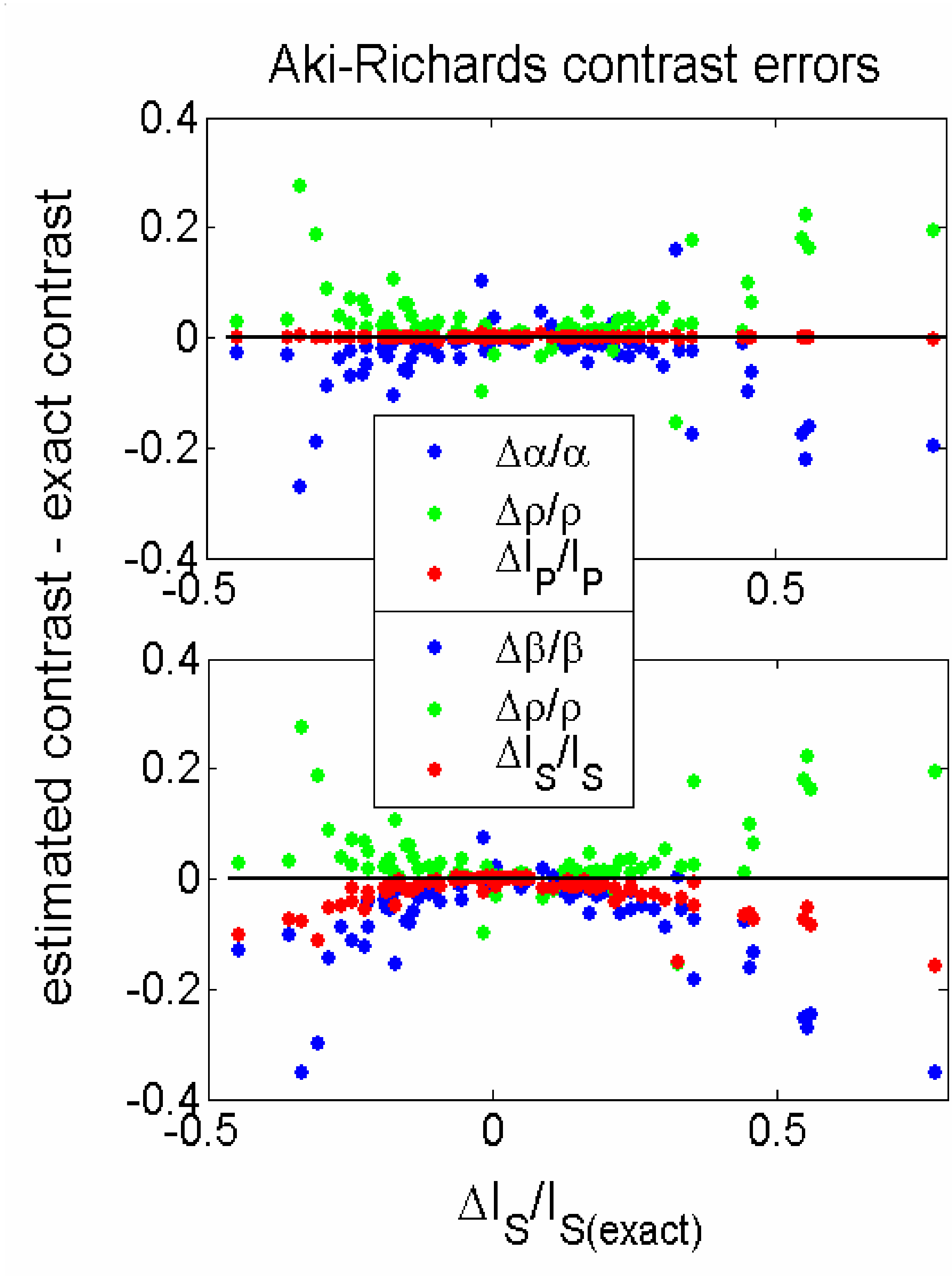

FIG. 2: Errors in $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, $\Delta\rho/\rho$, $\Delta I_P/I_P$, and $\Delta I_S/I_S$, as obtained from synthetic R_{PP} amplitudes by theAki-Richards theory.

The second point of interest is the thorough cancellation of errors apparent in the P-impedance contrast. As $\Delta\alpha/\alpha$ is usually larger than $\Delta\rho/\rho$, this indicates that the latter will normally have the larger percentage error, as is widely observed in 3-parameter inversions. There is also some cancellation of errors in summing $\Delta\beta/\beta$ and $\Delta\rho/\rho$ to obtain $\Delta I_S/I_S$ (or equivalently in differencing $\Delta\beta/\beta$ and $\Delta\alpha/\alpha$ to obtain $\Delta(\beta/\alpha)/(\beta/\alpha)$ or the Poisson ratio contrast), but it is not nearly as complete. It is in the S-impedance then that we would be most anxious to find improved methodology. Thus we next consider other inversion approaches.

In Figure 3 we plot the errors in $\Delta I_S/I_S$ obtained by the Aki-Richards method from three different data types. The red points are the same as those in Figure 2b, but their trend is more obvious on this scale. They are compared with values from inversion of R_{PS} amplitudes, and from joint inversion. The latter methods show more scatter in their trends, but also exhibit less error overall. To illustrate, the average %-error of the three result sets are 8.6% for R_{PP}, 8.2% for R_{PS}, and 7.2% for joint inversion. Thus we see a modest improvement when converted-wave data is employed.

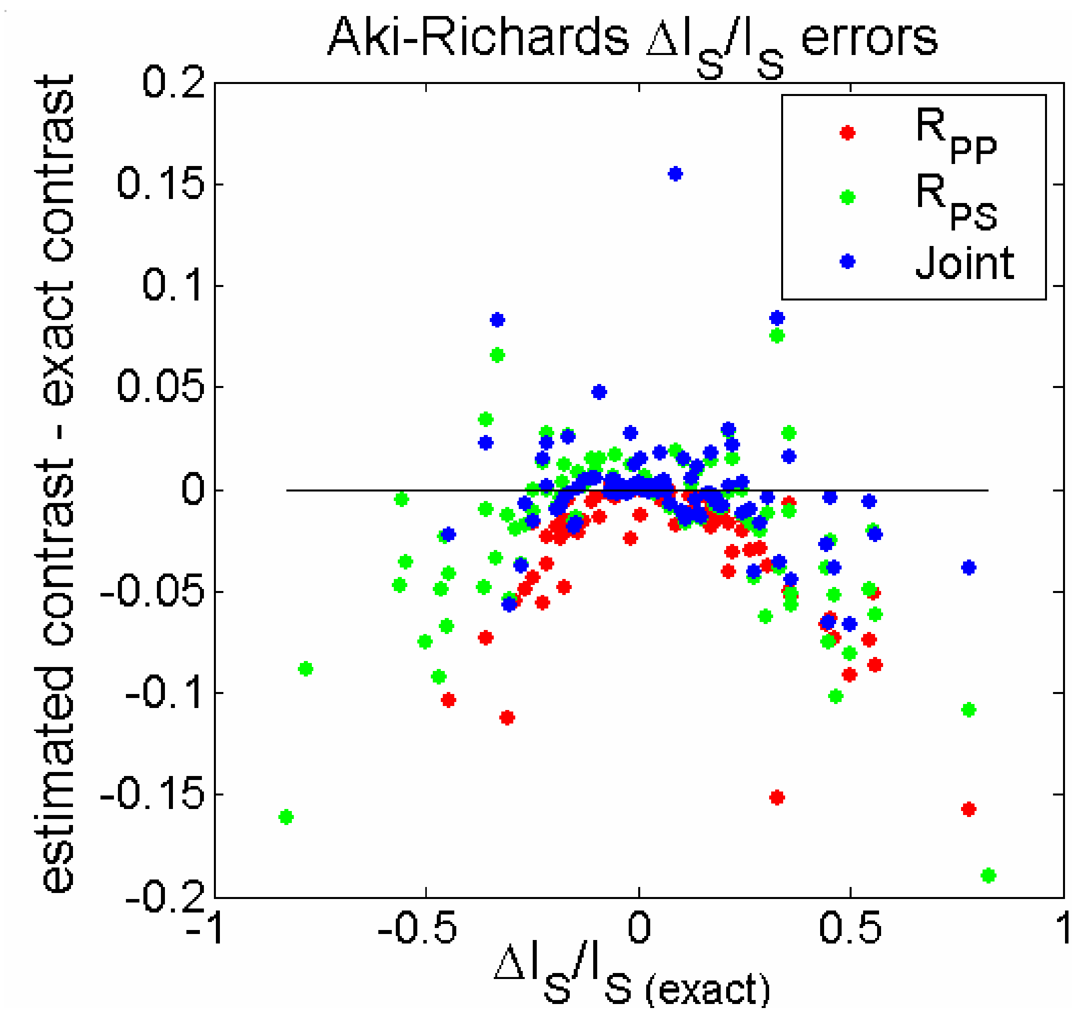

FIG. 3: Error in $\Delta I_S/I_S$, as obtained by the Aki-Richards theory from synthetic R_{PP} amplitudes, R_{PS} amplitudes and joint data.

Next we analyze results from alternate theoretical methods, with results displayed in Figures 4 and 5. The pseudo-linear results are similar in their trends to the Aki-Richards results, but the R_{PS} inversion result is somewhat more accurate. The average %-errors in this case are 13% for R_{PP}, 3.2% for R_{PS}, and 3.6% for joint inversion. The pseudo-quadratic result shown in Figure 5 is different than the others in appearance, as its principle error should be of cubic order rather than quadratic. Its average %-errors are 2.0% for R_{PP}, 0.22% for R_{PS}, and 0.50% for joint inversion, so once again R_{PS} yields the best overall values.

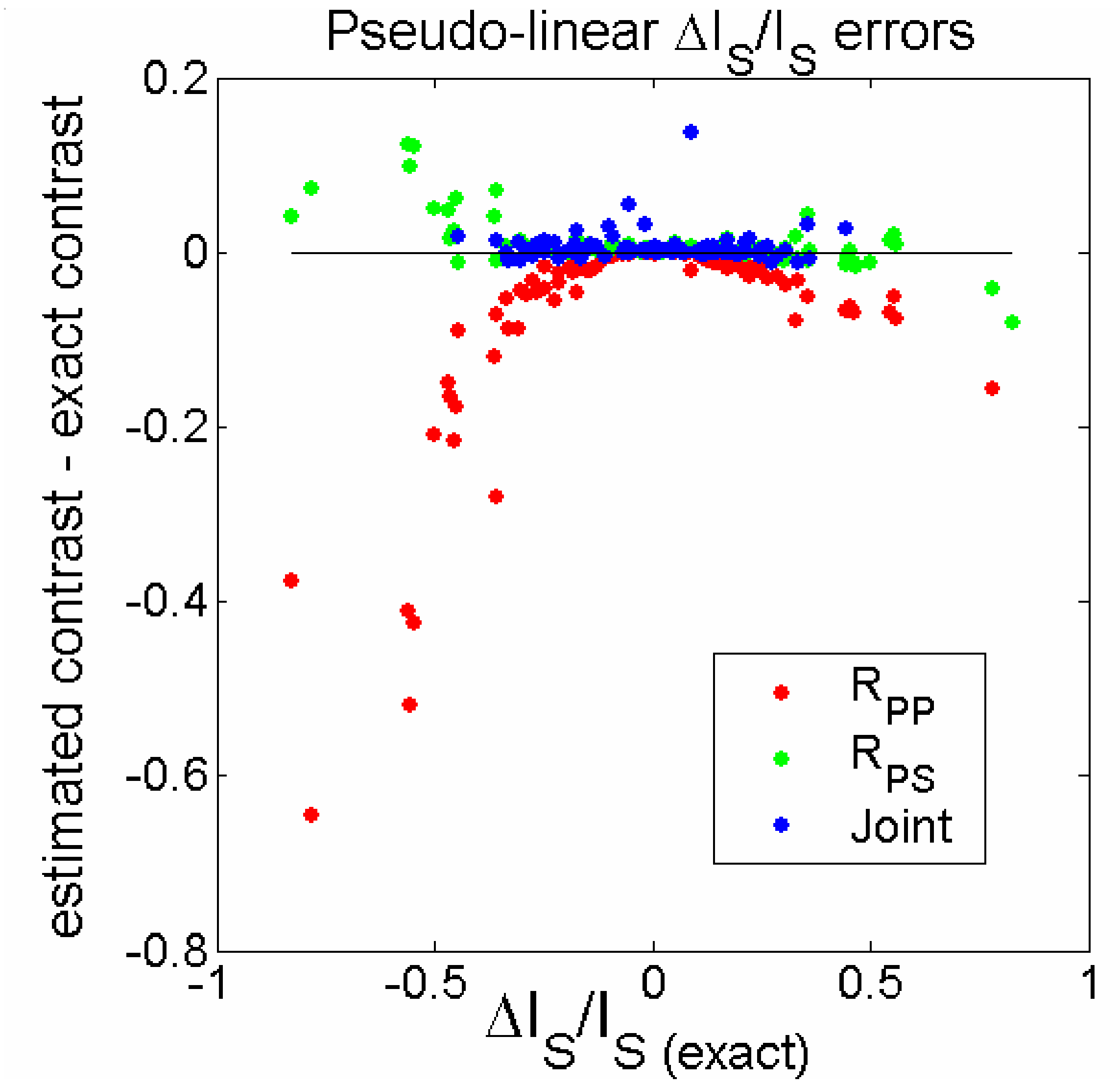

FIG. 4: Error in $\Delta I_S/I_S$, as obtained from synthetic R_{PP} amplitudes, R_{PS} amplitudes and joint data by the pseudo-linear theory.

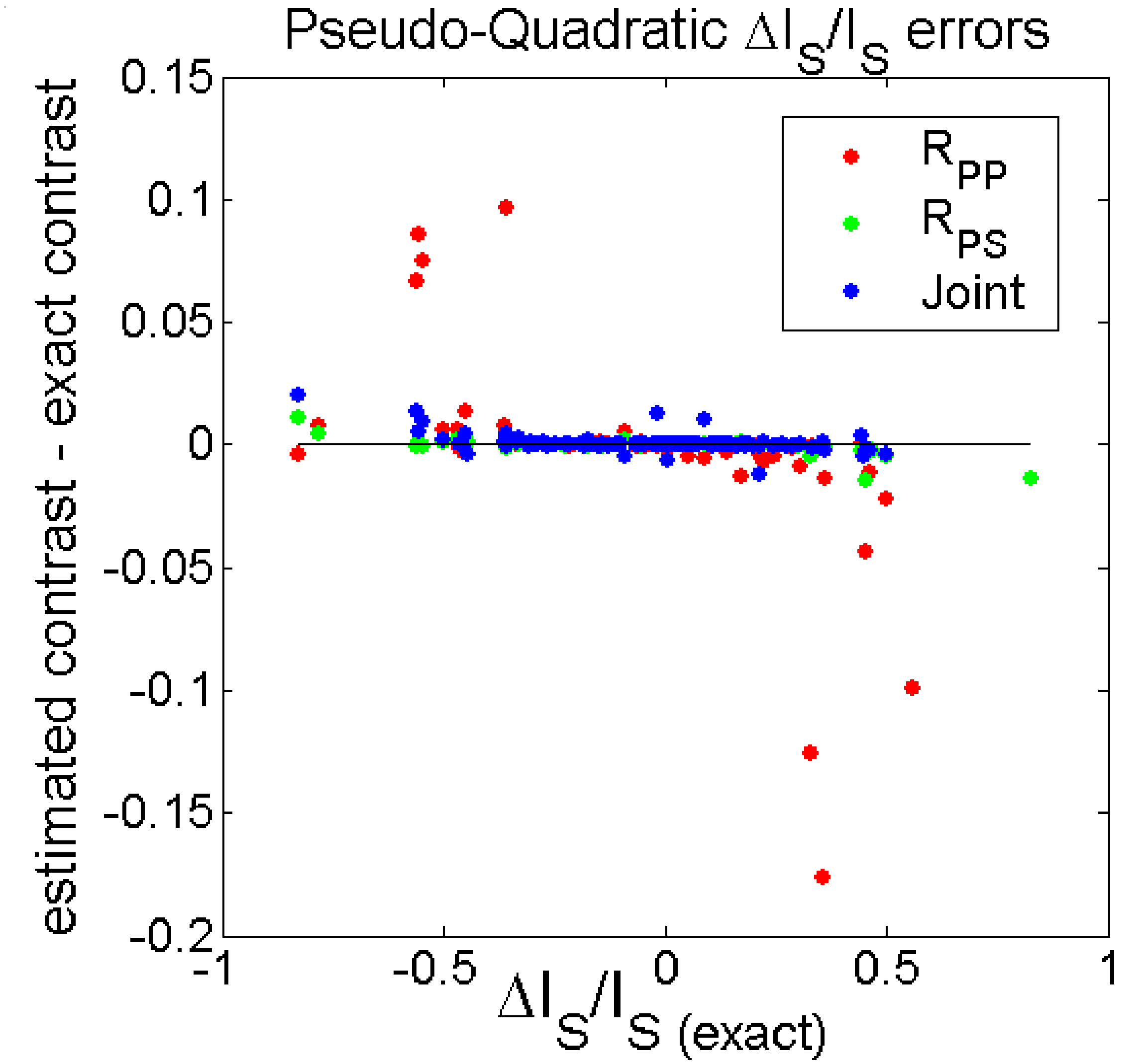

FIG. 5: Error in $\Delta I_S/I_S$, as obtained from synthetic R_{PP} amplitudes, R_{PS} amplitudes and joint data by the pseudo-quadratic theory.

The R_{PS} inversion results for all three methods are displayed on a logarithmic scale in Figure 6. Visually one can see that the pseudo-linear method is more accurate than Aki-Richards, and pseudo-quadratic is more accurate still.

DISCUSSION AND CONCLUSIONS

We are seeking to address the question of obtaining improved estimates of impedance, particularly the shear-wave impedance. We have found that inversion of R_{PS} and joint amplitudes yields more accurate results than inversion of R_{PP}. We have also found that methods such as the pseudo-linear and pseudo-quadratic techniques, which have been derived as part of this study, can extract higher accuracy from the R_{PS} and joint inversion than can the traditional Aki-Richards method. This is encouraging from a practical point of view as the pseudo-linear method can be easily implemented in place of the Aki-Richards method, requiring identical inputs.

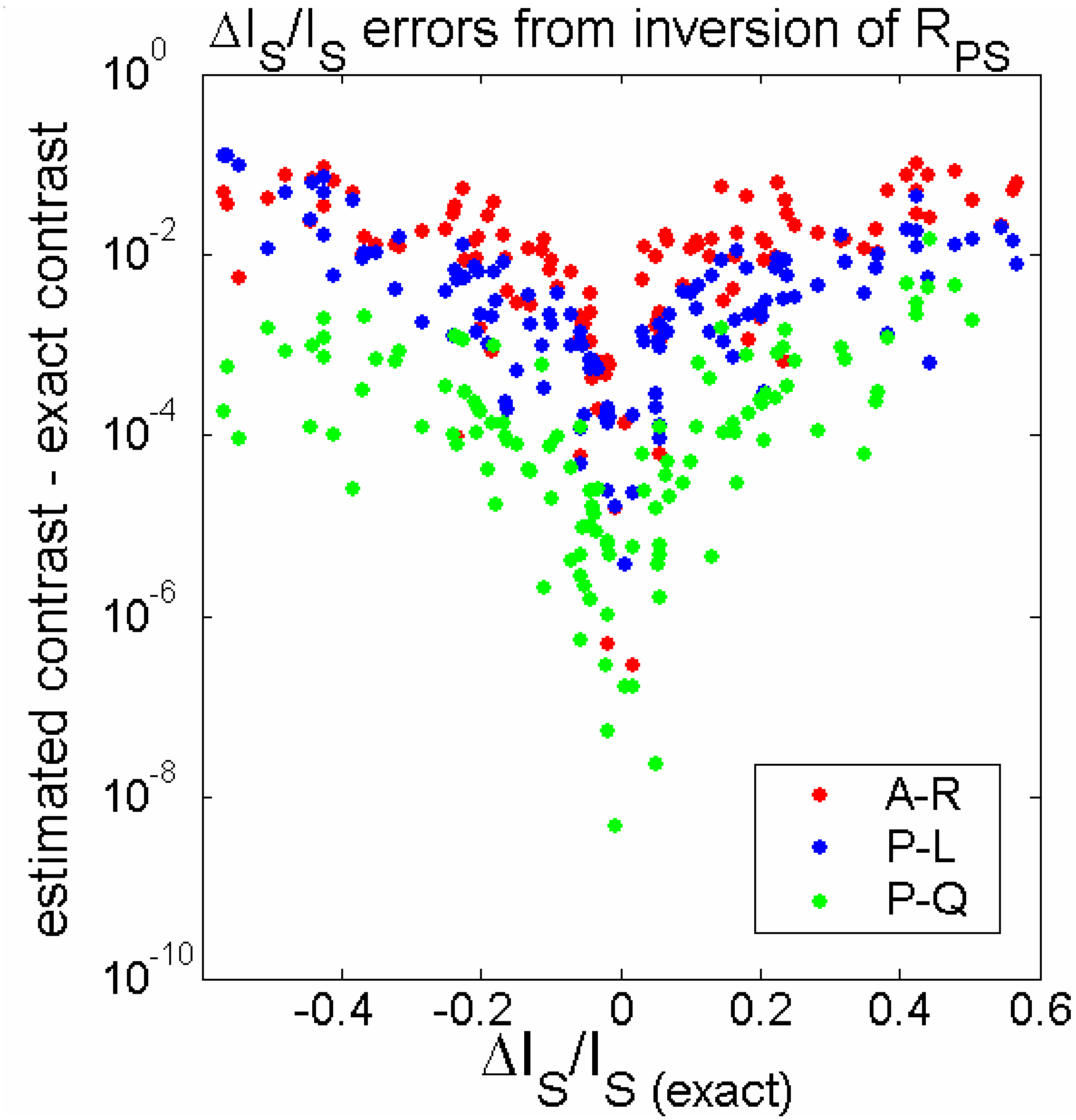

FIG. 6: Error in $\Delta I_S/I_S$, as obtained from synthetic R_{PS} amplitudes by various inversion methods.

The work in this study suggests several further directions of inquiry. A strong dependence of the errors upon $\Delta I_S/I_{S,exact}$ has been noted. This suggests other possibilities, such as the development of empirical corrections (Ursenbach, 2003a) to $\Delta I_S/I_S$ values obtained by inversion. Another important target of AVO inversion is the density contrast. Figure 7, analogous to Figure 6, shows a calculation of this quantity from converted-wave amplitudes. The pseudo-quadratic method shows considerable promise for this purpose. (The Aki-Richards result may also be compared to the density contrast displayed in Figure 2, which shows the converted-wave result to have more scatter than that of the compressional-wave.) It will also be of considerable interest to assess the degree to which the conclusions of this study hold when noise is added to various components of the inversion. Further investigation of all these issues is justified by the great value of more accurately delineating fundamental rock properties in exploration targets.

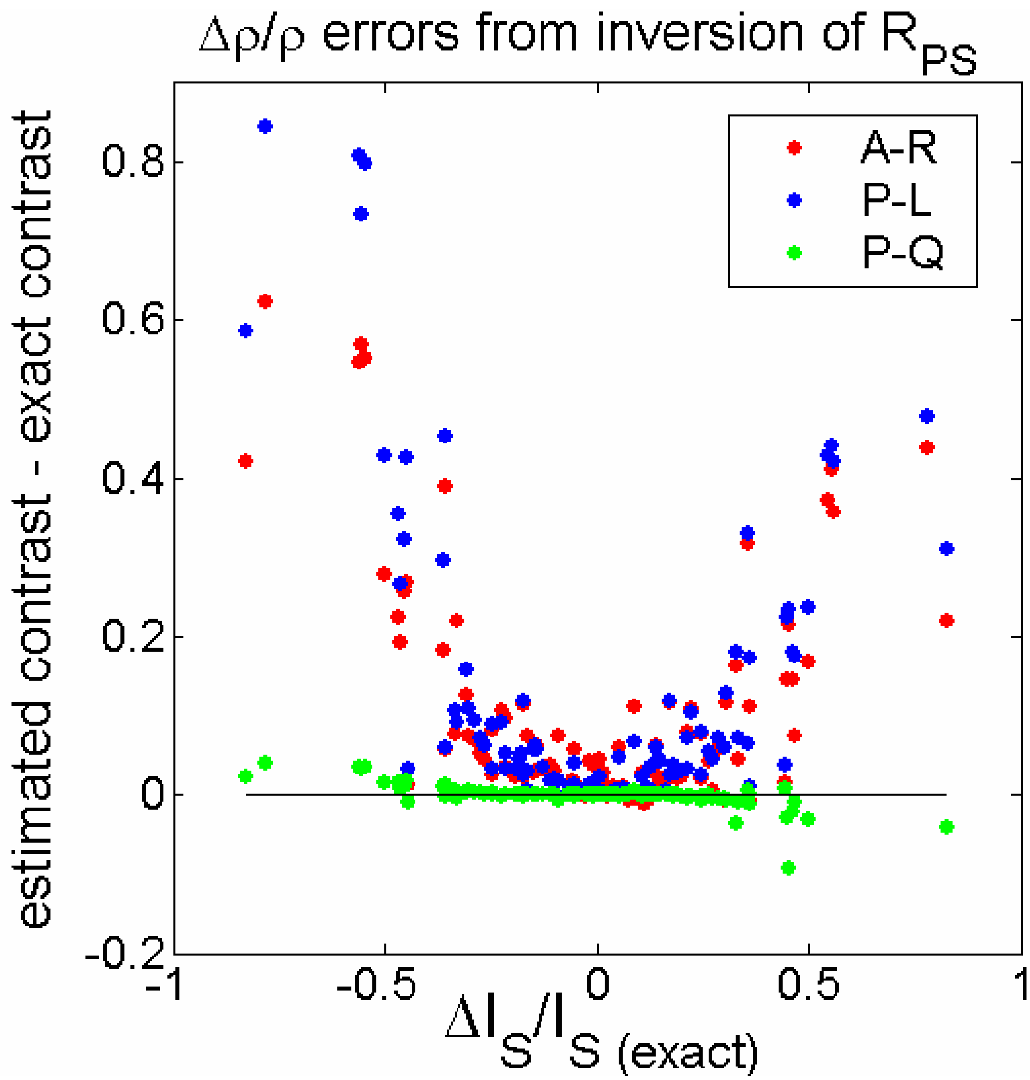

FIG. 7: Error in $\Delta\rho/\rho$, as obtained from synthetic R_{PS} amplitudes by various inversion methods.

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