Examining the phase property of the nonstationary Vibroseis wavelet

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SUMMARY

We have observed that Vibroseis wavelets behave very much as if they are minimum phase. This was discovered by applying a minimum-phase Wiener deconvolution to the separated Vibroseis-VSP downgoing waves and observing that the result is effectively a band-limited spike. Our observation was further confirmed by the similarity between Vibroseis-VSP downgoing waves and their minimum-phase equivalents. This finding contradicts the conventional assumption; therefore, it is necessary to investigate the reason for this phase property. By a simple model test, we show that both stationary and nonstationary minimum-phase filters result in an effective minimum-phase wavelet. If the Klauder wavelet is broadband, the phase of the wavelet embedded in the trace will be determined mainly by the minimum-phase factors including instrument and receiver response, far-field differential operators, and earth filtering. Furthermore, we applied a minimum-phase Gabor deconvolution to the correlated Vibroseis data and sweep-removed Vibroseis record, with the residual wavelet being close to minimum phase. For the synthetic data, the deconvolved traces from both approaches are consistent with the input reflectivity. For a real shot-gather, there are few differences between the deconvolved gathers from these two methods. These comparisons further confirm that the nonstationary wavelet embedded in the correlated surface Vibroseis seismic data is effectively minimum phase.

INTRODUCTION

In the 1980s and earlier, the wavelet embedded in correlated Vibroseis data was assumed to be the zero-phase autocorrelation of the sweep (see Brötz et al., 1987; Bickel, 1982). This assumption has been challenged by Sallas (1984) and Baeten and Ziolkowski (1990) who argued that the far-field wavelet is not an autocorrelation function but the crosscorrelation between the pilot sweep and the time derivative of the ground force, which can be estimated from a weighted sum of the vertical acceleration of the base plate and the reaction mass. Thus, the embedded Vibroseis wavelet for propagation in an elastic medium is not zero phase. This argument ignores attenuation. Others (e.g., Brittle, 2001) have stated that the presence of earth-attenuation results in a mixed-phase wavelet that is the convolution of the Klauder wavelet and the earth-attenuation minimum-phase filter, but this assumption is not verified. Therefore, the phase property of the wavelet embedded in the Vibroseis trace is still an unsolved problem. Fortunately, the phase of the wavelet can be examined empirically by studying the result of the minimum-phase deconvolution or the minimum-phase equivalent of the observed wavelet.

In a time-domain Wiener spiking deconvolution, if the wavelet is minimum phase and the reflectivity is white, the output of the deconvolution will resemble a band-limited zero-phase impulse located at the wavelet arrival time. Conversely if the output of this

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deconvolution is close to the band-limited zero-phase impulse, we can infer that the wavelet is effectively minimum phase. Alternatively, the difference between the observed wavelet and its minimum-phase equivalent shows how close the observed wavelet is to the minimum-phase one.

In this investigation, first the phase property of the Vibroseis wavelet is examined on the directly observed wavelet. We then propose an uncorrelated Vibroseis trace model and show how the Klauder wavelet and the other minimum-phase filters affect the phase of the embedded wavelet.

REAL DATA EXAMPLE AND PHASE EXAMINATION

Rosedale, Alberta

Provided by EnCana, the first dataset consists of five Vibroseis source points recorded simultaneously into VSP and surface spreads. Receivers were positioned in the borehole between 322 and 1820 m depth at an interval of 20 m. An additional 78 receivers were placed on the surface between 30 and 2310 m from the borehole at 30-m intervals. The five source points were located 27, 430, 960, 1350, and 1700 m from the borehole and a 12-s, 10–96 Hz linear sweep was used. Sixteen-second long, uncorrelated surface records and VSP records were recorded at a 2-ms sample rate.

The downgoing Vibroseis wavelets (DVWs) were directly estimated from the correlated VSP data using standard wavefield separation methods and are shown, time-aligned and normalized, in Figure 1a. Wiener spiking deconvolution was then applied to the DVWs. The Wiener inverse operator was designed with data from 0.2 to 0.3 s of Figure 1a. Operator length was 0.12 s and the stabilization factor was 0.0001. Figure 1b shows that the normalized, deconvolved wavelets are nearly identical approximations to band-limited zero-phase wavelets. This suggests that the observed DVWs are effectively minimum phase.

Pikes Peak, Saskatchewan

This experiment was conducted by Husky Energy over the Pikes Peak heavy-oil field in west-central Saskatchewan. Receivers were placed in a borehole at 7.5 m intervals from 27 m to 514.5 m. The vibration point was 23 m from the well and the sweep was linear from 8 to 200 Hz. Figure 2a shows the DVWs isolated from the correlated data, comparable to Figure 1a. Though the results of Wiener deconvolution on these wavelets were also symmetrical, band-limited impulses, we choose to show an alternate test of minimum phase. In Figure 2b, we show the minimum-phase equivalent wavelets (MPEs) for each observed wavelet in Figure 2a. These were calculated by inverting the operator obtained by running the Levinson algorithm on the DVWs, using all autocorrelation lags, with a white-noise stabilization factor of 0.0001. The MPEs are nearly identical to the observed DVWs; and to demonstrate this, we show the difference plot in Figure 2c. The differences were calculated after careful alignment of the wavelets using a crosscorrelation. Figure 2d shows that the correlation coefficients of the MPEs with DVWs are close to unity which means DVWs are effectively minimum phase.

Ross Lake, Saskatchewan

This experiment was conducted by Husky Energy and CREWES over the Ross Lake oilfield in southwest Saskatchewan. Receivers were placed in a borehole at 7.5 m intervals from 198 m to 1165 m. The vibration point was 54 m from the well and the sweep was linear from 8 to 180 Hz. Figure 3a shows the downgoing wavelets isolated from the correlated data, comparable to Figure 1a. Though the results of Wiener deconvolution on these wavelets also gave symmetrical, band-limited impulses, we choose again to show an alternate test of minimum phase. In Figure 3b we show the MPE for each observed wavelet in Figure 3a. The MPEs are nearly identical to the observed DVWs; and to demonstrate this, we show the difference plot in Figure 3c. The differences were calculated after careful alignment of the wavelets using a cross correlation. Again, the very small differences indicate that the DVWs are actually minimum phase.

A THEORETICAL PERSPECTIVE

An uncorrelated Vibroseis trace model, without multiples, can be written as

$$X_{ob} = W_v * I_{ns} * R_{ec} * \cdots * D_{iff} * Q_{filt} \bullet r, \tag{1}$$

where W_v represents either the Klauder wavelet or the wavelet resulting from frequency domain sweep deconvolution (Brittle and Lines, 2001); I_{ns} and R_{ec} are instrument and receiver responses, respectively; D_{iff} is the far-field differential operator (Baeten and Ziolkowski, 1990); Q_{filt} is the nonstationary earth filter; r is the reflectivity; and the ellipsis (…) symbolizes any other possible filters required to make our theory more realistic. We explicitly assume that all terms on the right-hand side of equation (1) are minimum phase except for W_v which is zero phase and r which is random phase. Also symbol $*$ stands for stationary convolution while $\bullet$ represents nonstationary convolution. Working in a local window, the amplitude spectrum of X_{ob} is approximately

$$\left|\hat{X}_{ob}\right| = \left|\hat{W}_v\right|\left|\hat{I}_{ns}\right|\left|\hat{R}_{ec}\right|\cdots\left|\hat{D}_{iff}\right|\left|\hat{Q}_{filt}\right|, \tag{2}$$

where the hat (^) indicates the Fourier transform and we assume that the amplitude spectrum of r is unity. This represents the amplitude spectrum of the embedded wavelet in our model and is what a perfect deconvolution algorithm would estimate. The deconvolution algorithm would then calculate a phase by the Kolomogorov technique which gives

$$\phi_{\min} = H\left(\ln\left|\hat{W}_v\right|\right) + H\left(\ln\left|\hat{I}_{ns}\right|\right) + \cdots + H\left(\ln\left|\hat{Q}_{filt}\right|\right). \tag{3}$$

where H denotes the Hilbert transform. Equation (3) is also the phase of the MPE associated with the embedded wavelet. Since, by assumption, all filters are minimum phase except the first, which is zero phase, this differs from the true phase only by the first term. It follows that we can explain our observations if

$$\left|H\left(\ln\left|\hat{W}_v\right|\right)\right| << \left|H\left(\ln\left|\hat{I}_{ns}\right|\right)+\cdots+H\left(\ln\left|\hat{Q}_{filt}\right|\right)\right|. \tag{4}$$

This is in fact usually true because $\left|\hat{W}_v\right| = 1$, and hence $\ln\left|\hat{W}_v\right| = 0$, for all frequencies except those near the end of the sweep. The effect of the end of the sweep is not large because the Hilbert transform is effectively a local operator.

Therefore, we expect, from theoretical grounds, that the phase of the minimum-phase equivalent should be nearly equal to the phase of the Vibroseis wavelet because:

- The Vibroseis effect is contained in a filter that has a broad-band, unit amplitude spectrum and is zero phase.

- The Hilbert transform will calculate a very small phase from such a result.

- All other filters involved are minimum phase or nearly so.

Now we merge all the stationary minimum-phase filters and write an effective minimum-phase wavelet as

$$W_{eff} = W_v * W_{\min} \tag{5}$$

where $W_{\min}$ denotes merged minimum-phase equivalent. Figure 4 shows the amplitude and phase spectra of $W_{\min}, W_v$ and W_{eff} for simple assumed forms for the first two. It indicates that the Klauder wavelet makes a very small contribution to the phase of the convolved result which is effectively a minimum-phase equivalent.

To investigate the influence of the nonstationary earth filter, here we build an uncorrelated Vibroseis record modelled as the nonstationary convolution of the pilot sweep, the constant-Q attenuation function, and reflectivity, and written as (adapted from Margrave et al., 2003)

$$x_d(t) = \frac{1}{2\pi}\int\left[\hat{s}(\omega)\int\alpha(\tau,\omega)r(\tau)e^{-i\omega\tau}\,d\tau\right]e^{i\omega\tau}\,d\omega\,, \tag{6}$$

where $x_d(t)$ represents the uncorrelated trace, $\hat{s}$ denotes the spectrum of the sweep, r is reflectivity, and α is the Q attenuation function (Margrave et al., 2003).

When the resulting wave travels in a horizontally layered, constant-Q medium, with Q equal to 60 for all the layers, it will be attenuated, reflected, and finally recorded on the surface. This process can be simulated by the nonstationary convolution of the sweep, the constant-Q attenuation function and the reflectivity as shown in Equation (6). Figure 5 shows the procedure described by Equation (6).

Usually we correlate $x_d(t)$ with the sweep to get a correlated trace, or do frequency domain sweep deconvolution (FDSD) on $x_d(t)$ to remove the sweep (Brittle and Lines, 2001). Figure 6 shows the traces and their amplitude spectra after crosscorrelation and

FDSD. Since the sweep is linear, two synthetic traces from the different approaches are actually identical, which means that the wavelets embedded in these two traces are similar to each other. The difference between the two amplitude spectra is caused by numerical errors in calculation.

The Gabor deconvolution (Margrave and Lamoureux, 2002) is now applied to the sweep-removed synthetic Vibroseis trace. Figure 7 shows the nonstationary wavelet estimate from the Gabor deconvolution and the impulse response of the earth filter. Here, we found that the shape of the wavelet estimate is quite close to that of the minimum-phase earth filter. It indicates that the amplitude spectrum of the reflectivity has less influence on the wavelet estimate in Gabor deconvolution.

We next applied Gabor deconvolution to both the sweep-removed and the correlated synthetic Vibroseis traces. Figure 8 shows the deconvolved traces and reflectivity. The results from both approaches are quite close to the input reflectivity and do not show large, spurious, phase rotations. It implies that the wavelet embedded in the correlated data is actually nonstationary minimum-phase as shown in Figure 7.

Figures 9a and 9b show the results of Gabor deconvolution on the traces from the crosscorrelation and the FDSD respectively. Figure 9c shows the difference between these data. If the wavelet in the sweep-removed data is minimum-phase, it can be inferred from the small and random differences shown in Figure 9c that the nonstationary wavelet embedded in the correlated surface Vibroseis data is close to being minimum phase.

DISCUSSION

These results may seem confusing since they seem to contradict theoretical expectations. It is a fundamental point of signal theory that a composite signal, formed as a convolution of two primitive signals, can only be minimum phase if both of the primitive signals are minimum phase. The uncorrelated data model expressed by equation (6) is conceptually $x_d = s * \alpha \bullet r$, where s is the sweep, α is the earth filter, r is the reflectivity. It follows immediately that the correlated trace is $x_c = w * \alpha \bullet r$, where w is the zero-phase Klauder wavelet. Therefore, it seems inescapable that correlated Vibroseis data cannot be minimum phase. Yet, our observations speak for themselves. The reconciliation may lie in the fact that a perfect impulse is mathematically both zero and minimum phase. That is, if the amplitude spectrum is unity for all frequencies, then the natural logarithm is zero and the Hilbert transform gives zero. As a sweep becomes more and more broadband, its corresponding Klauder wavelet (autocorrelation) must approach a spike. On the other hand, the earth filter is a strong, nonstationary, minimum-phase process. It seems plausible that the minimum-phase nature of the earth filter plays a much more important role in shaping the Vibroseis wavelet than the zero-phase Klauder wavelet. If this is the case, we do not yet understand why our final example seems to be getting less minimum-phase with increasing depth. This may have something to do with a progressively decreasing highest signal frequency, something we did not try to account for in our computation of the minimum-phase equivalents. While the Vibroseis wavelet cannot, in theory, be minimum phase, our experimental evidence suggests that it is so in a practical sense.

CONCLUSIONS

We have presented a combination of synthetic and real field experiments that support the conjecture that the embedded wavelet found in correlated Vibroseis data is, for practical purposes, minimum phase. This implies that Vibroseis data does not require a phase correction to agree with the minimum-phase assumption in a typical deconvolution algorithm.

ACKNOWLEDGEMENTS

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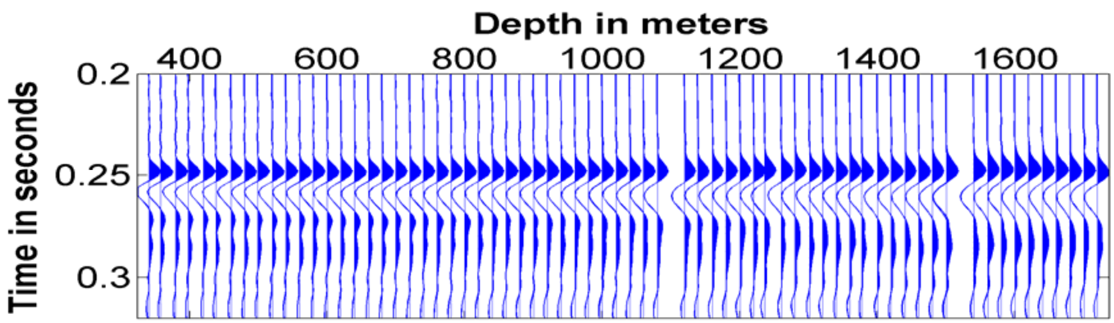
a)



b)

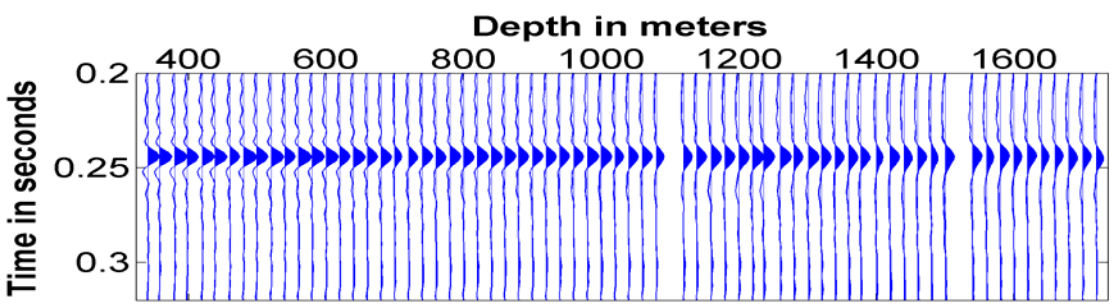


FIG. 1. a) Directly observed wavelets (normalized to peak amplitude), and b) the result of minimum-phase spiking deconvolution on the wavelets of a).

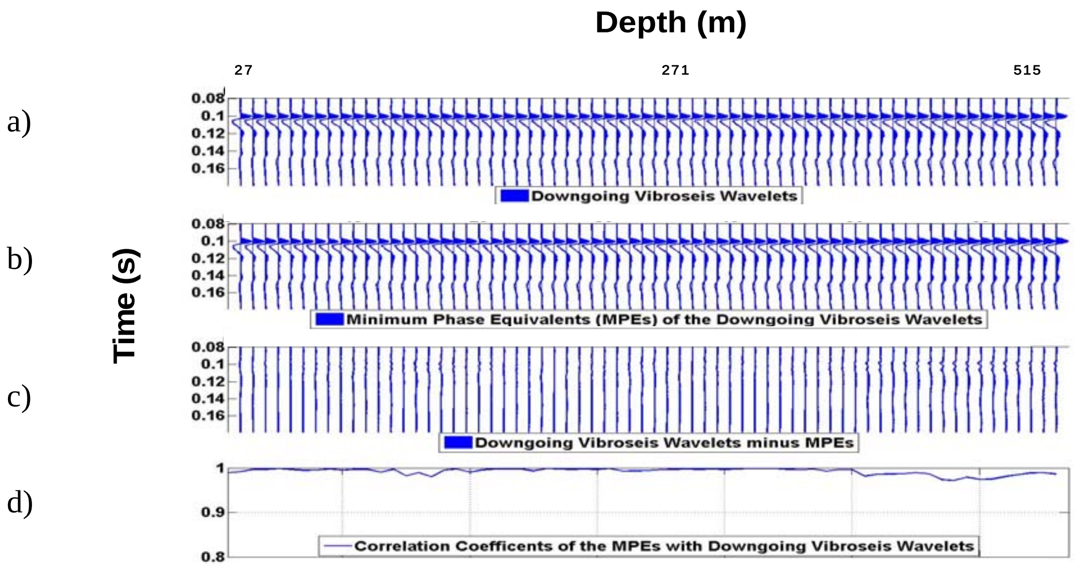


FIG. 2. a) Directly observed wavelets in VSP downgoing waves from Pikes Peak (normalized to peak amplitude); b) the minimum-phase equivalents (MPEs) of the wavelets shown in a); c) the difference between a) and b); and d) the correlation coefficients of the MPEs with downgoing Vibroseis wavelets.

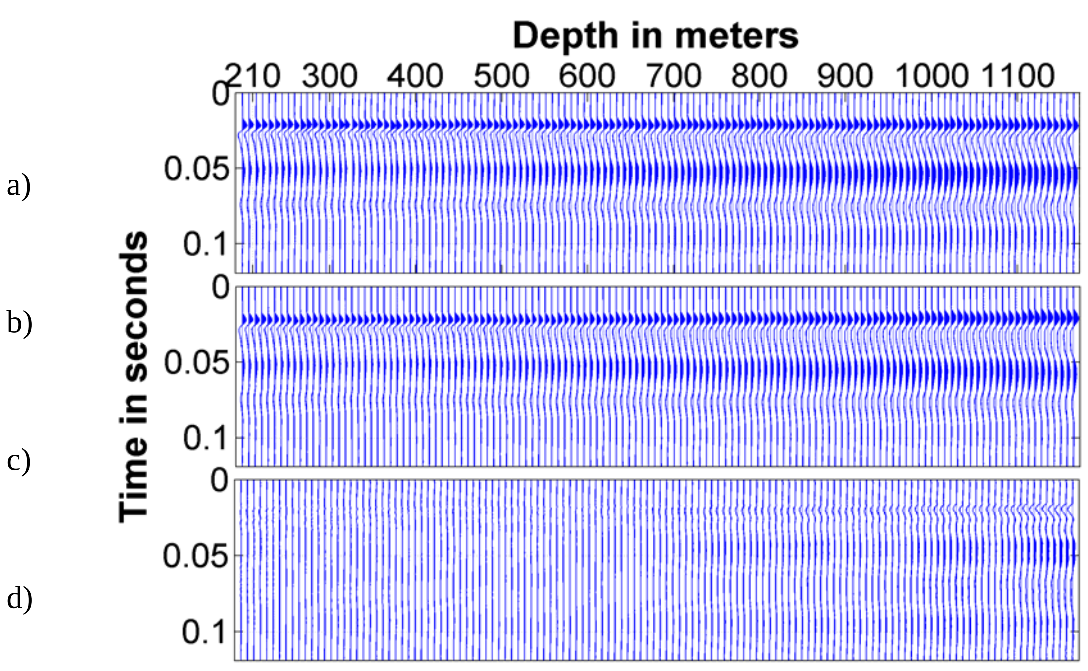


FIG. 3. a) Directly observed wavelets in VSP downgoing waves from Ross Lake (normalized to peak amplitude); b) the minimum-phase equivalents of the wavelets shown in a); and c) the difference between a) and b).

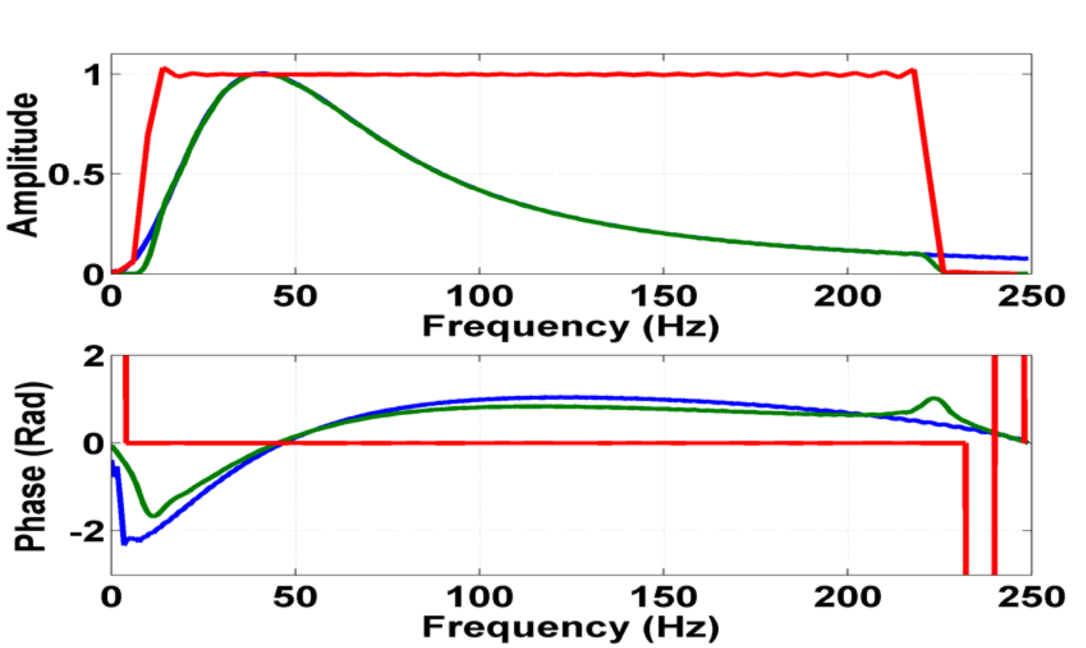


FIG. 4. Spectra of the Klauder wavelet (red), minimum-phase wavelet (blue), and effective minimum-phase wavelet (green).

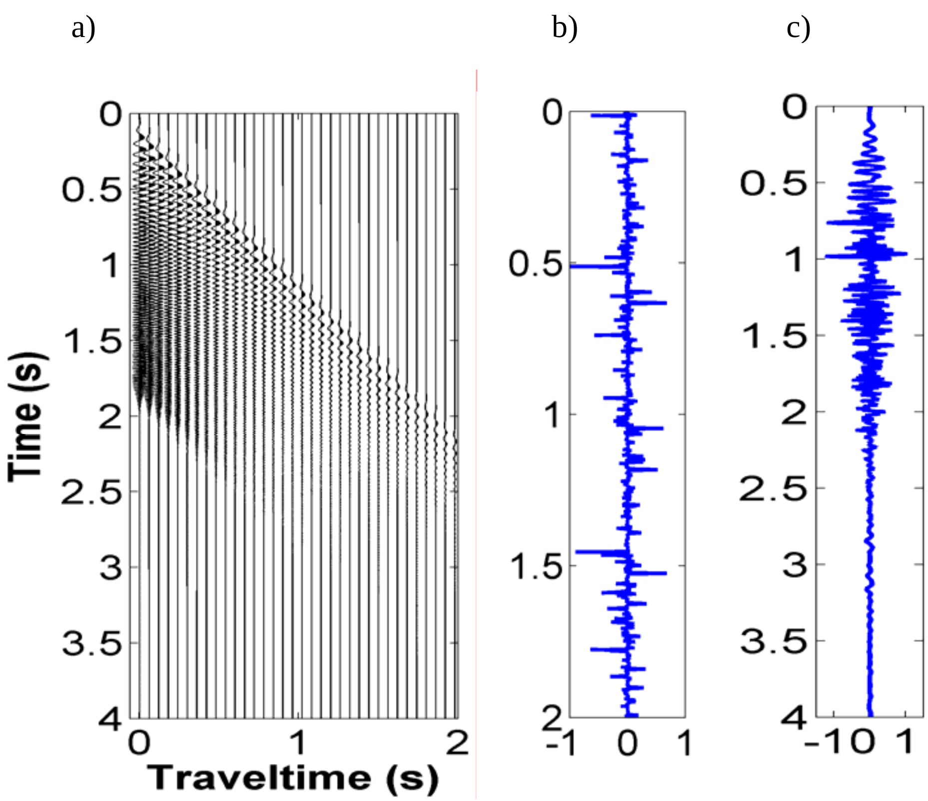


FIG. 5. a) The attenuated sweep, b) reflectivity, and c) uncorrelated trace.

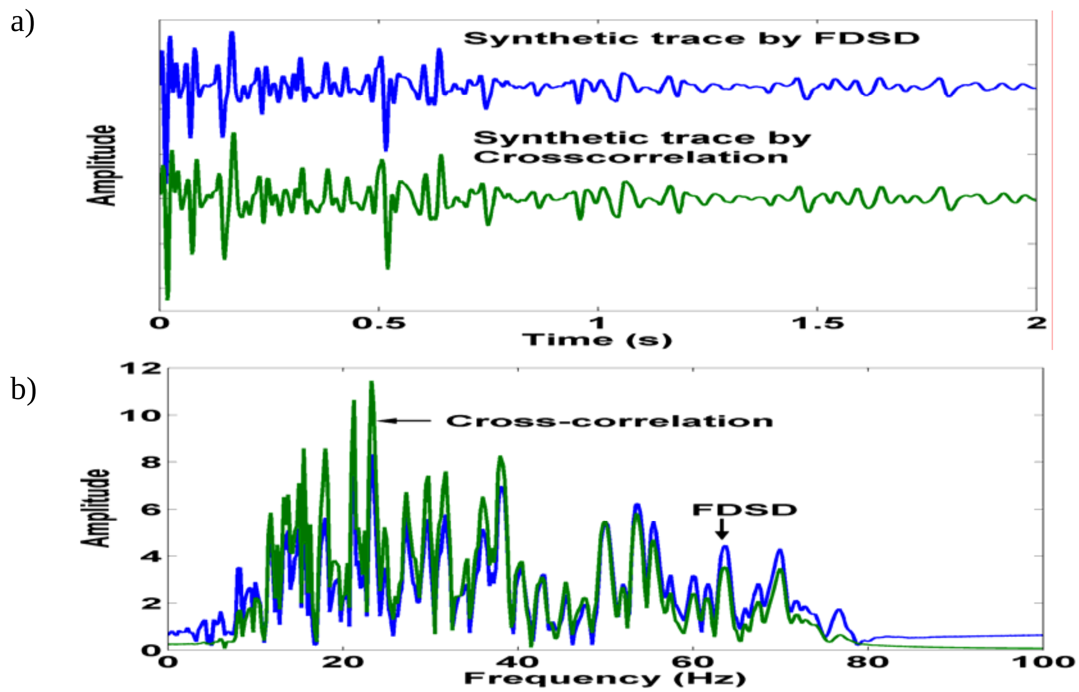


FIG. 6. a) Synthetic traces by FDSD and crosscorrelation, and b) amplitude spectra of the synthetic traces.

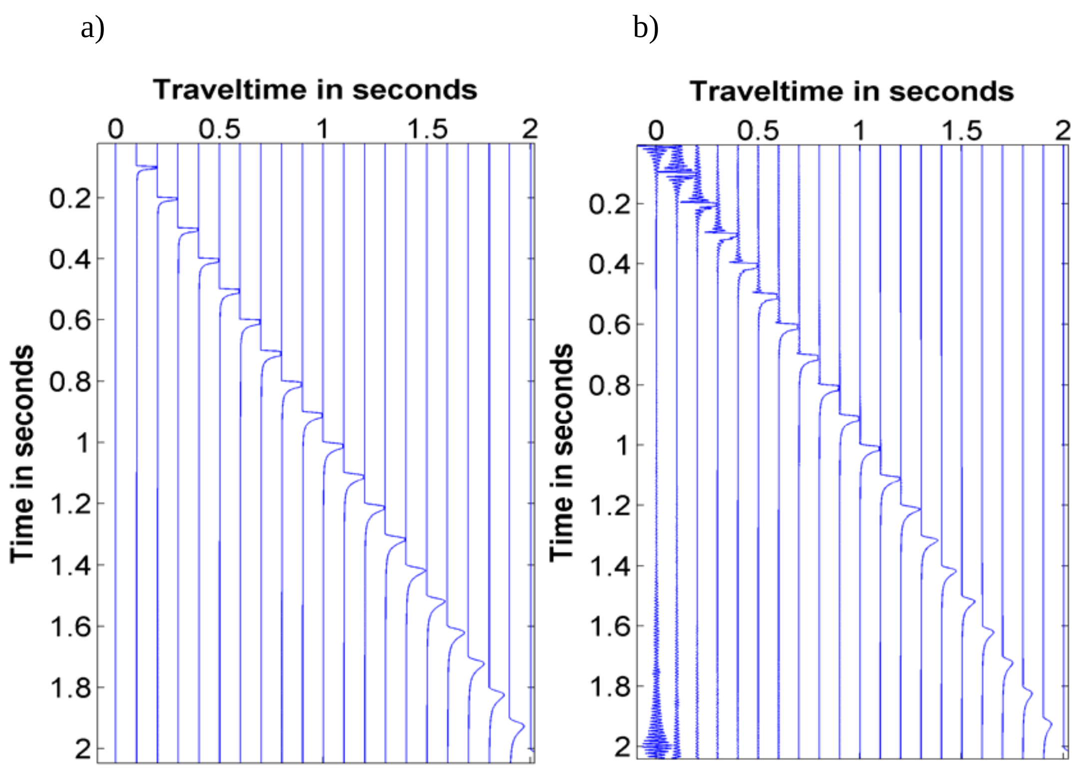


FIG. 7. a) Impulse response of the forward Q filter, and b) the wavelet estimates on the sweep-removed trace.

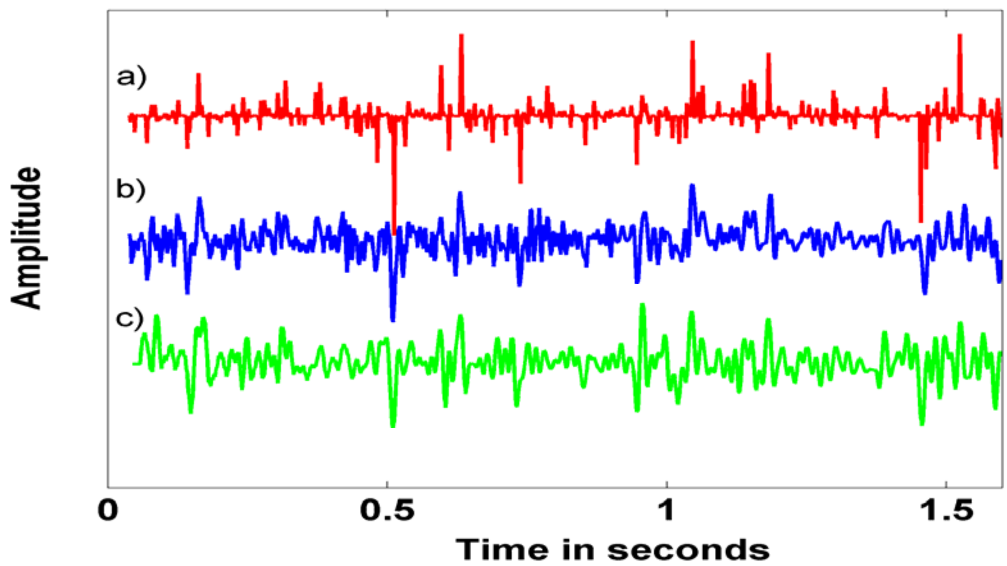


FIG. 8. a) The reflectivity, b) the trace after the Gabor deconvolution on the sweep-removed data, and c) the trace after the Gabor deconvolution on the correlated data.

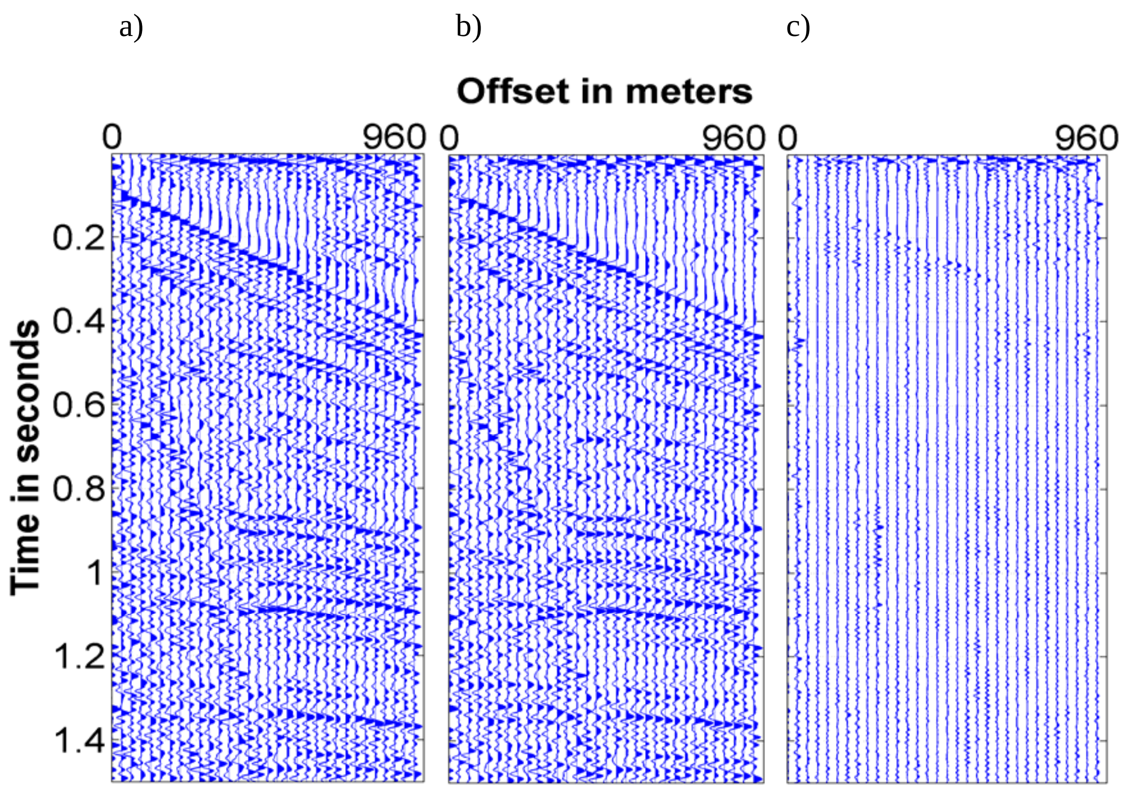


FIG. 9. a) The gather after minimum-phase Gabor deconvolution of the correlated data; b) the gather after minimum-phase Gabor deconvolution of the uncorrelated data; and c) the differences of the data shown in a) and c).

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