Converted wave resolution as a function of bandwidth

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ABSTRACT

A 1D-algorithm for the computation of trace envelopes when attenuation is present has been programmed. With this algorithm it is confirmed that adding low-frequency bandwidth can increase resolution. However, beyond a critical depth, adding low frequencies decreases resolution in these computations. A second, more realistic algorithm includes geometrical spreading and a noise floor. Assuming that any signal above the noise floor can be restored to a flat spectrum, pulse widths are computed from boxcar windows. Comparing the resulting pulse widths for C-waves and P-waves shows that adding low-frequency bandwidth improves resolution. Also, resolution crossover depths are moved deeper.

INTRODUCTION

The impact of attenuation on converted wave resolution has been investigated by Bale and Stewart (2002) and also by Deffenbaugh et al. (2000). It is observed that, with increasing depth, high frequencies are increasingly attenuated. Because of this high frequency loss, bandwidth is narrowed with increasing depth and resolution deteriorates. The idea of adding bandwidth at the low frequency end of the spectrum in order to counteract attenuation controlled bandwidth loss at the high end of the frequency spectrum has been investigated by Martin and Stewart (1994) as well as by Bale and Stewart (2002). In this study we seek to provide computational tools that allow one to estimate resolution that could be gained by adding low frequencies. The practical question of simultaneous low frequency noise contamination is not considered at this stage. Martin and Stewart (1994) point out that, with the advent of 24-bit recording systems, there is sufficient dynamic range available to recover signal when high-amplitude noise is present.

RESOLUTION CROSSOVER DEPTH

Deffenbaugh et al. (2000) define a resolution crossover depth for situations where $\boldsymbol{Q}_p > \boldsymbol{Q}_s$. They state that C-waves offer better resolution than P-waves at shallow targets before attenuation becomes significant. They also give an equation for the calculation of this crossover depth. Their equation implies a frequency of zero Hz for the lower limit of their seismic band, which is not too useful for investigating the role of lower frequencies. The purpose of this study is to develop computational tools that can be used to determine pulse width (wavelet resolution) as a function of bandwidth, which in turn is a function of depth. Then, comparing pulse width as a function of depth for C-waves and P-waves, the resolution crossover depth can be found. The prediction by Martin and Stewart (1994) as well as Bale and Stewart (2002) of more compact wavelets and enhanced C-wave resolution because of added low-frequency content can thus be put to the test.

COMPUTATIONAL ALGORITHM

The original plan had been to develop a rule of thumb for estimating pulse width from bandwidth as a function of depth. When assuming a homogeneous earth for simplicity, attenuation alone causes the loss of high frequencies with increasing depth. Attenuation is the result of anelasticity, which also leads to velocity dispersion of seismic waves. An equation for this frequency dependence of seismic velocities is given by Aki and Richards (1980):

$$v(\omega) = v_{ref}\left(1 + \frac{\ln\left(\omega / \omega_{ref}\right)}{\pi Q} - \frac{i}{2Q}\right) \tag{1}$$

Because $v(\omega)$ is part of the exponent when calculating attenuation, this exponent is different at every frequency point and no simplified equation could be found. The approach taken here is to compute signal strength as a function of depth, one frequency point at a time, followed by an inverse Fourier transform. Offset is taken to be constant in these computations. The roles of depth and offset can also be interchanged. Pulse width is measured at one half of maximum amplitude of trace envelopes. These computations are carried out for C-waves and P-waves independently because velocities are always different and normally Q-factors differ as well. Because these computations are done one frequency point at a time, it is easy to set upper and lower frequency limits and even spectral shaping can be introduced.

PULSE WIDTHS FOR ATTENUATED SPECTRA

Figure 1 shows trace envelopes, for a surface source with flat inband spectrum, at 1000m depth computed firstly, for a bandwidth of 1Hz to 80Hz and secondly, for a bandwidth of 10Hz to 80Hz. The two envelope maxima are aligned for easier comparison and $Q = 50$ is assumed. The larger bandwidth gives a narrower pulse, as expected. Figure 2 displays the equivalent result for a depth of 4000m. The wideband pulse has a steeper leading edge than the narrowband pulse, as before. However, the overall pulse width is increased for increased bandwidth. In other words, adding low frequencies decreases resolution at 4000m depth. The reason for this counter-intuitive result appears to be attenuation controlled high frequency roll off. The larger the depth, the steeper the roll off and higher frequencies contribute less and less to the resulting pulse shape. At a given depth, this roll off is steeper at lower frequencies. As a consequence, beyond a critical depth, adding n Hz at the low-frequency band edge effectively removes m Hz at the upper band edge because of relative attenuation and m exceeds n. Note that, for the above computations, no spectral shaping is applied, no noise is added and geometrical spreading is ignored.

PULSE WIDTHS FOR FLAT SPECTRA

For a more realistic model, a noise floor needs to be introduced as was done by Garotta and Granger (2001). The frequency f_{up} at which the seismic signal disappears below this noise floor will decrease with increasing depth. The algorithm assumes that a flat spectrum can be reconstructed between the lower band edge f_{low} and f_{up}. From this boxcar window a pulse width as a function of depth is computed for C-waves and P-waves. Figure 3 shows the results for the parameters indicated and assuming 500m constant offset. Resolution crossovers are evident and, as expected, the crossover is at larger depths for the low-frequency enhanced situation. The flat part at shallow depths of these curves is caused by an upper frequency cut-off of 80Hz. Clearly, when going deeper into the earth, attenuation starts to limit resolution of C-waves much sooner than for P-waves. The resolution advantage of C-waves at shallow depths can also be seen. Figure 3 demonstrates that, by adding low frequency bandwidth, the resolution crossover is pushed to larger depths. For the computation of Figure 4 the upper frequency cutoff is changed to 150Hz and f_{low} as well as Q_S are increased. The surprising result in Figure 4 is the lack of any resolution crossover for the wider bandwidth. At the time of writing it is not at all clear if this result is caused by a programming bug.

CONCLUSIONS

The 1D-algorithm (no geometrical spreading) programmed for the computation of trace envelopes shows that the advantage of adding low-frequency bandwidth is increased resolution. However, because of increasing attenuation at higher frequencies, beyond a critical depth adding low frequencies actually decreases resolution. A more realistic algorithm also programmed includes geometrical spreading and a -40dB noise floor. Also, it is assumed that signal above the noise floor can be restored to a flat spectrum. Pulse widths are computed from resulting boxcar windows and compared for C-waves and P-waves. The same shallow depth C-wave resolution advantage mentioned in the literature is found here, together with resolution crossovers. As predicted in the literature (Martin and Stewart, 1994; Bale and Stewart, 2002), adding low-frequency bandwidth improves resolution. The additional low frequencies also move resolution crossover depths deeper. There is some doubt whether or not a special case without resolution crossover (C-wave resolution exceeds P-wave resolution for all depths computed) is for real, or just a programming error.

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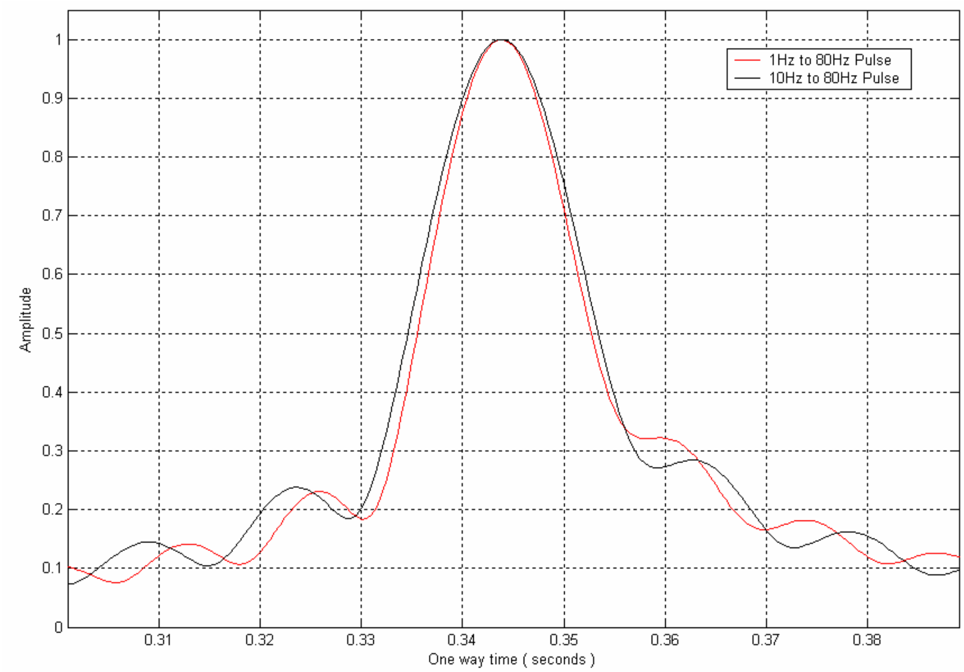

FIG. 1. Pulse comparison (aligned boxcar, 1000m depth, Q = 50).

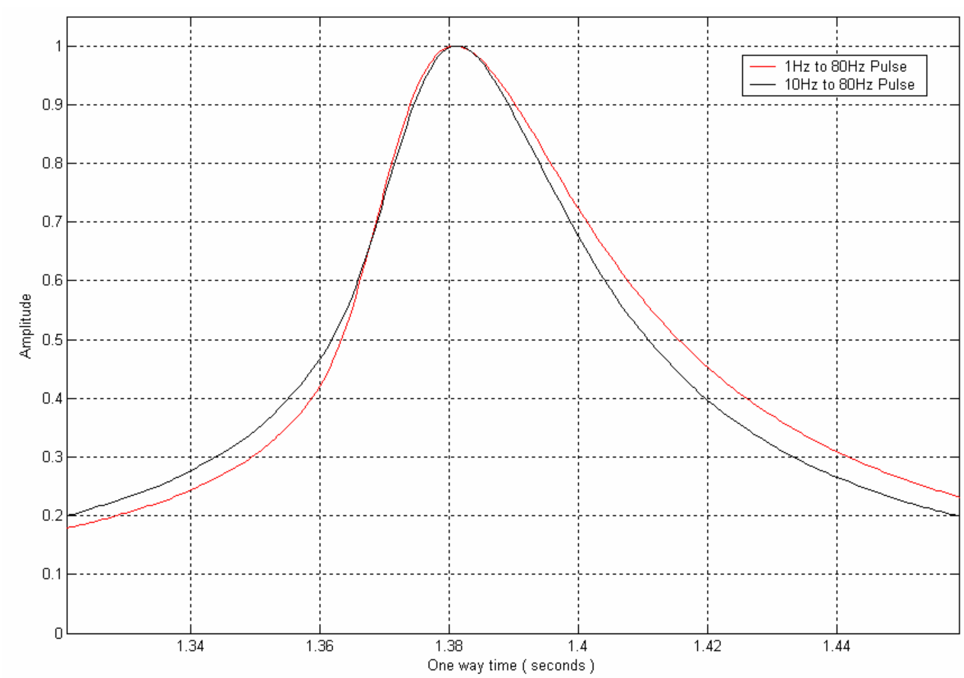

FIG. 2. Pulse comparison (aligned boxcar, 4000m depth, Q = 50).

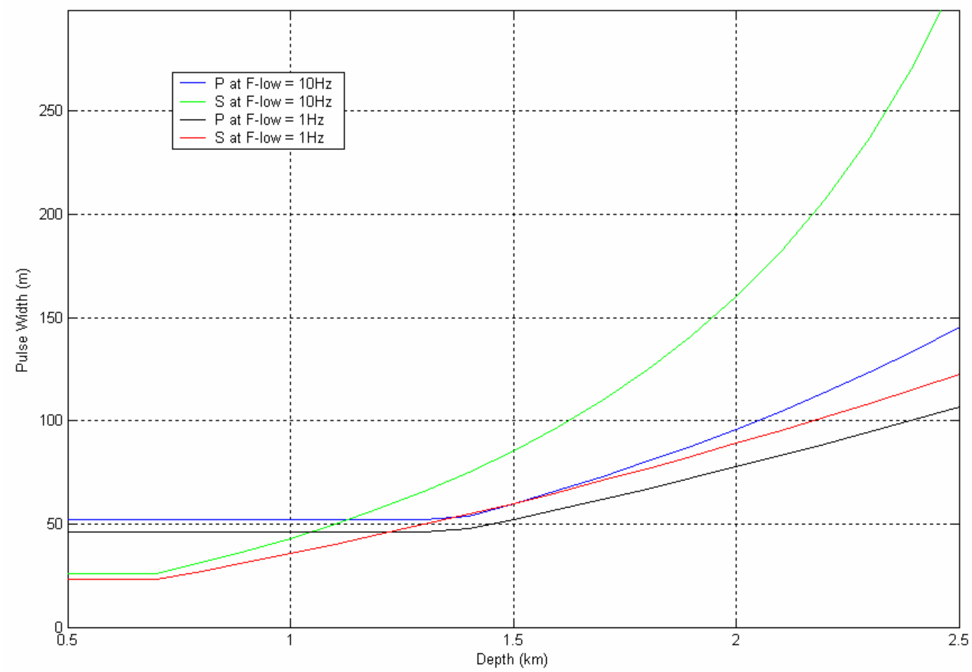

FIG. 3. -40dB noise floor sin(x)/x resolution as function of depth ($Q_P = 80$, $Q_S = 40$, $F_{max} = 80$Hz).

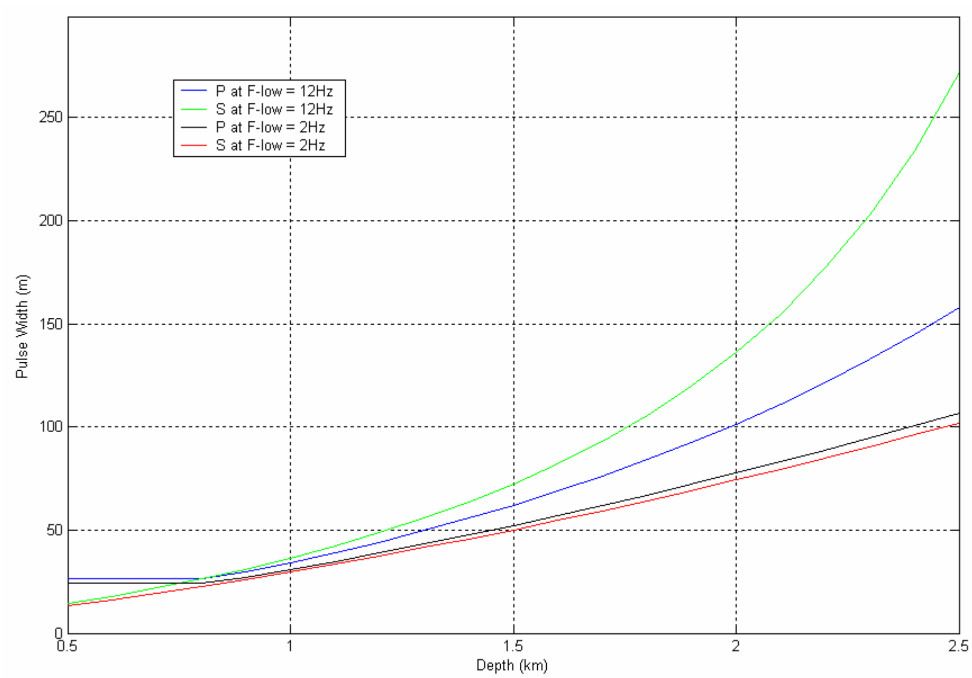

FIG. 4. -40dB noise floor sin(x)/x resolution as function of depth ($Q_P = 80$, $Q_S = 50$, $F_{max} = 150$Hz).