Connecting statics deconvolution and seismic interferometry

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ABSTRACT

In earlier work, we described a method for removing near-surface effects from seismic data which relaxes some of the inherent approximations of the traditional statics correction model. The complete method is known as 'raypath dependent statics' and has been demonstrated on two sets of real seismic data. An important part of the technique is what is known as 'statics deconvolution', in which 'statics distribution functions' are estimated for each seismic trace in a data set. The removal of its unique statics distribution function from each input seismic trace via deconvolution is the essence of this method. The deconvolution can be implemented using either a match filter or an inverse filter derived from the statics distribution function.

Recent work in the relatively new field of seismic interferometry has showed how to remove near-surface effects from seismic data by applying match filters or inverse filters to input traces, and constructing 'virtual source' gathers whose traces show no near-surface effects. In this report, we show the mathematical relationship of this work to the statics deconvolution concept, and we illustrate both approaches on an arctic data set for comparison.

The insight gained from making this connection should assist us in further development of methods to correct seismic data for near-surface effects.

INTRODUCTION

Statics Deconvolution

Correcting seismic reflection data for the effects of the earth's near-surface remains one of the more important issues in seismic data processing. Many techniques have been developed for dealing with this problem, some based on the static shift model, where the near-surface effects are approximated by travel time delays; some based on the convolution model, where near-surface effects are approximated by amplitude and phase variations. Our 'statics deconvolution' approach (Henley 2004, 2005, 2006) is an attempt to combine the two concepts into a single method, unified by a more realistic statics model.

The usual statics model makes the simplifying assumption that the transit of a reflection wavefront through the near-surface layers can be described by a simple time delay of that wavefront. Correcting for near-surface effects, in this instance, reduces to simply time-shifting each seismic trace by an amount determined by the sum of delays at the source and receiver locations. This approximation works well for a large proportion of land seismic data. Difficulties in determining the exact arrival time of wavefronts, coupled with the possibility of multiple arrivals for each wavefront (due to multi-pathing or near-surface scattering) led to the introduction of the concept of a "statics distribution function" describing the near-surface effects for each surface location on a seismic line (Rothman 1984, 1986), (Henley 2004). There are two ways to think of this function, not necessarily mutually exclusive. In one interpretation, the statics distribution function can be considered to describe the probability of a wavefront arrival as a function of delay time. Here, the highest peak of the function marks the most probable delay of a wavefront; and the width of the function shows the degree of timing uncertainty associated with that particular wavefront arrival. If this interpretation is accepted, the function values must all be positive, since they are considered to be probability values, and the integral of the function must be unity. The alternative interpretation considers each sample of the distribution function to be an estimate of the fractional part of a wavefront's amplitude arriving at the delay associated with that sample. In this view, negative values are acceptable, since a wavefront can be scattered from nearby interfaces with negative reflectivity; but the integral must still be unity, since the distribution function must account for all of the amplitude associated with the wavefront.

The use of the statics distribution function is what distinguishes the statics deconvolution method from other techniques. Although Rothman (1984, 1986) described the statics distribution function and how to estimate it, he elected to use the statistical concept to motivate the 'simulated annealing' statics method; and he only used the peak values and widths of the distribution functions as input. For the statics deconvolution method, we assert that all the samples in the distribution function can be considered to be 'reflectivity' spikes of varying amplitude describing the direct arrival of a reflection event as well as all its scattered and/or multi-pathed arrivals. The removal of the distribution function from a seismic trace can be accomplished by one of several deconvolution techniques. We have tested two of them: match filtering and inverse filtering. In the first case, we attempt to shrink the distribution function's length in time by deriving a match filter which ideally collapses the spike series into a single spike at zero delay time. The second method, which seems to work better on real data, is to derive an inverse filter for the distribution function. In the current work, we consider only the match filter approach for comparison to interferometry.

In order to deconvolve statics distribution functions, we must first determine them. We follow Rothman's approach by using trace cross-correlations as bandlimited estimates of statics distribution functions. Also following Rothman, we 'improve' the estimated functions by raising their samples to a power. This whitens the functions and narrows their peaks without adding new peaks. The modified functions can, themselves, be used as match filters on their respective traces; or a new match filter can be derived for each function, choosing a spike at zero time as the ideal to which to match the distribution function. Alternatively, a broadband inverse filter can be derived for each distribution function. Applying either the match filters or the inverse filters to their respective seismic traces by convolution ideally removes the effects of the embedded statics distribution functions.

To obtain an estimate of a source static distribution, for example, we first create a receiver pilot trace ensemble by mixing an NMO-corrected receiver ensemble over a specified number of traces. The mixing (summation) tends to average the statics distribution functions attributable to the different source locations to a symmetric distribution (a bandlimiting function), leaving a net phase contribution due only to the receiver statics distribution function common to all the input traces. Cross-correlating a raw trace from the original ensemble with this pilot trace will then yield a function which contains a bandlimited estimate of the individual trace *source* static distribution function, since the receiver statics distribution function from the trace will autocorrelate with the same receiver statics function from the pilot trace and contribute no net phase difference.

Seismic Interferometry

A special issue of Volume (71.4) of Geophysics (71.4, 2006) deals with seismic interferometry, which has processing applications in the areas of redatuming, imaging and migration of seismic data. A number of papers of interest are contained within this volume. To keep matters as simple as possible, only the paper by Bakulin and Calvert (2006) will be considered in this report as it contains material consistent with what is presented in this report.

In their paper, the VSP problem is considered from the perspective of time reversal methods to produce a downward continued seismic data set using what they term virtual sources (VS's) at the downhole geophone locations. The motivation for this is to remove the effects of complex near surface structure from the seismic data. What is sought from the processing applied is a redatumed receiver set from which the effects of the complex (highly laterally inhomogeneous) surface layer have been removed.

What is considered here is a similar type of problem with the exception that both the sources and receivers lay at the surface. Some of the concepts presented in Bakulin and Calvert (2006) will be used, the major difference being the design of a match filter needed to remove the near surface effects due to a geologically variable surface layer. Bakulin and Calvert (2006) use the initial portions of VSP traces to capture the near-surface variations of transmitted downgoing seismic wavefronts, while we use trace windows of deep reflections to capture the near-surface variations of both downgoing (source side) and upcoming (receiver side) wavefronts.

THEORETICAL OVERVIEW

The standard definition of a seismic trace resulting from a surface shot at point α and surface receiver at point β is given by

$$T_{\alpha\beta}(t) = w(t) * \Re_{\alpha\beta}(t) \tag{1}$$

where

$$\begin{aligned}
&T_{\alpha\beta}(t) \text{ – seismic trace recorded at the surface,}\\
&w(t) \text{ – source wavelet, and}\\
&\Re_{\alpha\beta}(t) \text{ – the earth's impulse response (reflectivity).}
\end{aligned} \tag{2}$$

The number of receivers per channel is N. Although not a requisite at this point, the constraint that the wavelet is band limited will be imposed. Further, as indicated by its lack of subscripts, it is constant at all source locations, i.e., for all channels. The sources or receivers, α and β are such that $\alpha \in [1, N]$ and $\beta \in [1, N]$.

Assume now that the reflectivity may be factored into three independent and distinct entities in the following manner

$$\Re_{\alpha\beta}(t) = s_\alpha(t) * R_{\alpha\beta}(t) * r_\beta(t) \tag{3}$$

In equation (3), the newly specified reflectivity, $R_{\alpha\beta}(t)$, may be thought of as a ray theoretical reflection response of a complex inhomogeneous medium, that is, comprised of only the reflection and transmission coefficients (losses), in addition to the time delays, of a subset of the infinite number of rays, sufficient to adequately approximate the full wave solution. This is known as a *partial ray expansion*. In other words, $R_{\alpha\beta}(t)$ is defined such that it is functionally independent of the heterogeneities of the complex surface layer in the geological model being considered. For $R_{\alpha\beta}(t)$ to have any meaning within this context, the model must have some geometrical definable structure in the form of interfaces where the elastic parameters are discontinuous, which contribute the reflection and/or transmission losses. These underlying layers are assumed to be plane, or reasonably close approximations thereof, and have minimal lateral variations in elastic parameters (velocities and densities). Ray theory and extensions thereof can, within certain limitations, provide solutions to this type of ray propagation problem. Consequently, the two additional functions, $s_\alpha(t)$ and $r_\beta(t)$, used in redefining the original reflectivity, $\Re_{\alpha\beta}(t)$, contain all of the relevant propagation information of the wave field *not* contained in $R_{\alpha\beta}(t)$. That is, $s_\alpha(t)$ is a function of the generally complex geological structure of the downward propagating sections of the rays, comprising the partial ray expansion, to their respective reflectors, together with source statics and other phenomena related to the source – reflector propagation of energy, such as geometrical spreading. In an analogous manner, the quantity $r_\beta(t)$ includes the dynamic property information, and receiver statics, required to propagate energy from the reflection points within the medium to the receiver.

Similar to the approach of Bakulin and Calvert (2006), we define one kind of virtual seismic trace as a sum over shots of the convolution of raw seismic traces with time-reversed seismic traces at the common shot positions. This operation defines a so-called time-reversal experiment (redatuming), described by Bakulin and Calvert (2006), over an aperture determined by the spread of shot positions. This virtual trace, exclusive of source wavelets, can be written as in Equation (4).

$$\Re_{\alpha\beta}(t) = \sum_{k=1}^{N} \Re_{k\alpha}(-t) * \Re_{k\beta}(t) \tag{4}$$

Expanding this equation in terms of surface functions, as in Equation (3), we get

$$\hat{\Re}_{\alpha\beta}(t) = \sum_{k=1}^{N} s_k(-t) * R_{k\alpha}(-t) * r_\alpha(-t) * s_k(t) * R_{k\beta}(t) * r_\beta(t) \tag{5}$$

If we now include the source wavelets in the equation, we get

$$\mathrm{T}_{\alpha\beta}(t) = w(-t) * w(t) * \sum_{k=1}^{N} s_k(-t) * R_{k\alpha}(-t) * r_\alpha(-t) * s_k(t) * R_{k\beta}(t) * r_\beta(t) \tag{6.a}$$

Rearranging the terms, we can carry the wavelets outside the summation, since they are assumed constant for the seismic survey.

$$\begin{aligned}\mathrm{T}_{\alpha\beta}(t) = &\left[w(-t) * w(t)\right] * \\ &\sum_{k=1}^{N}\left[s_k(-t) * s_k(t)\right] * \left[R_{k\alpha}(-t) * R_{k\beta}(t)\right] * \left[r_\alpha(-t) * r_\beta(t)\right]\end{aligned} \tag{6.b}$$

We can also carry the receiver surface functions outside the summation, since they are independent of the shot index, *k*.

$$\begin{aligned}\mathrm{T}_{\alpha\beta}(t) = &\left[r_\alpha(-t) * r_\beta(t)\right] * \left[w(-t) * w(t)\right] * \\ &\sum_{k=1}^{N}\left[s_k(-t) * s_k(t)\right] * \left[R_{k\alpha}(-t) * R_{k\beta}(t)\right]\end{aligned} \tag{6.c}$$

In the above equation it should be noted that terms can be paired into zero phase autocorrelations except for the first term outside of the summation, $\left[r_a(-t) * r_b(t)\right]$, the receiver balancing (statics) function for traces with common source k and receivers α and β, and the reflectivity functions $R_{k\alpha}(-t)$ and $R_{k\beta}(t)$. We assume, however, that the deep earth reflectivity varies only slowly laterally, so that the cross-correlation of functions $R_{k\alpha}(-t)$ and $R_{k\beta}(t)$ contributes negligible phase. Thus, the major phase contribution to $\mathrm{T}_{\alpha\beta}(t)$ comes from the receiver balancing function $\left[r_a(-t) * r_b(t)\right]$.

Rather than equation (6.c) suppose another type of virtual trace[1] is constructed as a sum over shots of match – filtered traces, which has the form

[1] The most basic definition of a match filter is crosscorrelating input of an arbitrary linear system with the resulting output. It has long been known that this crosscorrelation synthesizes the impulse response of a linear system using some random signal as an input.

$$\hat{\mathrm{T}}_{\alpha\beta}(t)=\sum_{k=1}^{N}F_{k\alpha\beta}(t)*\mathrm{T}_{\alpha\beta}(t)\,. \tag{7}$$

The effect of the match filter, $F_{k\alpha\beta}(t)$ is only to remove the phase differences between the receiver balancing functions $r_{\alpha}(t)$ and $r_{\beta}(t)$. As the seismic traces have been constrained to be band limited this match filter can be taken as cross – correlation of traces, given by

$$F_{kab}(t)=\mathrm{T}_{ka}(-t)*\mathrm{T}_{kb}(t) \tag{8}$$

Substituting this formula into equation (7) yields the expression

$$\hat{\mathrm{T}}_{\alpha\beta}(t)=\sum_{k=1}^{N}\mathrm{T}_{k\alpha}(-t)*\mathrm{T}_{k\beta}(t)*\mathrm{T}_{k\alpha}(t)=\sum_{k=1}^{N}\left[\mathrm{T}_{k\alpha}(-t)*\mathrm{T}_{k\alpha}(t)\right]*\mathrm{T}_{k\beta}(t)\,. \tag{9.a}$$

This equation can be expanded to explicitly show the contributions to phase:

$$\begin{aligned}\hat{\mathrm{T}}_{\alpha\beta}(t)&=\sum_{k=1}^{N}\left[\mathrm{T}_{k\alpha}(-t)*\mathrm{T}_{k\alpha}(t)\right]*\mathrm{T}_{k\beta}(t)\\&=\left[w(-t)*w(t)\right]*w(t)*\left[r_{\alpha}(-t)*r_{\alpha}(t)\right]*r_{\beta}(t)*\\&\quad\sum_{k=1}^{N}\left[s_{k}(-t)*s_{k}(t)\right]*s_{k}(t)*\left[R_{k\alpha}(-t)*R_{k\alpha}(t)\right]*R_{k\beta}(t)\end{aligned} \tag{9.b}$$

As a result of the average over shot locations, k, the phase of $s_k(t)$ will be averaged out and the only significant surviving phase (excluding the earth reflectivity $R_{k\beta}(t)$) will come from $r_{\beta}(t)$ and $w(t)$. Now $w(t)$ is the same for all shots and as a consequence contributes no net phase.

Finally, suppose we create an ensemble (gather) consisting of virtual traces constructed according to Equation (9b), where the common surface location for the gather is β. Since the phase-contributing surface function $r_{\beta}(t)$ will then be the same for every trace in the ensemble, the ensemble will exhibit no differential, or inter-trace phase differences (statics), but only a net phase for the entire gather, attributable to the receiver surface function $r_{\beta}(t)$. We will have corrected our input data for the shot surface functions (statics).

We assert, without proof, that using as input a complete set of virtual traces created according to Equation (9b), we could repeat the same virtual trace procedure. In this case, we sum over receivers instead of shots, to create a new set of virtual traces corrected for both source and receiver surface functions.

Static deconvolution

Starting with equation (3) and convolving a wavelet emanating from the source at position k ($k \in [1, N]$), we get, for a seismic trace

$$\mathrm{T}_{k\beta}(t) = w(t) * s_k(t) * R_{k\beta}(t) * r_\beta(t) \tag{10}$$

To remove $s_k(t)$ or $r_\beta(t)$, either an inverse filter or a matched filter is applied for each function. As an example, for $s_k(t)$

$$\hat{\mathrm{T}}_{\alpha\beta}(t) = \mathrm{F}_{\alpha\beta}(t) * \mathrm{T}_{\alpha\beta}(t) \tag{11}$$

where.

$$\mathrm{F}_{\alpha\beta}(t) = \mathrm{T}_{\alpha\beta}(-t) * \sum_{k=1}^{N} \mathrm{T}_{k\beta}(t) \tag{12}$$

In equation (12) the sum is a shot gather pilot trace so that

$$\hat{\mathrm{T}}_{\alpha\beta}(t) = \mathrm{T}_{\alpha\beta}(-t) * \mathrm{T}_{\alpha\beta}(t) * \sum_{k=1}^{N} \mathrm{T}_{k\beta}(t)\,. \tag{13}$$

Were we to expand the term inside the summation in Equation (13) into its surface functions and earth reflectivity function, we would find the source surface functions $s_k(t)$ averaging out due to the summation, as in Equation (9), leaving only $r_\beta(t)$ to contribute to phase.

Once again, we can create a trace gather consisting of traces with common surface location β. As before, the traces in this gather will exhibit no inter-trace phase differences due to surface functions; but the entire gather will have a net phase from the common receiver surface function for location β.

Similarly, we assert that the virtual traces created according to Equation (13), which have had the source surface functions removed, can be further subjected to new match filters similar to those described by Equation (12). This step will create new virtual traces which have had both source and receiver surface functions removed.

Cross correlation has been used to construct the match filter in both equations (9) and (13) primarily to show the mathematical similarities. In practice, numerical methods are employed to design an optimum match filter in both instances.

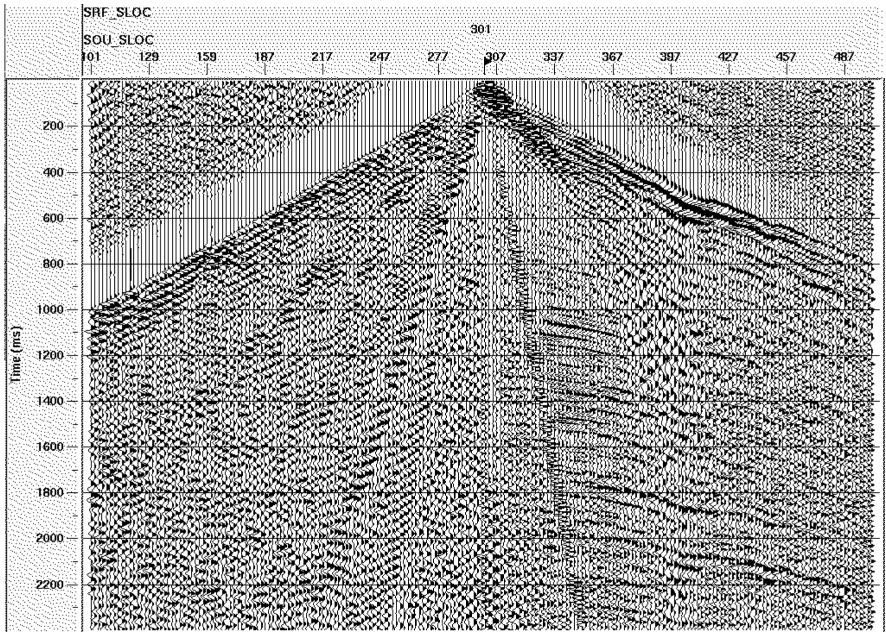
Comparing interferometry and statics deconvolution

Comparing equations (9) and (13), we see that they differ mainly in the order of the summation and the correlation. In statics deconvolution, we sum traces into a pilot trace to correlate with each raw trace, whereas in interferometry, we cross-correlate raw traces, then sum the correlations to form an operator. As similar as the techniques seem in theory, we need to confirm their similarity with a field trial.

AN EXAMPLE

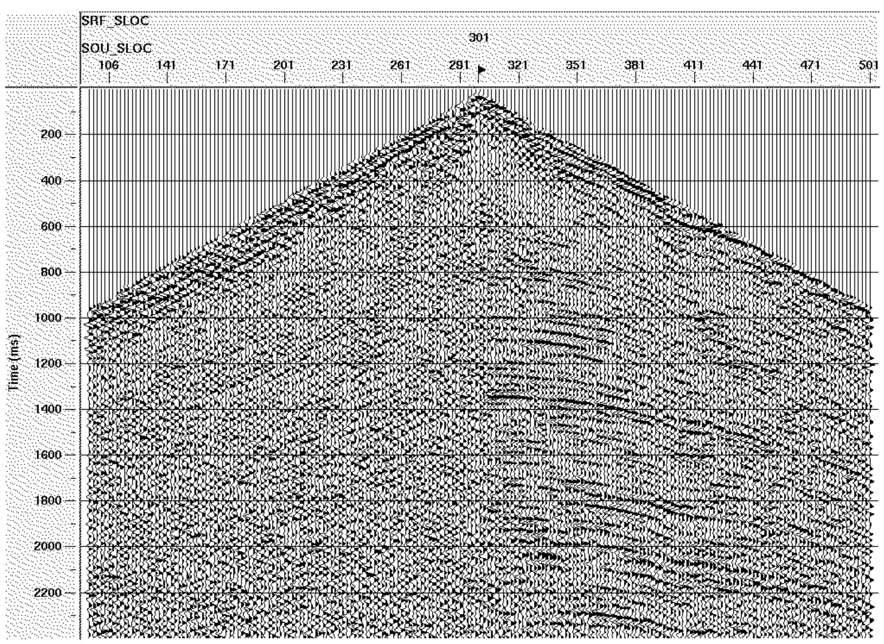
Three years ago, we tested statics deconvolution on a set of reflection data from Hansen Harbour in the MacKenzie Delta area. One reason for choosing these data is the fact that the receiver spread is only 50 stations long and is affected by only minor statics. Receiver coupling is very good and contributes nothing to near-surface effects. The source spread, however, was several times as long as the receiver spread and exhibited significant static and surface coupling variations along its length. This means that for this line we are able to test surface correction schemes by only solving for source correction functions…we don't need to worry about the receiver side. Hence, for our demonstration, we created virtual receiver gathers according to Equation (9) and static-deconvolved receiver gathers according to Equation (13). Without further processing, we then corrected these sets of gathers for moveout and stacked them.

A complication for this line, however, is the presence of considerable coherent noise, in the form of ice flexural wave noise, exhibited by any traces for which the source and receiver were both located on floating ice. This creates large dynamic range variations on traces and contaminates a significant portion of the input data with relatively intractable noise. Figure 1 shows an example of a receiver gather from the Hansen Harbour data. Noise is so strong on this gather that reflections are very difficult to see on the left side of the spread. Figure 2 shows the same receiver gather, but after extensive filtering and deconvolution (Henley 2004). With the noise attenuated, it is now easier to see reflections and to see that near-surface variations are visible as statics on the cleaner traces.



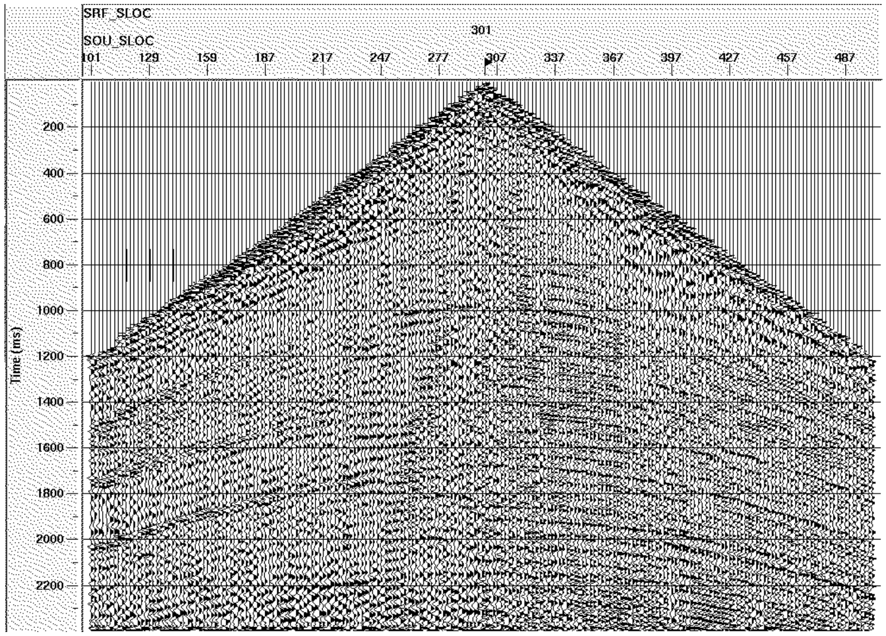
Raw Hansen Harbour receiver gather

FIG. 1. Raw receiver gather from Hansen Harbour showing severe coherent noise contamination and serious statics (right side of the spread).



Hansen Harbour receiver gather after radial filter and deconvolution

FIG. 2. Hansen Harbour receiver gather after extensive noise attenuation and deconvolution.

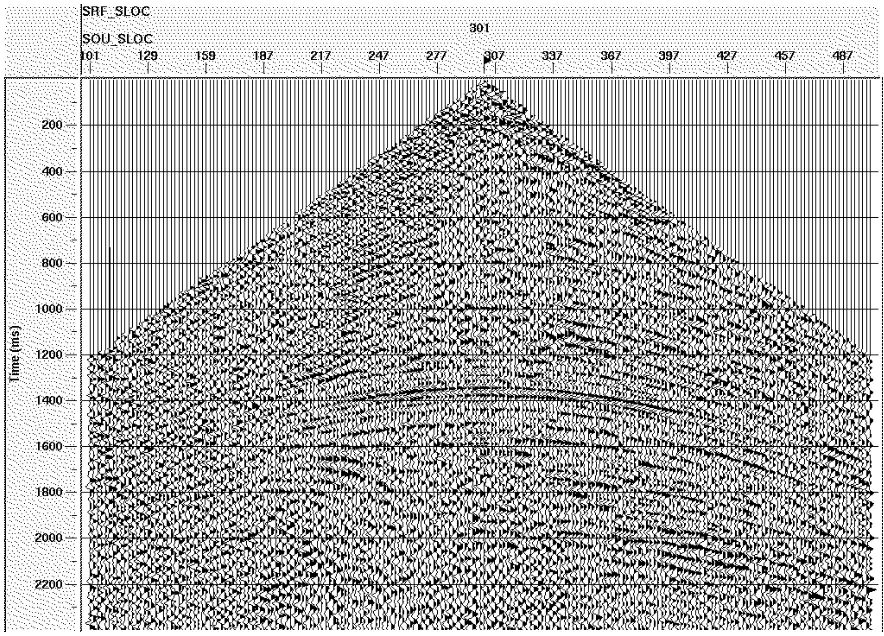


Hansen Harbour receiver gather after statics deconvolution

FIG. 3. Statics deconvolution applied to the receiver gather shown in Figure 2. Note that the statics deconvolution removes statics *and* enhances event coherence even on the poor data side of the spread.

Figure 3 shows the same receiver gather after statics deconvolution. To create pilot traces, we removed NMO from the gather and did a 100 trace running mix before restoring the NMO. The starting point for this operation was the filtered gather shown in Figure 2.

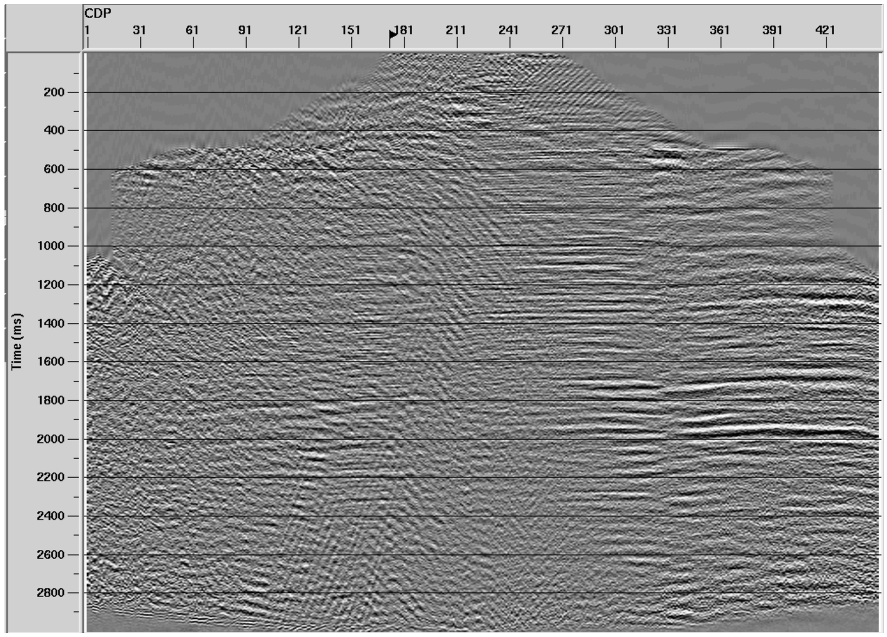
For comparison, Figure 4 shows a virtual receiver gather corresponding to the gather in Figures 1, 2, and 3. It should be noted that the virtual gather was constructed from raw shot gathers, with *no noise attenuation*. The gathers in both Figures 3 and 4 confirm that the near-surface source effects (at least those varying from trace to trace) have been removed from both gathers. The bandwidths of the two results differ, but the starting point for the traces in Figure 3 was a receiver gather that had already had coherent noise attenuated and had been whitened, whereas the virtual receiver gather process used raw data with no noise attenuation.



Hansen Harbour virtual receiver gather

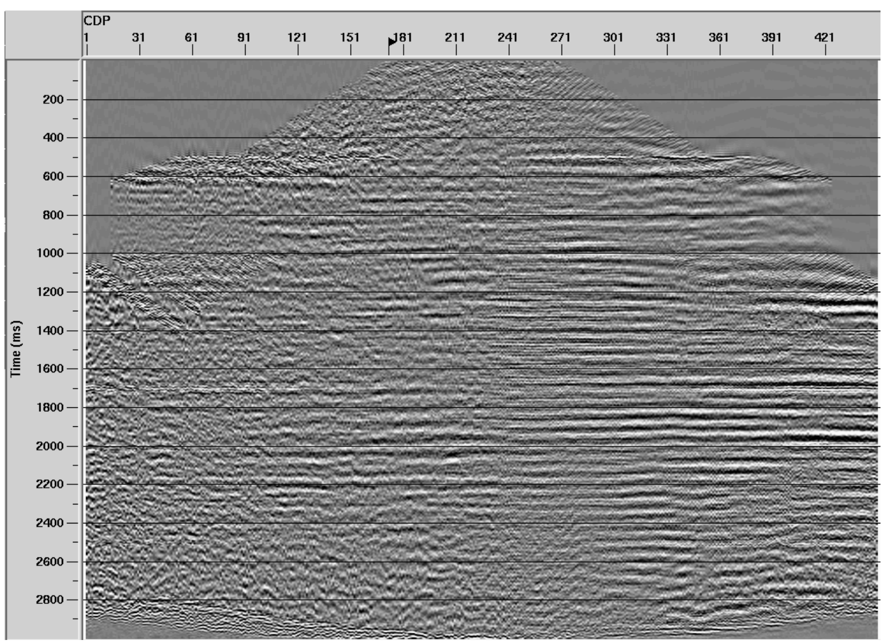
FIG. 4. Virtual receiver gather constructed according to equation (9). Note that even though this gather has a different appearance from Figure 3, the statics have been corrected and event coherence improved by this procedure, as well.

As a final comparison, Figures 5, 6, and 7 are the stacked sections generated from the full sets of gathers corresponding to Figures 1, 3, and 4, respectively. Recall that because the compact receiver spread for this survey exhibits no significant inter-spread surface function phase differences (statics), we can stack the traces from either the statics deconvolution process or the virtual trace process after only NMO correction, with no further surface corrections. There are significant bandwidth and coherence differences between Figures 6 and 7, but we expect that some of those are related to the selection of trace window used for the statics deconvolution process (often the entire trace) compared to that used for virtual trace creation (centred on prominent reflections).



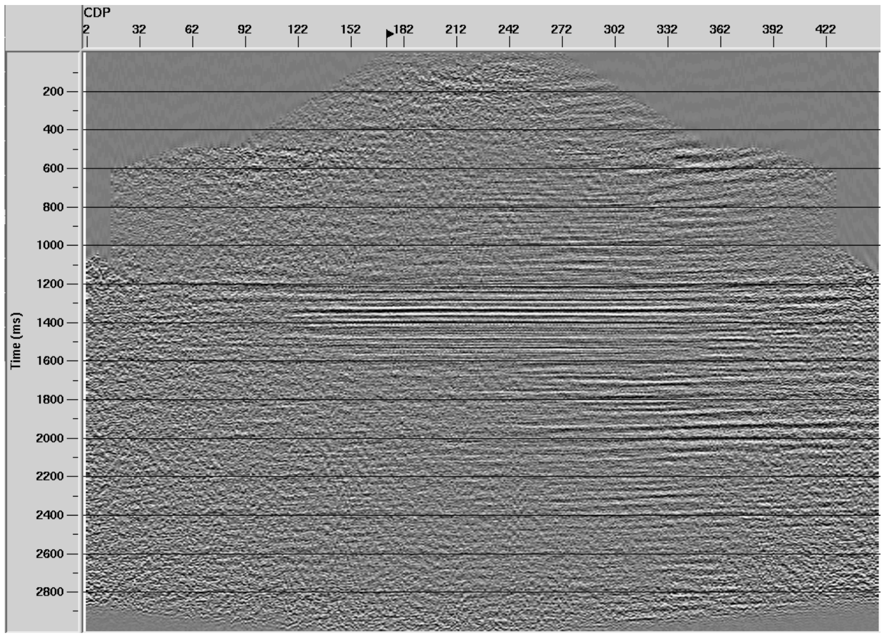
Hansen Harbour stack—no filtering, no statics

FIG. 5. Full CDP stack of all the raw receiver gathers for Hansen Harbour survey. Note the static bust on the right side of the section and the ice wave contamination left of centre.



Hansen Harbour stack—radial filtered, statics deconvolved

FIG. 6. Full CDP stack of the static deconvolved receiver gathers from Hansen Harbour. There are no static busts, and noise is much attenuated (the raw gathers were radial-filtered).



Hansen Harbour stack—virtual receiver gathers, no filtering

FIG. 7. Full CDP stack of virtual receiver gathers constructed for Hansen Harbour. It is interesting to note that this stack is broader in band and emphasizes different reflection sequences than the stack in Figure 6. Also, the ice wave noise is greatly attenuated, even though it was never explicitly addressed—a possible desirable side effect of the match filtering.

CONCLUSIONS

We have shown that our statics deconvolution method is strikingly similar to an extension of one of the more prominent interferometric methods used for imaging seismic data, both mathematically, and in real life when applied to actual seismic field data. We have not explored the effects of aperture, but have used all available data in our summations when doing the interferometry. Having only discovered the connections of statics deconvolution to interferometry quite recently, we hope to explore the relationship more thoroughly in the future.

An unexpected result that we also hope to explore more thoroughly is the effective attenuation of ice flexural wave noise by the match filtering procedure used to create virtual traces.

One of the projects we hope to initiate in order to explore these methods is the construction of a small model in which we can isolate specific effects in order to test various concepts and their ability to detect these effects and correct for them.

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APPENDIX

Bakulin and Calvert (2006)

$$D_{\alpha\beta}(t) = \sum_{k=1}^{N} S_{k\alpha}(-t) * S_{k\beta}(t) \tag{1}$$

$$S_{k\beta}(t) = w_k(t) * \tilde{S}_{k\beta}(t) \tag{2}$$

$$N_{k\beta}(t) = S_{k\alpha}(-t) * \tilde{S}_{k\beta}(t) = w_k(-t) * \tilde{S}_{k\alpha}(-t) * \tilde{S}_{k\beta}(t) \tag{3}$$

$$\hat{R}_{\alpha\beta}(t) = \sum_{k=1}^{N} N_{k\beta}(-t) = w(-t) * \sum_{k=1}^{N} \tilde{S}_{k\alpha}(-t) * \tilde{S}_{k\beta}(t) \tag{4}$$

$$D_{\alpha\beta}(t) = w(t) * w(-t) * \sum_{k=1}^{N} \tilde{S}_{k\alpha}(-t) * \tilde{S}_{k\beta}(t) \tag{5}$$

$$D_{\alpha\beta}(t) = \sum_{k=1}^{N} S_{k\beta}(-t) * M(t) * S_{k\alpha}^{-1}(t) \tag{6}$$