Estimating the location of a microseismic event when using a vertical array of receivers

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ABSTRACT

The accuracy of using a vertical array of receivers is investigated for locating microseismic events. Noise was added to the clock-time of the receivers and the errors in estimating the source location displayed. The errors vary with orientation and distance from the receiver array and the size of the receiver array.

INTRODUCTION

The location of a microseismic event can be estimated with an array of receivers in a well. The receivers are typically three-component (3C), and one receiver can estimate a location using the arrival direction of the wavefield and the time difference between the arrival times of the P and S waves. Detection and interpretation of the P and S wave arrivals can be a difficult task; and the estimation of the azimuthal direction may be inaccurate.

An alternative approach is to use only the first arrival times to estimate the source location. Three collinear and equally spaced receivers can be used to locate a microseismic source in cylindrical coordinates (r, z, θ), where the radial distance r and depth z can be estimated. However, the azimuthal component θ cannot be estimated. This approach assumes that the velocity is constant or that it can be assumed to be constant with the RMS assumption. A vertical array of receivers will contain a number of combinations of three collinear and equally spaced receivers. Each combination can be used to compute a unique estimate of the source. The mean of all these estimates will produce a more accurate estimate of the source locations.

The z axis can be arbitrary as in a deviated well, but I will assume it to be vertical.

METHOD

Estimating the number of combinations

Consider a vertical array of 7 equally spaced receivers with a receiver interval h, numbered from 1 to 7. Combinations with an interval of h, 2h and 3h can be formed as indicated below:

```
1   Rec.. 1  2  3
2   Rec.. 1  3  5
3   Rec.. 1  4  7
4   Rec.. 2  3  4
5   Rec.. 2  4  6
6   Rec.. 3  4  5
7   Rec.. 3  5  7
8   Rec.. 4  5  6
9   Rec.. 5  6  7
```

For an array of N_{rec} receivers, there will be N_{comb} combinations of three equally spaced receivers. The equation for an odd number of receivers is:

$$N_{comb-odd} = \frac{\left(N_{rec} - 1\right)^2}{4} = \frac{N_{rec}^2 - 2N_{rec} + 1}{4}, \tag{1}$$

and for an even number of receivers it is:

$$N_{comb-even} = \frac{N_{rec} * \left(N_{rec} - 2\right)}{4} = \frac{N_{rec}^2 + 2N_{rec}}{4}. \tag{2}$$

The following shows the number of receivers and a corresponding number of combinations. Notice that the number of combinations increases somewhat proportional to the square of the number of receivers:

```
NrecAry =   3    4    5    6    7    8    9   10   11   12   13   14   15   16   17   18   19   20   21
Ncomb   =   1    2    4    6    9   12   16   20   25   30   36   42   49   56   64   72   81   90  100
```

Error in estimating the clock-time of the receivers

The trace containing the microseismic event is assumed to be from a hydrophone or a directional recreation of the raypath from a three components receiver. Under ideal conditions, the clock time at the receivers can be measured very accurately, even though the sampling of the data is discrete. Cross-correlation of the recorded wavelet with the same wavelet yields a zero-phase wavelet with a peak that can be accurately interpolated. Even though the correlating wavelet may not be known accurately, the distortion in the cross correlation will be the same or similar for traces recorded close to each other. This timing error can be considered an error in the clock-time and will have no influence on the estimation of the source location.

Random noise that is independent from receiver to receiver is usually added to the amplitudes of the recorded time samples. This creates a bias on the cross-correlation and will shift the peak from trace to trace, causing a jitter or time error on the arrival clock-time. These errors in clock-times will produce a corresponding error in the estimated source location.

I will assume that the error in the clock-times can be represented by a Gaussian distribution with a standard deviation close to the recording error. I will use a standard deviation (SD.) of the noise in the range of 1.0 ms to 0.1 ms. This distribution may not be the best to use as extreme values can occur, but at this point it is the only one available. Coherent noise can produce an error in the measured clock-times but will be ignored.

Radom noise is added to the recorded times of each receiver in the array. All combinations of receivers will be used to compute estimates of the source location, and a mean value of the estimate will be computed for this trial. This process will be repeated with new noise values added to the receiver times. The distribution of the trial solutions will be used to identify the sensitivity of the estimated source location using an array of receivers.

Estimating the source location using three collinear receivers

In a companion paper (Bancroft et al. 2009), the source location (r_0, z_0, t_0) using three receiver clock-times t_1, t_2, and t_3 is given to be:

$$t_0 = \frac{t_1^2 - 2t_2^2 + t_3^2 - 2t_h^2}{2\left(t_1 - 2t_2 + t_3\right)}$$

$$r_0 = sqrt\left[v^2\left(t_1 - t_0\right)^2 - \left(z_1 - z_0\right)^2\right] \tag{3}$$

$$z_0 = \pm sqrt\left[v^2\left(t_1 - t_0\right)^2 - \left(x^2 + y^2\right)\right],$$

where h is the distance between receivers, v is the velocity, and $t_h = vh$.

Examples of one source and a vertical array of three receivers

The following figure shows the estimated source locations for one defined source location and a vertical array of three receivers. One hundred trials were conducted with 1.0 ms noise added to the receiver clock-times. The mean of the estimated solution is shown with a black "+". Figure 1a shows the results of valid solutions while Figure 1b shows the results that have all non-legitimate solutions removed (10 in this case), and then the outliers removed with a 10 percent alpha-trim mean.

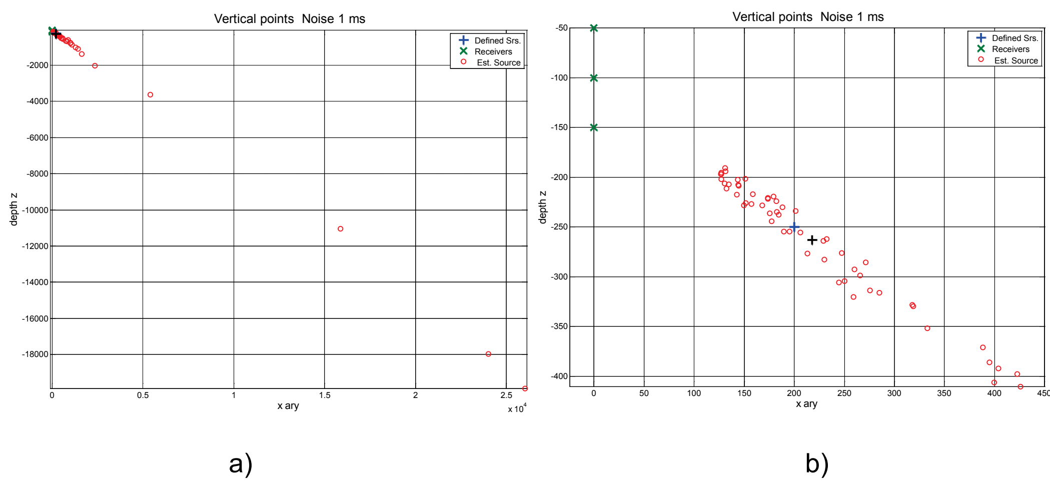
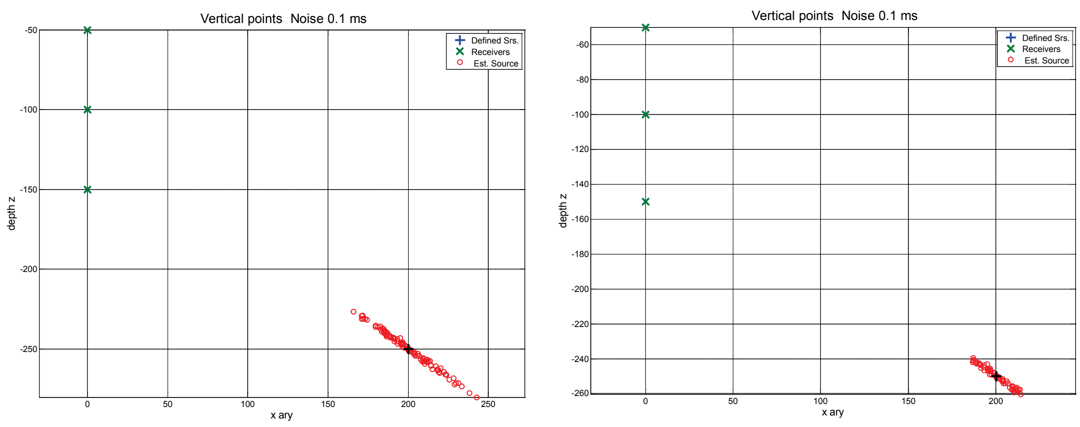


FIG. 1. Estimated source locations for 100 noisy time measurements on three receivers, a) raw data and b) with an alpha-trim mean. SD. of the noise is 1.0 ms.

The standard deviation (SD.) on the radial location was 86 m and the depth 62 m. The amplitude of the noise was reduced to 0.1 ms, producing the results in Figure 2. The corresponding SD.s were 8.4 and 5.6 m.

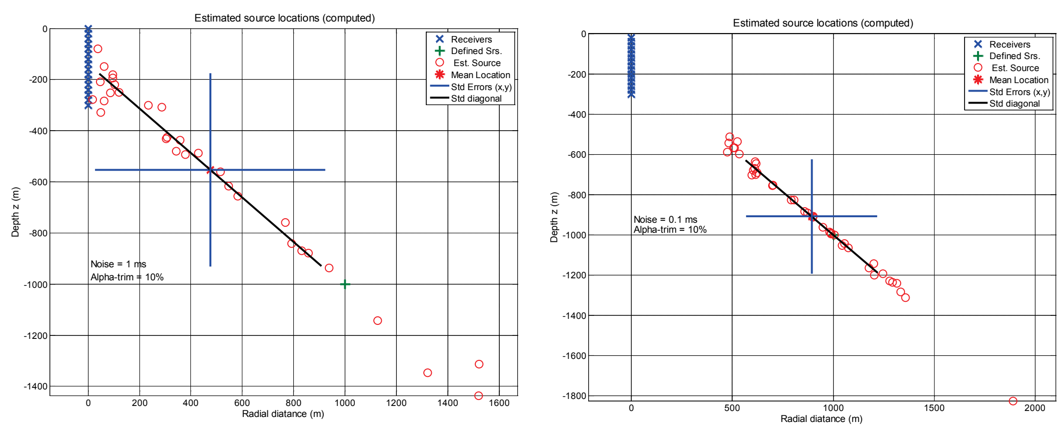


a) b)

FIG. 2. Estimated source locations for 100 noisy time measurements on three receivers, a) raw data and b) with an alpha-trim mean. SD. of the noise is 0.1 ms.

Examples of one source and a vertical array of many receivers

I will now use a vertical array with 16 receivers with a 20 m interval. The noise in Figure 3a has a SD. of 1.0 ms, and in (3b) it has a SD. of 0.1 ms.. The source is located at (1000, -1000) and there are 56 possible combinations. Figure 3a had only 31 valid solutions which were then alpha-trimmed for the mean computation and display. Figure 3b had a SD. of 0.1 ms and still had only 40 valid solutions.



a) b)

FIG. 3 Sixteen receivers with 56 possible combinations with a) a SD. on 1.0 ms and b) a SD. of o.1 ms.

The intersection of the large blue "+" sign in Figure 3 shows the average of the estimated source locations and their length of the SD. for *r* and *z*. The black line is the SD. of the length of the vectors from the central receiver to the estimated source location.

An alternate method for estimating the source location is based on the examples of Figure 3b but are now displayed with vectors drawn from the centre receiver to the estimated source location. Note that the intersection of the vectors provides an estimate of the source location. Consequently, a least squares solution was developed to estimate the best fit (r, z) for the intersection of the vectors.

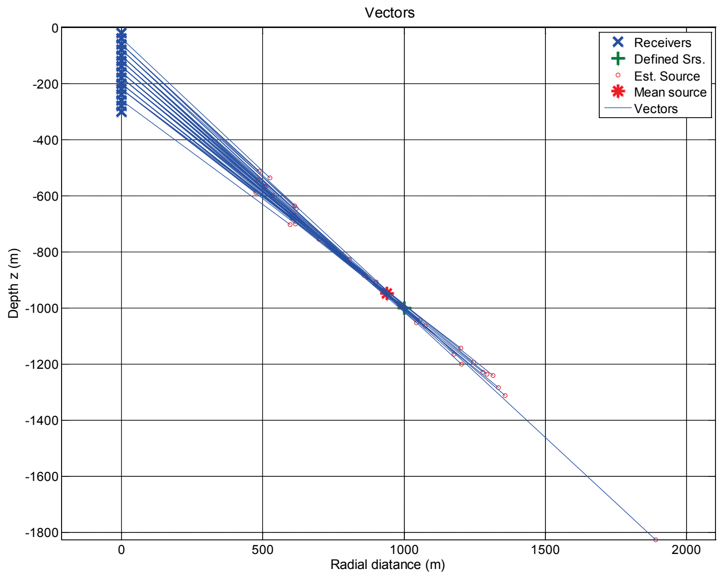


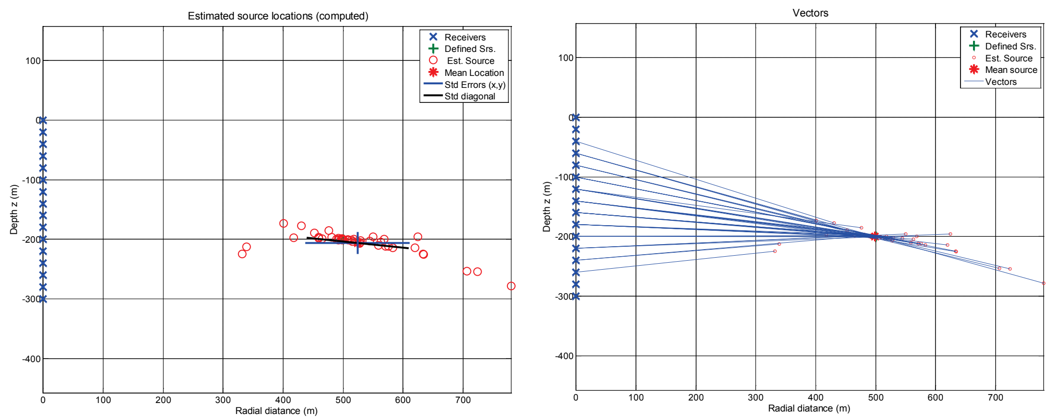
FIG. 4 Results of Figure 3 are plotted with vectors from the center of each combination to the estimated location.

The following table shows the defined source location with the estimated locations based on the analytical solutions, and the vector intersection. The data show the vector solution to be superior. That is usually the case when the solutions are well behaved.

	Radial Est.	Error	Z Est.	Error
Point	1000.00		-1000.00	
Point	893.05	106.95	-908.21	-91.79
Vector	938.86	61.14	-948.51	-51.49

The following figures show the results when the source is located at (500, -200) and the SD. of the noise at 0.1 ms. In this case, the vector solution is much more accurate.

	Radial Est.	Error	Z Est.	Error
Point	500.00		-200.00	
Point	524.19	-24.19	-206.04	6.04
Vector	498.53	1.47	-199.78	-0.22

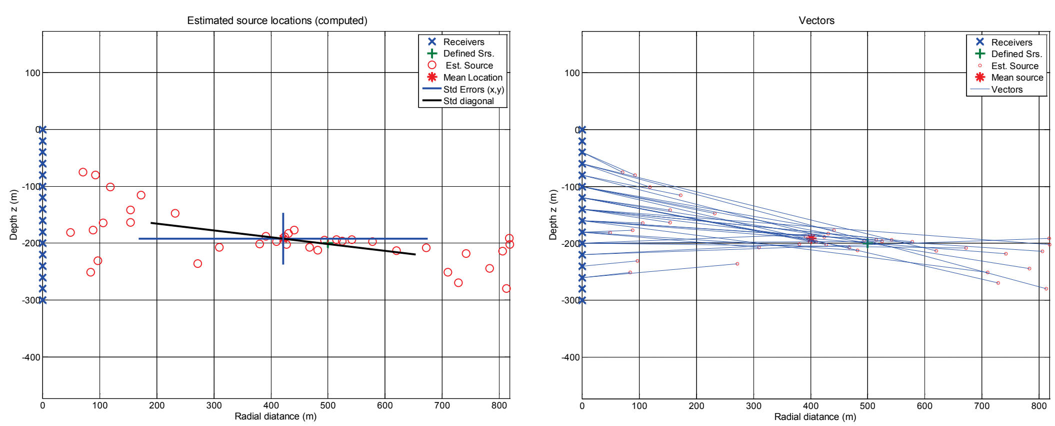


a) b)

FIG. 5 The source is located at (500, -200) a) showing the analytic solutions and b) the vectors with a SD. of 0.1 ms.

The geometrical configuration of Figure 5 is repeated but with a SD. of 1.0 ms.

	Radial Est.	Error	Z Est.	Error
Point	500.00		-200.00	
Point	421.57	78.43	-192.01	-7.99
Vector	401.21	98.79	-191.64	-8.36



a) b)

FIG. 6 The source is located at (500, -200) a) showing the analytic solutions and b) the vectors with a SD. of 1.0 ms.

Stability of the two methods

One hundred trials of the vertical array with one source were conducted to evaluate the stability of the source locations. The means of the 100 trials were similar to the means of one trial, and the SD.s of the 100 trial means was smaller than the errors above and did not include the true source location.

One hundred trials with noise = 1.0 ms:

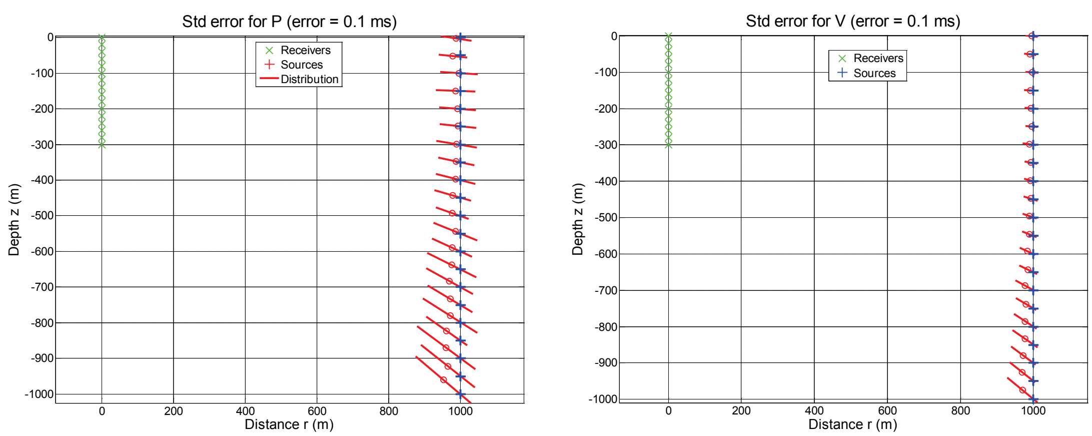
	Radial Est.	R. SD.	Z Est.	D. Std
Point	500.00		-200.00	
Point	432.38	60.11	-192.17	7.55
Vector	416.36	47.77	-191.81	6.00

One hundred trials with noise Std = 0.1 ms:

	Radial Est.	R. Std	Z Est.	D. Std
Point	500.00		-200.00	
Point	497.45	17.05	-199.68	2.04
Vector	499.55	4.66	-199.98	0.52

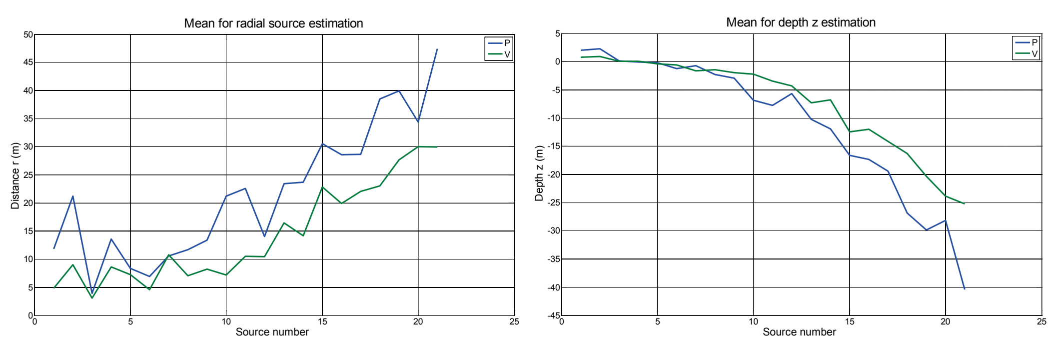
Various source locations

The source location in now varied, and at each location the estimated location and its radial distribution are plotted. Displays for the point "P" and vector "V" solutions are shown separately in Figure 7. Plots comparing the radial and depth SD.s are also shown in Figure 8. The first two images locates the sources at R = 1000 m, and varying from depths of z = 0 to 1000 m with a SD. of 0.1 ms. Figures 9 and 10 compare the results for various noise levels and offsets. In Figure 11, the length of the vertical array of receivers is doubled while the number of receivers remains the same. In Figure 12, the receiver array has only 8 receivers that are spread over the same distance.



a) b)

FIG. 7 Vertical array of receivers and source locations showing the estimated location and the radial SD., a) for the "P" estimate and b) the "V" estimate. The noise is 0.1 ms.



a) b)

FIG. 8 Comparison of the P and V solutions for a) the radial and b) the depth estimates.

The SD. of the noise in increased to 1.0 ms, and the poor results are shown in Figure 9.

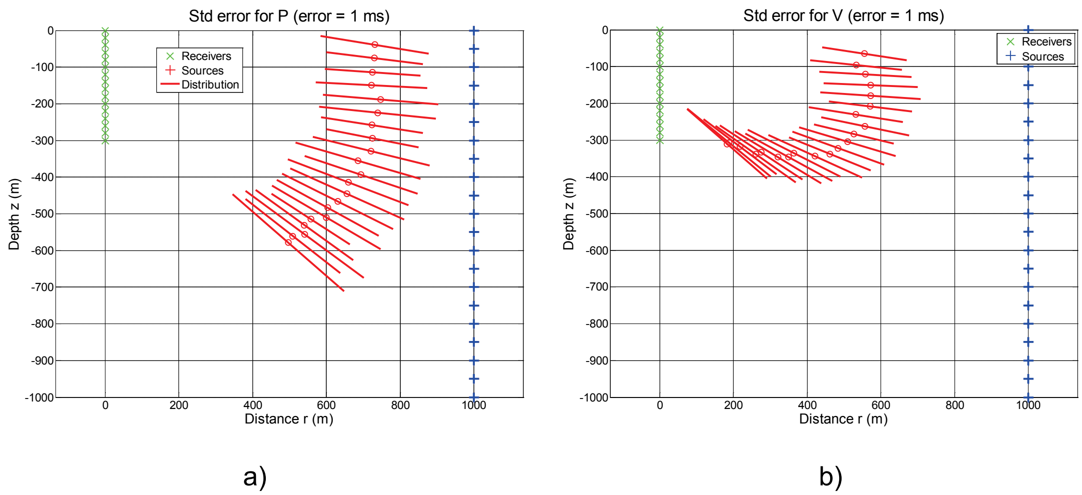


FIG. 9 Vertical array of receivers and source locations showing the estimated location and the radial SD., a) for the "P" estimate and b) the "V" estimate. The noise is 1.0 ms.

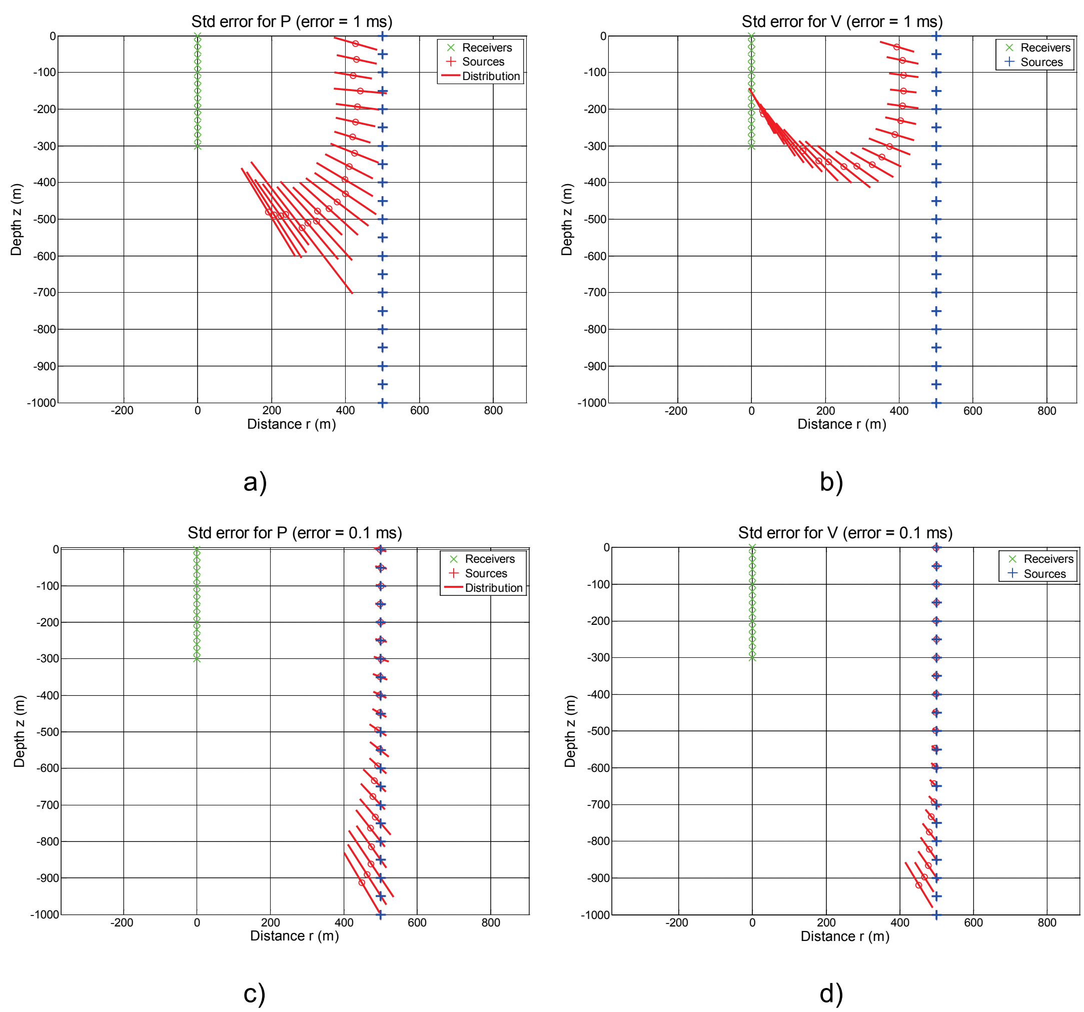


FIG. 10 Noise with SD.s of 1.0 ms a) and b) and 0.1 ms c) and d) at source location r = 500 m.

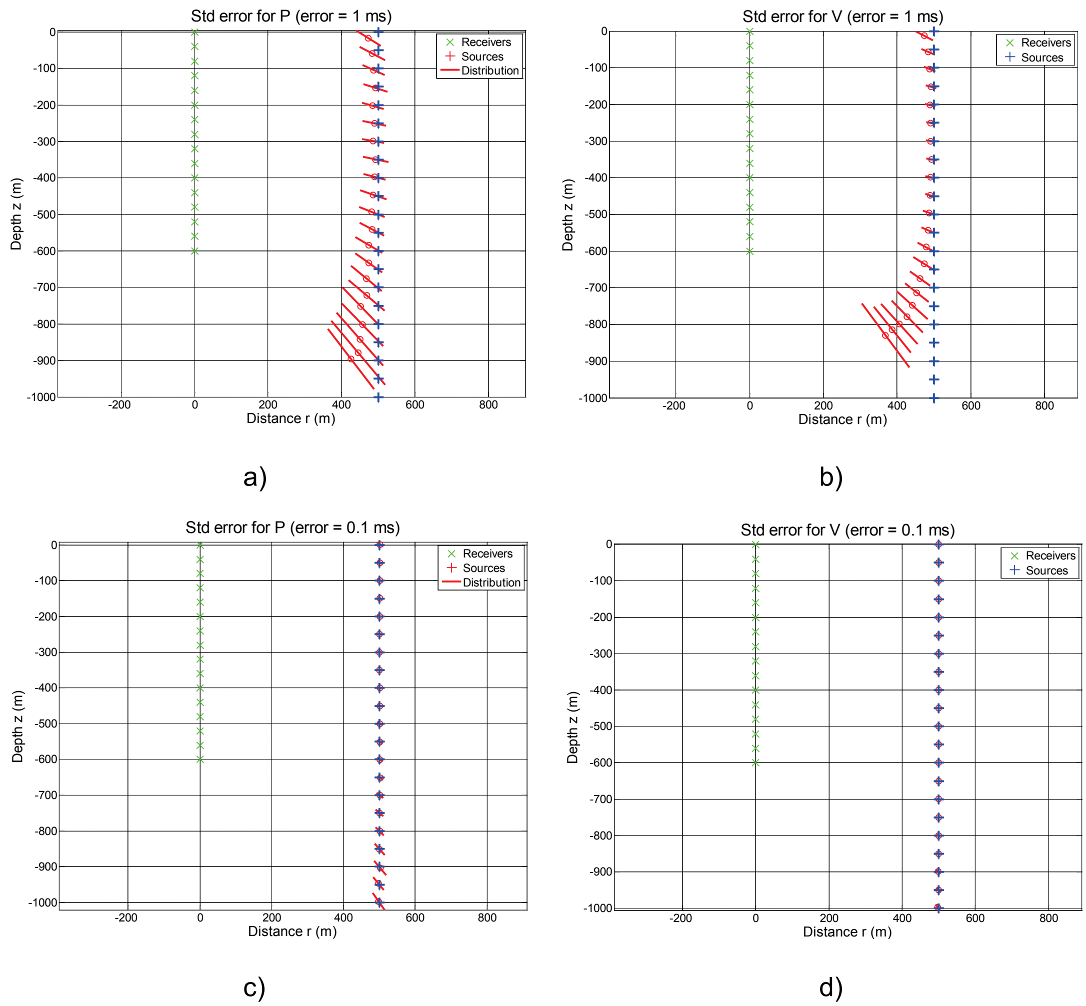


FIG. 11 Noise with SD.s of 1.0 ms a) and b) and 0.1 ms c) and d) at source location *r* = 500 m. In this figure, the vertical receiver array has doubled in size, but the number of receivers remains the same

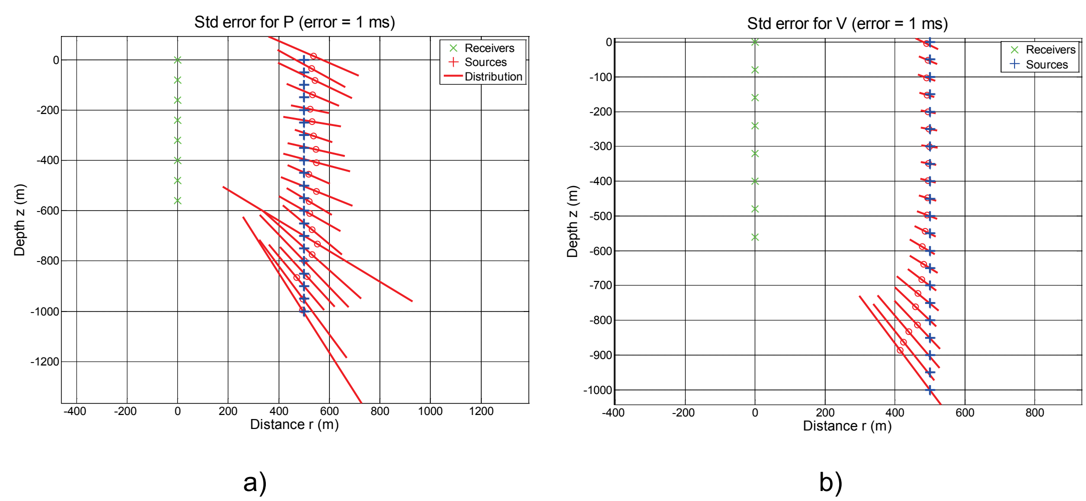


FIG. 12 Noise with SD.s of 1.0 ms a) P and b) V, where the number of receivers is reduced to 8 with a spacing of 80 m.

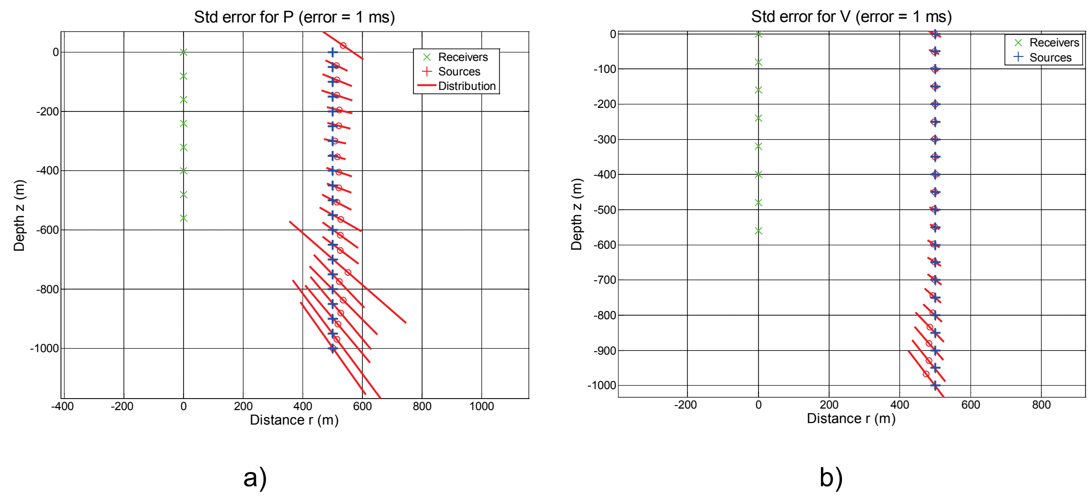


FIG. 13 Properties are identical to those in Figure 12, but now the velocities have been reduced from 3000 m/s to 1500 m/s, to simulate the difference between the first arrival time for the P wave and the S wave.

Figure 13 shows the results when changing the velocity from 3000 m/s to 1500 m/s, simulating the difference in the arrival times for P and S-waves. Except for the velocity, the parameters are identical to those in Figure 12. Using the S-wave, first arrival times may produce greater accuracy in estimating the source location than when using the P-wave first arrival times.

COMMENTS AND RESULTS

Error and sensitivity plots were shown for estimating source locations of microseismic events using a vertical array of receivers. Noise was added to the clock-time of the receivers, and the errors in estimating the source location was displayed. The errors vary with orientation and distance from the receiver array and the size of the receiver array.

The raypath traveltimes (delta times) for the S-wave are approximately twice as long as the traveltimes for the P-wave. The defined traveltime errors for the S-wave will be relatively smaller then the p-wave, and should produce a more accurate estimation of the source location.

ACKNOWLEDGEMENTS

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REFERENCES

Bancroft, J.C., 2009, Sensitivity measurements for locating microseismic events, CREWES Research Report.