Taking steps toward generating angle-domain common-image gathers

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ABSTRACT

The amplitude of the output of any true-amplitude migration can be used to estimate the angular-dependant reflectivity. However, explicit information regarding the reflection angle will be missing after a standard true-amplitude migration. In an amplitude-versus-angle (AVA) study, the explicit relation of the reflectivity function to the reflection angle is needed. This motivates us to work toward having the reflectivity function in the angle-domain from migrated data.

This short note summarizes the steps that we will investigate towards generating angle-domain common-image gathers (ADCIGs). Two approaches to generate ADCIGs are considered. In the first method, a shot-domain common-image gather (SDCIG), generated from true-amplitude common-shot Kirchhoff migration (Bleistein et al., 2001) is converted to the ADCIG; a postmigration mapping from surface-domain (shot-coordinate) to angle-domain. Based on the a priori knowledge of opening angle the calculated reflectivity function is placed in an angle bin. In standard common-shot migration and migrating *nshot* the reflectivity function for each subsurface point is stored in a vector of size $nz \times nshot$ (*nz*: number of depth samples). In this method the reflectivity function will be stored in an $nbin \times nz$ array where the *nbin* is number of angle bins. In the second approach, a 2.5D version of common-opening angle migration discussed in Bleistein and Gray (2002) for constant velocity is presented.

FIRST METHOD: POSTMIGRATION MAPPING

The common-shot migration/inversion (M/I) method of Bleistein et al. (2001) is considered to generate the SDCIGs for two-and-one-half dimensional (2.5D) data. The 2.5D formulas allow for migration of a single 2D line of data that take into account many aspects of 3D wave propagation. The following are the two 2.5D reflectivity function formulas used for common-shot M/I of Bleistein et al., (2001) for constant background velocity,

$$\beta(x;x_s) = \frac{4x_3}{\sqrt{2\pi c^3}} \int dx_g \frac{\sqrt{r_s + r_g}\sqrt{r_s}}{r_g^{3/2}} \cos\theta \int \sqrt{\omega} d\omega \, e^{-i\omega(r_s+r_g)/c + i\pi/4} u(x_s, x_g, \omega), \quad (1)$$

$$\beta_1(x;x_s) = \frac{2x_3}{\sqrt{2\pi c}} \int dx_g \frac{\sqrt{r_s + r_g}\sqrt{r_s}}{r_g^{3/2}} \int \sqrt{\omega} d\omega \, e^{-i\omega(r_s+r_g)/c + i\pi/4} u(x_s, x_g, \omega) \quad (2)$$

where x is a subsurface point, x_s is the source location of the common-shot gather, x_g is all the receiver points of the common-shot gather, c is the background velocity at the subsurface point x, r_s and r_g are ray distances from x to the source and receiver, 2θ is the opening angle between r_s and r_g, and $u(x_s, x_g, w)$ is the data generated at the source at x_s and recorded at the receiver located at x_g. Simplifying the formulas for the reflectivity functions $\beta(x,x_s)$ and $\beta_1(x,x_s)$ results in

$$\beta(x;x_s) = \int \frac{2\cos\theta}{c} \mathrm{W}(x_s,x_g)\, D(x,x_s,x_g) dx_g \ , \tag{3}$$

$$\beta_1(x;x_s) = \int \mathrm{W}(x_s,x_g)\, D(x,x_s,x_g) dx_g , \tag{4}$$

with $D(\xi)$ being a filtered version of the data, defined as

$$D(x,x_s,x_g) = \int i\omega \, d\omega \, e^{-i\omega(r_s+r_g)/c} u(x_s,x_g,\omega), \tag{5}$$

and W is the weighting function related to the amplitude.

The reflectivity function $\beta_1(x,x_s)$ gives the reflectivity magnitude when x is a specular point, and matches the conventional definition of reflectivity as follows

$$\text{peak of } \beta_1(x,x_s) \cong R(x,x_s). \tag{6}$$

However the reflectivity $\beta_1(x;x_s)$ hides information about the opening angle at the specular point while angle information is implicitly contained within the reflectivity $\beta(x;x_s)$. The reflectivity functions, $\beta(x,x_s)$ and $\beta_1(x,x_s)$, differ only by a factor of $2\cos\theta / c$. This difference allows us to estimate the $\cos\theta$ term from the ratio of the outputs without having to determine the specular source-receiver pair that produces the value of the related opening angle. For 2D synthetic data, Bleistein et al. (2001) showed that the comparison between numerical estimates for $\cos\theta$ of the specular opening angle and its theoretically determined counterpart is of high accuracy. Only for very far offsets is there a falloff in accuracy. This concept is explored in another paper in this volume (Sharma and Margrave, 2009). We will use this idea to estimate the opening angle from the ratio of the two reflectivity functions. We will perform some accuracy test on such an estimation before application to the first method. The opening angle estimates will be used in sorting reflectivities into angle bins.

For our proposal, the common-shot M/I will be applied to every shot and with only one migration pass both reflectivity functions $\beta(x;x_s)$ and $\beta_1(x;x_s)$ will be computed. The magnitude of the reflectivity is provided by $\beta_1(x;x_s)$) and the reflection angle information is obtained from the ratio of two the reflectivity functions. SDCIGs will be generated as information regarding the reflectivity function $\beta_1(x;x_s)$ and the reflection angle[1] θ is available. Now using a postmigration mapping the SDCIGs, whose traces are indexed by the lateral distance between image location and source location, will be mapped to ADCIGs whose traces are indexed by the opening angle at the image reflectors.

[1] Half of the opening angle for isotopic media is the reflection angle.

The following describes the postmigration mapping scheme. Consider the generated SDCIG at the subsurface point x, and the trace related to the first shot. For a particular depth, the magnitude of reflectivity ($\beta_1(x,z)$) and the reflection angle θ (for example, 10°) is available as we described. To map this point from the shot-coordinate-domain into the angle-domain, this reflectivity will be stored in an array of different sized angle bins. Consider a 5-bin storage for angle bins as $(1^\circ - 3^\circ)$, $(3^\circ - 5^\circ)$, $(5^\circ - 8^\circ)$, $(8^\circ - 13^\circ)$ and $(13^\circ - 35^\circ)$. For this example the reflectivity will be stored in the fourth angle bin. This mapping procedure of the point x at a particular depth is depicted in Figure (1).

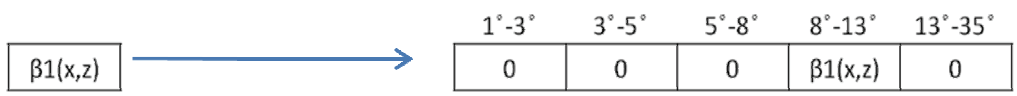

Figure 1: An example of the postmigration mapping of a subsurface point of (x,z).

With this mapping, the trace related to shot one in the SDCIG with the size of $nz \times 1$, will be mapped into matrix of size $nbin \times nz$ (here $nbin$ = 5) representing the angle gather. Figure (2) illustrates this mapping for one trace of SDCIG at the point x. For the SDCIG at point x, this mapping to the angle-domain is repeated from every trace. Thus, the ADCIG at the point x is updated after mapping each trace. Assume we have 100 shots. The SDCIG at point x has the size $100 \times nz$, while the ADCIG has the size $nbin \times nz$, as shown in Figure (3).

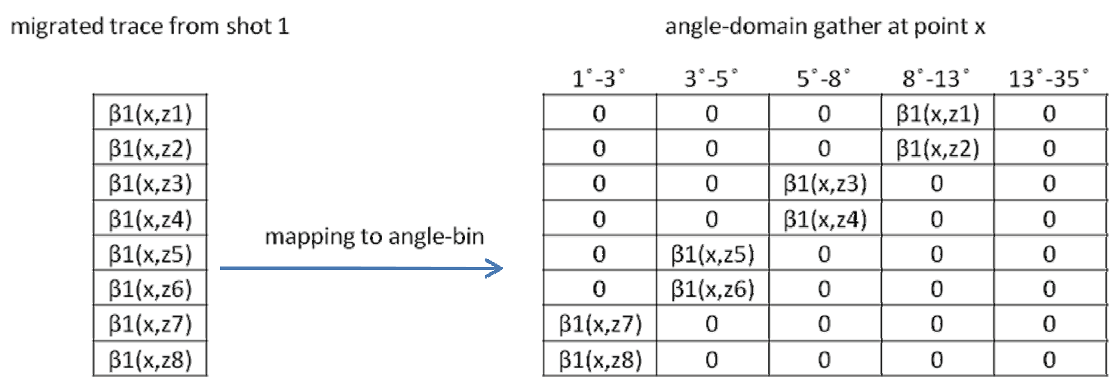

Figure 2: An example of the postmigration mapping for a trace with the size of $nz \times 1$ $(nz = 8)$, to angle domain with the size of $5 \times nz$.

At some depths, the reflectivity from several trace might be mapped to the same angle bin. In that case, the average of that reflectivity will be stored. This postmigration mapping might be accurate enough in some qualitative AVO analysis, but will not be accurate enough for a quantitative amplitude analysis. This mapping due to averaging amplitude in angle-bins, will not preserved the true amplitude nature of the migrated traces from the true amplitude common-shot migration. In that case, increasing the number of angle bins might be considered as a remedy. For example, having 1° angle bin increments up to some maximum angle could be implemented. The next step in generating the ADCIGs, is to think of a true amplitude common-angle migration. In this regard, a simplified version of the common-opening-angle migration/inversion of Bleistein and Gray (2002) for 2.5D data will be our second method.

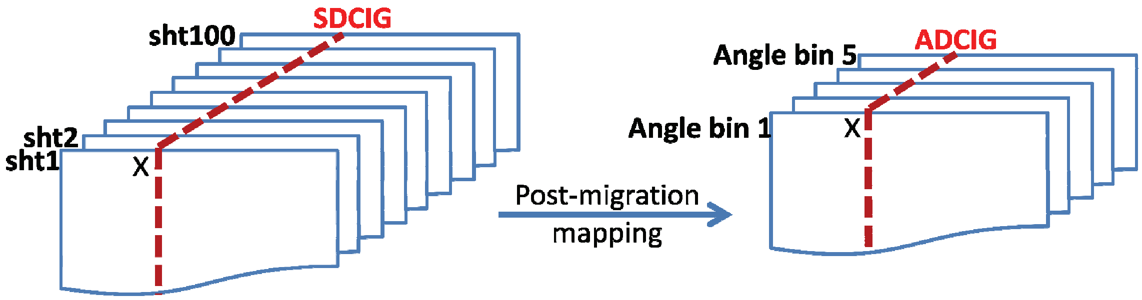

Figure 3: Postmigration mapping of a SDCIG at the point x to the ADCIG at that point.

SECOND METHOD: TRUE AMPLITUDE COMMON-OPENING ANGLE MIGRATION OF BLEISTEIN AND GRAY (2002)

Bleistein and Gray (2002) derived a common-opening angle M/I for the general 3D case. Their formula is derived as an integral over opening angle at an output point at depth. Due to difficulties in numerical implementation of an integral over opening angle, a transform from the subsurface coordinate (opening angle) to a surface coordinate was presented by Bleistein et al. (2005). We will follow their 2D transform to calculate the reflectivity as a function of opening angle.

Consider true-amplitude common-shot migration of 2.5D data. As mentioned earlier in the report, the reflectivity function $\beta(x, x_s)$ is actually the conventional reflectivity at the image point x from common-shot migration with the source at x_s which will be called $R(x; x_s)$ to make a clear definition of reflectivity. Therefore, the expression for $R(x; x_s)$ for 2.5D data, with the assumption of constant velocity, is

$$R(x;x_s) = \frac{2x_3}{\sqrt{2\pi c}} \int dx_g \frac{\sqrt{r_s + r_g}\sqrt{r_s}}{r_g^{3/2}} \int \sqrt{\omega} d\omega \, e^{-i\omega(r_s+r_g)/c + i\pi/4} u(x_s, x_g, \omega). \qquad (7)$$

The peak value of this reflectivity function is the angle-dependent reflection coefficient at the specular incident angle. Thus, we can equivalently write the reflectivity as $R(x;\theta)$, with θ being the specular incident angle. Now, for a fixed reflection angle θ and an angle increment of Δ, take an average of the reflectivity function as follows:

$$\overline{R}(x;\theta) = \frac{1}{\Delta} \int_{\theta-\Delta/2}^{\theta+\Delta/2} R(x;\theta')d\theta' = \frac{1}{\Delta} \int_{x_s} R(x:x_s) \times \left| \frac{dx_s}{d\theta'} \right|^{-1} dx_s. \qquad (8)$$

For which the Jacobian of this transform is derived in Bleistein et al., (2005) as,

$$\left|\frac{dx_s}{d\theta'}\right|^{-1} = 8\pi \frac{\cos\alpha_s}{c_0} A^2(x, x_s). \tag{9}$$

The quantity α_s is emergence angle at the source location, v_s is surface velocity at the shot location, and $A(x, x_s)$ is the WKBJ Green's function amplitude for a ray at the subsurface point x due to the source at x_s. Substituting the Jacobian in equation (9) into equation (8), gives the reflectivity function as a function of angle,

$$\overline{R}(x;\theta) = \frac{8\pi}{\Delta} \int_{x_s} \frac{\cos\alpha_s}{c_0} A_s^2 R(x; x_s) dx_s. \tag{10}$$

For constant velocity, a very good approximation for the WKBJ Green function amplitude is

$$A(x, x_s) \cong \frac{1}{4\pi r_s}. \tag{11}$$

Substituting $R(x; x_s)$ from equation (7) and the WKBJ amplitude from equation (10) results in

$$\begin{aligned}\overline{R}(x;\theta) = \frac{x_3}{\Delta\pi\sqrt{2\pi}} \int dx_s \frac{\cos\alpha_0}{c_0 r_s^2} \int dx_g \frac{\sqrt{r_s + r_g}\sqrt{r_s}}{\sqrt{c} r_g^{3/2}} \times \\ \int \sqrt{\omega} d\omega\, e^{-i\omega(r_s + r_g)/c + i\pi/4} u(x_s, x_g, \omega)\end{aligned} \tag{12}$$

Formula (12) will be implemented in a MATLAB based code to calculate common-angle M/I directly.

CONCLUSIONS AND FUTURE WORK

This short note was to inform the CREEWES sponsors the direction of our current research. The report describes the steps that will be taken in our next year's research, directed towards generating ADCIGs. The two mentioned methods will be implemented in MATLAB. The accuracy and the feasibility of the application will be investigated and tested.

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