Reflection and Transmission coefficients for SH wave in Plane Wave Domain

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ABSTRACT

The calculation of reflection and transmission coefficients at any interface is an important problem of elastodynamic theory. Historically, reflection and transmission coefficients have been obtained in several domains according to their importance. It is known that for isotropic case reflection and transmission coefficients depend on the acoustic impedance contrast and angle of incidence. The angle of incidence can be computed by using the cross product of two known unit normals. In this approach, derived reflection and transmission coefficients will be in plane wave domain. By deriving reflection and transmission coefficients in plane wave domain for 3D media, determination of dip and azimuth of interface are avoided, and, thereby, we avoid ray tracing ray tracing and exposure to caustics especially in anisotropic media. The problem that I solve, in this regard, is the problem of the special case of dipping interface and how to rotate the plane wave coordinate system from that determined by the computational grid, and the system determined by a dipping interface. Classical reflection and transmission coefficients in plane wave coordinates are worked out for reflectors aligned with the computational grid. For non-aligned reflectors, those with dip and azimuth, computation of effective reflection and transmission coefficients is not straight forward, for this the coordinate system must be rotated. To do this, a normal for each individual plane wave based on local velocity and vector cross product of this normal with the normal to reflector are computed. This cross product yields a ray parameter that presently is used to compute corresponding reflection and transmission coefficients for a given plane wave. The importance of this approach is the automatic adaptation of the reflection and transmission coefficients expression to a special case of dipping interface. These coefficients can then be used to scale the amplitude component of plane wave extrapolation across a reflector as is done in seismic forward modeling. Another importance of reflection and transmission coefficients in plane wave domain, is their use in Rayleigh Sommerfeld Modeling(RSM) of seismic data. In line traces and cross line traces are required in order to model the plane wave inputs. Presently, the problem associated with data acquisition is studied here by changing the number of cross line traces.

INTRODUCTION

Two methods, namely, Rayleigh Sommerfeld(RS) and Kirchhoff methods have been adopted for forward modeling in seismic exploration. Both the methods are originated from optical diffraction theory (Ersoy, 2007). RSM has been adopted in seismic instead of Kirchhoff modeling because of its superiority in terms of computing time with a large data. Rayleigh Sommerfeld Modeling(RSM) is known as 3D modeling technique. In the past, RSM was limited to laterally varying but angle independent reflection(R) and transmission coefficients. This work was elaborated by Cooper and Margrave for RSM with AVO. Ray tracing was used to compute incident angle at reflecting interface. Presently, an approach is proposed for RSM with AVO in plane wave domain in order, to avoid ray tracing and associated problem with it like caustics especially in anisotropic media, and to make RSM applicable for dipping interface. Basic geometry and result of Rayleigh-Sommerfeld theory are described by Margrave (Margrave and Cooper, 2007). Following the Margrave, Rayleigh Sommerfeld modeling (RSM) can be described in three major steps

1. Extrapolation of source wave field from the source to a datum reflector
2. Multiplication of the extrapolated wavefield in (1) with a reflectivity function
3. Extrapolation of (2) back to the surface.

In frequency domain,wavefield contains two terms known as amplitude, phase and described as

$$\Psi(\omega) = A(\omega)\, e^{i\phi(\omega)} \tag{1}$$

where ψ is the wavefield, A is the amplitude and ϕ is the phase term. Wavefield extrapolation can be done by phase shift operator (Margrave, 2008). At any boundary, the incident wavefield is partially reflected from and transmitted across the boundary. Reflection and transmission coefficients play an important role in order to obtain reflected and transmitted wavefield. Since the amplitudes of reflected and transmitted wavefield are obtained by multiplying the R and T coefficients with the incident wavefield. Thus, to accomplish RSM in plane wave domain, R and T coefficients are required in plane wave domain. Generally in past, reflection and transmission coefficients are given in terms of angle. Presently, an approach is described which is used to derive the reflection and transmission coefficients in plane wave domain.

METHODOLOGY

Presently, an approach, of going from spatial frequency domain into plane wave domain with out $\tau - p$ transformation, delineated. The preceding argument describes the importance of this approach in order to derive RSM in plane wave domain.
It is known that wavefield extrapolation in the $x - t$ domain involves two dimensional convolution(Berkhout, 1982). Time can be replaced by temporal frequency and now convolution, in one dimension only, is involved in extrapolation. The space variable x is replaced by the spatial frequency variable k_x by a second Fourier transformation and only multiplication is involved for extrapolation in the $k_x - \omega$ domain. Now, an approach is invoked at this place in order to transform the wavefield from $x - \omega$ domain into plane wave domain. The mathematically description is given in next paragraph.
The monochromatic wavefield can be transformed from space domain into spatial frequency domain as

$$\varphi\left(\vec{k}\right)_{\omega} = \int \Psi(\vec{x})_{\omega}\, e^{\left(i\vec{k}\bullet\vec{x}\right)} \vec{dx}, \tag{2}$$

where $\bullet$ is the dot product, equation (2) can be written as

$$\varphi\left(\vec{k}\right)_{\omega} = \int \Psi(\vec{x})_{\omega}\, e^{\left(i\omega\frac{\vec{k}}{\omega}\bullet\vec{x}\right)} \vec{dx}, \tag{3}$$

now using relation $\vec{k} = \omega\vec{p}$, where $\vec{p}$ is the slowness vector equation (3) is written as

$$\varphi(\vec{p})_{\omega} = \int \Psi(\vec{x})_{\omega}\, e^{(i\vec{p}\bullet\vec{x})} \vec{dx}. \tag{4}$$

Thus using equations (2), (3), (4) monochromatic wavefield can be described in plane wave domain without using $\tau - p$ transformation.
The importance of this approach in RSM is described presently. According to Rayleigh Sommerfeld diffraction theory the source wavefield at any observation point can be described as (Margrave and Cooper, 2007)

$$\Psi(x = P) = \frac{1}{4\pi} \int_s \Psi_0(\vec{x_s}) W(\vec{x_p} - \vec{x_s}) \rho(s) ds, \tag{5}$$

where P is the observation point and $\vec{x_p} = (x_p, y_p, z_p)$, $\vec{x_s} = (x_s, y_s, z_s)$ are the coordinates of the screen (reflector) and observation point, respectively. W is the z derivative of the Green's function, $\rho(s = (x_s, y_s))$ is the reflectivity function and $d_s = d_x d_y$. Since equation (5) is convolution, it can be described in Fourier domain(Margrave and Cooper, 2007) as

$$\Psi(x) = \frac{1}{4\pi} \int_s \hat{W}\left(k_x, k_y, z_p - z_s\right) \hat{\Psi_0 \rho}\left((k_x, k_y, z_s\right) e^{ik_x x + ik_y y} dk_x dk_y. \tag{6}$$

where 'hats' indicate 2D Fourier transform over x and y. As per equation (6) Rayleigh Sommerfeld modeling is just phase shift migration backwards(Margrave and Cooper, 2007). It is revealed from above equations that Fourier transformation takes place in RSM. Now using transformation by equations (2), (3), and (4), RSM can be described in plane wave domain. Further, to accomplish the RSM it is required to obtain the reflection and transmission coefficients in plane wave domain. Now, the approach is discussed which make it possible.

Plane wave domain approach in order to obtain R and T coefficients

Historically, the calculation of reflection and transmission coefficients for a plane waves on a free surface and welded contact interface was obtained by Zeoppritz(Borejko, 1996). This work was elaborated by Aki and Richards(Aki and Richards, 1980). The analytic expressions of reflection and transmission coefficients are known in term of the incident angle. The angle of incidence is the angle that of the incident and scattered plane make with the normal to the plane reflector. The plane of incidence can be represented by the unit normal vector to the plane wave in the propagation direction and can be computed as (Ferguson and Margrave, 2008)

$$\hat{\mathbf{p}} = \frac{p_1 \hat{\mathbf{i}} + p_2 \hat{\mathbf{j}} + q \hat{\mathbf{k}}}{\sqrt{p_1^2 + p_2^2 + q^2}}, \tag{7}$$

where p_1, p_2 are the input plane wave parameters and q is the vertical slowness in the incident medium and p_1, p_2, q are coupled according to a relation derived from the dispersion relation as

$$q = \frac{1}{v} \sqrt{1 - (vp_1)^2 - (vp_2)^2}. \tag{8}$$

where v is the velocity of the incident medium. The unit normal associated with reflecting subsurface plane can be computed as

$$\hat{\mathbf{a}} = \sin\theta\cos\phi \hat{\mathbf{i}} + \sin\theta\sin\phi \hat{\mathbf{j}} + \cos\theta \hat{\mathbf{k}}. \tag{9}$$

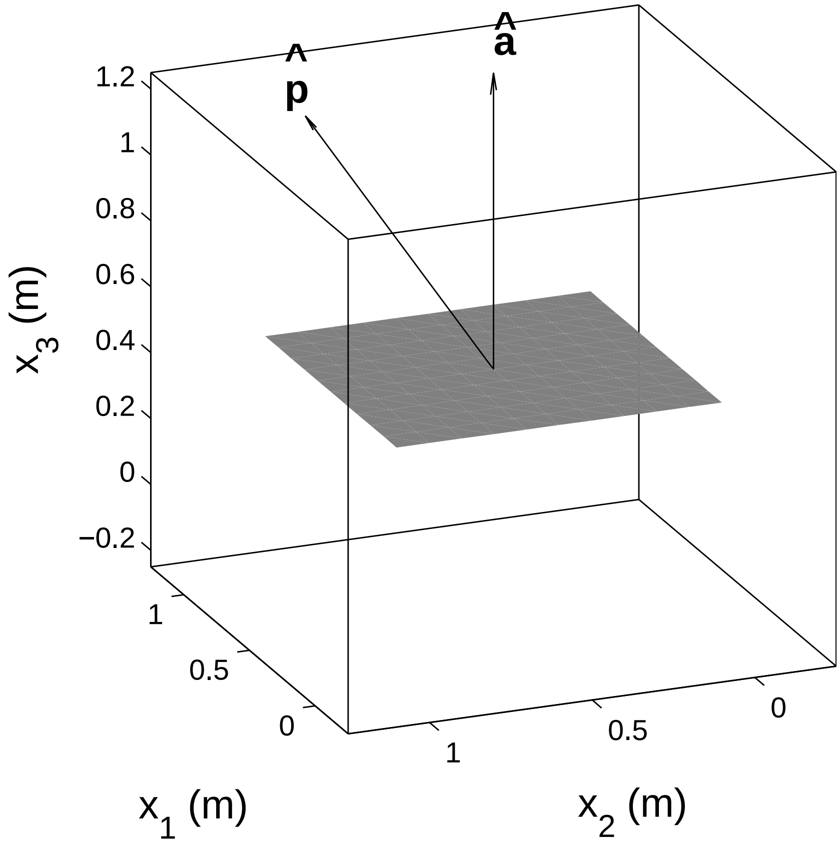

FIG. 1. Schematic representation of unit normals to plane wave and horizontal interface,the angle of incidence θ_I is the angle between normal$\hat{\mathbf{p}}$ and $\hat{\mathbf{a}}$. $\mathbf{x_1}$, $\mathbf{x_2}$ are the horizontal coordinate axis and $\mathbf{x_3}$ is the vertical axis.

where θ is the dip and ϕ is the azimuth of the normal to the interface. These two unit normals are shown in Figure 1. Figure 1 shows the two unit normal vector, $\hat{\mathbf{p}}$, normal to the plane wave in propagation direction, and $\hat{\mathbf{a}}$, normal to the horizontal interface shown by shaded plane. Now, following the simple vector calculus, the cross product of these two unit normal vectors is used to obtain the angle of incidence θ_I as

$$\sin\theta_I = |\hat{\mathbf{p}} \times \hat{\mathbf{a}}|, \tag{10}$$

The sine of angle of incidence is related to slowness along the interface as

$$p_I = \frac{sin\theta_I}{v} = |\hat{\mathbf{p}} \times \hat{\mathbf{a}}|\sqrt{p_1^2 + p_2^2 + q^2}. \tag{11}$$

where p_I is the slowness along the interface $($ray parameter$)$. Thus, the angle of incidence is obtained according to equation (10). After obtaining the angle of incidence from equation (11), this value is substituted in the known analytic expression (Kennett, 2001) in order to obtain R and T coefficients in plane wave domain.

3D R and T coefficients for SH wave

It is known that evaluating the reflection and transmission coefficients are beneficial to interpret the field records for lithology, fluid content etc by generating synthetic seismograms (Upadhyay, 2004). Incident, reflected, and transmitted plane P and S waves make a pertinent system. The assumption of two dimensional plane waves ensures us to discuss two separate groups of waves (Slawinski, 2003). These groups are the coupled P and SV waves, and the SH waves. The analytic expression of reflection and transmission coefficients for three dimensional plane waves in elastic media were given by Borejko (Borejko, 1996). Generalized ray- integral representation of pertinent waves were used in that paper.Generalized ray integral representation of SH wave for dipping structure was given by Zieger and Pao (Pao et al., 1984). Further, assuming isotropy, the standard '2D' formulas (Krebes, 2004) can be used for any plane reflector regarding its 3D orientation. According to seismic reflection theory, when an incident plane wave encounters the discontinuities in the properties at a horizontal interface between two homogeneous layers, there both the phenomenas: reflection from the boundary and transmission through the boundary take place. The boundary conditions, the continuity of displacement and traction, are considered at the boundary to obtain the amplitude information of reflected and transmitted waves. After applying these boundary conditions, reflection and transmission coefficients for SH wave are known in terms of angle (Krebes, 2004). These expressions are transformed into plane wave coordinates by estimating the angle of incidence using equation (11) and written as

$$R_{SH} = \frac{\rho_1 v_1^2 q_1 - \rho_2 v_2^2 q_2}{\rho_1 v_1^2 q_1 + \rho_2 v_2^2 q_2}, \tag{12}$$

and

$$T_{SH} = \frac{2\rho_1 v_1^2 q_1}{\rho_1 v_1^2 q_1 + \rho_2 v_2^2 q_2}, \tag{13}$$

where ρ_1, v_1 are the density and velocity of the incident medium, respectively. ρ_2, v_2 are the density and velocity of the refracted medium, respectively and q_1 is described as

$$q_1\left(\vec{p}\right) = \frac{1}{v_1}\sqrt{1 - \left(v_1 p_I\right)^2}, \tag{14}$$

and q_2 is described as

$$q_2\left(\vec{p}\right) = \frac{1}{v_2}\sqrt{1 - \left(v_2 p_I\right)^2}. \tag{15}$$

Dipping interface problem

Above expression of reflection and transmission coefficients can be used for a special case of dipping interface problem. In this case normal to the interface(see the Figure 2) would be different from horizontal one and can be computed from equation (9). Figure 2 shows the tilted interface and â is normal to this interface. An assumption, that normal to the interface lies in plane of propagation, is considered here. This constraint is applied on the equation (9). This assumption ensures that SH wave is still decoupled from P and SV waves (Sten and Wysession, 2002). Now, ray parameter for each individual plane wave is computed according to equation (11) and used in equations (12) and (13) in order to obtain

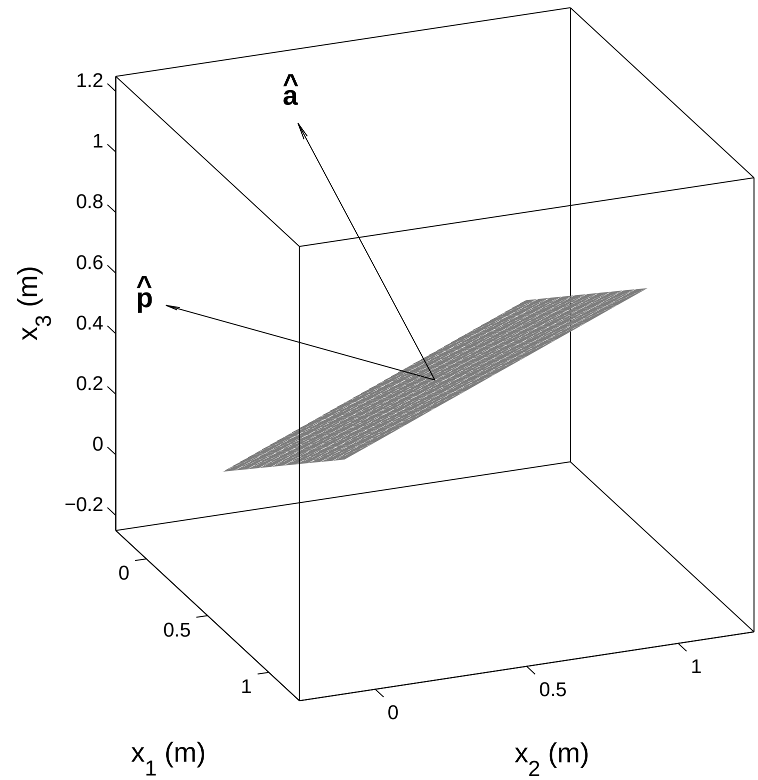

FIG. 2. Schematic representation of unit normals to plane wave and tilted interface,the angle of incidence θ_I is the angle between normal$\hat{\mathbf{p}}$ and $\hat{\mathbf{a}}$. $\mathbf{x_1}$, $\mathbf{x_2}$ are the horizontal coordinate axis and $\mathbf{x_3}$ is the vertical axis.

R and T coefficients for dipping interface, respectively.

In line and cross line slices of R and T coefficients can be obtained using $p_2 = 0$, $p_1 = 0$,respectively. Recalling equation (11) for $p_2 = 0$, the ray parameter is the same as horizontal slowness(p_1) in 2D case and for $p_1 = 0$ it would be p_2. Further, in line and cross line reflection and transmission coefficients can be obtained using equations (12) and (13) with different q_1 and q_2 from the equations (14) and (15). These equations are deduced and given as

$$q_1\left(\vec{p}\right) = \frac{1}{v_1}\sqrt{1-\left(v_1 p_1\right)^2}, \tag{16}$$

and

$$q_2\left(\vec{p}\right) = \frac{1}{v_2}\sqrt{1-\left(v_2 p_1\right)^2}, \tag{17}$$

for in line, and

$$q_1\left(\vec{p}\right) = \frac{1}{v_1}\sqrt{1-\left(v_1 p_2\right)^2}, \tag{18}$$

and

$$q_2\left(\vec{p}\right) = \frac{1}{v_2}\sqrt{1-\left(v_2 p_2\right)^2}. \tag{19}$$

for cross line.

For the zero slowness, the reflection and transmission coefficients are same for both in line and cross line cases and independent from the frequency. These are given as

$$R_{SH} = \frac{\rho_1 v_1 - \rho_2 v_2}{\rho_1 v_1 + \rho_2 v_2}, \tag{20}$$

and

$$T_{SH} = \frac{2\rho_1 v_1}{\rho_1 v_1 + \rho_2 v_2}. \tag{21}$$

respectively.

EXAMPLES

Now to explore the reflection and transmission coefficients change with p_1 and p_2, an example in which $v_1 = 1500m/s$ and $v_2 = 2500m/s$ with same density across the reflector is considered. Equations (12) and (13) are used to compute the reflection and transmission coefficients. Figure 3 shows 3D real and imaginary part of reflection and transmission coefficients, respectively. Presently, 512 in line and cross line traces are used for a particular frequency, 40Hz. Here, horizontal reflector is used. Figure 4 shows the in line and the cross line slice of reflection and transmission coefficients that are based on equations ((16) and to (19)). At zero slowness, the reflected wave has an amplitude -0.25$($as predicted by equation (20)$)$, whereas the transmitted wave amplitude is 0.75. As horizontal slowness increases to larger value, the amplitude of the transmitted wave increases and the reflected amplitude approaches to zero. Recalling equation (12), the reflected amplitude would be zero when horizontal slowness p_1 is equal to $1/\sqrt{v_1^2 + v_2^2}$ i.e 0.000343 s/m (see the appendix for mathematical manipulation). Despite crossing a significant change in velocity, there is no reflected wave for a plane wave at this horizontal slowness while transmitted wave has amplitude 1. Further, as slowness value increases larger to this slowness, the amplitude of transmitted wave continues to increase. This amplitude of transmitted wave increase due to an increase in the horizontal orientation of the transmitted wave. At the critical slowness, the transmitted wave would be horizontal. This implies that the vertical slowness in second medium will be zero. According to equation (17), this occurs at $p_1 = 1/v_2 = .0004$m/s. At this value of slowness, the amplitude is 2 for the transmitted SH wave and 1 for reflected wave. Beyond the critical slowness, there is no transmitted wave in lower layer and q_2 is imaginary in this situation. They are known as evanescent waves and their amplitudes decay with depth. Due to the imaginary vertical slowness, the reflection and transmission coefficients become complex beyond the critical slowness. Once the coefficients become complex, the shape of reflected pulse and transmitted pulse is modified (Kennett, 2001). Following this theory, there will be a distortion of the reflected and transmitted pulses at $p_1 > 1/v_2$ as depicted in Figure 4 . Same description can be used for cross line slices of the reflection and transmission coefficients.

Figure 5 shows the real and imaginary part of the 3D reflection and transmission coefficients when interface is dipping. In line and cross line slice of the reflection and transmission coefficients are shown in Figure 6. In Figure 6 the region, in between negative and positive critical slowness, is shifted and not symmetrical about zero slowness. In this figure the reflected and transmitted amplitudes are not getting values 1 and 2 respectively at positive critical slowness. The reason of this discrepancy is the discussed in the next paragraph.

The same number of in line and cross line traces are used by now. Presently, the number

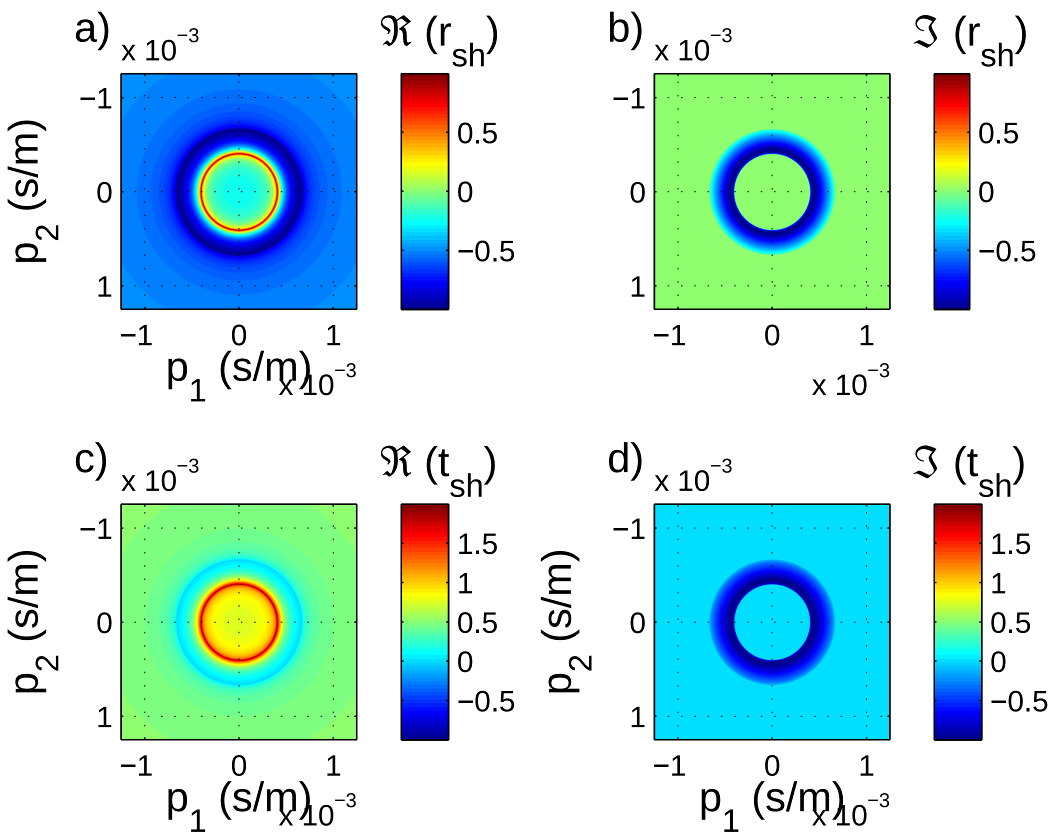

FIG. 3. a) Real part of reflection coefficient. b) Imaginary part of reflection coefficient. c) Real part of transmission coefficient. d) Imaginary part of transmission coefficient. 512 in line and 512 cross line traces are used.

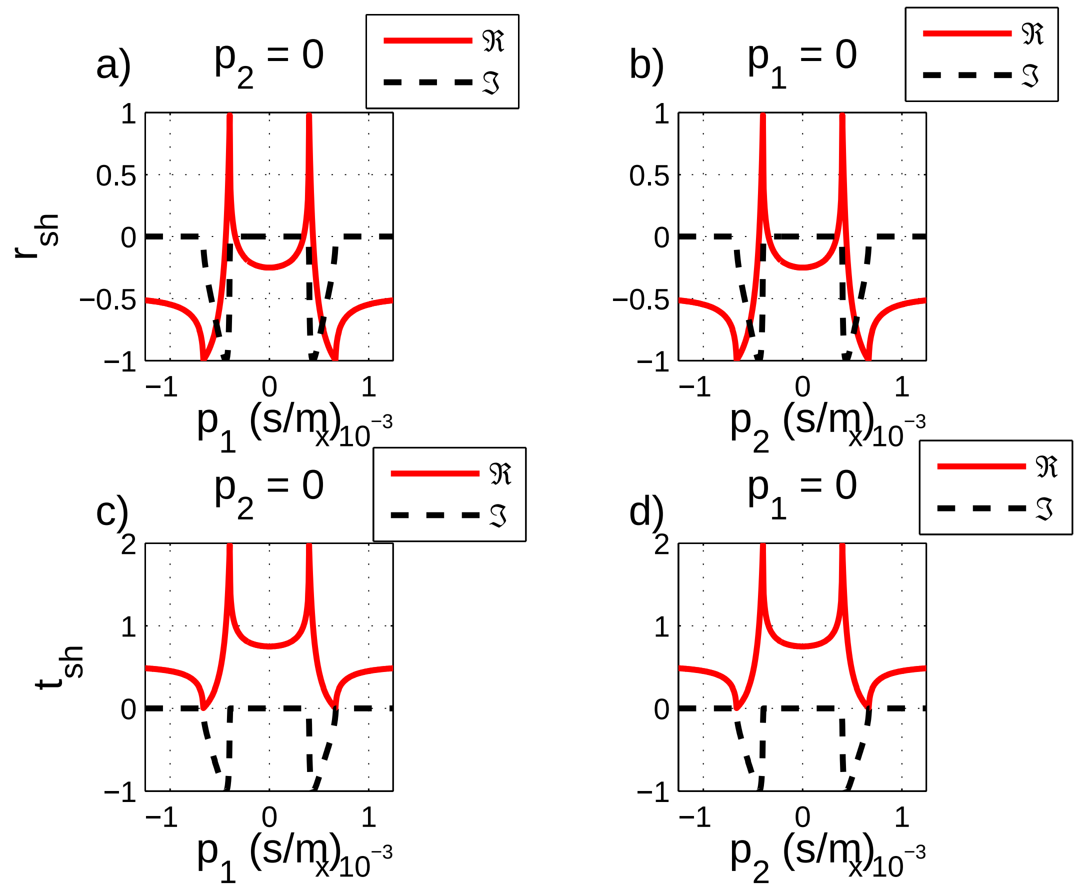

FIG. 4. a) In line slice of reflection coefficient. b) Cross line slice of reflection coefficient. c) In line slice of transmission coefficient. d) Cross line slice of transmission coefficient.

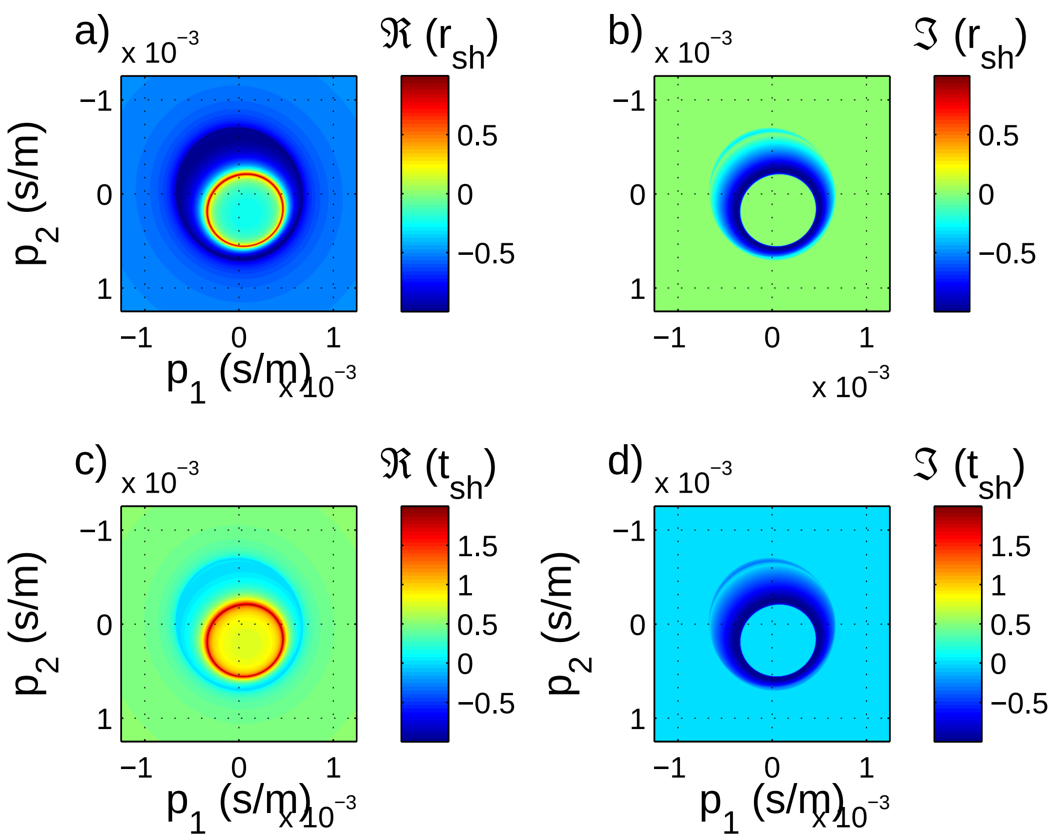

FIG. 5. a) Real part of reflection coefficient. b) Imaginary part of reflection coefficient. c) Real part of transmission coefficient. d) Imaginary part of transmission coefficient for dipping interface.

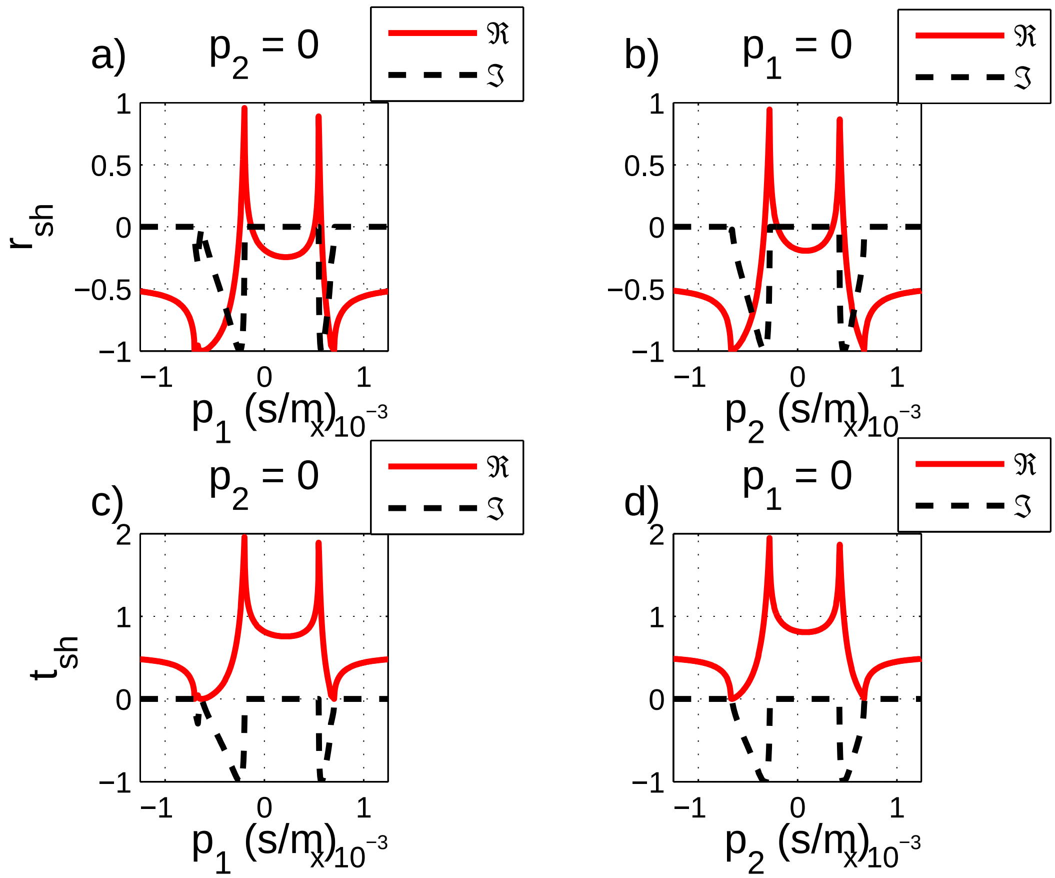

FIG. 6. a) In line slice of reflection coefficient. b) Cross line slice of reflection coefficient. c) In line slice of transmission coefficient. d) Cross line slice of transmission coefficient for dipping interface.

of in line traces is fixed and different number of cross line traces are used in order to see the effect of this change on the reflection and transmission coefficients. Figures 7 and 8 are obtained with 512 in line traces and 128 cross line traces. The description for in line slice of the reflection and transmission coefficients is same as given previously. Here, the cross line slices of the reflection and transmission coefficients are considered. For cross line slice, the reflected amplitude is -0.25 and transmitted amplitude is 0.75 at zero slowness(as predicted by the equations (20) and (21)). In this case, the extracted reflected and transmitted amplitudes are not as predicted by equations (12) and (13) at critical slowness. The reason is that there is no sample point in MATLAB at critical slowness in this case. The sample rate depends on the number of traces inversely and it is 0.000019 s/m along the slowness axis. In this case, near to the slowness$(0.000343s/m)$ at which reflected amplitude is expected to be zero, the obtained slowness in MATLAB is 0.00035 s/m while the occurred slowness near to critical slowness is 0.0004039 s/m. The obtained sample point just before to 0.0004039 s/m is 0.00039 s/m. Now the interpolation is used in MATLAB to get the values at all the points in between two sample points. According to theory, the amplitude values of the reflected and transmitted waves at these two sample points are not such that the expected result is obtained at critical slowness using interpolation, this is because, near to critical slowness, there is large difference between the values of the reflected and transmitted amplitude for closely sample points. Thus, due to interpolation used by MATLAB, the reflected and transmitted amplitudes are not getting the expected values at critical slowness.

Figures 9 and 10 show the real and imaginary part of the 3D reflection and transmission coefficients, and in line and cross line slices of coefficients with same input as previous but for dipping interface(dip$=15°$), respectively. These figures show some shifting along slowness axis and are not symmetric about zero slowness. Here, in line and cross line slices show inconsistency in the reflected and transmitted amplitudes at positive and negative Nyquists. This is again attributed to the problem associated with MATLAB programming. In programming, the negative Nyquist is used as per theory but positive Nyquist is taken as the difference of the positive Nyquist and sample rate. It is known that sample rate depends on the number of traces inversely. The less number of traces means coarse sample rate and coarse sample rate means a considerable difference between positive Nyquist and negative. Thus, there is the difference in the values of the reflected and transmitted amplitude occurring at the positive and negative Nyquists. Now, the figures for the reflection and transmission coefficients are drawn with same input parameters as previous model but different number of cross-line traces. Presently, only 8 cross-line traces are used. Figures 11 and 12 demonstrate the obtained results for this case. As can be seen, in cross line slices of the reflection and transmission coefficients, the reflected amplitude and transmitted amplitude is -0.25 and 0.75 at zero slowness(as predicted by equations (20) and (21)). The values of the reflected amplitude and transmitted amplitude are decreasing and increasing respectively as slowness increases for the reflected and transmitted but in this case the pattern of the reflection and transmission coefficients is differing very much from expected one . Here, the sample rate is 0.000312 s/m that is very coarse while the difference between the critical slowness and the slowness at which reflected amplitude is zero, is .000057 s/m. It shows that there is no point between these two slownesses whereas there is big difference occurred in the amplitudes at these slownesses. Presently, MATLAB uses the interpolation to get the values of amplitude at the slownesses in between two sample points. Thus, the

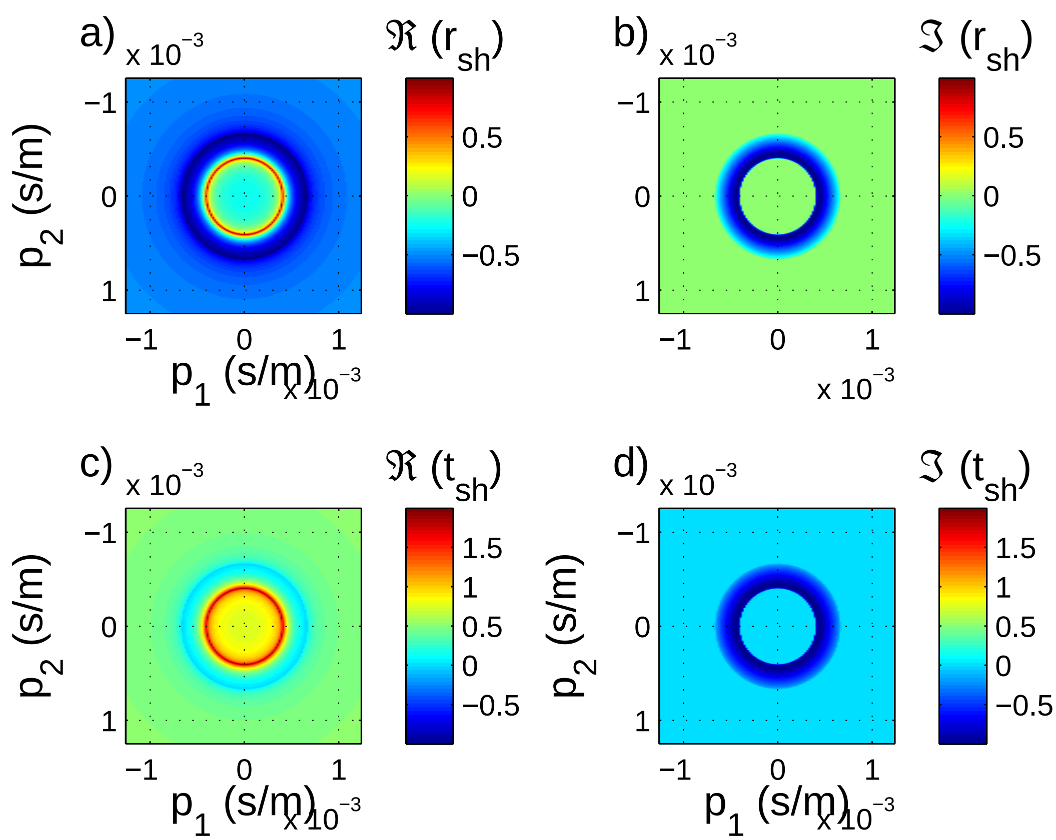

FIG. 7. a) Real part of reflection coefficient. b) Imaginary part of reflection coefficient. c) Real part of transmission coefficient. d) Imaginary part of transmission coefficient for horizontal interface. 512 in line and 128 cross line traces are used.

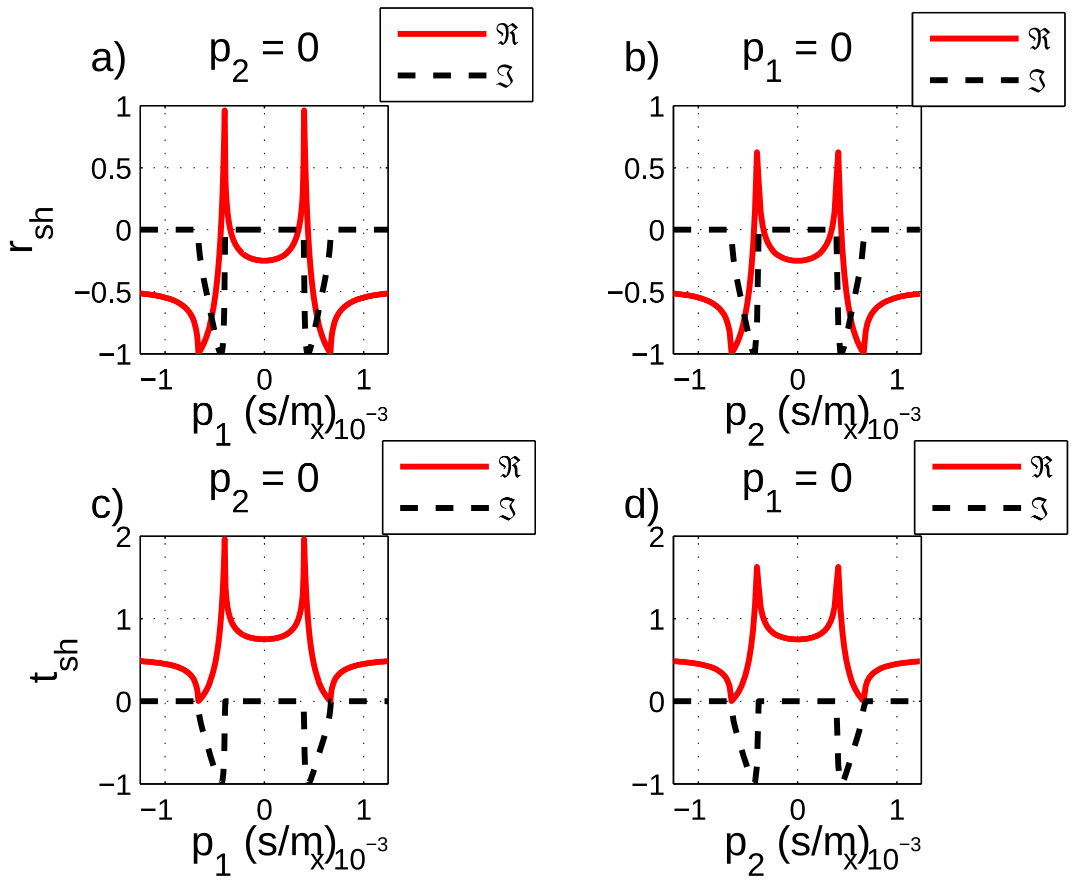

FIG. 8. a) In line slice of reflection coefficient. b) Cross line slice of reflection coefficient. c) In line slice of transmission coefficient. d) Cross line slice of transmission coefficient for horizontal interface

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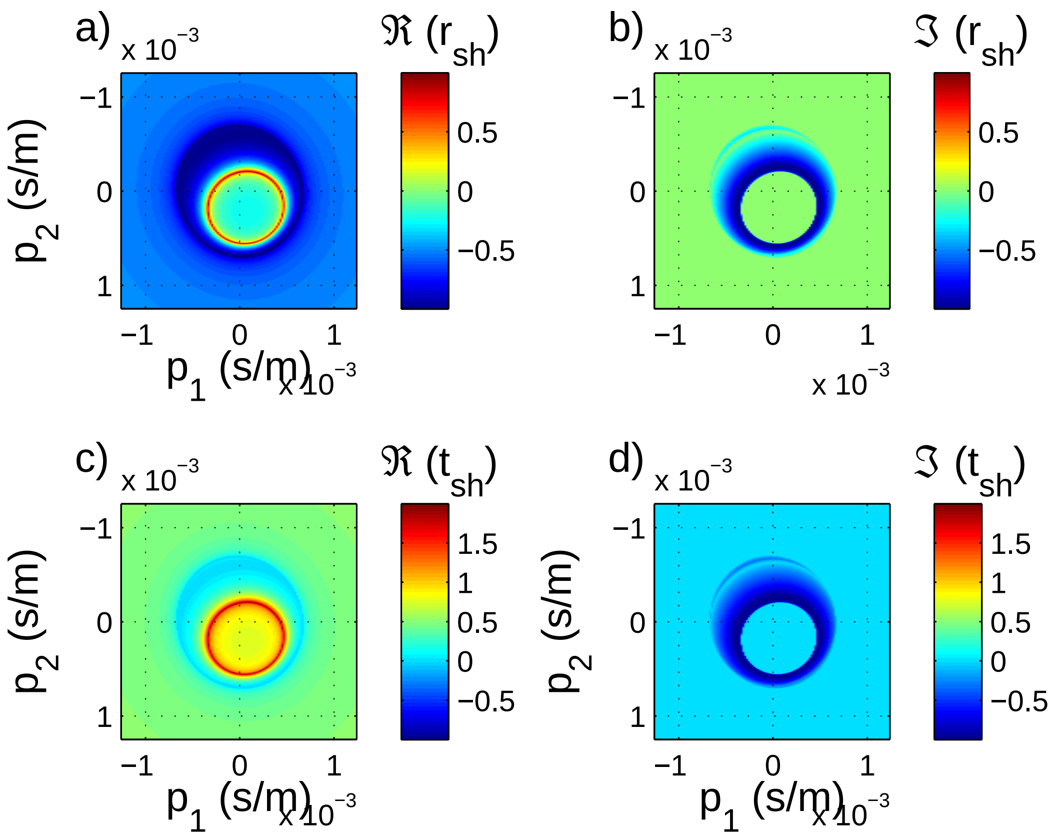

FIG. 9. a) Real part of reflection coefficient. b) Imaginary part of reflection coefficient. c) Real part of transmission coefficient. d) Imaginary part of transmission coefficient for dipping interface.

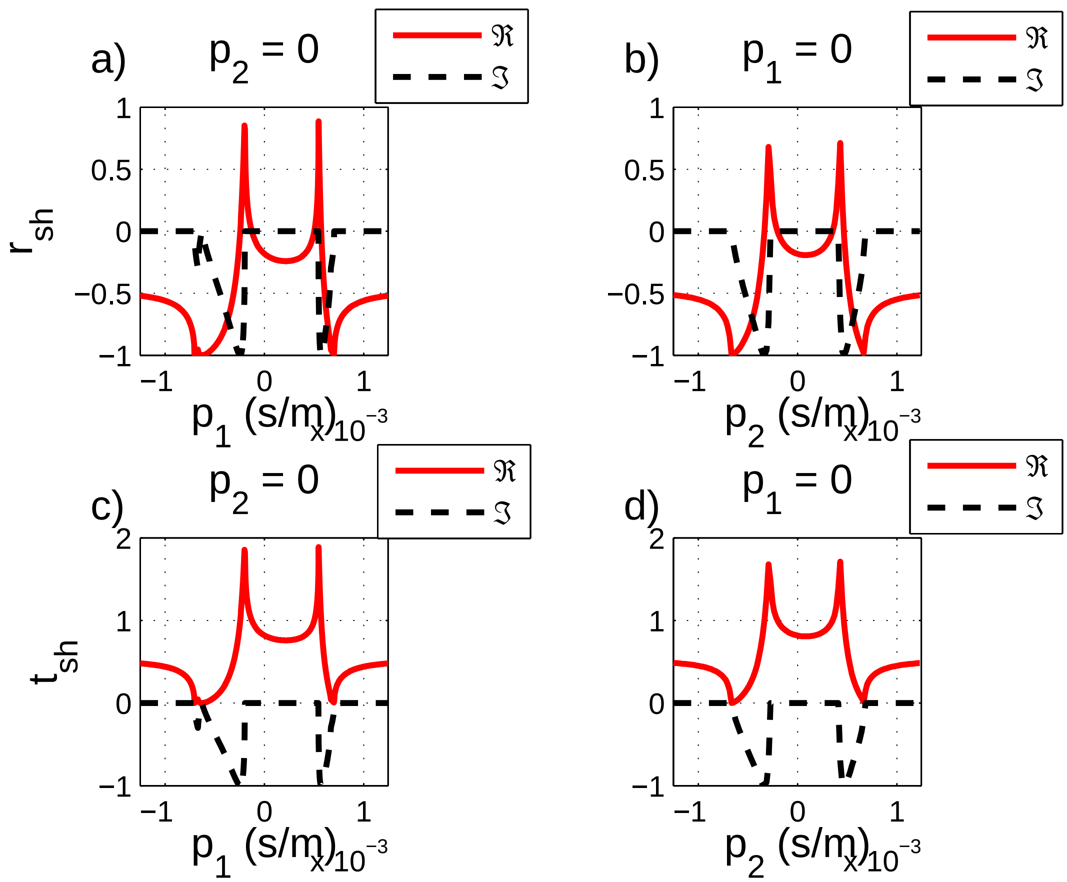

FIG. 10. a) In line slice of reflection coefficient. b) Cross line slice of reflection coefficient. c) In line slice of transmission coefficient. d) Cross line slice of transmission coefficient for dipping interface.

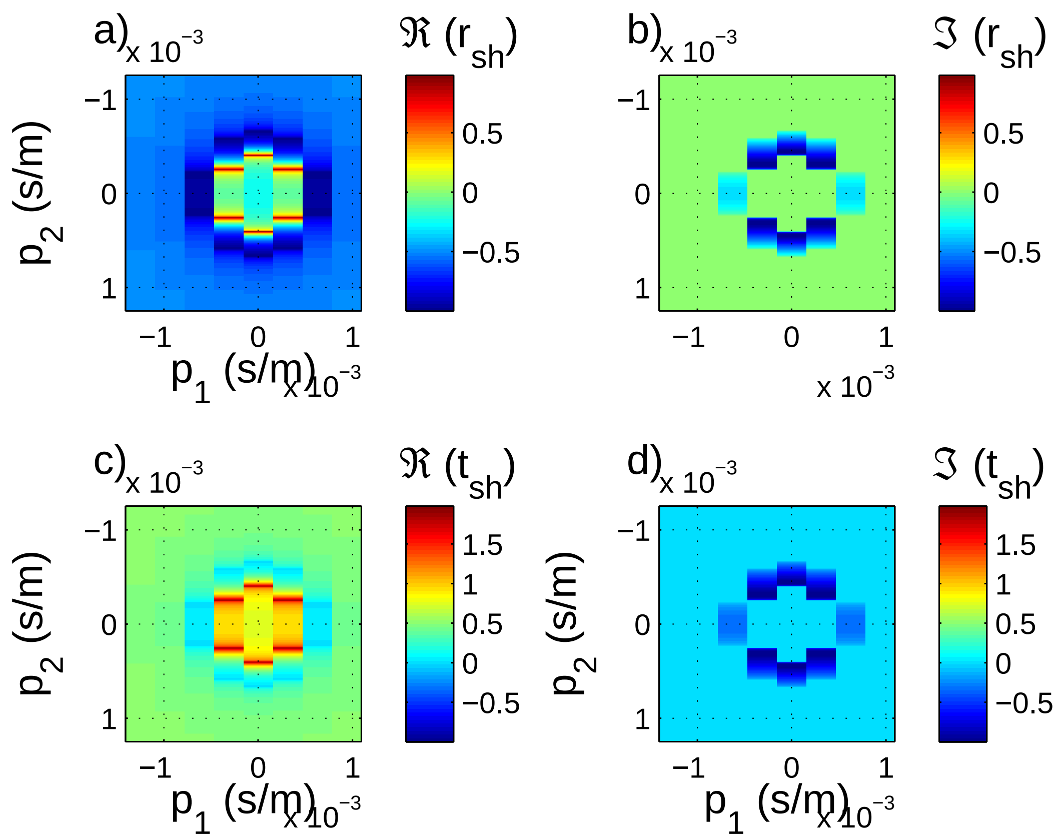

FIG. 11. a) Real part of reflection coefficient. b) Imaginary part of reflection coefficient. c) Real part of transmission coefficient. d) Imaginary part of transmission coefficient for horizontal interface. 512 in line and 8 cross line traces are used.

obtained results are very far away from expected ones. These Figures(13, 14) are for dipping interface. In this case the cross-line slice gives the distorted picture of the amplitude of the reflected and transmitted waves.

DISCUSSION AND CONCLUSIONS

The angle of incidence is obtained by using the cross product of two known unit normals. This value of incident angle is used in analytic expressions of R and T coefficients. Thus,the reflection and transmission coefficients have been obtained in plane wave domain. First of all reflection and transmission coefficients have been obtained for the reflector aligned with computational grid. Presently, it has been shown that obtaining reflection and transmission coefficients for the reflector non-aligned with computational grid is easy. The importance of this approach is the automatically adaptation of R and T expressions to the special dipping interface case. The power of this is that no ray tracing is required. The reflection and transmission coefficients obtained for same number of in line traces but different number of cross line traces, show the deviation from the expected one. It reveals the problem associated with data acquisition and make it required to acquire the data correctly.

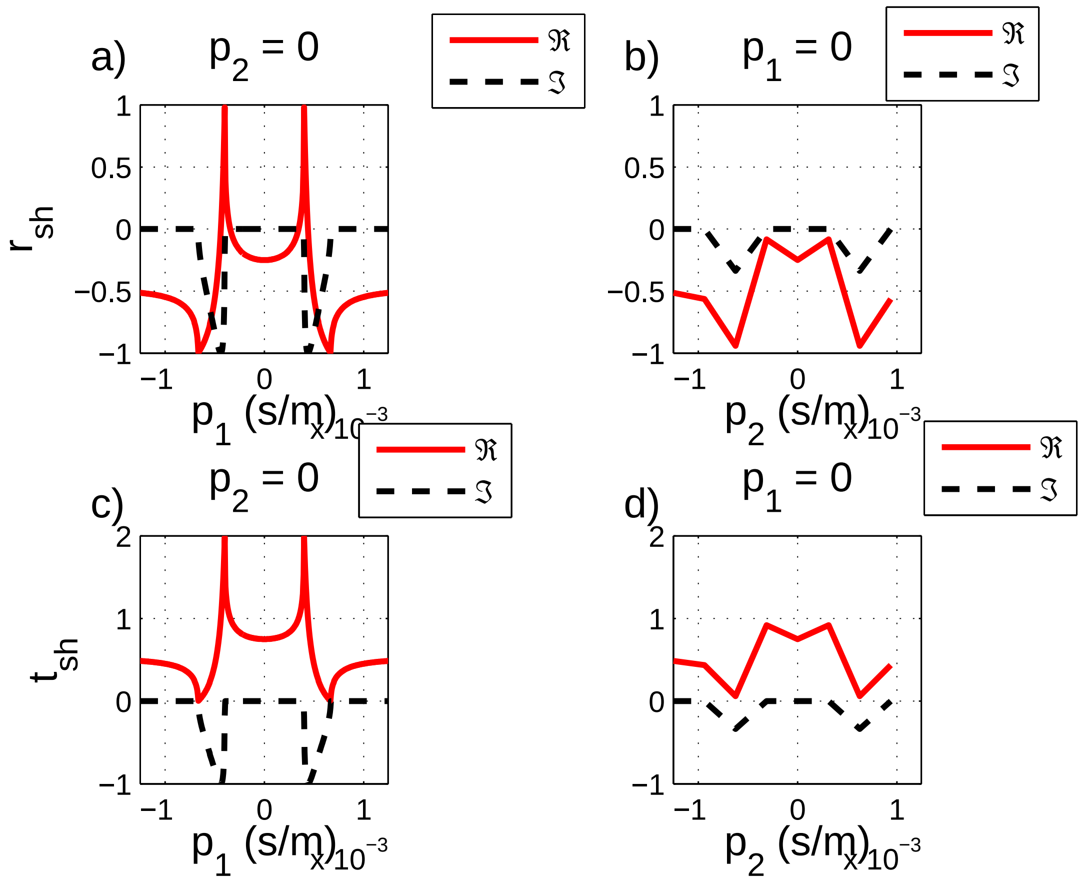

FIG. 12. a) In line slice of reflection coefficient. b) Cross line slice of reflection coefficient. c) In line slice of transmission coefficient. d) Cross line slice of transmission coefficient for horizontal interface.

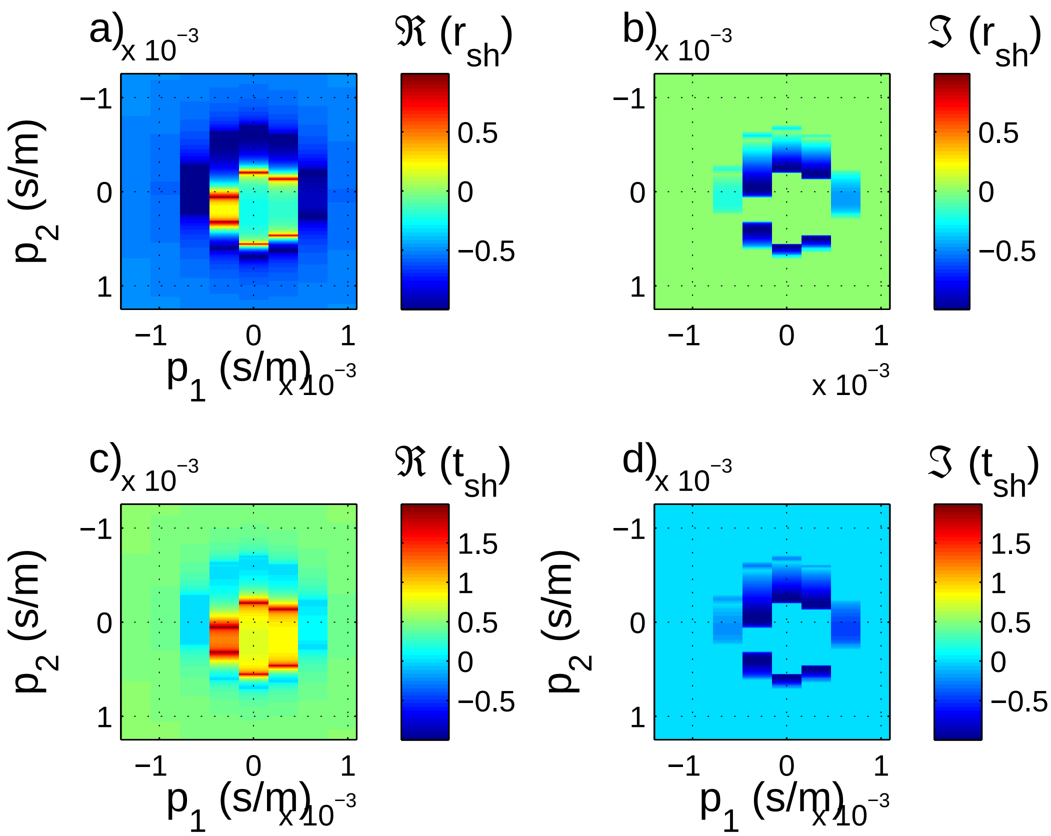

FIG. 13. a) Real part of reflection coefficient. b) Imaginary part of reflection coefficient. c) Real part of transmission coefficient. d) Imaginary part of transmission coefficient for dipping interface. 512 in line and 8 cross line traces are used.

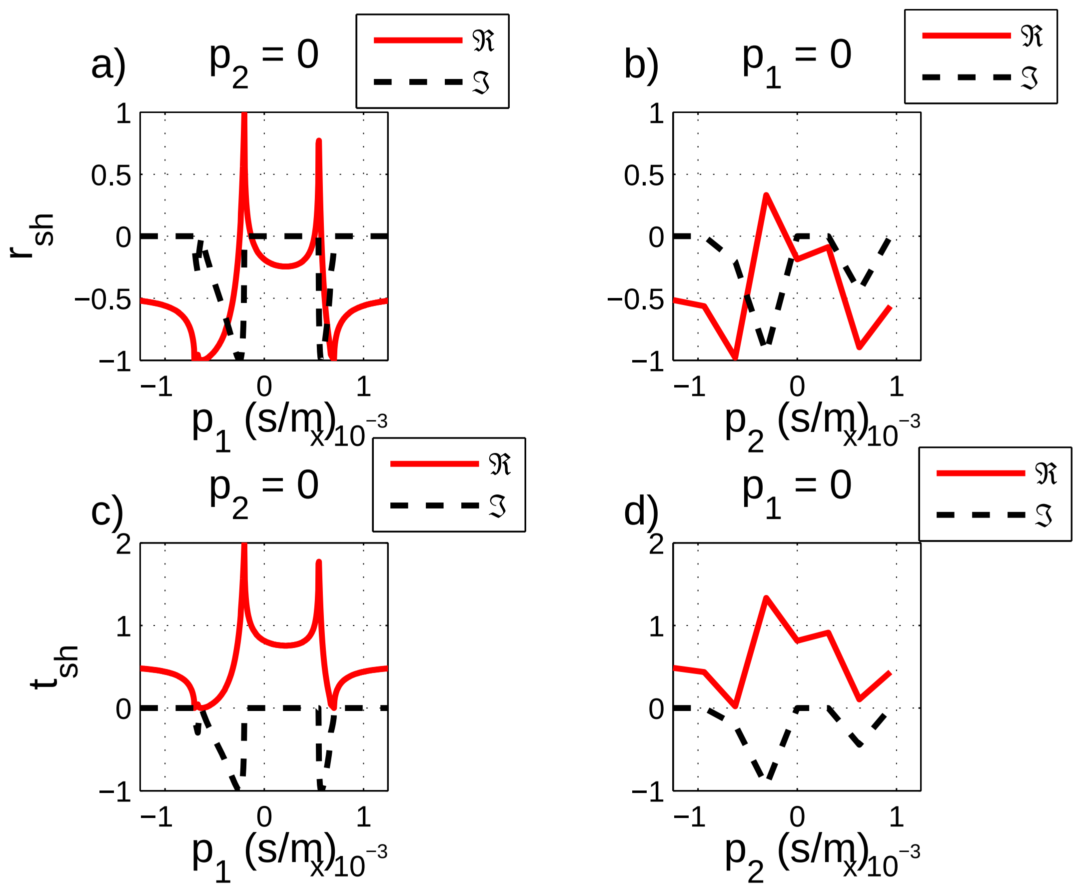

FIG. 14. a) In line slice of reflection coefficient. b) Cross line slice of reflection coefficient. c) In line slice of transmission coefficient. d) Cross line slice of transmission coefficient for dipping interface.

Cross line slice does not show any effect of dipping interface only in line slice does.

FUTURE WORK

1. Rayleigh Sommerfeld Modeling with AVO.
2. RSM for dipping interface.In this paper, only a special case of dipping interface is discussed here but in future I will try to solve the problem of a generalized case of dipping interface. For this the idea is to carry the information of polarization vector of vector wavefield through the wavefield propagation. By knowing the polarization vector at the earth surface, this information can be carried to the reflector in wavefield extrapolation. If reflector is dipping, then compute the component of vector wavefield in a rotated coordinate system that coincides with dipping interface. Now carry this information of vector wavefield back to the earth surface, and then rotate coordinate system again. Anisotropic case will be considered in future.

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APPENDIX

TO FIND THE VALUE OF SLOWNESS AT WHICH REFLECTED AMPLITUDE WOULD BE ZERO

From equation (12) reflection coefficient R_{SH} is zero when

$$\rho_1 v_1^2 q_1 = \rho_2 v_2^2 q_2, \tag{A-1}$$

Where, in this example, density (ρ) is the same across the interface. Further, constant ρ simplifies equation (A-1) so that

$$v_1^2 q_1 = v_2^2 q_2. \tag{A-2}$$

Using equations (16) and (17), replace q_1 and q_2 in equation (A-2) to get

$$v_1^2 \left(1 - (v_1 p_1)^2\right) = v_2^2 \left(1 - (v_2 p_1)^2\right) \tag{A-3}$$

where we have squared both sides.
Solution p_1 for equation (A-3) corresponds to the ray parameter where R_{SH} is zero,

$$p_1 = \frac{1}{\sqrt{v_2^2 + v_1^2}} \tag{A-4}$$

ENERGY FLUX FOR REFLECTED AND TRANSMITTED WAVES

Earlier it is seen that the transmission coefficent exceeds 1 at critical slowness. The energy transported by the traveling wave is considered to see how it occurs because energy must be conserved. For a harmonic SH plane wave, the flux of energy per unit wavefront, E, in the direction of propagation is the product of the energy density(energy per unit area) and the velocity (Sten and Wysession, 2002)

$$E = A^2 \omega^2 \rho \beta / 2 \tag{A-5}$$

where A is the amplitude of wave and ρ is the density of medium. Since welded contact of two medium is considered, there is no energy to be accumulated at the interface. According to law of energy conservation, energy flux of the incident wave would be equal to the reflected and transmitted waves energy fluxes. Energy fluxes for the constituent waves are

$$E_I = \omega^2 \rho_1 \beta_1 \cos j_1 dx/2, \tag{A-6}$$

for the incident SH wave

$$E_R = R_{SH}^2 \omega^2 \rho_1 \beta_1 \cos j_1 dx/2, \tag{A-7}$$

for the reflected wave, and

$$E_T = T_{SH}^2 \omega^2 \rho_2 \beta_2 \cos j_2 dx/2, \tag{A-8}$$

for the transmitted wave,where an incident wave has unit amplitude, dx is the element of the interface, j_1, j_2 are the angle of incidence and transmission, respectively. These fluxes satisfy the conservation of energy

$$E_I = E_R + E_T, \tag{A-9}$$

and this expression can be written in terms of the incident, reflected and transmitted energy fluxes as

$$\rho_1\beta_1\cos j_1 = R_{SH}^2\rho_1\beta_1\cos j_1 + T_{SH}^2\rho_2\beta_2\cos j_2, \tag{A-10}$$

In terms of the vertical slowness, equation (A-10) is

$$\rho_1\beta_1^2 q_1 = R_{SH}^2\rho_1\beta_1^2 q_1 + T_{SH}^2\rho_2\beta_2^2 q_2, \tag{A-11}$$

It is seen from in line and cross line slices of the R and T coefficients (Figure 4), at zero slowness the reflection and transmission coefficients are -0.25 and 0.75, respectively, while density is the same across the boundary and velocitiesβ_1=1500m/s and β_2=2500 m/s are used. To verify this result in terms of energy conservation, q_1 and q_2 are replaced by $1/\beta_1$ and $1/\beta_2$,respectively at zero slowness (normal incidence, j_1 =0 and j_2 =0), and equation (A-11) becomes

$$1 = R_{SH}^2 + T_{SH}^2\frac{\beta_2}{\beta_1} \tag{A-12}$$

Using the value of variables used in above equation we get $1 = (-0.25)^2 + (0.75)^2 * 2500/1500$ from equation (A-12) and it reveals that energy is conserved at zero slowness. Now we consider the critical slowness case where reflected and transmitted amplitudes are 1 and 2, respectively. To verify this result the vertical slowness in medium 2, q_2, is zero($j_1 =$ critical and $j_2 = \pi/2$). Recalling the equation (13), the transmission coefficient goes to 2 as slowness approaches the critical value but according to equation (A-8), the energy of the transmitted wave vanishes at this slowness because wavefront factor $cos j_2$ is vanished here. Thus, energy is conserved here also.