Low wavenumber reflectors

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ABSTRACT

A numerical modelling environment was created to accurately evaluate reflections from a 1D interface that has a smooth transition shape between two different velocities. The objective was to accurately propagate a chirp through the transition zone, where the size of the transition zone is similar to the wavelengths within the chirp.

Transition zones in the shape of a raised cosine were evaluated using the incident and reflected chirps to estimate the shape and amplitudes of the transition zones. Reasonable matches were obtained.

INTRODUCTION

The reflection of energy from transition zones with a continuous or smooth change in impedance is dependent on the frequency content of the seismic energy. An abrupt transition will reflect and transmit all frequencies of the wavelet with amplitudes that are numerically predictable. Gradual transition zones, where the size of the smooth transition zone is much greater than the wavelengths within the wavelet will not reflect energy and it will be completely propagated into the new medium, however, there will be an amplitude change in the wavelet to preserve energy.

Very accurate modelling of the wave equation was required to preserve the polarity and shape of a chirp as it was transmitted within two mediums. The objective was to prevent visible distortion of the transmitted chirp, which implied that grid distortion be kept below 0.1% of the peak amplitudes before and after the chirp. This was achieved by using grid dimensions of 1 meter and a time sample intervals of 0.25 millisecond for a chirp that contains frequencies within a typical seismic bandwidth from 10 to 70 Hz (Bancroft 2010). The finite difference solution to the wave-equation used a seven point approximation for the second derivative.

Model

An example of a snapshot of a chirp within the first medium is shown in Figure 1a. The velocity in the mediums is 1,000 and 2,000 m/s, giving reflection amplitudes of 1/3, and a transmission amplitude of 2/3. The red line represents the impedance change within the medium and the blue line the particle displacement in the depth or z direction. A poor modelling algorithm was used to propagate the energy in (b) and show the chirp in the first medium, reversed in time from (a), and the transmitted chirp in the second medium. Note the considerable grid dispersion at the high frequency end of the chirp. Accurate modelling is illustrated in (c) when there is a smooth transition over a range of 4 m, well below the wave-numbers within the chirp, to prevent aliasing. Also note the correct polarity of the reflected chirp in (c) relative the incorrect polarity in (b).

To evaluate the noise, the amplitudes of the figures were increase significantly to ensure the grid dispersion noise was below the desired threshold of 0.1%.

Comments on particle displacement

The blue line representing the chirp represents particle displacement and is positive, for example to the right, representing downward measurement of depth as positive. The reflected wave travels in a negative direction and a geophone will record a negative movement, opposite that of the downgoing wave. The transition in (b), used a step function which produced the wrong amplitude polarity and is often assumed to be correct. The transition in (c) used a four point smoother to prevent aliasing, and produced the correct polarity. The abrupt transmission in (b) is aliased relative to the finite difference approximation. Simulation involving shear or transverse waves, such as a wave on a string, will maintain the same polarity relative to particle displacement.

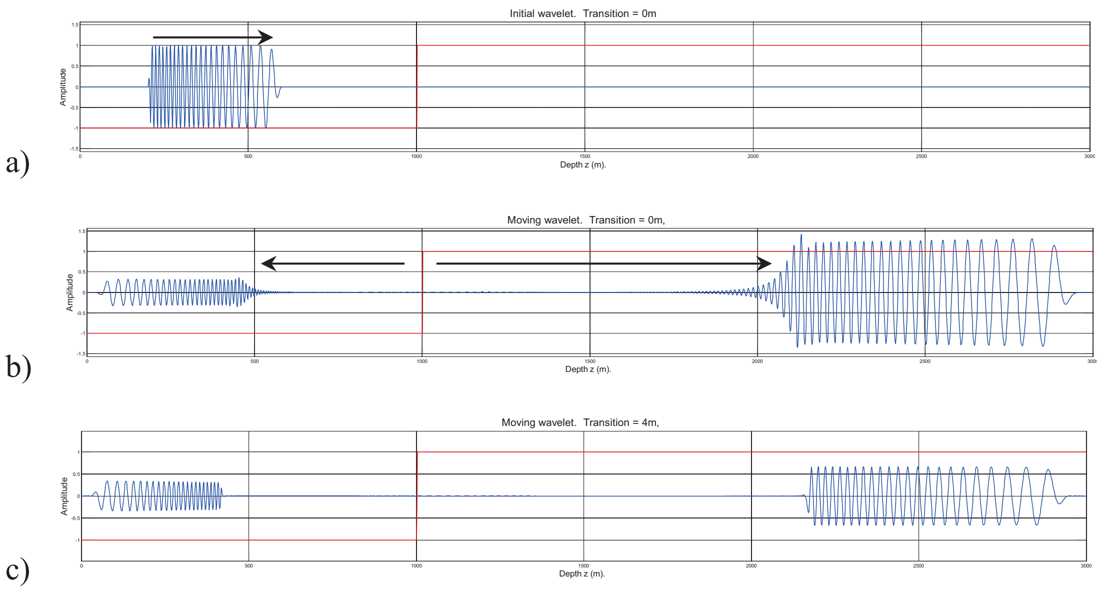

FIG. 1. Examples of snapshots of wavelets within a two layered medium, a) the initial chirp in the first medium, b) after reflection and transmission using a poor quality modelling algorithm, b) a high quality algorithm (Bancroft 2010).

RECOVERING THE TRANSITION SHAPE

A one half cycle of a raised cosine was used to model the velocity transition from the first medium with a velocity of 1,000 m/s to the second medium with a velocity of 2,000 m/s. The large velocity contrast was used to produce large reflection amplitudes that were of similar to those of the transmitted amplitudes for ease of viewing. The shape of the transition velocities was a raised half cosine. The derivative of this shape is a half sine and is used as an approximation to the reflectivity. This is not an exact representation, but serves as an indication of the size of the transition zone.

I assume a very simple seismic model where the incident wavelet (chirp) $\boldsymbol{x}(t)$ is convolved with the reflectivity $\boldsymbol{r}(\boldsymbol{t})$ to produce the reflection $\boldsymbol{s}(t)$. The reflectivity is recovered by deconvolving the reflection wavelet with the incident wavelet. I perform this process in the frequency domain with simple division of dividing $\boldsymbol{S}(f)$ by $\boldsymbol{X}(f)$. The chirp used covered a frequency band of 10 to 70 Hz, but there are some very small amplitudes that extend all the way to the Nyquist frequency of 2,000 Hz. However, the noise in the higher frequencies of the reflected wave ***S***(*f*), and much greater and the division by ***X***(*f*) produces excessive noise in ***R***(*f*). Consequently, it was necessary to add a high cut filter to the reflection spectrum to attenuate this noise. A very small numerical value was added to the incident spectrum to prevent a divide by zero.

RESULTS

The following figures show the results of estimating the shape of the reflectivity for various widths of the transition zone. The width of the transition zones vary from 5 m to 150 m, well beyond the wavelengths of the chirp that was used. The frequencies if 10 to 70 Hz represent wavelength of 100 m to 15 m. In the figures showing the reflectivity, the blue line represents the estimated reflectivity, and the red line is the half sine representation of the actual reflection. The alignment is not exact, but is there as a reference for the shape. The amplitude of the half sine was scaled to the amplitude of the reflection.

A modulated chirp, shown in Figure 2, used a carrier frequency of 40 Hz, modulated with 30 Hz, to give the frequency range from 10 to 70 Hz.

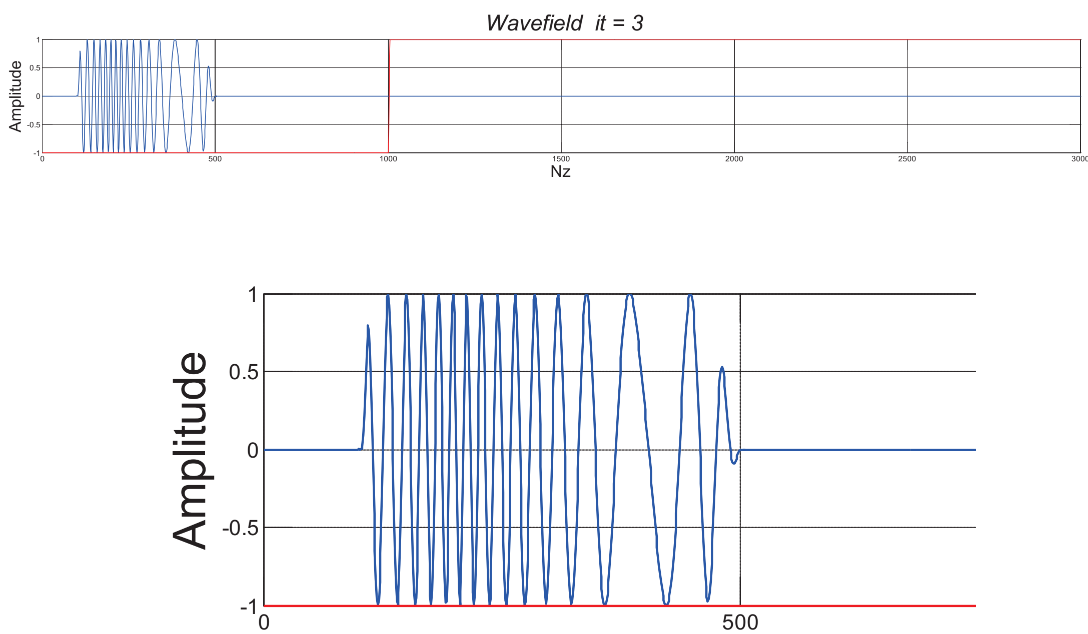

FIG. 2 Snapshot of the chirp within the model and its zoom to show the detail.

Figure 3 shows a snapshot of the reflected and transmitted chirps for a small reflector transition width of 5 m. With this short transition, all frequencies in the chirp are reflected. Figures 4 through 12 show the estimated shape of the reflector with transition widths that range from 5 m. to 150 m.

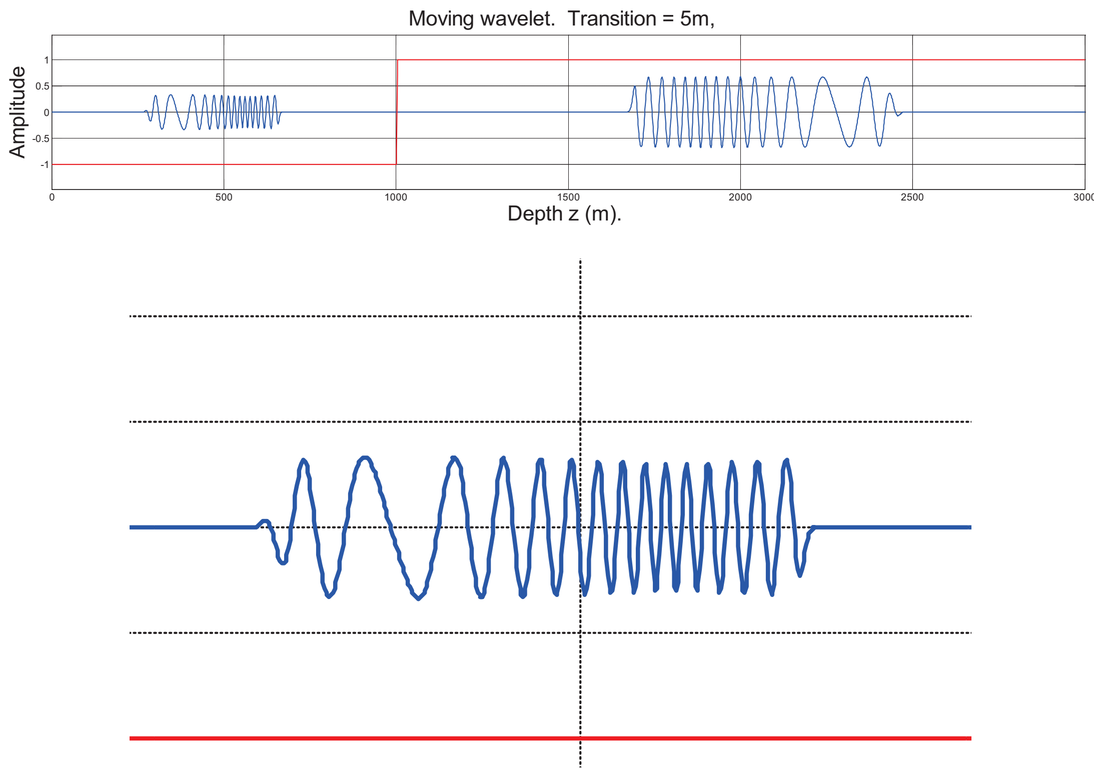

FIG. 3 Snapshot of the reflected and transmitted waves for a transition interval of 5m. Also shown is a zoom of the reflected chirp.

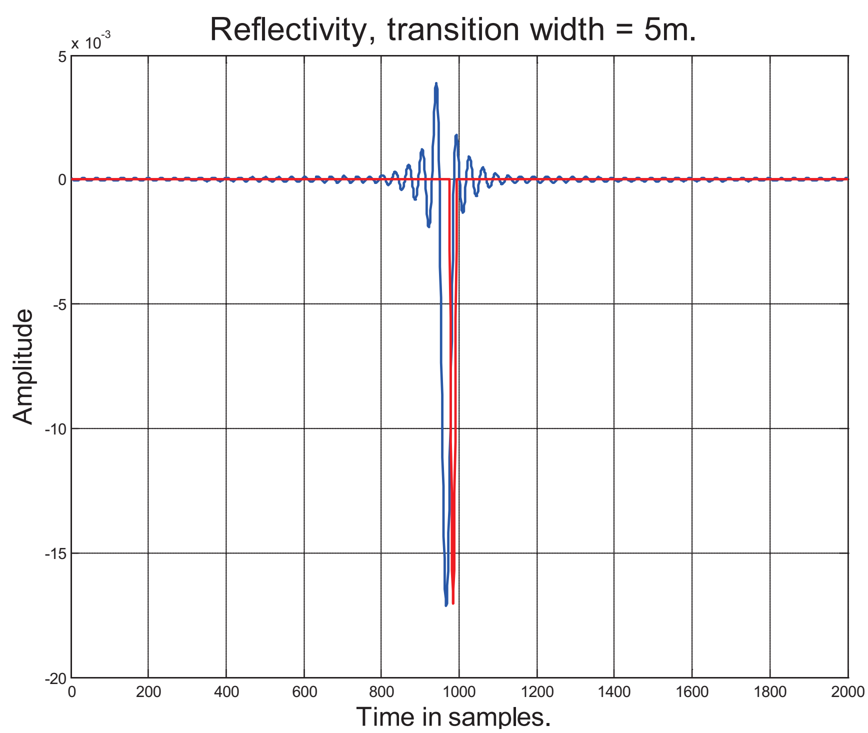

FIG. 4 Reflectivity for a 5 m transition interval.

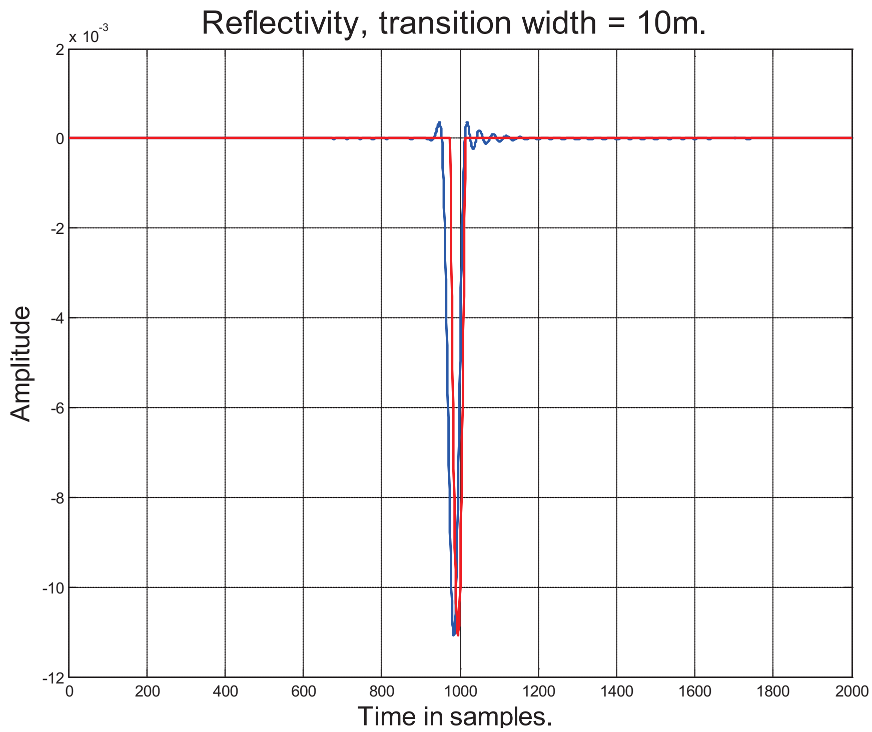

FIG. 5 Reflectivity for a 10 m transition interval.

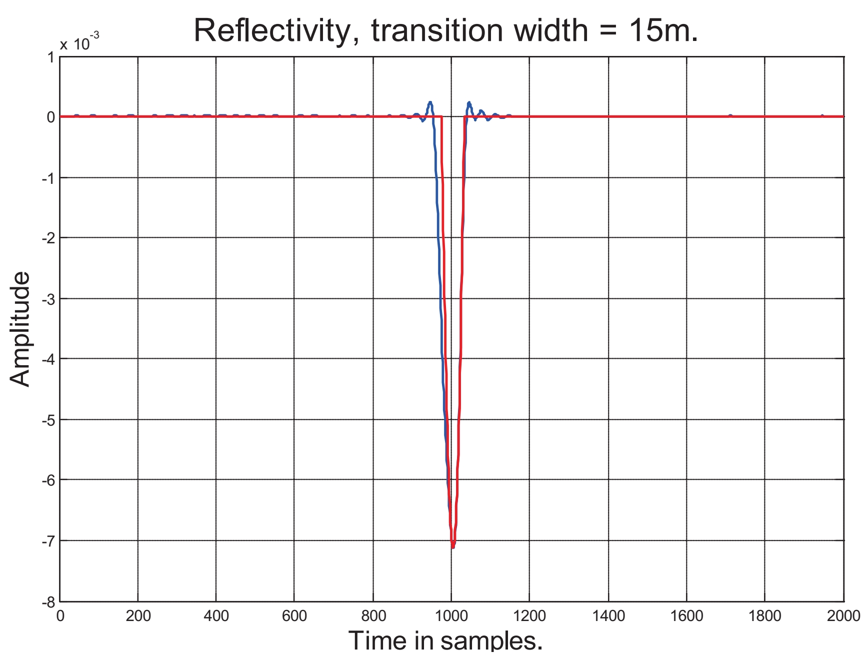

FIG. 6 Reflectivity for a 15 m transition interval.

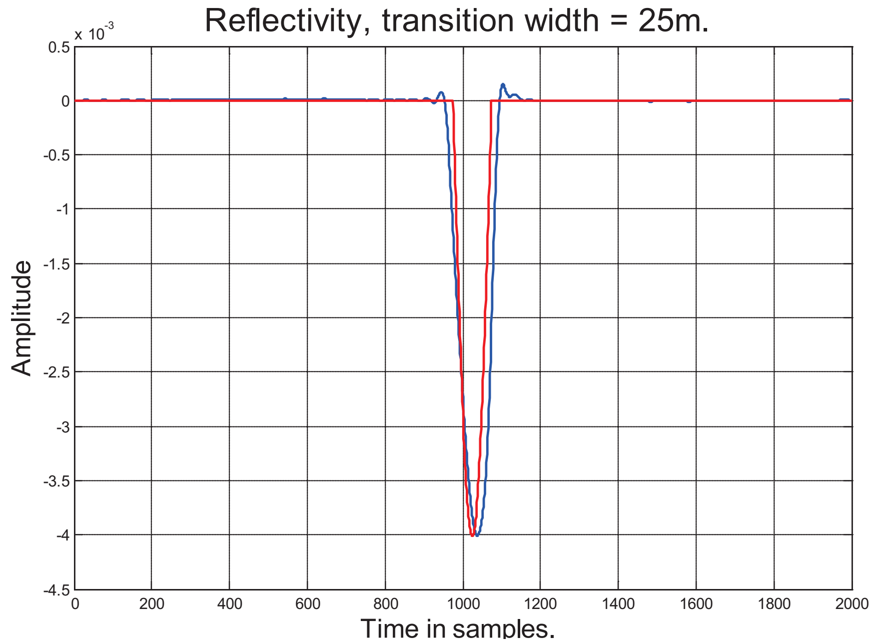

FIG. 7 Reflectivity for a 25 m transition interval.

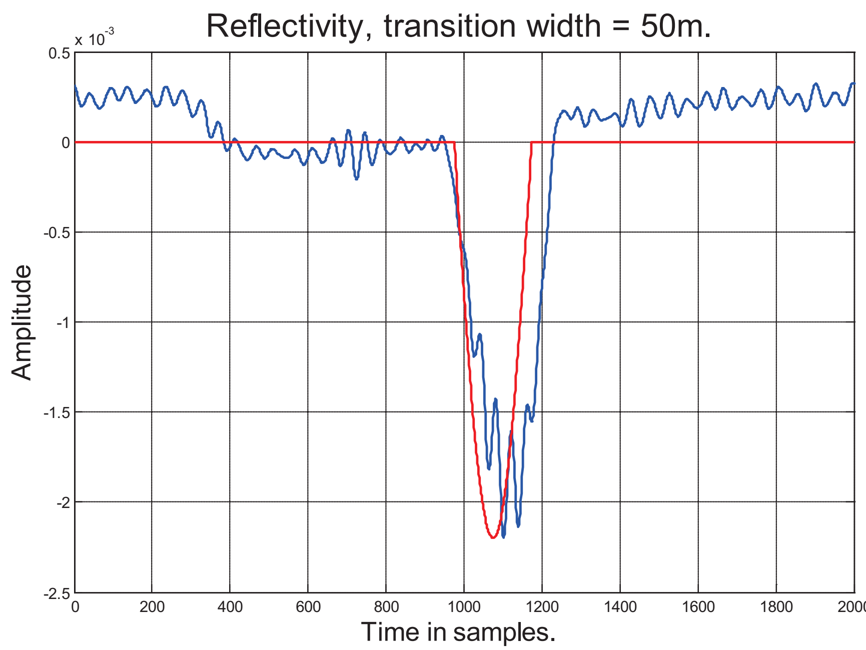

FIG. 8Reflectivity for a 50 m transition interval.

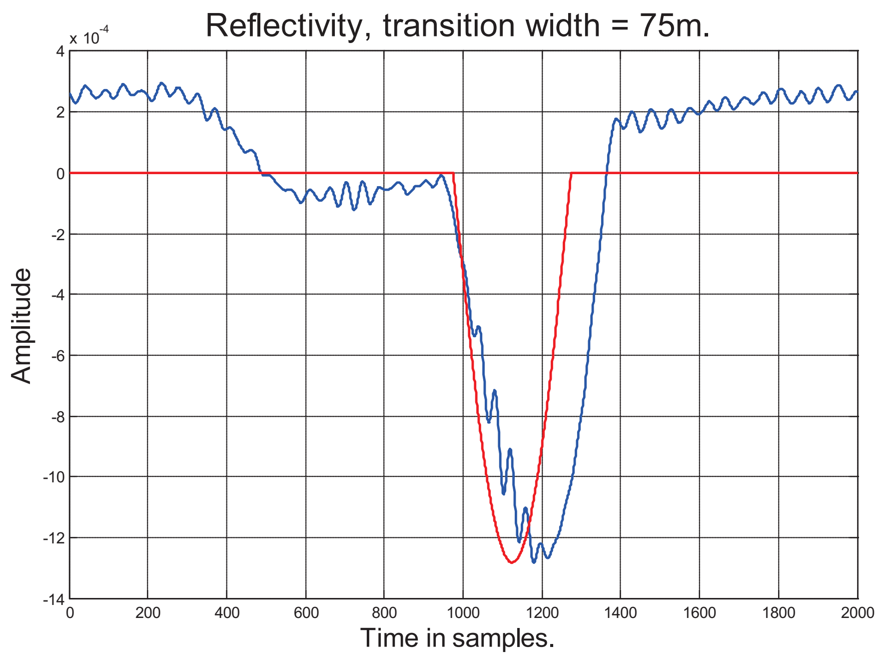

FIG. 9 Reflectivity for a 75 m transition interval.

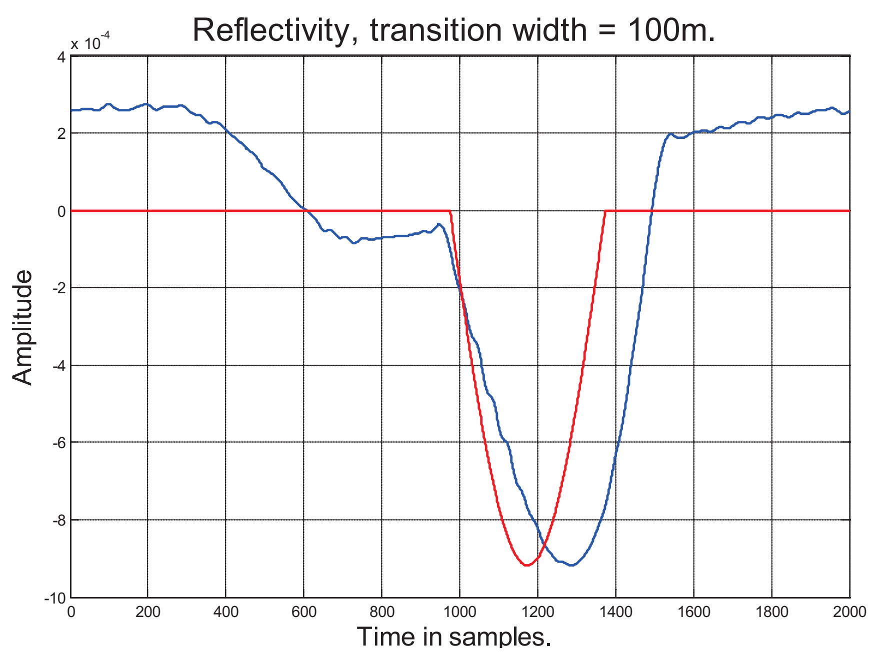

FIG. 10 Reflectivity for a 100 m transition interval.

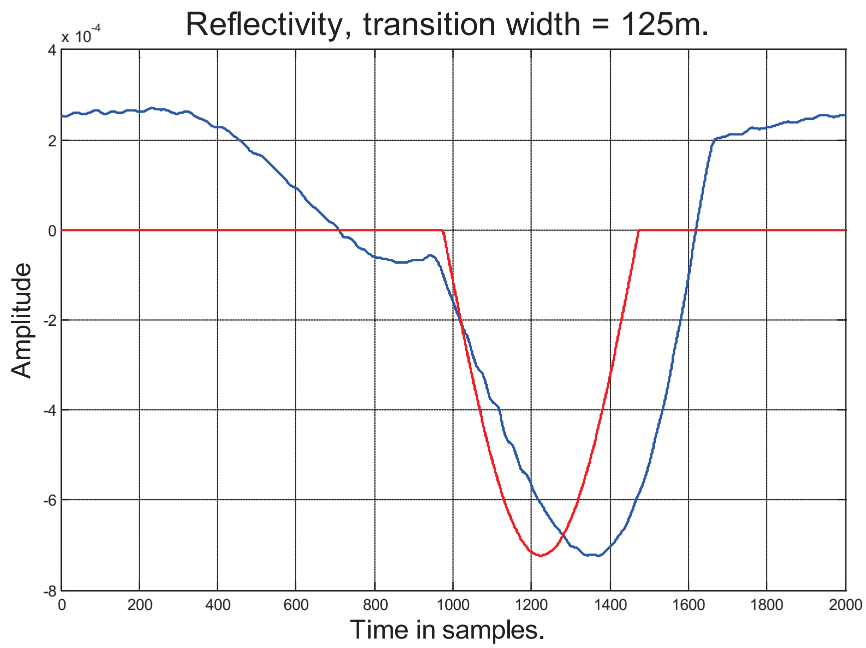

FIG. 11 Reflectivity for a 125 m transition interval.

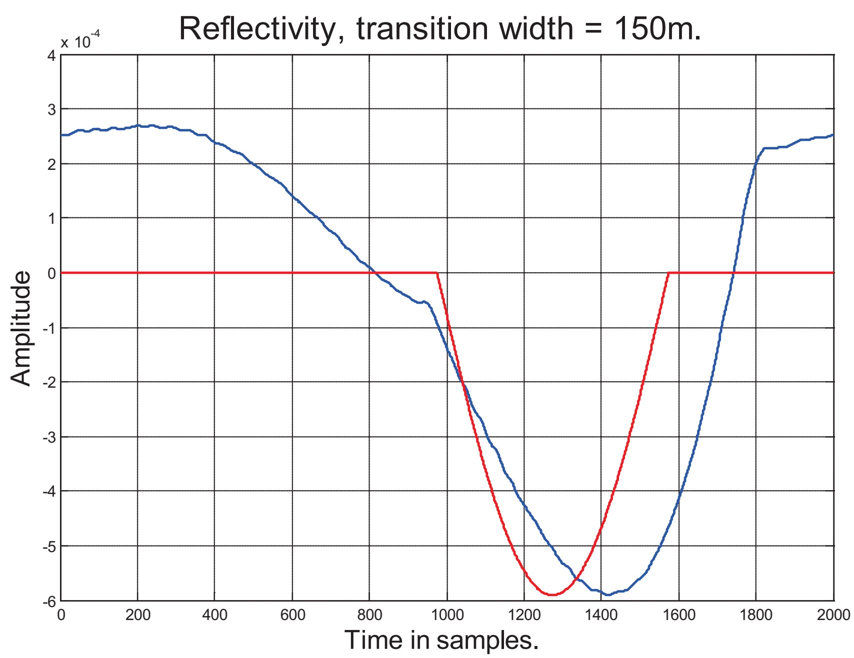

FIG. 12 Reflectivity for a 150 m transition interval.

DISCUSSION OF THE RESULTS

When the transmission zone is small, the energy of the reflection is largest giving a good signal to noise ration of the estimated transition zone as illustrated in Figure 3. However, the width of the estimated transition zone is limited by the bandwidth of the chirp.

When the width of the transition is within the wavelengths of the chirp, there is some attenuation of the reflected energy as the higher frequencies are passed through to the second layer.

At the larger transition zones, most of the energy is transmitted and little is reflected. Consequently there are poor signal to noise ratios, especially at the lower frequencies. At this point, no attempt has been made to modify the lower frequencies. Figure 13 shows a snapshot of the transmitted and reflected energy for a transition width of 75 m which is the same width as the maximum wavelength of the chirp. There is very little reflected energy, consequently any noise will appear relative large to the theoretical reflection energy. The amplitude spectra are shown in Figure 14 with (a) the amplitude spectrum of the incident chirp to 1,000 Hz. Parts (b), (c), and (d), show zoomed spectra for the incident chirp, the reflected chirp, and the computed reflection profile. The horizontal axis for (b), (c), and (d) has a maximum of 50 samples, representing a maximum of 100 Hz.

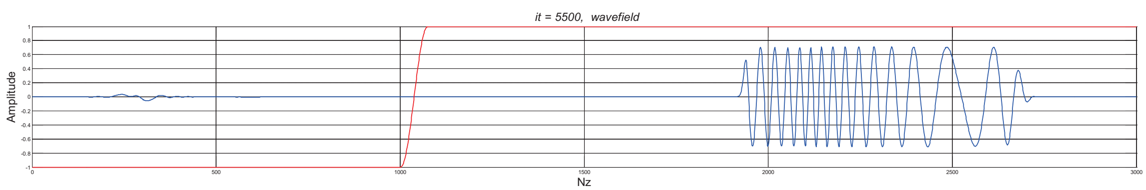

FIG. 13 Reflection with transition width equal to 75 to illustrate the loss of reflection energy.

CONCLUSIONS

A high quality 1D modelling algorithm was used to evaluate the reflection and transmission of energy from a reflection interface that has a smooth transition interval. Test were conducted with various widths of the transition interval and the estimated size and shape compared favorably with the actual shape, especially when the size of the transition zone was within the range of the wavelengths of the chip.

SOFTWARE

`2011-Matlab\ChirpReflections\ChirpVerticalHighOrder.m`

ACKNOWLEDGEMENTS

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REFERENCES

Bancroft, J. C., 2010, Investigating reflections from smooth transition boundaries. CREWES Report-Vol. 22

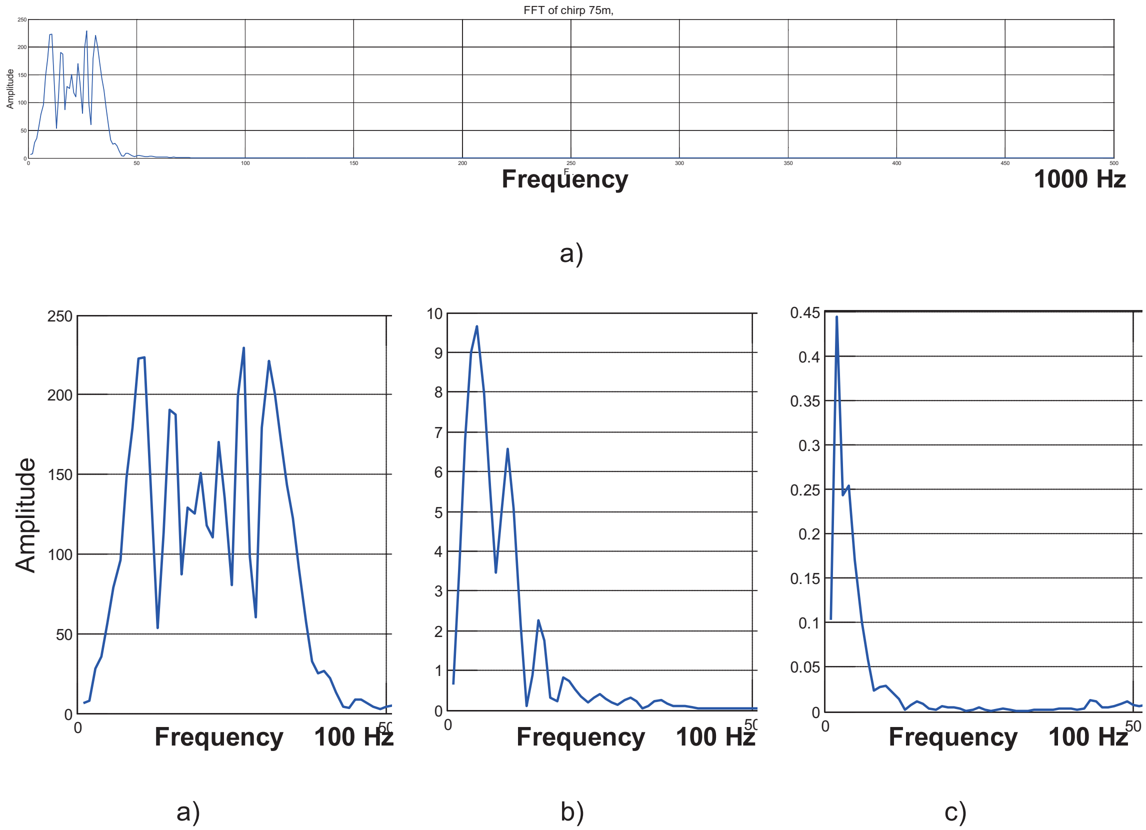

FIG. 14 Amplitude spectra a) for the incident chirp, the zoomed spectra for b) the incident chirp, c) the reflected chirp, and d) the computed reflectivity.