Full waveform inversion of anelastic reflection data: an analytic example

Chris Bird and Kris Innanen

ABSTRACT

Full waveform inversion is taking an increasing important role in exploration seismology. As this role continues to grow, we must develop our understanding of the basic nature of full waveform inversion. We develop an analytic example of anacoustic full waveform inversion for a simple attenuating Earth model. The first gradient of full waveform inversion is calculated for analytic, one-dimensional zero-offset data of the attenuating model. The result is analyzed and we attempt to develop an understanding into how full waveform inversion will produce the proper model in this attenuating environment.

INTRODUCTION

Full waveform inversion (FWI) is taking on an increasingly important role in seismic exploration. As this role grows, we must develop (1) implementations and algorithms, and (2) our understanding of the basic nature of FWI. In this paper we are concerned with the second of these issues. Particular issues associated with inversion of anelastic media have been discussed by Hicks and Pratt (2001) and Malinowski et al. (2011). Here we add to these discussions by developing an analytic example of the first iteration of anacoustic FWI. The first gradient of full waveform inversion is calculated for analytic data of a very simple attenuating Earth model. This simple Earth model consists of an elastic overburden overlaying an attenuative target with the interface between the layers occurring at a depth of 300m. The source and receiver are colocated at the surface and so the analytic data calculated is one dimensional and normal incidence. The analytic data for this simple model is used to calculate the first step of the gradient function for full waveform inversion. The gradient is analyzed and we attempt to draw insight into how full waveform inversion will reconstruct the proper model in this setting.

ANALYTIC DATA AND INITIAL MODEL

In this example the actual medium is a two layer model in which both layers have velocity c_0. There is a contrast in Q, however, as the first layer is elastic and the second layer is anelastic with a Q value of 40. Therefore there is a single primary in the data. We define the perturbations in wavespeed (a_C) and Q (a_Q) as

$$a_C = 1 - \frac{c_0^2}{c_1^2}$$
$$a_Q = \frac{1}{Q}$$

Figure 1(a) shows the simple attenuating Earth model. The blue line in Figure 1 corresponds to the perturbation in velocity, a_C, and the red line corresponds to the perturbation in Q, a_Q,. Notice that since there is no contrast in the acoustic properties of the medium and so the profile of a_C is constant at zero. However, the profile of a_Q jumps at $300m$, which corresponds to the contrast in Q at that depth. Also, the starting model to be used in the calculation of the gradient is that of a homogeneous acoustic model with velocity c_0, which is shown in Figure 1(b). In Figure 2 a_C and a_Q are plotted as a function of depth. For a source and receiver colocated at the surface the analytic, one-dimensional normal incidence data for this Earth model can be written as

$$D(\omega) = \frac{1}{i2K_0} + R(\omega)\frac{e^{i2K_0 z_1}}{i2K_0},$$

where $K_0 = \frac{\omega}{c_0}$, z_1 is the depth to the reflector and since there is no acoustic impedance contrast, the reflection coefficient is given approximately by (Innanen, 2011; Bird et al., 2010)

$$\begin{aligned} R(\omega) &\approx -\frac{1}{2}a_Q F(\omega) \\ &\approx -\frac{1}{2}a_Q \left(\frac{i}{2} - \frac{1}{\pi}\log(\omega/\omega_r)\right) \end{aligned}$$

FULL WAVEFORM INVERSION OF ANALYTIC DATA

A well-known result of the theory of FWI is that the gradient is given by the equation (Tarantola, 1984; Pratt, 1999; Margrave et al., 2010)

$$g(z) = \int_{-\infty}^{\infty} d\omega \omega^2 G(0,z,\omega) G(z,0,\omega) \delta P^*(0,0,\omega) \tag{1}$$

where $g(z)$ is the first step of the gradient, $G(0, z, \omega)$ is the Green's function for a wave traveling from source position $z_s = 0$ to depth z and $G(z, 0, \omega)$ is the Green's function for a wave traveling from source position $z_s = z$ to a receiver at $z = 0$. $\delta P^*(0, 0, \omega)$ is the complex conjugate of the data residuals. In order to calculate the data residuals, we need another Green's function, $G(0, 0, \omega)$ which is the analytic data we obtain from our initial model. The needed Greens functions can be written as

$$\begin{aligned} G(0,0,\omega) &= \frac{1}{i2K_0} \\ G(z,0,\omega) &= \frac{e^{iK_0 z}}{i2K_0} \\ G(0,z,\omega) &= \frac{e^{iK_0 z}}{i2K_0} \end{aligned}$$

We can therefore write the data residuals as

$$\delta P(0,0,\omega) = D(\omega) - G(0,0,\omega) = R(\omega)\frac{e^{i2K_0 z_1}}{i2K_0},$$

taking the complex conjugate we obtain

$$\delta P^{*}(0,0,\omega)=-\frac{1}{2}a_Q\left(\frac{i}{2}+\frac{1}{\pi}\log(\omega/\omega_r)\right)\frac{e^{-i2K_0z_1}}{i2K_0},$$

now we can calculate the gradient

$$\begin{aligned}
g(z) &= \int_{-\infty}^{\infty} d\omega\omega^2 G(0,z,\omega)G(z,0,\omega)\delta P^{*}(0,0,\omega)\\
&= \int_{-\infty}^{\infty} d\omega\omega^2\left[\frac{e^{iK_0z}}{i2K_0}\right]^2\left[-\frac{1}{2}a_Q\left(\frac{i}{2}+\frac{1}{\pi}\log(\omega/\omega_r)\right)\frac{e^{-i2K_0z_1}}{i2K_0}\right]\\
&= \frac{c_0^2}{4}\int_{-\infty}^{\infty} d\omega\frac{e^{i2K_0(z-z_1)}}{i2K_0}\left[\frac{i}{4}a_Q+a_Q\frac{\log(\omega/\omega_r)}{2\pi}\right]\\
&= \frac{ic_0^3}{32}a_Q\int_{-\infty}^{\infty} d(2K_0)\frac{e^{i2K_0(z-z_1)}}{i2K_0}+\frac{c_0^2}{8\pi}a_Q\int_{-\infty}^{\infty} d\omega\frac{e^{i2K_0(z-z_1)}}{i2K_0}\log(\omega/\omega_r)\\
&= \frac{ic_0^3}{32}a_QH(z-z_1)+\frac{c_0^2}{8\pi}a_Q\int_{-\infty}^{\infty} d\omega\frac{e^{i2K_0(z-z_1)}}{i2K_0}\log(\omega/\omega_r)\\
&= g_1+g_2
\end{aligned} \tag{2}$$

where $H(z-z_1)$ is a Heaviside function. The first term in the gradient, g_1, is a step function which turns on at the depth of the reflector z_1, but notice that there is a complex i in front of the Heaviside and therefore the step is wholly imaginary. This seems intuitively correct as we know that in order to model attenuation we let wavenumber or velocity have an imaginary component. Also, the second term, g_2, contains an $\frac{e^{i2K_0(z-z_1)}}{i2K_0}$, which when integrated alone would produce a Heaviside function which steps at the appropriate depth of the reflector, z_1. However, there is a $\log(\omega/\omega_r)$ also contained in the integral which acts as filter and hence performing the integral should yield a step function which has been in some way filtered. The integral g_2 was evaluated numerically. Figure 1(c) shows the wholly imaginary step function g_1 vs depth. Figure 1(d) shows the absolute value of g_2, it also has a step at z_1. However, it is not a sharp step but rather has a droopy like appearance. This is due to the filtering that the $\log(\omega/\omega_r)$ performs on the step function. This result is of course not correct, we should expect the final outcome of full waveform inversion to produce a real part of the model which is zero (as shown by the blue curve in Figure 1(a)).

CONCLUSIONS

The first step of the gradient for full waveform inversion was calculated on analytic data for a simple Earth model. This simple Earth model consisted of an elastic overburden overlaying an attenuative target. One dimensional, normal incidence data was calculated for this model and was used to calculate the first step of the gradient function. It was found that the gradient predicted an imaginary step function located at the depth of the attenuative target. This imaginary step function seems to be an intuitively correct result as introducing absorption into the wave equation usually involves allowing the wavespeed to have an imaginary component. Because of dispersion, the real part of the gradient was a step function which is filtered by $\log(\omega/\omega_r)$. The insight we gain is in seeing what the next iterations must accomplish if they are to converge to the correct answer. The imaginary part appears to be moving well towards the right answer, whereas the dispersion and its effect on the real part has left much remaining work to be done to reconstruct the correct result. Since both are here related to the same parameter, we suggest and will consider as a matter of future research the idea of using the former to help condition the latter.

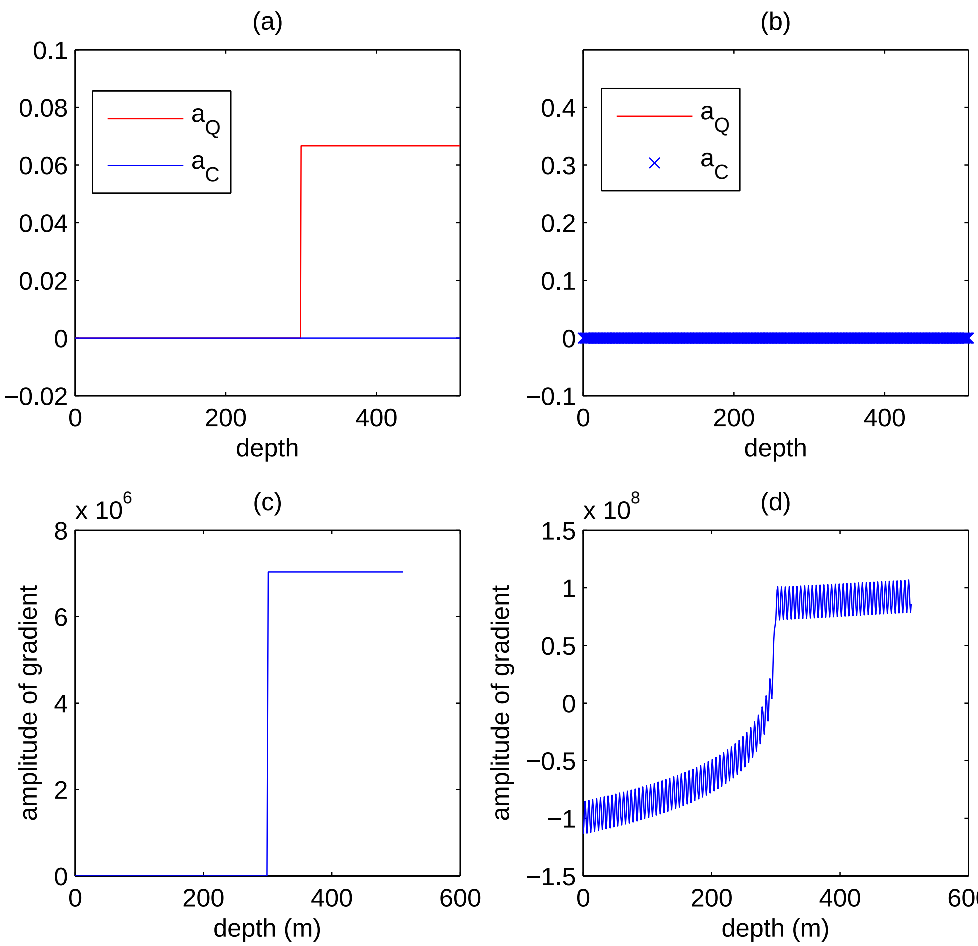

FIG. 1. In (a) a simple attenuative Earth model consisting of an elastic overburden overlaying an anelastic target. The is a contrast in Q at 300m. In (b) the starting model for FWI is homogeneous and perfectly acoustic. In (c) the imaginary part of the gradient g_1 and in (d) the real part of the gradient g_2

REFERENCES

Bird, C., Innanen, K., Naghizadeh, M., and L, L., 2010, Determination of anelastic reflectivity: how to extract seismic avf information: CREWES Annual Report, **22**, No. 4.

Hicks, G., and Pratt, G., 2001, Reflection waveform inversion using local descent methods: estimating attenuation and velocity over a gas-sand deposit: Geophysicsl, **66**, No. 2, 598–612.

Innanen, K., 2011, Inversion of the seismic AVF/AVA signatures of highly attenuative targets: Geophysics, **76**, No. 1, R1–R14.

Malinowski, M., Operto, S., and Ribodetti, A., 2011, High-resolution seismic attenuation imaging from wide-aperture onshore data by visco-acoustic frequency-domain full-waveform inversion: Geophysical Journal International, **186**, No. 3, 1179–1204.

Margrave, G. F., Ferguson, R. J., and Hogan, C. M., 2010, Full waveform inversion with wave equation migration and well control: CREWES Annual Report, **22**, No. 4.

Pratt, R. G., 1999, Seismic waveform inversion in the frequency domain, part 1: Theory and verification in a physical scale model: Geophysics, **64**, No. 3, 888–901.
URL `http://link.aip.org/link/?GPY/64/888/1`

Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, **49**, No. 8, 1259–1266.
URL `http://link.aip.org/link/?GPY/49/1259/1`