A particle view of dispersive wave propagation

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ABSTRACT

We have shown elsewhere that simple seismic wave phenomena may be modeled with sets of notional particles which drift freely and collide. To extend this modeling idea to incorporate attenuation, we merely replace each single particle with a large number of particles, each moving with a velocity drawn from a suitable distribution. Numerical examples demonstrate the qualitative correctness of this model; quantitatively it is supported by arguments due to Bickel (1993), who points out that the constant Q impulse response is equivalent to one of the (one-sided) probability density functions from which we have drawn particle velocities.

INTRODUCTION

In last year's CREWES report we discussed the possibility of describing simple seismic wave experiments (such as zero-offset and walk-away VSP surveys) in terms of colliding particles, as opposed to classically propagating waves (Innanen, 2010). Each propagating waveform, or event, was identified as a particle with a mass and a momentum, and these properties were used to discuss scalar reflection, transmission, and propagation phenomena in layered media.

In order to pursue a practical use of such a description (assuming there is one), we will require that the model be capable of describing more than just these simplest of wave behaviours. In this short note we describe how to extend the particle/collision view of a seismic experiment to incorporate another common seismic phenomenon: wave attenuation and dispersion.

The approach is to consider a waveform to be not one particle, but a large number of them, and to assign to each particle a probability of propagating with a given velocity. The waveform at a certain location and time will then be proportional to the number of particles which have reached that location in that time, given these probabilities.

Tracking these particles as locations and times change, we find that the totality of the particles and their distributions closely resembles attenuative and dispersive wave propagation in 1D. We also find that the type of information we must supply to invoke the model is quite similar to that needed for standard wave dispersion: a distribution of velocities (in this case, particle velocities) and a fixed reference.

Interestingly, the same probability density functions which supply causal/physical type attenuating pulse shapes under this particle model, have already been pointed to as being formally connected to attenuation models. For instance, a classic constant-Q impulse response is mathematically identical to a Pareto-Levy probability distribution (Bickel, 1993)*. We sense, therefore, that a particle-based attenuation model may be establishing a useful link between a tangible (though notional) physical idea—particles that move with a range of velocities—and some of the formal mathematics of seismic attenuation.

FORMULATION

We will formulate the problem of a pulse propagating in 1D, i.e., in the direction of increasing distance z as time t increases. To begin, let the pulse be arranged such that it passes $z = 0$ at time $t = 0$, propagating in the direction of positive z. This is illustrated at the top of Figure 1. The standard wave interpretation of this pulse is that it represents a propagating disturbance in an otherwise quiet displacement, acceleration, or pressure field.

Let us instead interpret this "spike" shaped function as a close clustering of a large number, say N, of particles of unit mass, all drifting freely to the right. The top plot could either be seen as an actual image of this cluster, or simply as a plot of the relative number of particles found at all times with z fixed at $z = 0$.

Now, if all N particles drift with the same speed, at greater depths the plots would illustrate the same "spike" shape shifted to the right by greater amounts. However, our intent is to assign different velocities to the particles. Some of the particles would then arrive at a given depth z earlier than others, and the spike shape would spread out as z increased. If the distribution of velocities were assigned correctly, this spreading would occur in manner like that illustrated in the bottom three panels of Figure 1.

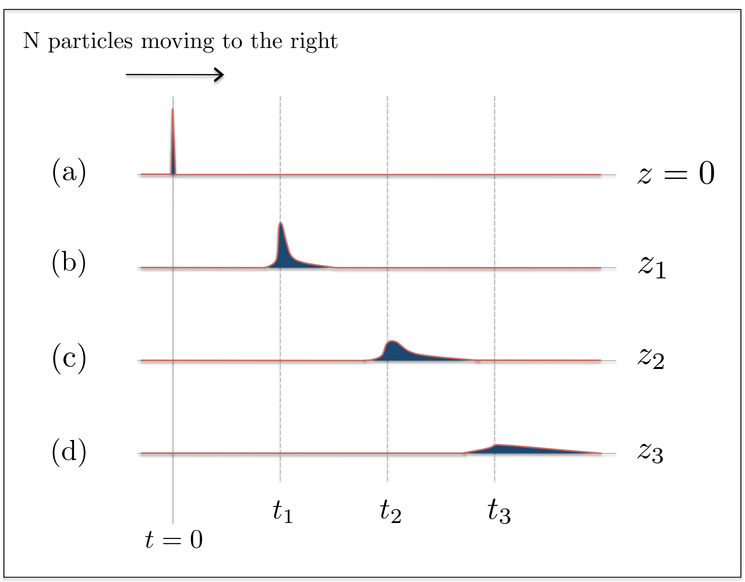

FIG. 1. Illustration of an attenuating pulse propagating in the positive z direction as t increases. The pulse shape is interpreted as a clustering of a large number of particles drifting freely to the right. If the particles, which in (a) are seen to pass $z = 0$ at $t = 0$ as a group, are assigned different velocities based on a suitable probability distribution function, the resulting evolution of the particle clusters (b–d) reproduces attenuative/dispersive wave propagation.

*The Pareto-Levy distribution is argued by Bickel to make physical sense since it was first derived as a general form for all diffusive processes, of which attenuative propagation is a particular example.

Let c_R be the fastest speed any particle can take on. Then any one of the N particles might take on the speed

$$c = c_R - \Delta c, \qquad \Delta c \geq 0. \tag{1}$$

Let the number of particles of the full N which deviate from c_R by Δc be

$$n_{\Delta c} = N \times p(\Delta c), \tag{2}$$

where p is a suitable probability density function.

Particles will arrive at locations z at many different times. At $t = t_1$, how many particles arrive at $z = z_1$? If a particle departs from $z = 0$ and $t = 0$ and arrives at z_1 at t_1, it must have moved at speed

$$c = \frac{z_1}{t_1}, \tag{3}$$

which, by equation (1), means it must have deviated from c_R by an amount Δc where

$$\Delta c = c_R - \frac{z_1}{t_1}. \tag{4}$$

So, the number of particles observed passing z_1 at t_1 is, by equation (2),

$$n_{\Delta c} = N \times p\left(c_R - \frac{z_1}{t_1}\right). \tag{5}$$

Equation (5) then is a prescription for plotting a distribution of particles in mid-drift, at a fixed time over all space z, or at a fixed z over all time (as in Figure 1a–d). The distribution mimics attenuative wave propagation, inasmuch as it is a translation of a realization of the chosen probability density function, which, per Bickel (1993), if chosen properly is the impulse response of a constant Q medium.

CAUSALITY AND PARTICLE VELOCITIES

In equation (1) the condition $\Delta c \geq 0$ leads to a kind of causality. Since c_R is the fastest any particle can travel, it will define the arrival time of the wave, and the further condition then ensures that no particle will arrive earlier than that.

When we choose a distribution for Δc to follow, $p(\Delta c)$, we must choose it to conform to the condition $\Delta c \geq 0$. That is, the probability that any particle arrives before the true arrival time associated with c_R must be zero to ensure a causal pulse.

Symmetric distributions, like the Gaussian, are therefore not allowable. The best distributions for our purposes are those which track numbers of occurences (and so are defined over positive valued outcomes), such as Poisson and Gamma distributions, and of course the Pareto-Levy distribution mentioned above. The choice of pdf and its parameters is akin to choosing the particular Q model.

EXAMPLES

Poisson velocity distribution

If we choose the Poisson distribution for the velocities of our particles, we are adopting a pdf of the form

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x \geq 0, \tag{6}$$

where λ is a parameter (Abramowitz and Stegun, 1972). We may then choose values for λ and c_R, and a range of Δc values, and plot the resulting pdf; we do this in Figure 2a.

We next apply equation (5). That is, we set N particles moving such that they all pass $z = 0$ at $t = 0$ going in the positive z direction, having assigned each particle a velocity drawn from the pdf in equation (6). The number of arriving particles, as functions of time, are plotted at three increasing depths z_1, z_2, and z_3 in Figure 2b. The similarity to a pulse propagating through an attenuative/dispersive medium is clear, though it is a qualitative similarity.

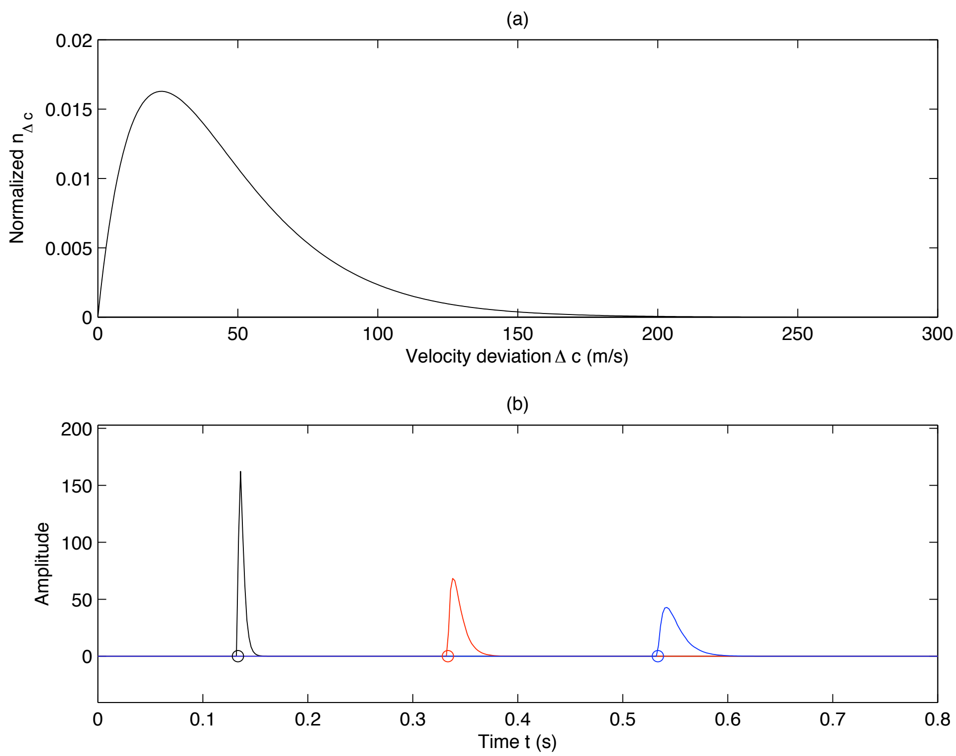

FIG. 2. (a) Normalized velocity distribution $Np(\Delta c)$ chosen as a Poisson distribution, with $\lambda = 1.0 \times 10^{-10}$ and Δc, the deviation from $c_R = 1500$m/s, ranging from 0-300m/s. (b) Resulting traces at depth z values of 200m (black), 500m (red) and 800m (blue). Coloured circles represent arrival times calculated using c_R.

Pareto-Levy velocity distribution

If we choose the Poisson distribution for the velocities of our particles, we are adopting a pdf of the form

$$p(x) = \left(\frac{1}{2\pi}\right) \int d\omega e^{i\omega x} \tilde{p}(\omega), \tag{7}$$

where

$$\tilde{p}(\omega) = e^{K(i\omega)^{\alpha}}. \tag{8}$$

Here K and α are parameters (Bickel, 1993). Choosing c_R, a range of Δc values, and the parameters, we construct a realization of the pdf in equation (7), and, again, applying equation (5), we allow particles with velocities drawn from the Pareto-Levy distribution to propagate to three representative depths. The number/occurence of particles at every time for the fixed depth is a trace; these are plotted in Figure 3b. Again we point to the similarity of the results to a propagating pulse in an attenuating medium.

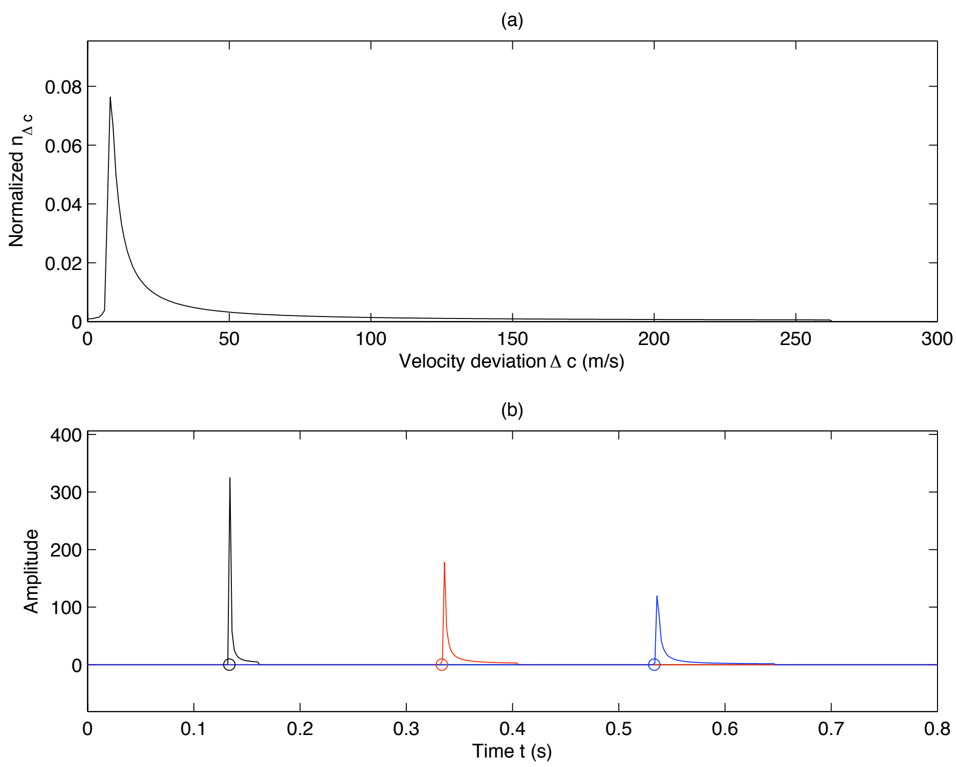

FIG. 3. (a) Normalized velocity distribution $Np(\Delta c)$ chosen as a Pareto-Levy distribution, with $K = -6.0$, $\alpha = 0.4$, and Δc, the deviation from $c_R = 1500$m/s, again ranging from 0-300m/s. (b) Resulting traces at depth z values of 200m (black), 500m (red) and 800m (blue). Coloured circles represent arrival times calculated using c_R.

CONCLUSIONS

We have been able to choose a reasonably small set of rules governing notional particles, which drift freely, or accelerate, have velocities drawn from suitable probability distributions, in such a way that phenomena of attenuative wave propagation are reproduced plausibly. This can be considered an addendum or extension of past work in which seismic waves have been modeled as sets of drifting and colliding particles.

ACKNOWLEDGMENTS

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