A framework for accurate approximation of difference reflection data from baseline to monitor survey in a time-lapse problem using AVO analysis

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ABSTRACT

Perturbation theory has been used widely in many applications in seismology, more recently for time-lapse problems. Time-lapse is a cost-effective approach for monitoring the changes in the fluid saturation and pressure over a period of time in a reservoir.The difference data during the change in a reservoir from the baseline survey to monitor survey are described through applying the perturbation theory. We defined a form for the difference reflection data, $\Delta R_{PP}(\theta)$, in order of physical change or baseline interface contrast and time-lapse changes. A framework for linear and non linear time-lapse difference data is formulated using amplitude variation with offset (AVO) methods. The linear forms are equivalent to those of Landro (2001) and higher order terms represent corrections appropriate for large contrasts. We conclude that in many plausible time-lapse scenarios increase in accuracy associated with higher order corrections is non-negligible.

INTRODUCTION

Behavior of a reservoir can change over time due to production or employing enhanced oil recovery techniques to restor formation pressure and improve the fluid flow. Monitoring theses changes, time-lapse monitoring, facilitates management of a reservoir and extends the useful life of an oilfield. In a time-lapse monitoring one or more seismic survey, monitor surveys, are acquired following the first survey, baseline survey, in a particular interval of time when geological/geophyical characteristics of a reservoir is changed. A time lapse, 4D seismic survey, compares repeated seismic surveys over months, years, or decades and adds the fourth dimension calendar, time, to the seismic data (Greaves and Fulp, 1987; Lumley, 2001). The measurable difference in the seismic trace between the baseline and monitor survey which is called difference data, can be in amplitude, frequency, polarity, or the location of the interfaces (Zhang, 2006; Innanen and Naghizadeh, 2010).

Changes in the pressure or fluid saturation in a reservoir can be an indicator to determine the difference data between the baseline and monitor survey. Time-lapse amplitude variation with offset (AVO) methods are applied to analyze these changes (Tura and Lumley, 1998; M. Landrø and Strønen, 1999). Time-lapse AVO methods indicate a none linear relationship between the pressure and saturation changes and P wave velocity change. Indeed, there is a highly non-linear relationship between relative P wave velocity change and the pressure change in a reservoir, demanding of obtaining higher order approximations of the relationship between seismic parameters and pressure changes. Also larger relative changes in P wave velocity can be the case in time-lapse problems which leads to higher error when the problem is linearly approximated, Figure 1 (Landrø, 2001).

The perturbation (scattering) theory can be used as a framework to model the difference data in a time lapse survey and first suggested by Zhang (2006). The baseline survey is set

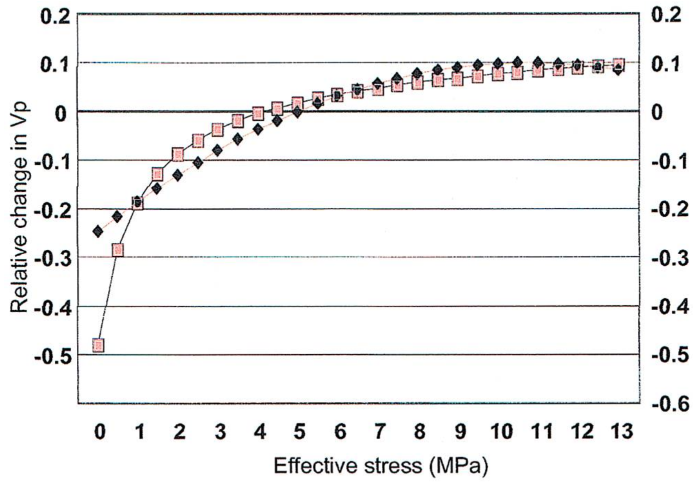

FIG. 1. Relationship between relative change in P-wave velocity versus changes in net pressure based upon a calibrated Gassmann Model (Landrø 2001) .

to be the background medium which goes under perturbation by the time of the monitor survey. The perturbation is presented here such that it quantifies the changes in P wave and S wave velocities and density form the time of the baseline to monitor survey .

The study described here focuses on applying the perturbation theory in time-lapse amplitude variation with offset (Time-lapse AVO) method to model a framework to describe the difference data from a baseline survey to monitor survey in a reservoir. Reflection coefficients are derived for the baseline and monitor survey using Zoeppritz equations to calculate the reflection coefficient for difference data.

Theory

Consider an incident P wave striking on the boundary of a planar interface between two elastic media with rock properties V_{P0}, V_{S0}, ρ_0 and V_{P1}, V_{S1}, ρ_1 as shown in Figure 2.

The amplitudes of these reflected and transmitted P and S waves can be calculated through setting boundary conditions. Normal and tangential components of the stress and displacement must be continuous across the interface. Setting these requirement in the problem leads to Zoeppritz equations which can be rearranged in a matrix form (Aki and Richards, 2002):

$$P \begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_P$$

where

$$P \equiv \begin{bmatrix} -X & -\sqrt{1-B^2X^2} & CX & \sqrt{1-D^2X^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-C^2X^2} & -DX \\ 2B^2X\sqrt{1-X^2} & B(1-2B^2X^2) & 2AD^2X\sqrt{1-C^2X^2} & AD(1-2D^2X^2) \\ -1+2B^2X^2 & 2B^2X\sqrt{1-B^2X^2} & AC(1-2D^2X^2) & -2AD^2X\sqrt{1-D^2X^2} \end{bmatrix}.$$

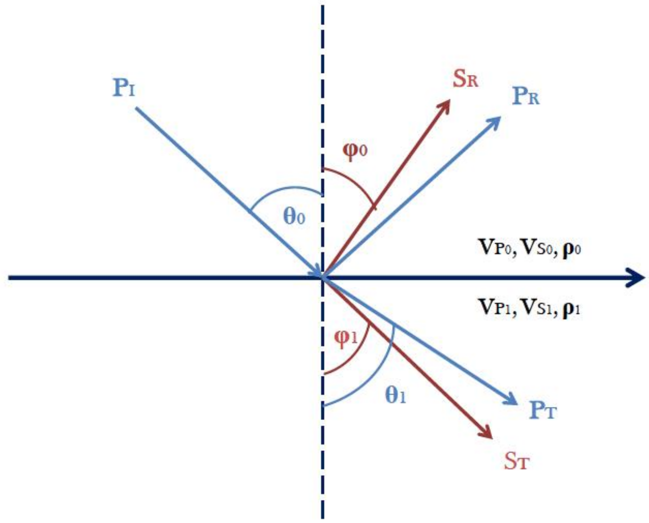

FIG. 2. Displacement amplitude of an incident P-wave with related reflected and transmitted P and S waves.

$X = \sin\theta_0$, θ_0 is the P-wave incident angle , and

$$b_P \equiv \begin{bmatrix} X \\ \sqrt{1-X^2} \\ 2B^2X\sqrt{1-X^2} \\ 1-2B^2X^2 \end{bmatrix}.$$

The ratio of elastic parameters are defined as:

$$A \equiv \frac{\rho_1}{\rho_0},\ \ B \equiv \frac{V_{S_0}}{V_{P_0}},\ \ B' \equiv \frac{V_{P_0}}{V_{S_0}},\ \ C \equiv \frac{V_{P_1}}{V_{P_0}},\ \ D \equiv \frac{V_{S_1}}{V_{P_0}},\ \ E \equiv \frac{V_{P_1}}{V_{S_0}},\ \ F \equiv \frac{V_{S_1}}{V_{S_0}}. \tag{1}$$

Reflection coefficients are determined by forming auxiliary matrices P_P and P_S and using Cramer's rule:

$$R_{PP}(\theta_0) = \frac{det(P_P)}{det(P)},\ \ R_{PS}(\theta_0) = \frac{det(P_S)}{det(P)}. \tag{2}$$

We next seek a way to expand these solutions about the contrasts across the interface.Now lets introduce perturbation parameters the wave experience traveling from medium one to two (Figure 3):

$$a_{VP} = 1 - \frac{V_{P_0}^2}{V_{P_1}^2},\ \ a_{VS} = 1 - \frac{V_{S_0}^2}{V_{S_1}^2},\ \ a_\rho = 1 - \frac{\rho_0}{\rho_1}. \tag{3}$$

The elastic parameters now, are expressed in terms of perturbations which measures the change in transmitted P and S waves velocities and densities from the first medium to the second medium (Innanen, 2011):

$$A = (1-a_\rho)^{-1},\ \ C = (1-a_{VP})^{-\frac{1}{2}},\ \ D = B \times (1-a_{VS})^{-\frac{1}{2}} = \frac{V_{S_0}}{V_{P_0}} \times (1-a_{VS})^{-\frac{1}{2}}. \tag{4}$$

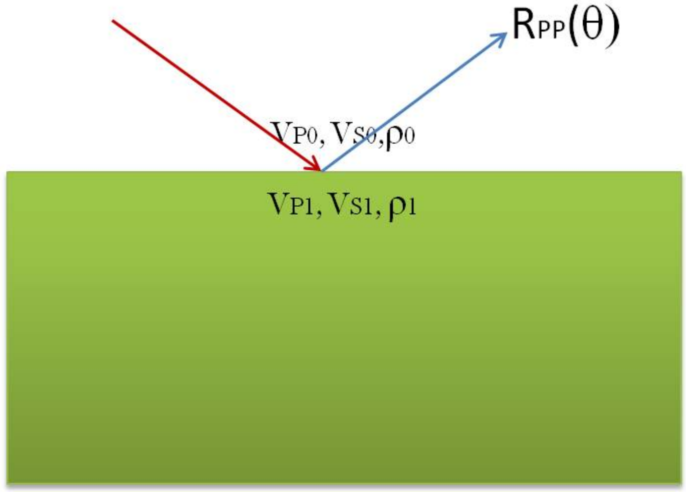

FIG. 3. Rock properties of the model.

These parameters are substituted in Zoeppritz matrix, P, in equation above. The elements of this new matrix now, are functions of a_ρ, a_{VP}, a_{VS}, and $\sin(\theta_0)$. Using Taylor's series:

$$
\begin{aligned}
(1-a_\rho)^{-1} &= 1 + a_\rho + a_\rho^2 + ... \\
(1-a_{VP})^{-\frac{1}{2}} &= 1 + \frac{1}{2}a_{VP} + \frac{1\times 3}{2\times 4}a_{VP}^2 + ... \\
(1-a_{VS})^{-\frac{1}{2}} &= 1 + \frac{1}{2}a_{VS} + \frac{1\times 3}{2\times 4}a_{VS}^2 + ...
\end{aligned}
\tag{5}
$$

and truncating the expansions of X, a_ρ, a_{VP}, a_{VS} after second order, Zoeppritz matrix is re-calculated as:

$$
P(1,:) = \begin{bmatrix} -X \\ -1 + \frac{1}{2}B^2X^2 \\ X + \frac{1}{2}Xa_{VP} + \frac{3}{8}Xa_{VP}^2 \\ 1 - \frac{1}{2}B^2X^2(1 + a_{VS} + a_{VS}^2) \end{bmatrix},
$$

$$
P(2,:) = \begin{bmatrix} 1 - \frac{1}{2}X^2 \\ -BX \\ 1 - \frac{1}{2}X^2(1 + a_{VP} + a_{VP}^2) \\ -BX(1 + \frac{1}{2}a_{VS} + \frac{3}{8}a_{VS}^2) \end{bmatrix},
$$

$$
P(3,:) = \begin{bmatrix} 2B^2X \\ B(1-2B^2X^2) \\ 2B^2X(1 + a_\rho + a_{VS} + a_\rho^2 + a_\rho a_{VS} + a_{VS}^2 \\ [(B(1-2B^2X^2)(1 + a_\rho + \frac{1}{2}a_{VS} + \frac{1}{2}a_\rho a_{VS} + a_\rho^2 + \frac{3}{8}a_{VS}^2) \\ -B^3X^2(2a_{VS} + 2a_\rho a_{VS} + 3a_{VS}^2)] \end{bmatrix},
$$

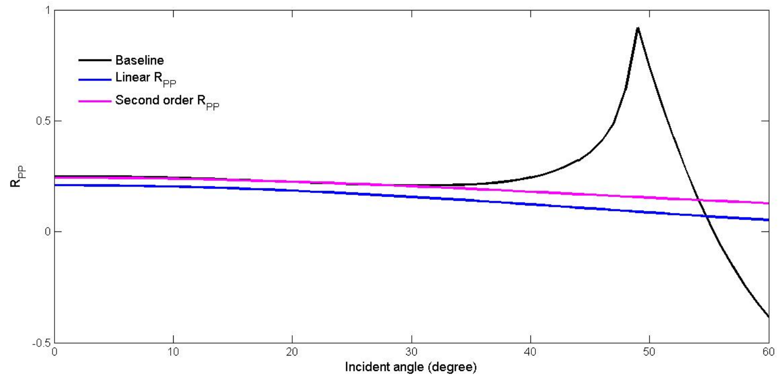

FIG. 4. R_{PP} with linear and second order approximation, Elastic incidence parameters: $V_{P0} = 3000m/s$, $V_{S0} = 1500m/s$ and $\rho_0 = 2.000gm/cc$; Baseline parameters:$V_{P_1} = 4000m/s$, $V_{S_1} = 2000m/s$ and $\rho_1 = 2.500gm/cc$.

and

$$P(4,:) = \left[\begin{array}{c} -1+2B^2X^2 \\ 2B^2X \\ [(1-2B^2X^2)(1+\frac{1}{2}a_{VP}+a_\rho+\frac{1}{2}a_\rho a_{VP}+a_\rho^2+\frac{3}{8}a_{VP}^2) \\ -2B^2X^2(a_{VS}+a_{VP}a_{VS}+a_\rho a_{VS}+a_{VS}^2)] \\ -2B^2X(1+a_{VS}+a_\rho+a_{VS}^2+a_\rho a_{VS}+a_\rho^2) \end{array}\right].$$

Exact $R_{PP}(\theta_0)$ which can be calculated by applying Cramer's rule, can be organized into terms that are first, second, etc. order in any of perturbation parameters, a_ρ, a_{VP} and a_{VS}:

$$R_{PP}(\theta_0) = R_{PP}^{(1)}(\theta_0) + R_{PP}^{(2)}(\theta_0) + ... \tag{6}$$

Having truncated exact R_{PP} beyond the first order and second order, $R_{PP}^{(1)}$ and $R_{PP}^{(2)}$ were calculated as:

$$\begin{aligned} R_{PP}^{(1)}(\theta_0) &= (\frac{1}{4}+\frac{1}{4}X^2)a_{VP} + (-2B^2X^2)a_{VS} + (\frac{1}{2}-2B^2X^2)a_\rho \\ R_{PP}^{(2)}(\theta_0) &= (\frac{1}{8}+\frac{1}{4}X^2)a_{VP}^2 + (B^3X^2-2B^2X^2)a_{VS}^2 + \\ &\quad (\frac{1}{4}-\frac{1}{4}BX^2-B^2X^2+B^3X^2)a_\rho^2 + (2B^3X^2-B^2X^2)a_\rho a_{VS} \end{aligned} \tag{7}$$

Where $X = \sin(\theta_0)$, and $B \equiv \frac{V_{S_0}}{V_{P_0}}$. As in Figure 4, the second order approximation R_{PP} is in a better agreement with the exact R_{PP} than the linear approximation in R_{PP}.

A framework for time-lapse AVO

We will consider two seismic experiments involved in a time-lapse survey, the baseline survey, followed by a monitoring survey. The P wave and S wave velocities and density change from the time of the baseline survey to monitoring survey (Figure 5). This pair of

FIG. 5. Rock properties of the model at the time of the baseline(left) and monitor (right) survey.

models is consistent with an unchanging caprock overlying a porous target during production.

In a time lapse study we need to consider two groups of perturbation parameters. First groups express the perturbation caused by propagating the wavefield from the first medium to the second medium in the baseline survey:

$$a_{VP} = 1 - \frac{V_{P_0}^2}{V_{P_{BL}}^2}, \;\; a_{VS} = 1 - \frac{V_{S_0}^2}{V_{S_{BL}}^2}, \;\; a_\rho = 1 - \frac{\rho_0}{\rho_{BL}}, \tag{8}$$

To account for the perturbation from baseline to monitor survey we define:

$$b_{VP} = 1 - \frac{V_{P_{BL}}^2}{V_{P_M}^2}, \;\; b_{VS} = 1 - \frac{V_{S_{BL}}^2}{V_{S_M}^2}, \;\; b_\rho = 1 - \frac{\rho_{BL}}{\rho_M}, \tag{9}$$

Elastic parameters are re-defined in terms of perturbations in P wave and S wave velocities and densities as:

$$\begin{aligned} &A = (1-a_\rho)^{-1} \times (1-b_\rho)^{-1}, \;\; C = (1-a_{VP})^{-\frac{1}{2}} \times (1-b_{VP})^{-\frac{1}{2}}, \\ &D = B \times (1-a_{VS})^{-\frac{1}{2}} = \frac{V_{S_0}}{V_{P_0}} \times (1-a_{VS})^{-\frac{1}{2}} \times (1-b_{VS})^{-\frac{1}{2}}, \end{aligned} \tag{10}$$

$\Delta R_{PP}(\theta_0)$ which is the difference data reflection coefficient is determined using AVO perturbation method (which is reviewed in the previous section). Linear and second order terms in the difference data reflection coefficient are as follows:

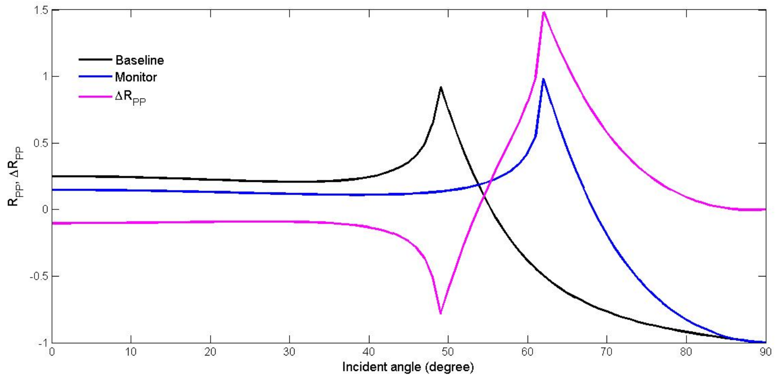

FIG. 6. R_{PP} for the Baseline and Monitor survey and ΔR_{PP}, Elastic incidence parameters: $V_{P0} = 3000m/s$, $V_{S0} = 1500m/s$ and $\rho_0 = 2.000gm/cc$; Baseline parameters:$V_{P_{BL}} = 4000m/s$, $V_{S_{BL}} = 2000m/s$ and $\rho_{BL} = 2.500gm/cc$; Monitor parameters:$V_{P_M} = 3400m/s$, $V_{S_M} = 1700m/s$ and $\rho_M = 2.375gm/cc$.

$$\begin{aligned}
\Delta R^{(1)}_{PP}(\theta_0) =&(\frac{1}{4} + \frac{1}{4}X^2)b_{VP} + (-2B^2X^2)b_{VS} + (\frac{1}{2} - 2B^2X^2)b_\rho \\
\Delta R^{(2)}_{PP}(\theta_0) =&(\frac{1}{8} + \frac{1}{4}X^2)b^2_{VP} + (B^3X^2 - 2B^2X^2)b^2_{VS}+ \\
&(\frac{1}{4} - \frac{1}{4}BX^2 - B^2X^2 + B^3X^2)b^2_\rho + (2B^3X^2 - B^2X^2)b_\rho b_{VS}+ \\
&(2B^3X^2 - 2B^2X^2)b_{VS}a_{VS} + (\frac{1}{4}X^2)b_{VP}a_{VP} + (2B^3X^2 - B^2X^2)a_\rho b_{VS}+ \\
&(2B^3X^2 - B^2X^2)b_\rho a_{VS} + (2B^3X^2 - \frac{1}{2}BX^2)a_\rho b_\rho
\end{aligned} \tag{11}$$

Where $X = \sin(\theta_0)$, and $B \equiv \frac{V_{S0}}{V_{P_0}}$. The third order terms in the difference data are provided in Appendix B.

For a numerical example, the data used by Greaves and Fulp (1987) are applied . The 3-D seismic survey had been provided over a period of 15 months on the Holt sand, reservoir. There was an increase in gas saturation which caused a measurable decrease in elastic parameter. The measurement showed a decrease about 5 percent in density, 15 percent to 35 percent in velocity. Figure 6 shows the R_{PP} for the baseline and monitor survey resembling 5 and 15 percent decrease in density and velocities respectively from the time of baseline survey and monitor survey. The exact difference data are compared with the linear and higher order approximations derived from the equations above (Figure 7). The second and third approximations are in a good agreement with the exact one for angles below the critical angle. As it is seen in the Figure 7, approximating the difference data with higher order terms is required.

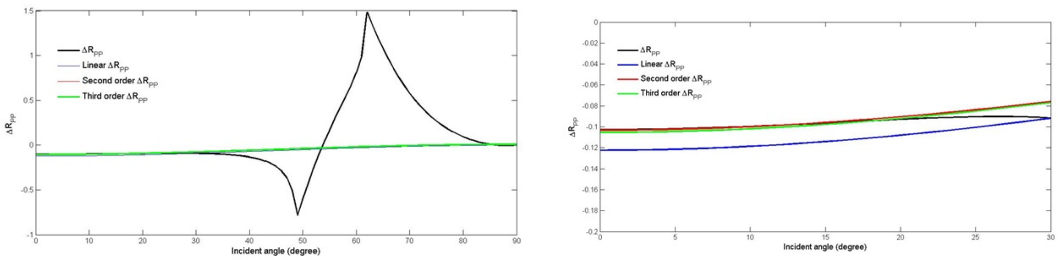

FIG. 7. ΔR_{PP} for the exact, linear, second order, and third order approximation. Elastic incidence parameters: $V_{P0} = 3000m/s$, $V_{S0} = 1500m/s$ and $\rho_0 = 2.000gm/cc$; Baseline parameters:$V_{P_{BL}} = 4000m/s$, $V_{S_{BL}} = 2000m/s$ and $\rho_{BL} = 2.500gm/cc$; Monitor parameters:$V_{P_M} = 3400m/s$, $V_{S_M} = 1700m/s$ and $\rho_M = 2.375gm/cc$.

Time-lapse AVO in terms of relative seismic parameter changes

Computing time-lapse difference data in terms of relative changes in seismic parameters is a more recognizable, and possibly more numerically accurate way. These relative changes are defined as follow:

$$\begin{aligned}\frac{\Delta V_P}{V_P} &=2 \times \frac{V_{Pb} - V_{P0}}{V_{Pb} + V_{P0}}\\ \frac{\Delta V_S}{V_S} &=2 \times \frac{V_{Sb} - V_{S0}}{V_{Sb} + V_{S0}}\\ \frac{\Delta \rho}{\rho} &=2 \times \frac{\rho_b - \rho_0}{\rho_b + \rho_0}\end{aligned} \tag{12}$$

for baseline perturbations and

$$\begin{aligned}\frac{\delta V_P}{V_P} &=2 \times \frac{V_{Pm} - V_{Pb}}{V_{Pm} + V_{Pb}}\\ \frac{\delta V_S}{V_S} &=2 \times \frac{V_{Sm} - V_{Sb}}{V_{Sm} + V_{Sb}}\\ \frac{\delta \rho}{\rho} &=2 \times \frac{\rho_m - \rho_b}{\rho_m + \rho_b}\end{aligned} \tag{13}$$

for time-lapse perturbations.a_{VP}, a_{VP}, a_{VP}, and ... can be expanded as appropriate series of relative changes as:

$$\begin{aligned}a_{VP} &=2\left(\frac{\Delta V_P}{V_P}\right) - 2\left(\frac{\Delta V_P}{V_P}\right)^2 + \frac{3}{2}\left(\frac{\Delta V_P}{V_P}\right)^3 - ...\\ a_{VS} &=2\left(\frac{\Delta V_S}{V_S}\right) - 2\left(\frac{\Delta V_S}{V_S}\right)^2 + \frac{3}{2}\left(\frac{\Delta V_S}{V_S}\right)^3 - ...\\ a_\rho &= \left(\frac{\Delta \rho}{\rho}\right) - \frac{1}{2}\left(\frac{\Delta \rho}{\rho}\right)^2 + \frac{1}{4}\left(\frac{\Delta \rho}{\rho}\right)^3 + ...,\end{aligned} \tag{14}$$

and

$$
\begin{aligned}
b_{VP} =& 2\left(\frac{\delta V_P}{V_P}\right) - 2\left(\frac{\delta V_P}{V_P}\right)^2 + \frac{3}{2}\left(\frac{\delta V_P}{V_P}\right)^3 - ... \\
b_{VS} =& 2\left(\frac{\delta V_S}{V_S}\right) - 2\left(\frac{\delta V_S}{V_S}\right)^2 + \frac{3}{2}\left(\frac{\delta V_S}{V_S}\right)^3 - ... \\
b_{\rho} =& \frac{\delta\rho}{\rho} - \frac{1}{2}\frac{\delta\rho}{\rho}^2 + \frac{1}{4}\frac{\delta\rho}{\rho}^3 + ...,
\end{aligned}
\tag{15}
$$

Substituting Equation (15) in the first equation in Equation(11) results in Landro's equation for linear term in difference data reflection. Agreement of our linear results with Landro's work also has been proven directly in Appendix A.

$$
\Delta R_{PP}^{(1)}(\theta_0) = \frac{1}{2}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta V_P}{V_P}\right) - \frac{2V_S^2}{V_P^2}\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_S}{V_S}\right)\sin^2\theta + \frac{\Delta V_P}{2V_P}\sin^2\theta \tag{16}
$$

The second order term difference data in terms of relative parameters is recalculated using the same process:

$$
\begin{aligned}
\Delta R_{PP}^{(2)}(\theta_0) =& \Gamma_{\delta V_P}\left(\frac{\delta V_P}{V_P}\right)^2 + \Gamma_{\delta V_S}\left(\frac{\delta V_S}{V_S}\right)^2 + \Gamma_{\delta\rho}\left(\frac{\delta\rho}{\rho}\right)^2 + \Gamma_{\delta\rho\delta V_S}\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right) \\
&+ \Gamma_{\Delta V_S\delta V_S}\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_S}{V_S}\right) + \Gamma_{\Delta V_P\delta V_P}\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta V_S}{V_S}\right) \\
&+ \Gamma_{\Delta\rho\delta V_S}\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right) + \Gamma_{\Delta V_S\delta\rho}\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right) + \Gamma_{\Delta\rho\delta\rho}\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta\rho}{\rho}\right)
\end{aligned}
\tag{17}
$$

where

$$
\begin{aligned}
\Gamma_{\delta V_P} &= \frac{1}{2} + \sin^2(\theta_0) \\
\Gamma_{\delta V_S} &= 4\left(\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - 2\left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0)\right) \\
\Gamma_{\delta\rho} &= \left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0) - \frac{1}{4}\left(\frac{V_{S_0}}{V_{P_0}}\right) \sin^2(\theta_0) + \frac{1}{4} \\
\Gamma_{\delta\rho\delta V_S} &= 2\left(2\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0)\right) \\
\Gamma_{\Delta V_S\delta V_S} &= 8\left(\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0)\right) \\
\Gamma_{\Delta V_P\delta V_P} &= \sin^2(\theta_0) \\
\Gamma_{\Delta\rho\delta V_S} &= 2\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0) \\
\Gamma_{\Delta V_S\delta\rho} &= 2\left(2\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0)\right) \\
\Gamma_{\Delta\rho\delta\rho} &= 2\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \frac{1}{2}\left(\frac{V_{S_0}}{V_{P_0}}\right) \sin^2(\theta_0)
\end{aligned}
\tag{18}
$$

The third order term is also recalculated based on relative parameters in Appendix B.

CONCLUSION

Employing the perturbation theory in many geophysical area such as time-lapse is worthwhile. Time-lapse measurements provide a tool to monitor the dynamic changes in subsurface properties during the time of the exploitation of a reservoir. Changes in the fluid saturation and pressure will have an impact in elastic parameters, such as P wave and S wave velocities and density, of subsurface which can be approximated by applying time-lapse AVO analysis methods. As changes in these parameters have a non-linear relationship with the fluid saturation and pressure changes in the reservoir, and also change in these parameters can be large in a time-lapse scenario; calculating the higher order terms for the difference data reflection coefficient is highly required. Having a framework to formulate the difference reflection data for ΔR_{PP}, the next step can be formulating the difference reflection data for ΔR_{PS}, and ΔR_{SS}.

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APPENDIX A

The result (only the linear term) were compared to the the difference data reflection coefficient derived by Landro (2001). The difference data reflection coefficient in Landro's paper is:

$$\Delta R_{PP}^{(1)}(\theta) = \frac{1}{2}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta V_P}{V_P}\right) - 2\frac{V_S^2}{V_P^2}\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_S}{V_S}\right)\sin^2\theta + \frac{\Delta V_P}{2V_P}\tan^2\theta \tag{19}$$

This equation can be rearranged as:

$$\Delta R_{PP}^{(1)}(\theta) = \frac{1}{2}(1+\tan^2\theta)(\frac{\Delta V_P}{V_P}) + (-\frac{4V_S^2}{V_P^2})\sin^2\theta(\frac{\Delta V_S}{V_S}) + (\frac{1}{2} - \frac{2V_S^2}{V_P^2}\sin^2\theta)(\frac{\Delta\rho}{\rho}) \tag{20}$$

For small θ:

$$1+\sin^2\theta = 1+\sin^2\theta+\sin^4\theta = 1+\sin^2\theta(1+\sin^2\theta) \sim 1+\frac{\sin^2\theta}{\cos^2\theta} = 1+\tan^2\theta \tag{21}$$

Here we used this approximation,

$$\frac{1}{\cos^2\theta} = \frac{1}{1-\sin^2\theta} \sim 1+\sin^2\theta \tag{22}$$

Substituting $X = \sin\theta$ and $B = \frac{V_S^2}{V_P^2}$ leads to:

$$\Delta R_{PP}^{(1)}(\theta) = \frac{1}{2}(1+X^2)(\frac{\Delta V_P}{V_P}) + (-4B^2X^2)(\frac{\Delta V_S}{V_S}) + (\frac{1}{2} - 2B^2X^2)(\frac{\Delta\rho}{\rho}) \tag{23}$$

Considering:

$$\begin{aligned}
b_{VP} =&1-\frac{V_{P_{BL}}^2}{V_{P_M^2}}=\frac{V_{P_M}^2-V_{P_{BL}}^2}{V_{P_M}^2}=\left(\frac{V_{P_M}+V_{P_{BL}}}{V_{P_M}}\right)\times\left(\frac{V_{P_M}-V_{P_{BL}}}{V_{P_M}}\right)\\
\sim&2\times\left(\frac{V_{P_M}-V_{P_{BL}}}{V_{P_M}}\right)\sim 2\times\frac{\Delta V_P}{V_P}\\
b_{VS} =&1-\frac{V_{S_{BL}}^2}{V_{S_M^2}}=\frac{V_{S_M}^2-V_{S_{BL}}^2}{V_{S_M}^2}=\left(\frac{V_{S_M}+V_{S_{BL}}}{V_{S_M}}\right)\times\left(\frac{V_{S_M}-V_{S_{BL}}}{V_{S_M}}\right)\\
\sim&2\times\left(\frac{V_{S_M}-V_{S_{BL}}}{V_{S_M}}\right)\sim 2\times\frac{\Delta V_S}{V_S}\\
b_\rho =&1-\frac{\rho_{BL}}{\rho_M}=\frac{\rho_M-\rho_{BL}}{\rho_M}=\frac{\Delta\rho}{\rho}
\end{aligned} \tag{24}$$

The equation (11) is equivalent to the equation (12), which shows the agreement of our linear approximation for the difference data with the one derived by Landro (2003):

$$\begin{aligned}
\Delta R_{PP}^{(1)}(\theta_0) =&(\frac{1}{4}+\frac{1}{4}X^2)b_{VP}+(-2B^2X^2)b_{VS}+(\frac{1}{2}-2B^2X^2)b_\rho\\
=&(\frac{1}{4}+\frac{1}{4}X^2)(2\times\frac{\Delta V_P}{V_P})+(-2B^2X^2)(2\times\frac{\Delta V_S}{V_S})+(\frac{1}{2}-2B^2X^2)\frac{\Delta\rho}{\rho}\\
=&\frac{1}{2}\left(\frac{\Delta\rho}{\rho}+\frac{\Delta V_P}{V_P}\right)-\frac{2V_S^2}{V_P^2}\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right)\sin^2\theta+\frac{\Delta V_P}{2V_P}\tan^2\theta
\end{aligned} \tag{25}$$

APPENDIX B

The third order term in difference data in time-lapse AVO is calculated as:

$$\begin{aligned}
\Delta R_{PP}^{(3)}(\theta_0) =&(\frac{15}{64}X^2+\frac{5}{64})b_{VP}^3+(\frac{7}{4}B^3X^2-2B^2X^2)b_{VS}^3+(\frac{1}{2}B^3X^2-\frac{3}{8}BX^2+\frac{1}{8})b_\rho^3\\
&+(\frac{1}{2}B^2X^2-B^3X^2)(a_{VP}a_{VS}b_\rho+a_{VS}a_\rho b_{VS}+a_{VS}b_{VP}b_\rho+a_\rho b_{VP}b_{VS}+b_{VP}b_{VS}b_\rho\\
&+a_{VP}b_{VS}b_\rho+a_{VP}a_\rho b_{VS})+(2B^3X^2-\frac{1}{2}B^2X^2)(a_{VS}b_{VS}a_\rho+a_{VS}b_{VS}b_\rho)\\
&+(\frac{3}{2}B^2X^2-\frac{1}{2}B^3X^2-\frac{1}{8}BX^2)(a_{VS}a_\rho b_\rho+a_\rho b_\rho b_{VS})+(\frac{1}{4}B^2X^2)(a_{VP}b_{VP}b_{VS}\\
&+a_{VP}a_{VS}b_{VP})+(\frac{1}{4}B^2X^2-\frac{1}{8}X^2-\frac{1}{16})(a_{VP}b_{VP}b_\rho+a_{VP}a_\rho b_{VP})-(B^3X^2)\\
&(a_{VP}a_{VS}b_{VS}+a_{VS}b_{VS}b_{VP})+(B^2X^2-B^3X^2-\frac{1}{4}BX^2-\frac{1}{8}X^2)(a_\rho b_{VP}b_\rho)+\\
&(\frac{13}{64}X^2-\frac{1}{64})(a_{VP}b_{VP}^2+b_{VP}a_{VP}^2)+(\frac{1}{2}B^2X^2-\frac{1}{2}B^3X^2-\frac{1}{8}BX^2-\frac{1}{16}X^2-\frac{1}{16})\\
&(a_{VP}b_\rho^2+b_{VP}a_\rho^2+b_{VP}b_\rho^2)-(\frac{1}{2}B^3X^2)a_{VP}b_{VS}^2+(\frac{1}{8}B^2X^2-\frac{1}{16}X^2-\frac{1}{32})\\
&(a_\rho b_{VP}^2+b_\rho b_{VP}^2+b_\rho a_{VP}^2)+(\frac{13}{4}B^3X^2-2B^2X^2)(b_{VS}a_{VS}^2+a_{VS}b_{VS}^2)\\
&+(2B^3X^2-\frac{3}{4}B^2X^2)(a_\rho b_{VS}^2+b_\rho b_{VS}^2+b_\rho a_{VS}^2)+(\frac{1}{8}B^2X^2)b_{VS}a_{VP}^2\\
&-(\frac{1}{2}B^3X^2)b_{VP}a_{VS}^2+b_{VP}b_{VS}^2)+(\frac{3}{4}B^3X^2+\frac{1}{4}B^2X^2-\frac{1}{16}BX^2)(b_{VS}b_\rho^2+a_{VS}b_\rho^2\\
&+b_{VS}a_\rho^2)+(2B^2X^2-\frac{1}{2}B^3X^2-\frac{5}{8}BX^2-\frac{1}{8})(a_\rho b_\rho^2+b_\rho a_\rho^2)
\end{aligned} \tag{26}$$

This third order term in difference data in time-lapse AVO is also recalculated in terms of relative parameters as:

$$\begin{aligned}
\Delta R_{PP}^{(3)}(\theta_0) =& 8\left(\frac{15}{64}X^2+\frac{5}{64}\right)\left(\frac{\delta V_P}{V_P}\right)^3+8\left(\frac{7}{4}B^3X^2-2B^2X^2\right)\left(\frac{\delta V_S}{V_S}\right)^3 \\
&+\left(\frac{1}{2}B^3X^2-\frac{3}{8}BX^2+\frac{1}{8}\right)\left(\frac{\delta\rho}{\rho}\right)^3+4\left(\frac{1}{2}B^2X^2-B^3X^2\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right) \\
&+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)+ \\
&\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta V_S}{V_S}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)]+ \\
&4\left(2B^3X^2-\frac{1}{2}B^2X^2\right)[\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)] \\
&+4\left(\frac{1}{4}B^2X^2-\frac{1}{8}X^2-\frac{1}{16}\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)] \\
&+2\left(B^2X^2-B^3X^2-\frac{1}{4}BX^2-\frac{1}{8}X^2-\frac{1}{8}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right) \\
&-8\left(B^3X^2\right)[\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_S}{V_S}\right)] \\
&+2\left(B^2X^2\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)] \\
&+2\left(\frac{3}{2}B^2X^2-\frac{1}{2}B^3X^2-\frac{1}{8}BX^2\right)[\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta\rho}{\rho}\right)+\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)] \\
&\left(B^2X^2\right)[\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)^2+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)^2+\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta V_P}{V_P}\right)^2] \\
&-4\left(B^3X^2\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta V_S}{V_S}\right)^2+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\Delta V_S}{V_S}\right)^2+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta V_S}{V_S}\right)^2] \\
&+8\left(\frac{13}{4}B^3X^2-2B^2X^2\right)[\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_S}{V_S}\right)^2+\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta V_S}{V_S}\right)^2] \\
&+2\left(\frac{3}{4}B^3X^2+\frac{1}{4}B^2X^2-\frac{1}{16}BX^2\right)[\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)^2+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)^2 \\
&+\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right)^2]+4\left(2B^3X^2-\frac{3}{4}B^2X^2\right)[\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)^2+\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta V_S}{V_S}\right)^2 \\
&+\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)^2]+8\left(\frac{13}{64}X^2-\frac{1}{64}\right)[\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\Delta V_P}{V_P}\right)^2+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta V_P}{V_P}\right)^2] \\
&+4\left(\frac{1}{8}B^2X^2-\frac{1}{16}X^2-\frac{1}{32}\right)[\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta V_P}{V_P}\right)^2+\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)^2+\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)^2] \\
&+2\left(\frac{1}{2}B^2X^2-\frac{1}{2}B^3X^2-\frac{1}{8}BX^2-\frac{1}{16}X^2-\frac{1}{16}\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)^2+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)^2 \\
&+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\Delta\rho}{\rho}\right)^2]+\left(2B^2X^2-\frac{1}{2}B^3X^2-\frac{5}{8}BX^2-\frac{1}{8}\right)[\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta\rho}{\rho}\right)^2 \\
&+\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta\rho}{\rho}\right)^2]
\end{aligned} \tag{27}$$

Where $X = \sin(\theta_0)$, and $B \equiv \frac{V_{S_0}}{V_{P_0}}$.