Re-expressing frequency dependent reflection and transmission coefficients for P-waves incident on porous fluid-filled media

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ABSTRACT

There are various parameterizations of reflection and transmission coefficients for poroelastic media. These expressions are also typically non-linear. From the perspective of petrophysical inversion, non-linearity can cause an undesired extra complexity to the problem which may cause an ineffective prediction of poroelasticity. Therefore, we would like to present a framework of equations based off of previous findings of normal incidence, frequency dependent reflection and transmission coefficients. A linearized form for these coefficients would provide a computationally faster measure of inverted poroelastic parameters. Future investigations of the accuracy and precision of these inverted parameters would show particular circumstances where these inversions may be useful.

INTRODUCTION

The purpose of this report is to provide different nomenclature for expressions that are provided by Gurevich et al. (2004). The nomenclature that would be used are definitions that can allow us to observe relative changes of properties of one homogeneous geological layer to the properties of another that is separated by a welded contact or interface. There is much research that analyzes these relative changes using difference-average or $\frac{\Delta h}{h}$ which is an expression that characterizes a difference in h divided by the average of h where h is some property of a two-layer model (Shuey, 1985; Aki and Richards, 2002; Smith and Gidlow, 1987). This analysis of relative change is also described as reflectivity. The expression provided by Gurevich et al. (2004) is non-linear and a linearized expression is currently being researched. In order to do this, we must redefine expressions provided by Gurevich et al. (2004) in terms of relative changes that are characteristic of poroelastic media. By following Russell et al. (2011) in which he shows a linearized poroelastic AVO expression that is in terms of fluid, shear modulus, and density. Modifying Gurevich's expression to fit these model parameters would be the first step. The next step would be to reformulate those poroelastic modelling parameters into some kind of parameter that can measure contrasting properties of a two layered medium. We can do this in terms of reflectivity as mentioned above but we would also like to present an expression in terms of perturbations. The full derivation of a linearized expression will not be shown here but instead, a framework of Gurevich's expressions will be put in place such that future analysis can be performed where linearized forms can be derived. The reason in which we would like to achieve linearized forms is because a cost-effective least-squares inversion method can be used to quantify poroelastic properties of the media at hand. As it stands currently, these reflection and transmission coefficients are non-linear and thus factors such as matrix stability in regards to a least-squares inversion plays an important role. It may be important to note that as linearized forms will be derived, an interchangeability from perturbation to reflectivity is possible and vice versa.

NORMAL INCIDENT R/T COEFFICIENTS

Porous-porous medium

One set of expressions derived by Gurevich et al. (2004) are used to calculate reflection and transmission coefficients of an incident P-wave in fluid-saturated media for a two-layer case. Unlike elastic behaviour, Gurevich et al. (2004) shows that there are two kinds of P-waves that result after an incident plane wave, $e^{i\omega t}$, disturbs a solid that bears fluid; a fast P-wave which can be defined as $v = \sqrt{(K + 4/3\mu)/\rho}$ and the Biot slow wave. These two types of compressional waves form the expressions for the corresponding reflection and transmission coefficients illustrated below.

The nomenclature that will be used to re-express these frequency dependent formulas are combinations of the variable naming used by Russell et al. (2011) and a naming convention that will use the subscript "0" describing a property of the upper medium and "1" for the lower medium. Figure (1) represents the two-layer case for porous media which

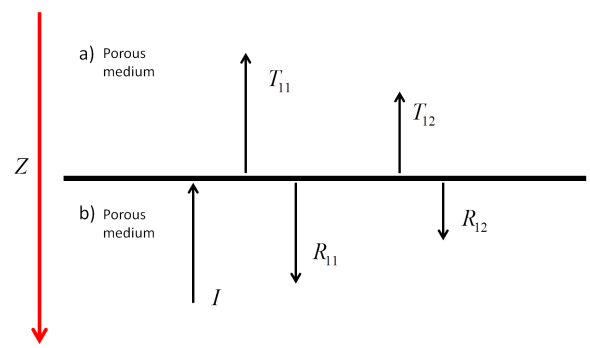

FIG. 1: A representation of an incoming P-wave in layer b. The reflection and transmission coefficients are for P-waves only and the subscripts indicate a fast P-wave (1) and Biot's slow wave (2).

also illustrates the types of reflections and transmissions that occur. For a fast P-wave, a corresponding reflection and transmission coefficient is produced which are labelled R_{11} and T_{11} respectively and the Biot slow P-wave also produces a reflected and transmitted P-wave labelled R_{12} and T_{12}. For the fast P-wave, the expressions for the reflection and transmission coefficient is shown as

$$R_{11}(\omega) = \frac{\rho_1 v_1 - (1 - X)\rho_0 v_0}{\rho_1 v_1 + (1 + X)\rho_0 v_0}, \tag{1}$$

and

$$T_{11}(\omega) = \frac{2\rho_1 v_1}{\rho_1 v_1 + (1 + X)\rho_0 v_0}. \tag{2}$$

At first glance, by setting $X = 0$, R_{11} and T_{11} are reverted back to their elastic forms. This implies that the X term contains all of the implicit physical poroelastic properties of the media, namely f and μ. This X term will determine poroelastic variations in fluid and shear modulus while the density terms that are outside of X will show the density variations. As shown by Gurevich et al. (2004), X is defined as

$$X(\omega) = \frac{H_1 (k_1)_{\text{fast}} \left(\frac{C_0}{H_0} - \frac{C_1}{H_1}\right)^2}{(k_0)_{\text{slow}} N_0 + (k_1)_{\text{slow}} N_1}, \tag{3}$$

This X term contains moduli and expressions that overlap with the work from Russell et al. (2011). This overlap will allow us to manipulate equation (3) to a form that is ready for linearization in a_f and a_μ. Other terms that do not overlap will be kept in their original notation as provided by Gurevich et al. (2004). Overlapping terms include H, C, and N where

$$H = K_{\text{dry}} + \frac{4}{3}\mu + f, \tag{4}$$

$$C = \alpha M, \tag{5}$$

and

$$N = \frac{C}{\alpha} - \frac{C^2}{H}. \tag{6}$$

This then leaves $(k)_{\text{slow}}$ and $(k)_{\text{fast}}$ which are expressions for wavenumbers corresponding to Biot's slow wave and fast P-wave respectively. These variables are left unchanged from Gurevich et al. (2004) and are described as

$$k_{\text{slow}} = \sqrt{\frac{i\omega\eta}{\kappa N}}, \tag{7}$$

where η represents steady-state shear viscosity of a pore fluid, κ is the steady-state permeability of a solid skeleton, $\omega = 2\pi f$ which is the angular frequency and

$$k_{\text{fast}} = \frac{\omega}{v}. \tag{8}$$

With equations (4) - (8), we can substitute these into (3) which yields

$$X(\omega) = \frac{\left(K_{1\text{dry}} + \frac{4}{3}\mu_1 + f_1\right)(k_1)_{\text{fast}}\left(\frac{\alpha_0 M_0}{K_{0\text{dry}}+\frac{4}{3}\mu_0+f_0} - \frac{\alpha_1 M_1}{K_{1\text{dry}}+\frac{4}{3}\mu_1+f_1}\right)^2}{\frac{N_0}{\sqrt{N_0}}\sqrt{\frac{i\omega\eta_0}{\kappa_0}} + \frac{N_1}{\sqrt{N_1}}\sqrt{\frac{i\omega\eta_1}{\kappa_1}}}, \tag{9}$$

where $\frac{N}{\sqrt{N}}$ may be written as

$$\frac{N}{\sqrt{N}} = \frac{M\left(1 - \frac{f}{K_{\text{dry}}+\frac{4}{3}\mu+f}\right)}{\sqrt{M}\sqrt{1 - \frac{f}{K_{\text{dry}}+\frac{4}{3}\mu+f}}}. \tag{10}$$

As we can see, equation (9) contains many instances of non-linearity. There are squared terms as well as square root terms in f_1 and μ_1 that can be expanded in series and is the most difficult part to expand in equations (1) and (2) since the density terms are not coupled with as much complexity. By substituting this newly formed equation for X back into R_{11} and T_{11}, expressions that are in terms of fluid, shear modulus, and density are now available for fast P-waves. For slow P-waves, the reflection and transmission coefficients are

$$R_{12}(\omega) = \frac{2\tilde{X}_1\rho_0 v_0}{\rho_1 v_1 + (1+X)\rho_0 v_0}, \tag{11}$$

and

$$T_{12}(\omega) = \frac{2\tilde{X}_0\rho_0 v_0}{\rho_1 v_1 + (1+X)\rho_0 v_0}. \tag{12}$$

These slow P-wave amplitudes differ from their fast P-wave counterparts by an additional $\tilde{X}_0$ and $\tilde{X}_1$. The following equations that represent these frequency dependent moduli are

$$\tilde{X}_0(\omega) = \frac{X}{\frac{H_0}{C_0}\left(\frac{C_0}{H_0} - \frac{C_1}{H_1}\right)}, \tag{13}$$

and

$$\tilde{X}_1(\omega) = \frac{X}{\frac{H_1}{C_1}\left(\frac{C_0}{H_0} - \frac{C_1}{H_1}\right)}. \tag{14}$$

After making the appropriate substitutions into (13) and (14), these become

$$\tilde{X}_0(\omega) = \frac{X}{\frac{K_{0\,\mathrm{dry}}+\frac{4}{3}\mu_0+f_0}{\alpha_0 M_0}\left(\frac{\alpha_0 M_0}{K_{0\,\mathrm{dry}}+\frac{4}{3}\mu_0+f_0} - \frac{\alpha_1 M_1}{K_{1\,\mathrm{dry}}+\frac{4}{3}\mu_1+f_1}\right)}, \tag{15}$$

and

$$\tilde{X}_1(\omega) = \frac{X}{\frac{K_{1\,\mathrm{dry}}+\frac{4}{3}\mu_1+f_1}{\alpha_1 M_1}\left(\frac{\alpha_0 M_0}{K_{0\,\mathrm{dry}}+\frac{4}{3}\mu_0+f_0} - \frac{\alpha_1 M_1}{K_{1\,\mathrm{dry}}+\frac{4}{3}\mu_1+f_1}\right)}, \tag{16}$$

which both equivalently have f_1 and μ_1 dependencies as X does. Finally, expressions for R_{11}, T_{11}, R_{12}, and T_{12} are expressed in terms of f, μ, and ρ.

Free fluid-porous medium

In the other set of expressions provided by Gurevich et al. (2004), the geologic case involves a porous layer overlying a free fluid. As we have seen for the previous case, similar observations are made again. For instance, the equation for R_{11} is defined by impedances of layers "0" and "1" where a $(1-Y)$ is embedded in the numerator and the denominator to allow for fluid compensated effects, when $Y=0$ the equations for R_{11} and T_{11} have been reduced to their elastic forms, and the fluid and shear modulus constants are contained within Y. Figure (2) represents this case where the incident P-wave pulse begins its propagation in layer b) and travels in the $-z$ direction. The only difference between this figure and the previous figure is the lack of R_{12}. One of the reasons for this difference is due to a lack of a R_{12} amplitude in the displacement vectors as shown by Gurevich et al. (2004). The expressions for the reflection and transmission coefficients for a fast P-wave are similarly constructed as equations (1) and (2) with the key difference lying in the modulus that contains all of the physical poroelastic information. This was expressed as X earlier and is expressed as Y here and the reflection and transmission coefficients are

$$R_{11}(\omega) = \frac{\rho_1 v_1 - (1-Y)\rho_0 v_0}{\rho_1 v_1 + (1+Y)\rho_0 v_0}, \tag{17}$$

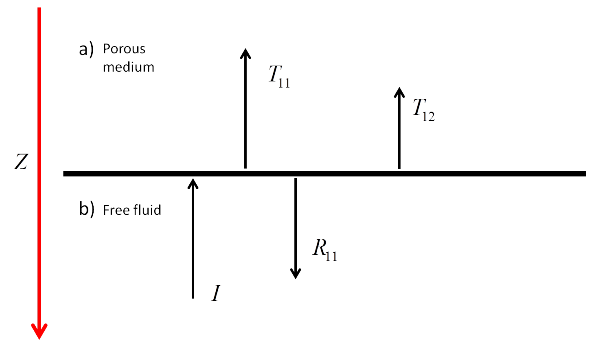

FIG. 2: A second representation of an incoming P-wave in layer b which is instead a free fluid. The reflection and transmission coefficients are for P-waves only and the subscripts indicate a fast P-wave (1) and Biot's slow wave (2).

and

$$T_{11}(\omega) = \frac{2\rho_1 v_1}{\rho_1 v_1 + (1+Y)\rho_0 v_0}, \tag{18}$$

where Y is described as

$$Y(\omega) = \left(\frac{C_0}{H_0} - 1\right)^2 \sqrt{-\frac{i\omega\kappa_0}{\eta_0 N_0}}\rho_1 v_1. \tag{19}$$

By careful inspection, one may notice that equation (19) is a reduced form of (3). This occurs based off of two observations. The first is based off of the relation $H_1 = C_1 = K_1$ which states that the bulk modulus (K) of the free fluid is equal to two other poroelastic constants (C and H). The second observation is to assume infinite permeability (κ) of the free fluid. In addition to these assumptions, we may explicitly define variables C, H, and N to obtain

$$Y(\omega) = \left(\frac{\alpha_0 M_0}{K_{0\text{dry}} + \frac{4}{3}\mu_0 + f_0} - 1\right)^2 \left(1 - \frac{f_0}{K_{0\text{dry}} + \frac{4}{3}\mu_0 + f_0}\right)^{-1/2} \sqrt{-\frac{i\omega\kappa_0}{\eta_0 M_0}}\rho_1 v_1. \tag{20}$$

Equation (9) contains many instances of non-linearity in f_1 and μ_1 but this is not the case for Y. In fact there are no dependencies on either f_1 or μ_1 in equation (20) but only one ρ_1 dependence. Thus it can be stated that for incidence P-waves travelling in a free fluid and interacts with a boundary of a porous medium, R_{11} and T_{11} only contain a density perturbation. Replacing the Y term in equations (17) and (18) allow us to obtain new expressions that are in terms of f, μ, and ρ. For the Biot slow wave, the transmission coefficient is

$$T_{12}(\omega) = \frac{2\tilde{Y}\rho_0 v_0}{\rho_1 v_1 + (1+Y)\rho_0 v_0}, \tag{21}$$

where the slow P-wave induces a modified poroelastic variable, $\tilde{Y}$, which is also a reduced expression, simplified from equation (13)

$$\tilde{Y}(\omega) = \frac{Y}{1 - \frac{H_0}{C_0}} = \frac{C_0}{H_0}\left(\frac{C_0}{H_0} - 1\right)\sqrt{-\frac{i\omega\kappa_0}{\eta_0 N_0}}\rho_1 v_1. \tag{22}$$

Since it appears that $\tilde{Y}$ also does not contain any dependence on a fluid or shear modulus perturbation, T_{12} is also only dependent on a density perturbation. Again, by explicitly replacing C, H, and N into forms that are in terms of f, and μ, $\tilde{Y}$ becomes

$$\begin{aligned}\tilde{Y}(\omega) = \frac{\alpha_0 M_0}{K_{0\text{dry}} + \frac{4}{3}\mu_0 + f_0}\left(\frac{\alpha_0 M_0}{K_{0\text{dry}} + \frac{4}{3}\mu_0 + f_0} - 1\right) \\ \times\left(1 - \frac{f_0}{K_{0\text{dry}} + \frac{4}{3}\mu_0 + f_0}\right)^{-1/2}\sqrt{-\frac{i\omega\kappa_0}{\eta_0 M_0}}\rho_1 v_1.\end{aligned} \tag{23}$$

DISCUSSION

These expressions now provide an alternate description of what is explained by Gurevich et al. (2004) which is a means to show how relative fluid motion within a skeleton framework behaves for incident P-waves. Observing reflection and transmission amplitudes for different frequencies display differing amplitudes. These different amplitudes can either increase or decrease in variation relative to a reference frequency based on poroelastic contrasts in C/H (Gurevich et al., 2004). This work on frequency dependent poroelastic reflections will be used in conjunction with other work that describes expressions for poroelastic reflections in an angle dependent manner.

CONCLUSION

Expressions for normal incidence, frequency dependent reflection and transmission coefficients have been provided to ultimately allow expansions in a_f, a_μ, and a_ρ. These series expansions will allow a linearized expression that will tailor to a least-squares inversion which is a computationally fast approach in predicting these unknown parameters. This is the next step in analysis of these poroelastic parameters of study. It appears that a direct inversion of a_f is useful for gas sand targets located in the subsurface (Russell et al., 2011). We would like to further explore how this parameter may change over frequency. Although we may only understand its character over a $0°$ angle reflection, the normal incidence reflection typically provides the most clarity of displacement with respect to its oblique counterparts.

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REFERENCES

Aki, K., and Richards, P. G., 2002, Quantitative seismology, 2nd ed.

Gurevich, B., Ciz, R., and Denneman, A. I. M., 2004, Simple expressions for normal incidence reflection coefficients from an interface between fluid-saturated porous materials: Geophysics, **69**.

Russell, B. H., Gray, D., and Hampson, D. P., 2011, Linearized AVO and poroelasticity: Geophysics, **76**, C19–C29.

Shuey, R., 1985, A simplification of the Zoeppritz equations: Geophysics, **50**, 609–614.

Smith, G., and Gidlow, P., 1987, Weighted stacking for rock property estimation and detection of gas: Geophysical Prospecting, **35**, 993–1014.