Finite-difference models with an internal water-bottom boundary condition

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ABSTRACT

The rationale for generating comparable physical and computer models is given. The requirement for a finite-difference model with one of its internal horizons conforming to a physical boundary condition is explained. The boundary condition is that of a water-bottom; where the acoustic waves in the water interact with the elastic waves in the solid material below. The condition is developed for use within the staggered-grid representation. Examples are given for a model that matches a physical model in water, and for some simple offshore seismic type models.

INTRODUCTION

Acquisition of data on models is often used to simulate data acquired on the real earth. This is done because a model can be designed to illustrate interesting conditions which then can be searched for in the real data. There are two types of model commonly used. The first is a miniature simulation of the earth, excited and recorded in miniature, and is called a physical model. The second is a more abstract method, where a computer is used to store the values of key physical rock parameters arranged in arrays assigned to evenly spaced positions in the earth. This is then stimulated externally and propagated, and the results stored at specified recording positions. This is computer modeling. The two methods have different strengths and weaknesses, but both types are done at the University of Calgary.

The authors of this paper decided that it would be valuable to enable computer models that could be closely compared to physical models. Where the two model results agreed, there would be more confidence in their accuracy. Where they did not agree, it might emphasize the strength of one model type and possibly indicate where improvements could be made with either type.

Physical models are usually elastic models, but they are often made with a non-elastic layer of water over the rigid zones of interest. This is done to reduce the problems associated with coupling and moving the sonic stimulators and sensors. A major barrier to close comparison of the two model types was then the difficulty of running a computer elastic model under a non-elastic layer.

There are many computer model systems that simulate an elastic earth, and there are many that simulate an acoustic (non-elastic) earth. These acoustic models are usually used as a lower cost, and in most cases a sufficiently accurate substitute, for pressure waves propagated in an elastic model.

For both elastic and acoustic computer models, methods for representation of physical boundaries have been developed. These are known as boundary conditions, and have been used for analytic (continuous) as well as computer models. For computer models they usually apply at the straight line model edges, although some attempts have been made to specify free surface conditions at a topographic surface (e.g. Manning, 2008). These boundaries are specified to affect conditions only inside the model, implications for wave front effects beyond the model being, of course, irrelevant.

There are two unique criteria of an internal water-bottom boundary condition: there must be essentially a free surface for shear displacements but continuity for pressure displacements; and both sides of the boundary are essential parts of the model. Also, it would be desirable to allow structure of the water-bottom, so that deep-sea seismic could be simulated.

PHYSICAL MODELS AT THE UNIVERSITY OF CALGARY

The best introduction to physical modelling at the University of Calgary is found in Wong et al. (2009). One of the more significant recent modeling reports was Mahmoudian, Margrave and Wong (2012).

THE COMPUTER MODEL AND MODIFICATIONS

The authors have written several papers covering staggered-grid finite-difference two-dimensional models with elastic parameters, the latest being Wong and Manning (2012). The basic technique was developed by Virieux (1986), and was implemented by one of us in the course of writing his thesis (Manning, 2007).

The first modification was to change the internal model loop structure so that the first loops were through X values, and the inner loops were for Z values. This was done with the assumption that most model layers would have single Z values for a given X, whereas a single Z value might well intersect a layer at many X values. The water bottom could then be specified as an array of depths at each X value. These depths were obtained from the geological definition file as the top of the first layer with $\mu > 0$.

The main modification was in the top of the elastic zone, just below the water-bottom. To begin with, all acceleration calculations for the pressure waves were completed throughout the model, as if there was no boundary. Grid cells spanning the boundary then had to be made free of shear stresses. The standard technique for doing this is to simulate this state with an elastic model which has no stress within the top cells, but in this case allowing compressional stress but no shear stress.

The series of X displacements directly above the boundary were temporarily replaced with new values which would make the cross-boundary cells free of shear stress. These displacements were then used to recalculate the uppermost elastic pressure accelerations, and were included when calculating the shear accelerations from the boundary to the bottom of the model.

First tests found that in many cases, instabilities developed along the water-bottom border. A smoothing filter applied to the X displacements directly under the border often was sufficient to cure this. The filter was 3 by 3 symmetric, with approximately 0.5 in the centre, and with the eight surrounding coefficients equal and scaled to make the sum of the coefficients equal to 1.0.

RESULTS

Figure 1 shows a comparison of a physical model and a computer model with a compatible set of parameters, although the physical model display has a 200 ms AGC. The water depth here is a pseudo 635 metres, and the elastic layer below is at 1143 metres and has a velocity of 2750 m/sec (pressure) and 1480 m/sec (shear). The third layer has slightly differing parameters between the physical and computer cases, but the combinations of depth and velocity were chosen to simulate the same zero offset time. (A lower computer model velocity helped to reduce dispersion). The last reflection in the computer model is from the model bottom, and is less deep than the physical model.

The types of events in Figure 1 are marked on the computer model with initials. P marks primary pressure waves. M marks a water bottom multiple. C marks converted waves that were converted back to pressure waves as they crossed the water bottom from below. H marks a head wave that peels off the primary water bottom reflection. This event shows a strong difference between the two models.

There is obviously a large amount of numerical dispersion in the computer model, although by coincidence it seems to be roughly matched by reverberations in the physical model. Corrections for the acoustic portion of the computer model would give a more satisfactory result.

Figure 2 shows a snapshot of the wave fronts as they interact with the water bottom of the above model. The transmitted pressure wave has advanced well beyond the point where it formed and now feeds energy back into a head wave with diminishing amplitude. The transmitted shear wave, on the other hand, has stayed in step with the incident wave and is reinforcing the reflection amplitude. The two waves are in step because the water velocity (1485 m/sec) is very close to the shear wave velocity in the water bottom (1480 m/sec). The reflection amplitude has been enhanced because it has essentially been reinforced by a head wave from a shear event.

Figure 3 shows a snapshot within a model which has an internal boundary with structure, in this case a constant slope of 50%. An explosion in the upper water zone creates only pressure waves in the acoustic medium. When these waves reach the internal boundary, their energy is transferred to both shear and pressure waves in the elastic medium.

Figure 4 shows a snapshot within a model which has an internal boundary with the same slope, but in the opposite direction. The same type of behaviour is shown.

Figure 5 is similar to Figure 4, but the internal boundary has a lesser slope of 37%. This slope is starting to show an instability which will eventually overwhelm the plot. This is a deficiency in the present software which must be addressed for production software.

CONCLUSIONS

The ability to make direct comparisons between physical models and computer models may make valuable contributions toward quality control and modelling enhancements.

Computer simulation of the water bottom boundary condition is worthwhile, but needs more effort to be reliable.

The large velocity contrast between water and elastic pressure waves will require more work on correction algorithms to reach an acceptable level of dispersion

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FIGURES

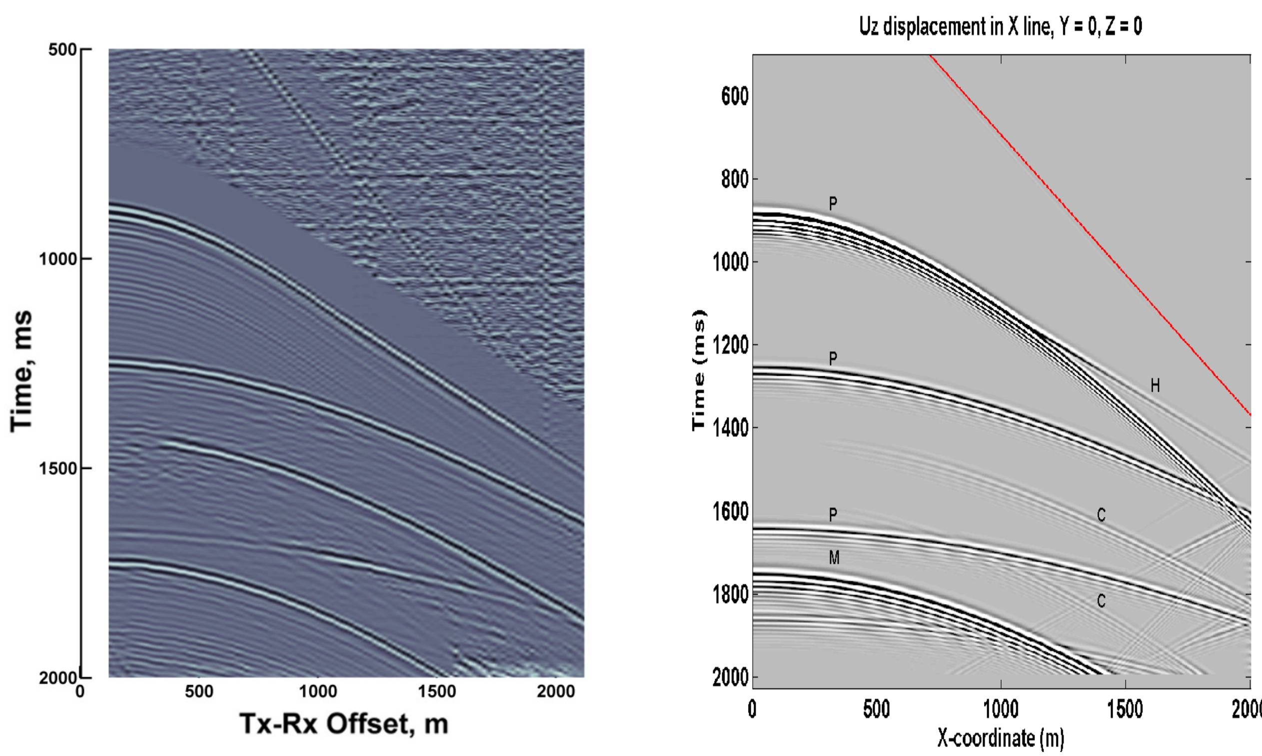

Figure 1: At left is a physical model with 200 ms AGC. At right is a numerical model with parameters of the layers approximately the same as those from the physical model. Primaries are marked P, a multiple marked M, a head-wave marked H, and converted wave events, reconverted, are marked C. The most obvious difference is the complete conversion of the first primary reflection into a head-wave in the physical model.

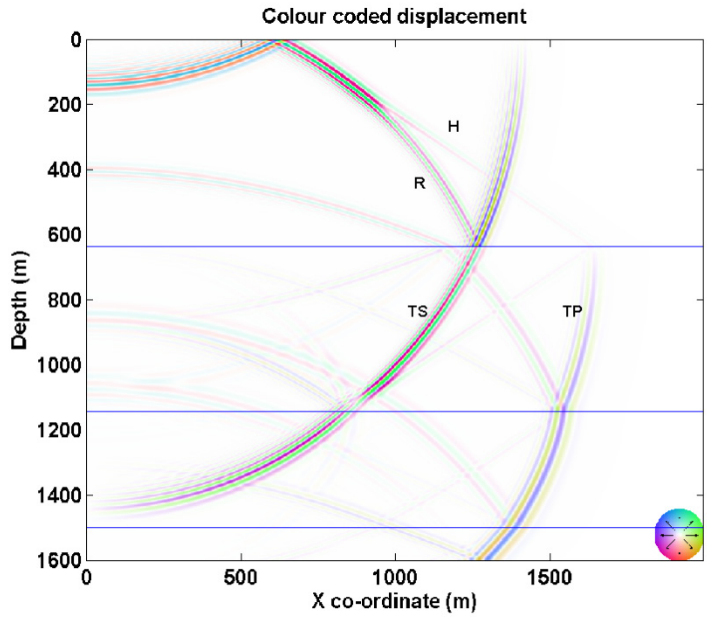

Figure 2: A snapshot of the computer model showing the wave fronts that will appear at the surface in Figure 1. The water bottom is at 635 m. R is the reflected wave, H is the head wave, TS is the transmitted shear, and TP is the transmitted pressure wave. The head wave has lower amplitude than the reflected wave.

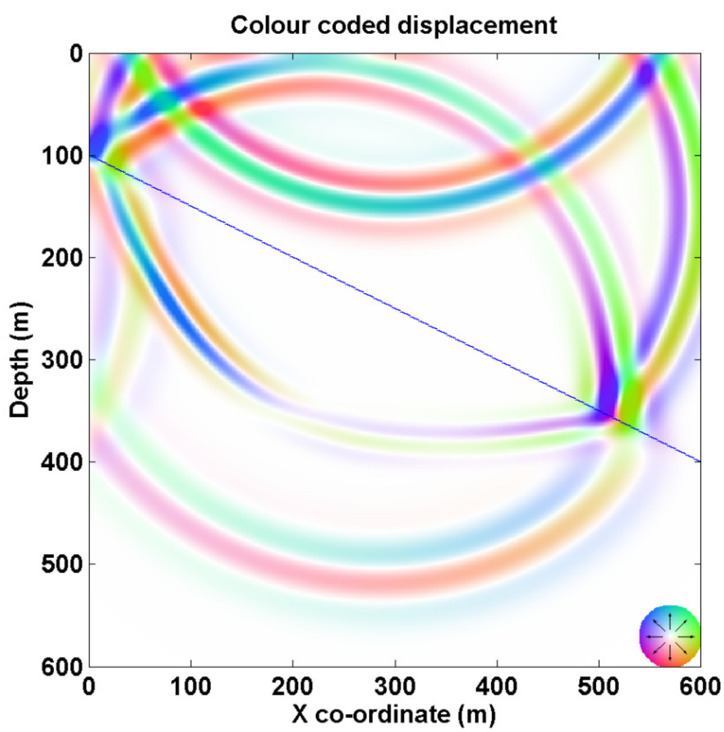

Figure 3: A snapshot of wave fronts from an explosive source in the water zone above the boundary which slopes down to the right. Note that above the boundary the wave fronts are simple arcs which were reflected from the surface and the boundary itself. Below is the elastic zone, where the waves are split into shear (slower) and pressure (faster).

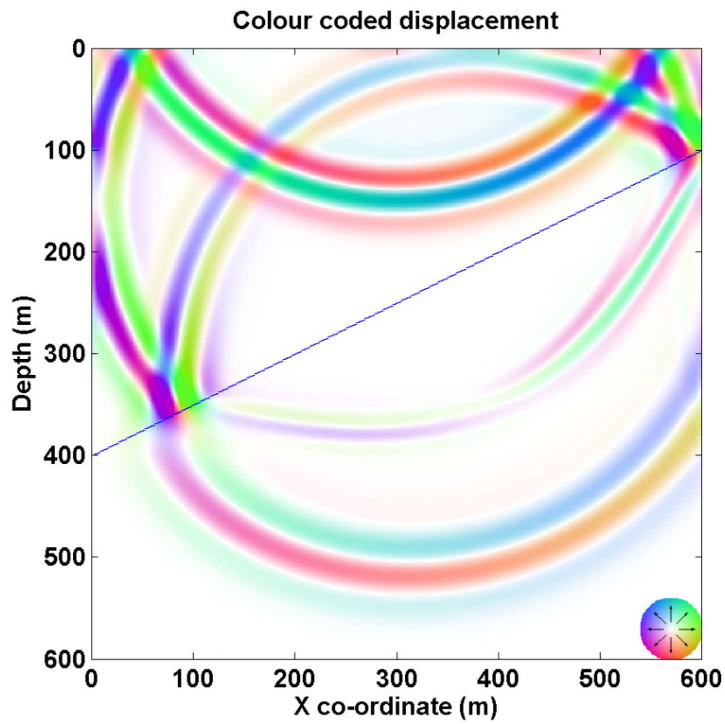

Figure 4: A wave fronts snapshot as in Figure 3, but with the boundary slope down to the left. The slope here is 50%

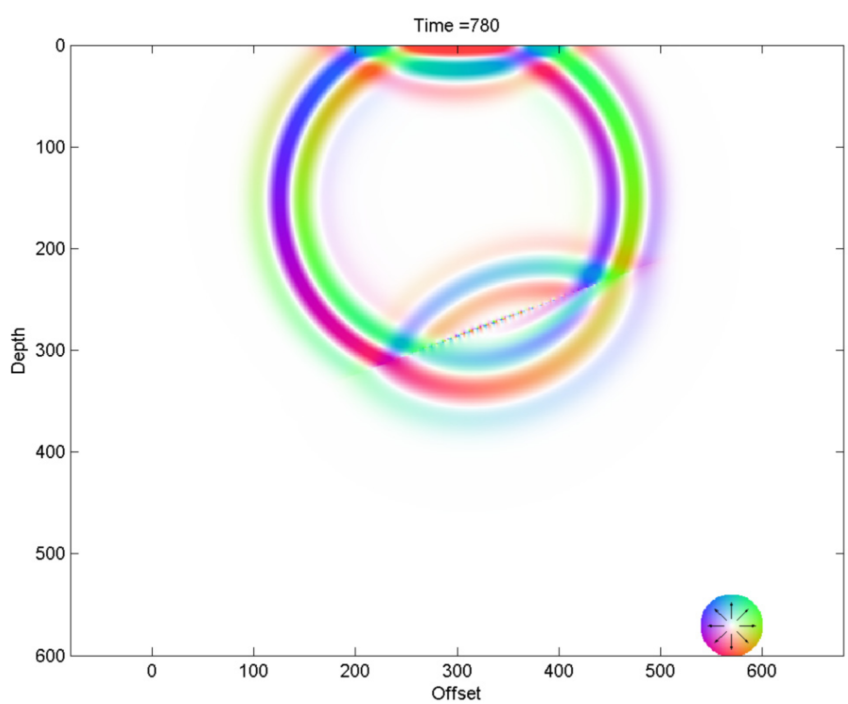

Figure 5: A wave front snapshot as in Figure 4, but with a slope of 37%. With this slope instability begins to appear which will gradually dominate the plot.