Exploring potential applications of Gaussian Ball Filters in Sharpe's Hollow Cavity Model

Christopher C. Petten*, Gary F. Margrave†

ABSTRACT

This report investigates the viability of using Gaussian Ball Filters as described by Aldridge et al. to account for variance in cavity radii with charge size in the Shapre Hollow Cavity Model. The dominant frequency of these filters were observed to decrease with increasing width of the Gaussian Ball; if it is assumed that this width corresponds to the charge size the results are similar to that of Sharpe's predictions. A cubic relationship between cavity radius and dominant frequency of the resulting spectra was estimated by Petten et al. in Sharpe's model. The Gaussian Ball model however appeared to display a square relationship between the dominant frequency and the width of the Gaussian filter. Several different wavelets were used in conjunction with the Gaussian Ball model to observe the effects of different wavelets. The results of this test showed that both minimum phase and Ricker wavelets produced frequency spectra that were similar in form to those produced by Sharpe's model as well as the Priddis and Hussary data examined by Petten et al. in 2012.

INTRODUCTION

In a previous study the viability of the Sharpe Hollow Cavity Model (SHCM) in modeling dynamite explosions was examined using test charge data obtained in Hussar and Priddis during the CREWES field experiments (Petten, 2012). The outcome of this study showed that the SHCM could make reasonably accurate predictions about key features of the frequency spectra obtained from real data. These features include the low-frequency rolloff present in dynamite spectra, the decrease in dominant frequency with larger charge sizes, and an increased amplitude response with larger charges. The SHCM assumes that the region of non-linear propagation of elastic waves emitted by a dynamite explosion can be contained within a theoretical hollow cavity of radius a. The radius of the cavity depends on the charge size, where larger charges are associated with larger cavity radii.

The relationship between cavity radius and charge size is still a poorly understood phenomenon because the cavity radius is something that emerges from the mathematical solution of the model. The key assumption in the SHCM is that waves behave linearly beyond the cavity and that elastic waves are produced by an arbitrary pressure pulse acting over the inside of the cavity as shown in Figure 1. In order to utilize the SHCM to accurately model dynamite explosions a link between charge size and cavity radius has to be established and justified mathematically. In the 2012 CREWES report published by Petten et al. a method for estimating a proportionality constant between charge size and cavity radius has been outlined; it did not however provide any sort of mathematical justification for the variance in charge size with cavity radius.

*University of Calgary
†University of Calgary

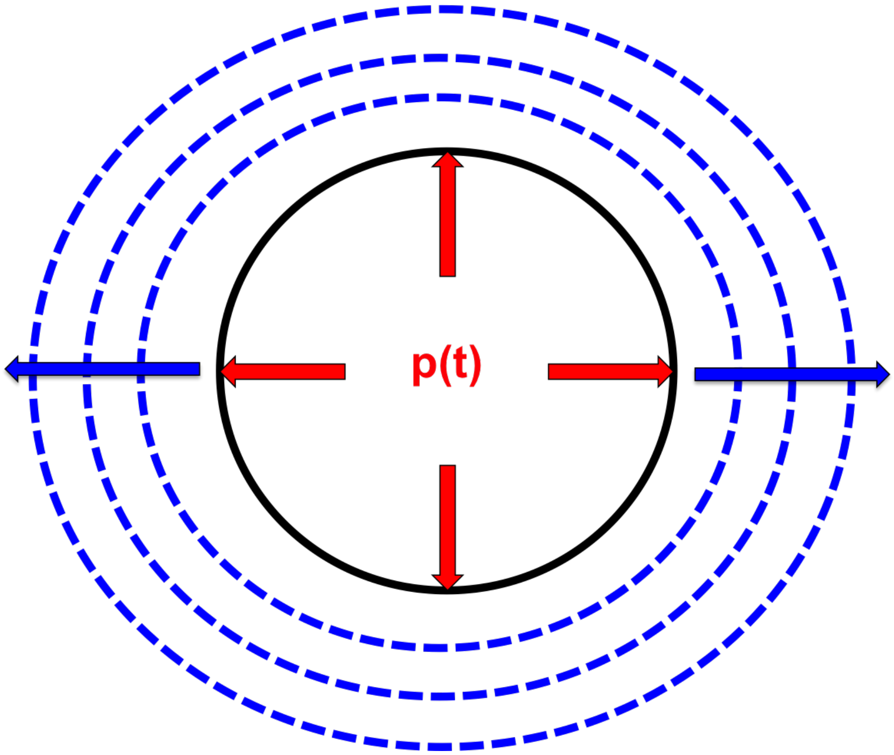

FIG. 1. Graphical depiction of the SHCM. The cavity (shown in black) is acted on by a pressure pulse $p(t)$ that is uniformly distributed over the interior of the cavity. This results in emission of spherical waves (shown in blue) which propagate outward from the surface of the cavity.

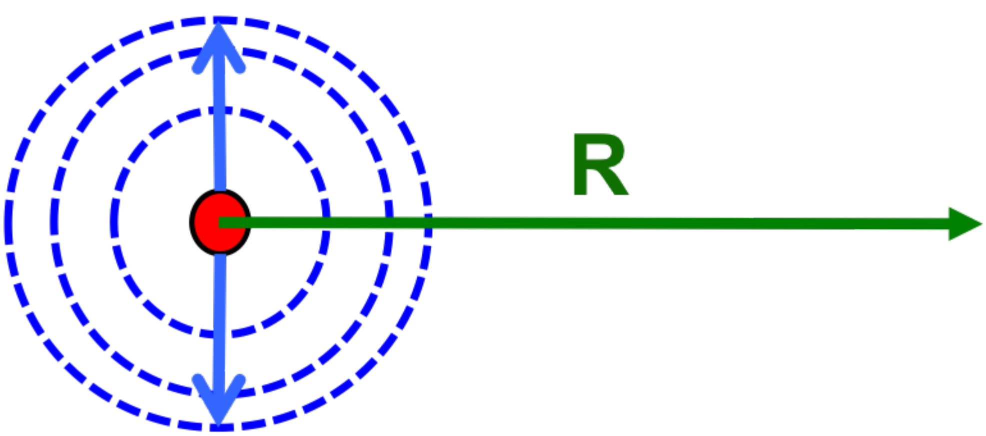

FIG. 2. Graphical depiction of the Gaussian Ball model. The point source (shown in red) acts as a finite point in space that instantaneously releases energy into the subsurface. The result is a series of body waves being emitted from the source that propagate outward in a radial pattern (shown in blue).

Aldridge provides a starting point for solving this problem using a series of Gaussian Ball Filters to account for the variance in charge size (Aldridge, 2011). If it is assumed that the width of the Gaussian Ball corresponds to the size of the charge then there may be a mathematical basis for varying cavity radii in the SHCM. Mathematically advancing this model could lead to increased accuracy in predictions made by the SHCM, which could greatly aid in the design of seismic surveys conducted with explosive pressure sources.

THEORY

The SHCM and the Gaussian Ball model make two fundamentally different assumptions about the nature of the source. Sharpe's model (shown in Figure 1) assumes that an explosive source is characterized by a time-varying boundary condition that acts uniformly over the interior surface of a cavity (Sharpe, 1942). Operating under this assumption the energy of an explosion is released into the subsurface over a finite period of time; this is usually in the form of some sort of energy decay over a very small time period, with larger explosions having more rapid energy release. Meanwhile, the Gaussian Ball model (shown in Figure 2) operates under the assumption that the energy is instantaneously released into the subsurface and acts as more an impulse pressure (Alrdridge, 2011). This model does not account for the region of space in close proximity to the source where the emitted waves do not behave linearly, and therefore assumes that waves propagate in a linear fashion throughout the entire medium. Thus, this model can only be applied to far-field approximations.

Aldridge has explained the Gaussian Ball model in great depth in his 2011 technical report so only a brief overview of the model will be covered in this particular report (Aldridge, 2011). The governing partial differential equations used to derive this model are as follows:

$$\frac{\partial v_i}{\partial t} - \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j} = \frac{1}{\rho}\left(f_i + \frac{\partial m_{ij}}{\partial x_j}\right) \tag{1}$$

and

$$\frac{\partial \sigma_{ij}}{\partial t} - \lambda\frac{\partial v_k}{\partial x_k}\delta_{ij} - \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) = \frac{\partial m_{ij}}{\partial t} \tag{2}$$

where: δ_{ij} is the Kronecker delta, ρ is the density of the medium, λ and μ are the Lamé parameters of the medium, f_i is the component of the force vector, and m_{ij} is the moment density tensor. The derivation of the Gaussian Ball model starts with the assumption that the explosive source can be treated as a point source in space, which can be approximated as a delta function at the point of origin. Under this assumption, the moment density tensor can be expressed as

$$m_{ij}(\mathbf{x}, t) = -M\left(\frac{\delta_{ij}}{\sqrt{3}}\right) w(t)\frac{1}{h^3}\exp\left(-\pi\frac{\|\mathbf{x} - \mathbf{x}_s\|^2}{h^2}\right) \tag{3}$$

where M is a magnitude scalar, $w(t)$ is a source activation wavelet, and h is the width of the Gaussian Ball. Note that the square-root term in this equation is a normalization term that implies that this is a three-dimensional explosion. By carrying out an integration over a volume integral an expression for the pressure from a point source is obtained, which can be represented as:

$$p(r, t) = M\left(\frac{1 - \frac{4}{3}\gamma^2}{4\pi\alpha^2 r}\right) w(t) \tag{4}$$

where r is the distance from the source to the receiver, α is the p-wave velocity, and γ is the ratio of p- to s-wave speed.

Equation 4 represents the pressure pulse response that results from a point source in space. After the pressure pulse has been obtained using Equation 4, a Gaussian Ball filter is applied to it in order to complete the model. The result is a time-domain convolution:

$$p(r,t)_{Gaussian} = p(r,t)_{Point} * G(r,t) \tag{5}$$

where $G(r,t)$ is the time-domain Gaussian Ball filter. The width of the Gaussian Ball, h, as well as the magnitude scalar, M, corresponds to the amount of energy that is released in the explosion. The expression for the Gaussian Ball filter in the frequency domain provided by Aldridge is:

$$G(w/w_c) = exp[-(w/w_c)^2] \tag{6}$$

where w_c is a characteristic frequency parameter given by

$$w_c = \frac{2\sqrt{\pi}\alpha}{h}. \tag{7}$$

Examination of Equations 4 through 5 shows that the general expression for the frequency spectra in the Gaussian Ball model can be expressed as

$$p_G(w) = p_p(w)G(w)M \tag{8}$$

where $p_G(w)$ is the frequency spectra for the pressure pulse obtained after application of a Gaussian Ball filter to an arbitrary pressure trace, $p_p(w)$ is the frequency spectra for the arbitrary pressure trace, and M is the magnitude scalar. Note that the choice of wavelet in this case is completely arbitrary so it is possible to subscribe a variety of different energy pulses to this model by modifying the input wavelet. Figure 3 shows an initial pressure pulse that has been modified by a Gaussian Ball using Equation 8. Figure 4 shows the frequency spectra of the same pulse. Observation of this pulse in the both the time and frequency domain shows that the Gaussian Ball modifies both the the amplitude and the dominant frequency of the pulse.

Figure 5 shows a series of Gaussian Balls with constant magnitudes and varying widths, in both the time and the frequency domain. Referring to Equations 6 and 7, the width of the Gaussian is inversely proportional to the w_c term, which is itself inversely proportional to the width of the Gaussian Ball, h. Therefore, the width of the Gaussian Ball filter is directly proportional to h, which corresponds to the charge size. Larger charges have higher h values and are therefore associated with wider Gaussian Ball filters. As the width of the Gaussian Ball diminishes in the time domain for smaller charges, it broadens in the frequency domain. This could provide a crucial link between charge size and cavity radius in the SHCM as this provides mathematical justification for cavity radius increasing with larger charge sizes. Also note that larger charges in the Gaussian Ball model are associated with higher magnitude values, represented as M in Equation 8. Therefore, using Equation 8 as a reference, larger charge sizes should produce a lower dominant frequency while having a larger amplitude response due to higher magnitude values. Conversely, smaller charges should have lower magnitude values but should produce higher dominant frequencies. These same observations were noted in the SHCM, so the two models could possibly be combined to make more accurate predictions about the nature of the frequency spectra that results from a dynamite explosion.

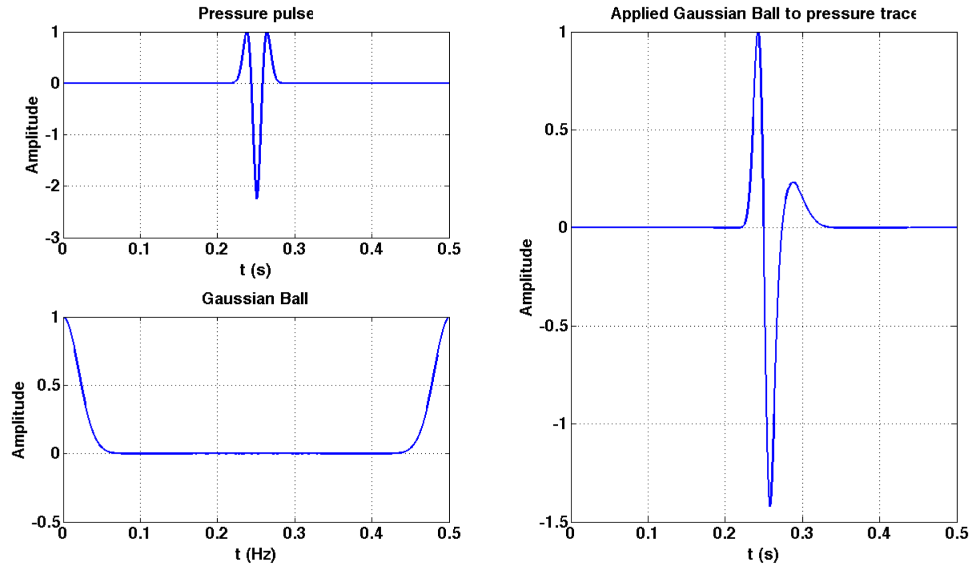

FIG. 3. Application of a Gaussian Ball filter to an arbitrary pressure pulse that was calculated with Equation 4 using a Ricker wavelet as the source activation waveform.

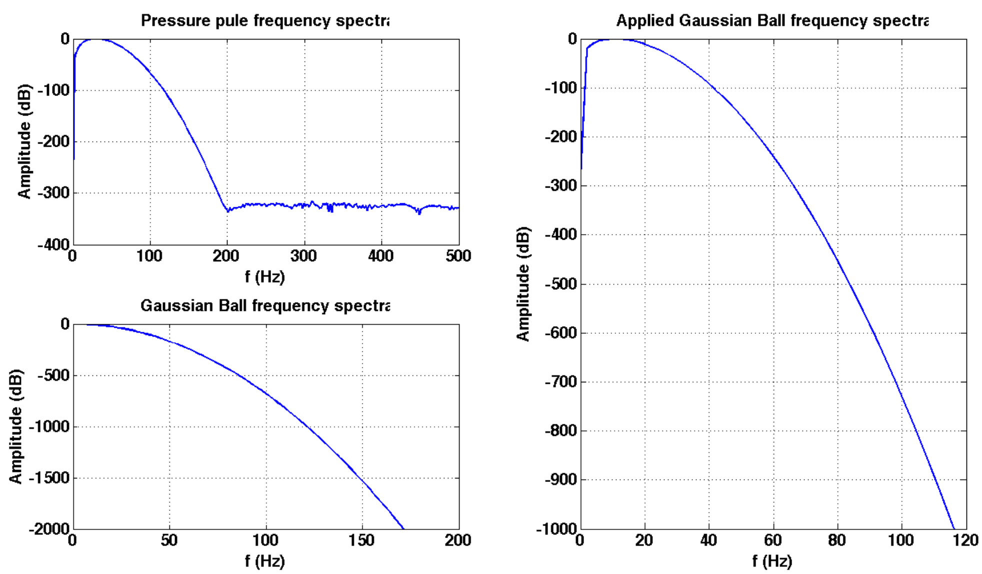

FIG. 4. Frequency spectra for the pulse shown in Figure 3. The dominant frequency of the trace has been changed with application of the Gaussian Ball filter.

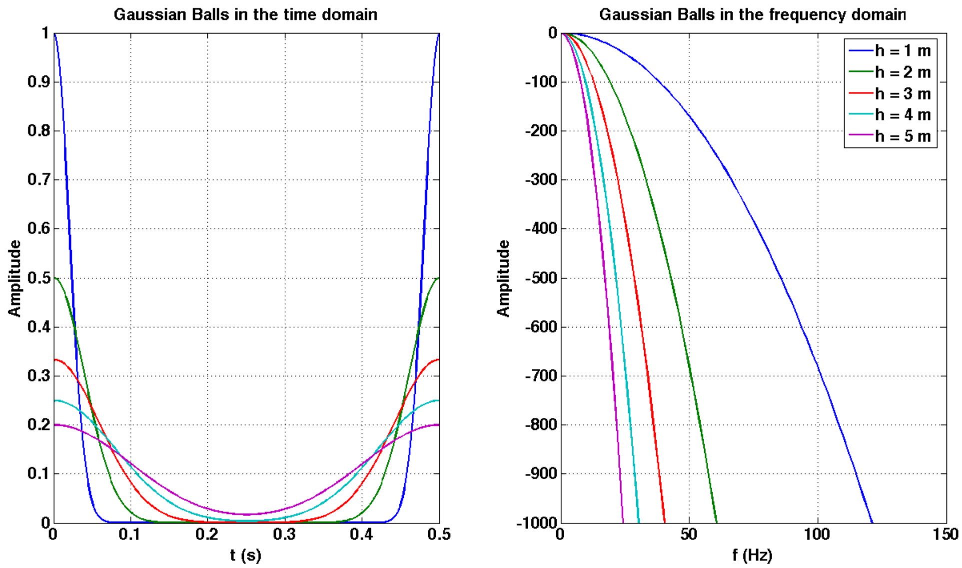

FIG. 5. A series of Gaussian Balls with varying widths, shown in both the frequency and time domain. Note that in this particular case the magnitude has been kept constant for each Gaussian Ball filter.

VARYING SOURCE ACTIVATION WAVEFORMS

As mentioned earlier, when using Equation 8 the choice of source activation wavelet used in the Gaussian Ball model is completely arbitrary. However, it is important that the wavelet represent the energy pattern chosen in the model represents something that would be realistic for an explosive source. There are three common types of wavelets used in modeling dynamite explosions which will be covered in this report: a Ricker wavelet, a minimum phase wavelet, and a decaying exponential wavelet. Each of these wavelets are reasonable representations of the energy radiated from an explosions since they contain a large amount of energy that diminishes over a set time interval.

The choice of magnitude also plays an important role when simulating an explosion using the Gaussian Ball model. In previous studies there was sufficient evidence to suggest that there was a cubic relationship between cavity radius, a, and charge size, m, in the SHCM (Petten, 2012). Therefore, in this study a cubic relationship will be assumed between the Gaussian Ball width, h, and the magnitude, M, such that

$$M = h^3. \tag{9}$$

Note that the magnitude in this model is simply a scale factor, so the choice is completely arbitrary. It is entirely possible to use other magnitudes as desired depending on the type of explosive source that is being used.

Ricker Wavelet

Figure 6 shows a Gaussian Ball model where a Ricker wavelet was used as the source activation waveform. Each Gaussian Ball in this example has been modified using Equation 9, so each filter has a different magnitude and width value. Observation of the convolved

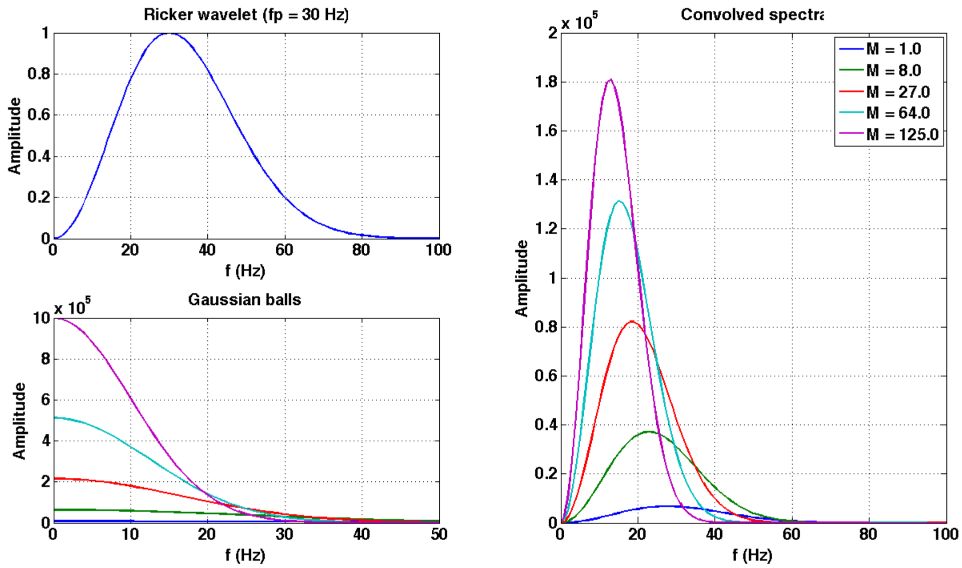

FIG. 6. Gaussian Ball model using a Ricker wavelet as the source activation wavelet. The wavelet in this case has been convolved with with several Gaussian Balls with varying width and magnitude.

spectra shows a distinct decrease in the dominant frequency with larger h and M values. In this model the ball width and the magnitude increase with larger charge sizes, so the results of this test are very similar to what was observed in the SHCM (Petten, 2012).

In order to examine a potential relationship between the observed dominant frequency and the ball width, a series of dominant frequencies were measured for Gaussian Balls of varying width and magnitude. The results of this experiment can be seen in Figure 7. Both a linear and a square data set were fit to the measured data; the square fit seemed to be the most appropriate fit for this particular data set.

Minimum Phase Wavelet

Figure 8 shows a Gaussian Ball model where the source wavelet used was a minimum phase wavelet. The Gaussian Balls used in this experiment were identical to those used in the Ricker wavelet experiment shown in Figure 6. Note that frequency spectra appear to be quite similar and that the same decrease in dominant frequency with larger charge sizes was observed in this case.

Exponentially Decaying Wavelet

Figure 9 shows a Gaussian Ball model where a decaying exponential has been used as the source activation wavelet. Once again, the Gaussian Balls used for this experiment where the same as those used in the Ricker wavelet experiment. In this case an amplitude increase with larger charges was observed however, there did not appear to be a change in the dominant frequency of the spectra. This does not match the results of the experiments carried out with the SHCM in previous studies, despite the fact that this activation source worked best in Sharpe's model (Petten, 2012). This phenomenon could prove to

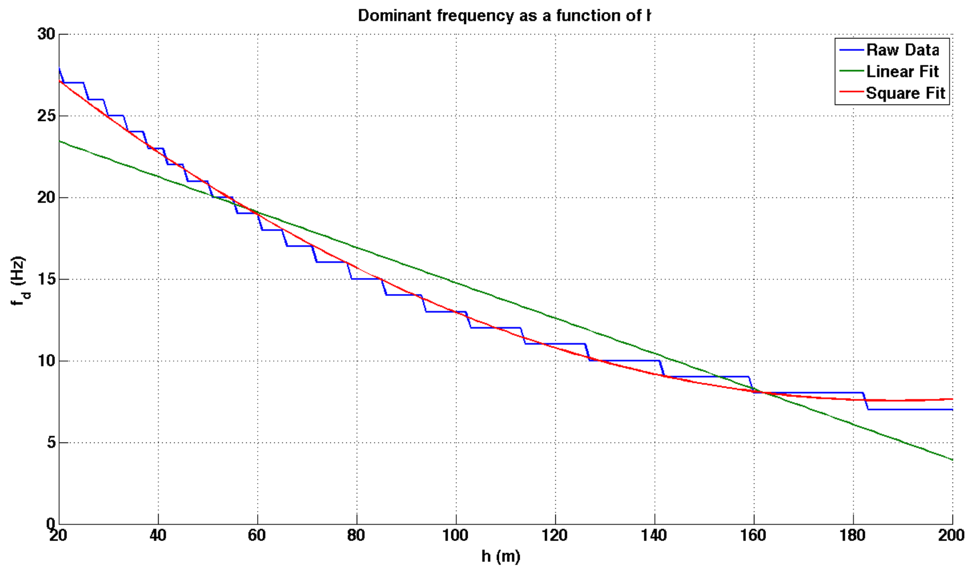

FIG. 7. Measured dominant frequency in the Gaussian Ball model as a function of the Gaussian Ball width. Note that a squared polynomial is the best fit for the data, suggesting that there is a square relationship between dominant frequency in the spectra and the Gaussian Ball width.

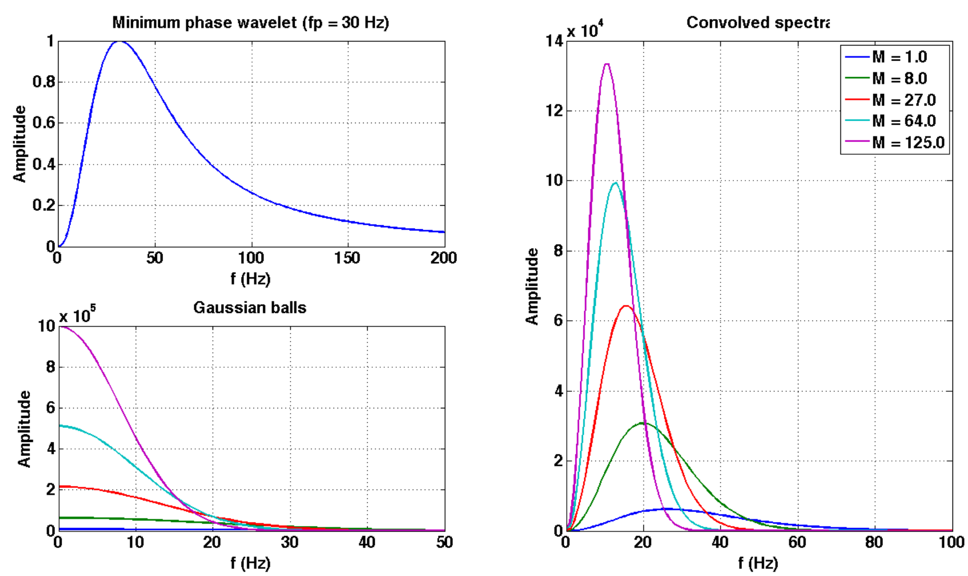

FIG. 8. Gaussian Ball model using a minimum phase wavelet as the source activation wavelet. The wavelet in this case has also been convolved with a series of Gaussian balls of varying magnitude and width. The final spectra appears to be very similar to that of the Ricker wavelet experiment shown in Figure 6.

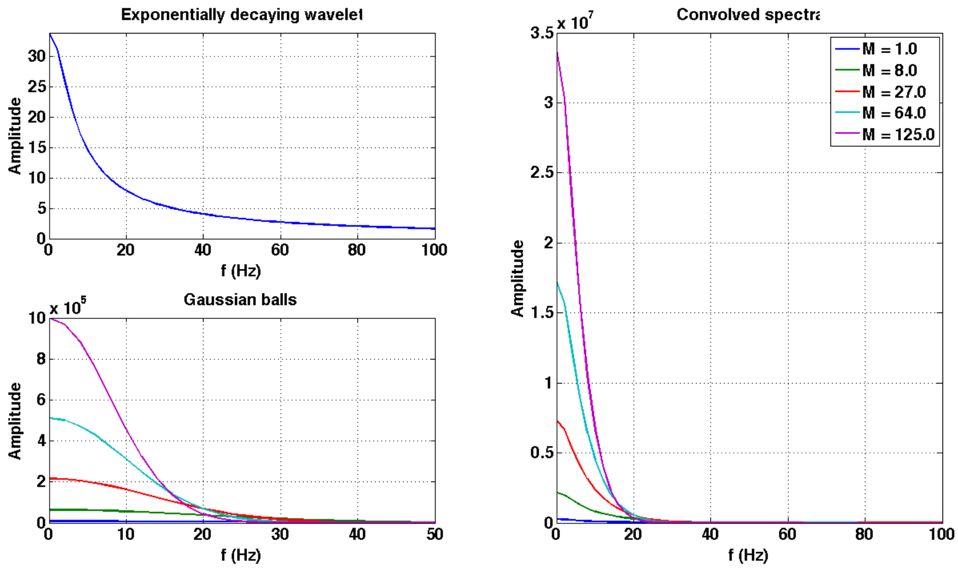

FIG. 9. Gaussian Ball model using a decaying exponential as the source activation wavelet. The Gaussian Ball in this experiment were also the same as those used in the Ricker wavelet experiment shown in Figure 8; however, the frequency spectra in this case appears to be drastically different than that of the Ricker and minimum phase wavelets.

be somewhat problematic when attempting to integrate the two models, however, further investigation into that matter is still required.

DISCUSSION

In a previous study involving the SHCM, it was established that in order to tie theoretical data to real data obtained in the field, several key features of the predicted frequency spectra must be present (Petten, 2012). As mentioned earlier, these features include: a decrease in dominant frequency for larger charges, a low-frequency roll-off in the lower end the spectrum, and a higher amplitude response for larger charges. The results from the Ricker and minimum phase wavelets shown in Figures 6 and 8 display each of these key features. This observation suggests that the Gaussian Ball model can be used in conjunction with the SHCM to improve surveys that use explosive pressure sources, provided that a Ricker or minimum phase wavelet is used as the source activation wavelet. The results of the exponentially decreasing wavelet experiment however, does not appear to have these features present. This suggests that the type of wavelet used in the Gaussian Ball model may be restricted when simulating explosions using this model.

In the case of the Ricker and minimum phase wavelets, the shrinking Gaussian Ball resulted in a higher dominant frequency. Since the width of the Gaussian Ball is tied directly to the charge size via the ball width, the dominant frequency is also related to the width of the filter. The change in dominant frequency can therefore be attributed to the shrinking or expanding of the Gaussian Ball in the time domain. This provides an important mathematical justification for the change in dominant frequency with cavity radius in the Sharpe model, as the width of the Gaussian Ball could potentially be tied to the cavity radius in Sharpe's model as well.

There appears to be a squared relationship between Gaussian Ball width and the dominant frequency, which is based on the results shown in Figure 7. Note however that the choice of magnitude in this study was completely arbitrary, so there currently does not exist a reliable means of linking magnitude to the dominant frequency. Further work needs to be done in order to establish this relationship with any degree of reliability.

CONCLUSIONS

Based on the results of this study, several conclusions can be made regarding the Gaussian Ball model and and its potential integration with the SHCM to improve the modeling of explosive pressure sources. The Gaussian Ball model produces similar results to the SHCM provided that a Ricker or minimum phase wavelet is used as the source activation wavelet. Other wavelets may also produce favorable results, however, they have not been investigated at this point in time. The decaying exponential does not appear to work with this model when trying to reproduce Sharpe's results, which could potentially limit this model when trying to use it in conjunction with the SHCM. The shift in dominant frequency can be attributed to a shrinking or expanding Gaussian Ball width in the in the time domain, which could be a crucial link between charge size and cavity radius in Sharpe's model. Finally, there appears to be a squared relationship between dominant frequency and Gaussian Ball width. Note however that further work on the relationship between magnitude and ball width needs to be further developed before this this conclusion can be confirmed with any degree of reliability.

REFERENCES

Aldridge, D. F., T. M. Smith, S. S. Collis, 2011, A Gaussian explosion seismic energy source: SEG Technical Program Expanded Abstracts, pp. 2997-3001.

Petten, C. P., G. Margrave, 2012, A brief comparison of the frequency spectra from the Hussar 2011 and Priddis 2012 test shoots and the theoretical predictions of the Sharpe Hollow Cavity Model: CREWES Research Report, Vol. 24, No. 75.

Petten, C. P., G. Margrave, 2012, Using the Sharpe Hollow Cavity Model to investigate power and frequency content of explosive pressure sources: CREWES Research Report, Vol. 24, No. 76.

Sharpe, J., 1942, The production of elastic waves by explosive pressure sources: I. Theory and empirical observations, **7**, No. 2, 144-154.