Series analysis of anisotropic reflection coefficients for inversion

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ABSTRACT

Azimuthal AVO analysis is typically performed using linearizations of the exact formula for anisotropic reflection coefficients. These approximations often make simplifying assumptions about the types of media on each side of an interface and fail at large angles, especially when there is a large contrast in elastic parameters across the interface. Since the larger angles of incidence are more sensitive to azimuthal anisotropy, this failure can cause poor estimates of azimuthal anisotropy. In order to better understand and reduce the nonlinearity that can adversely affect inversions using linearizations, we analyze higher-order terms of the reflection coefficients. We show that the nonlinearity for large contrasts and long offsets is significant, indicating the need to use exact reflection coefficients in many situations.

INTRODUCTION

This paper is motivated by another CREWES report in this volume (Kolb et al., 2014) in which we showed that due to inaccuracies of linearized equations at large incidence angles for a large contrast, elastic parameters were incorrectly estimated in the inversion. Innanen (2013) describes an approach for analyzing the lower orders of elastic AVO nonlinearity and computing the higher orders necessary to estimate reflection coefficients in the precritical region for isotropic media, and Innanen and Mahmoudian (2014) provide an example where these orders are non-negligible in physical modeling data. In AVO, a large amount of useful information can be obtained at smaller incidence angles so, depending on how much information is required, large offset data may be ignored. In azimuthal AVO, the effect of azimuthal anisotropy is greater at larger incidence angles for which the particle motion is becoming increasingly horizontal at the interface. Thus, the importance of accurately estimating large offset reflection coefficients is very great and should not be overlooked. Coming to grips with the nonlinearity of azimuthal AVO will impact the analysis and inversion of interpreted horizons; however in addition it is becoming clear that iterative full waveform inversion problems may be cast to more rapidly and intelligently converge if a significant degree of reflection strength nonlinearity is present.

EXPANSION OF ANISOTROPIC REFLECTION COEFFICIENTS

To create a series expansion of the anisotropic reflection coefficient, we apply the methodology from Innanen (2013) to the equation for the exact P-P reflection coefficient, R_{PP}, in VTI media from Graebner (1992). We use the notation for the coefficients from Rüger (2001) as we find it easier to follow, and Rüger (2001) mentions a misprint in Graebner (1992). It should also be noted that there is a misprint in Rüger (2001) as well; the stiffness coefficients in equation (4.26) of that paper should not be density normalized (c's

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instead of a's).

We perform the expansion by parameterizing the reflection coefficient in terms of the horizontal slowness, p, squared (p^2) and perturbations that measure contrasts across the layers at the interface. The first step we perform is parameterizing all the elastic parameters in terms of the parameters of the top layer. We do this by defining 5 weak contrast parameters:

$$\delta a_{11} = 1 - \frac{a_{11}^{(1)}}{a_{11}^{(2)}}, \delta a_{13} = 1 - \frac{a_{13}^{(1)}}{a_{13}^{(2)}}, \delta a_{33} = 1 - \frac{a_{33}^{(1)}}{a_{33}^{(2)}}, \delta a_{55} = 1 - \frac{a_{55}^{(1)}}{a_{55}^{(2)}}, \delta \rho = 1 - \frac{\rho^{(1)}}{\rho^{(2)}}, \quad (1)$$

where a_{11}, a_{13}, a_{33}, and a_{55} are density-normalized stiffness coefficients and the superscripts (1) and (2) refer to the upper and lower layers.

This allows us to define the second layer parameters in terms of the top layer parameters:

$$a_{11}^{(2)} = \frac{a_{11}^{(1)}}{1 - \delta a_{11}}, a_{13}^{(2)} = \frac{a_{13}^{(1)}}{1 - \delta a_{13}}, a_{33}^{(2)} = \frac{a_{33}^{(1)}}{1 - \delta a_{33}}, a_{55}^{(2)} = \frac{a_{55}^{(1)}}{1 - \delta a_{55}}, \rho^{(2)} = \frac{\rho^{(1)}}{1 - \delta \rho}. \quad (2)$$

This parameterization for the expansion is different than many parameterizations used for reflection coefficients in which the layer parameters are perturbed about the average or background layer parameters (e.g. Vavryčuk and Pšenčík, 1998; Rüger, 1998). The advantage in this approach is that one can perform a layer-stripping approach with this method since no information about the bottom layer is required in order to calculate its properties. Additionally, coupling between parameters is clearly demonstrated in the progressively higher order terms, whereas this nonlinearity is hidden within the parameterization of perturbations using a background medium. One additional advantage is that there is no ambiguity due to the choice of background velocities. The disadvantage of this method, however, is that it is less accurate to the same order because the coupling in the average medium parameterization is treated as a higher order phenomenon.

We start our expansion by expanding the vertical slownesses in terms of the weak contrast parameters defined in equation (1):

$$q_{\alpha}^{(1)} = \frac{1}{\sqrt{a_{33}^{(1)}}} - \frac{a_{13}^{(1)2} + 2a_{13}^{(1)}a_{55}^{(1)} + a_{33}^{(1)}a_{55}^{(1)}}{2\sqrt{a_{33}^{(1)}}(a_{33}^{(1)} - a_{55}^{(1)})}p^2 + \cdots ,$$

$$q_{\beta}^{(1)} = \frac{1}{\sqrt{a_{55}^{(1)}}} - \frac{(a_{13}^{(1)} + a_{55}^{(1)})^2 + a_{11}^{(1)}(a_{55}^{(1)} - a_{33}^{(1)}))}{2\sqrt{a_{55}^{(1)}}(a_{55}^{(1)} - a_{33}^{(1)})}p^2 + \cdots ,$$

$$q_\alpha^{(2)} = \frac{1}{\sqrt{a_{33}^{(1)}}} - \frac{a_{13}^{(1)2} + 2a_{13}^{(1)}a_{55}^{(1)} + a_{33}^{(1)}a_{55}^{(1)}}{2\sqrt{a_{33}^{(1)}}(a_{33}^{(1)} - a_{55}^{(1)})}p^2 - \frac{1}{2\sqrt{a_{33}^{(1)}}}\delta a_{33} + \cdots ,$$

$$q_\beta^{(2)} = \frac{1}{\sqrt{a_{55}^{(1)}}} - \frac{(a_{13}^{(1)} + a_{55}^{(1)})^2 + a_{11}^{(1)}(a_{55}^{(1)} - a_{33}^{(1)}))}{2\sqrt{a_{55}^{(1)}}(a_{55}^{(1)} - a_{33}^{(1)})}p^2 - \frac{1}{2\sqrt{a_{55}^{(1)}}}\delta a_{55} + \cdots . \qquad (3)$$

These series expansions are then used to create series expansions of the direction cosines of the polarization vectors which are used to create series expansions of the matrix M elements. Using Cramer's rule we then obtain a series expansion of R_{PP}. The first two orders are as follows:

$$R_{PP} = \frac{1}{4}(\delta a_{33} + 2\delta\rho) + \frac{a_{13}^{(1)}(a_{13}^{(1)} + a_{55}^{(1)})}{2(a_{33}^{(1)} - a_{55}^{(1)})}p^2\delta a_{13} - \frac{a_{33}^{(1)}(a_{13}^{(1)} + a_{55}^{(1)})^2}{4(a_{33}^{(1)} - a_{55}^{(1)})^2}p^2\delta a_{33} + \frac{1}{8}(\delta a_{33})^2$$
$$- \frac{(a_{13}^{(1)} + a_{33}^{(1)})^2 a_{55}^{(1)}}{4(a_{33}^{(1)} - a_{55}^{(1)})^2}p^2\delta a_{55} - \frac{a_{55}^{(1)}(a_{13}^{(1)} + a_{33}^{(1)})(a_{13}^{(1)} + a_{55}^{(1)})}{(a_{33}^{(1)} - a_{55}^{(1)})^2}p^2\delta\rho + \frac{1}{4}(\delta\rho)^2 + \cdots$$

This formulation allows a series expansion to be cut off at arbitrary order without making rock physics assumptions and can be rotated in the same way as in the Vavryčuk and Pšenčík (1998) paper to describe a monoclinic medium overlying another monoclinic medium. From this point it can be simplified as needed.

FORWARD MODELING COMPARISON OF EXPANSION METHODS

In this formulation we treat the weak contrast parameters as well as p^2 as orders in the series expansion and expand to fourth order. We use the estimated stiffnesses used in Kolb et al. (2014), originally from Mahmoudian (2013), to analyze the series expansion results. We compare our formulation to the formula from Vavryčuk and Pšenčík (1998) for the fast direction of the lower layer (simulated to be parallel to cracks) in Figure 1 and the slow direction of the lower layer (simulated to be perpendicular to cracks) in Figure 2. Although we test our formulation to fourth order against the formula from Vavryčuk and Pšenčík (1998) in first order, the formula from Vavryčuk and Pšenčík (1998) is more accurate for the synthetic model we use (see Figures 1 & 2). This is partly because Vavryčuk and Pšenčík (1998) use average medium properties while we are perturbing from the top layer's elastic properties, but also because we are counting θ terms (in p) as part of our order whereas this is not the case in Vavryčuk and Pšenčík (1998), and so these higher order terms in our expansion are wrapped into what is called first order in other perturbations.

Using up to fourth order terms allows for 4 orders of weak contrast parameters at normal incidence, 3 orders of weak contrast parameters in front of the p^2 term, and 2 orders of weak contrast parameters in front of the p^4 term. 1, p^2, and p^4 are similar to the Shuey formulation (Shuey, 1985) that is often used where the reflection coefficient is broken into 1, $\sin^2\theta$, and $\sin^2\theta\tan^2\theta$ which are 3 angle terms corresponding to progressively larger angles of incidence. Our fourth-order expansion can then be seen as approximating third order in the AVO gradient, and second order in the AVO curvature. A change in medium properties over the average medium properties is a series expansion in our small contrast parameters (as shown in Innanen (2013) for isotropy):

$$\begin{aligned} \frac{\Delta V_{P_V}}{V_{P_V}} &= \frac{1}{2}\delta a_{33} + \frac{1}{4}(\delta a_{33})^2 + \frac{5}{32}(\delta a_{33})^3 + \cdots \,, \\ \frac{\Delta V_{P_H}}{V_{P_H}} &= \frac{1}{2}\delta a_{11} + \frac{1}{4}(\delta a_{11})^2 + \frac{5}{32}(\delta a_{11})^3 + \cdots \,, \end{aligned} \tag{4}$$

etc. V_{P_V} refers to the vertical P-velocity and V_{P_H} refers to the horizontal P-velocity (there is only one since this formulation is for a single vertical plane). Because our fourth order expansion does not approximate the exact reflection coefficient as well as linearizations in terms of the average medium parameters, it is likely that the higher order terms in equation (5) have important contributions at large incidence angles.

CONCLUSIONS

The relatively rapid decay in approximation accuracy of our series expansion truncated at fourth order for large contrasts and large incidence angles demonstrates the large amount of nonlinearity in the anisotropic reflection coefficient. This result is in agreement with isotropic AVO theory in the precritical region for which there is also a large degree of increasing nonlinearity. For anisotropic series expansions, however, there are more elastic parameters as well as perturbation parameters, resulting in a much more complex, and less useful, result. Parameterizations using background medium properties are much better suited for modeling the reflection coefficient in these situations but that is only because the nonlinearity is hidden in the parameterization. There is still opportunity for further analysis of series expansions of anisotropic reflection coefficients, but as series terms of order four and higher become necessary to explain data amplitudes, in practice we begin to consider using exact reflection coefficients to model the nonlinearity instead.

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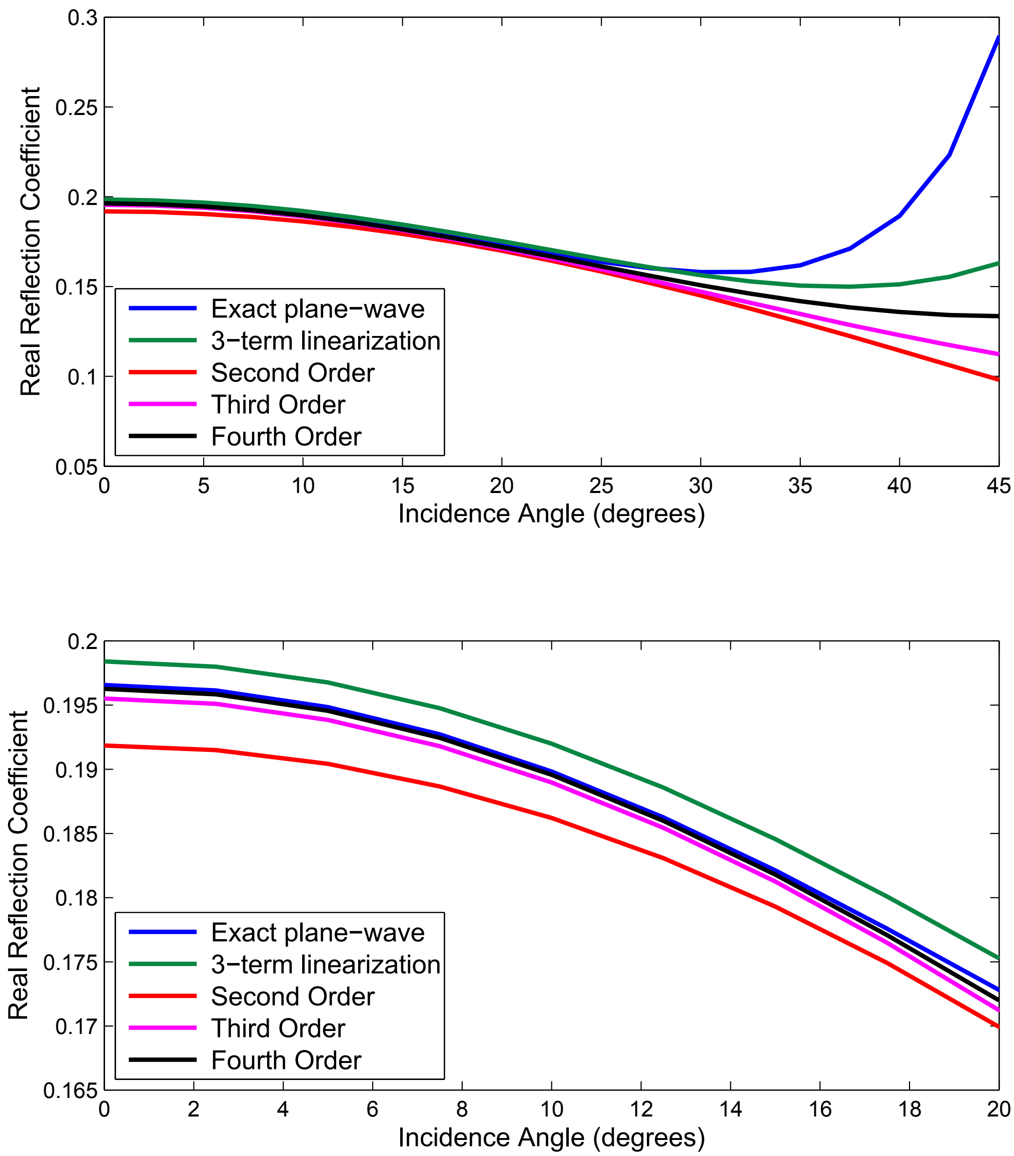

FIG. 1. Comparison of higher order terms from our series expansion to the exact plane-wave reflection coefficient as well as the 3-term linearization from Vavryčuk and Pšenčík (1998) for the fast direction (of the bottom layer) of our synthetic model with stiffnesses from Mahmoudian (2013). (top) 0 - 45 degrees angle of incidence (bottom) 0 - 20 degrees angle of incidence. The 3-term linearization using average medium properties approximates the plane-wave reflection coefficient better than our fourth order (lower order when not counting theta terms) at large angles of incidence for large contrasts.

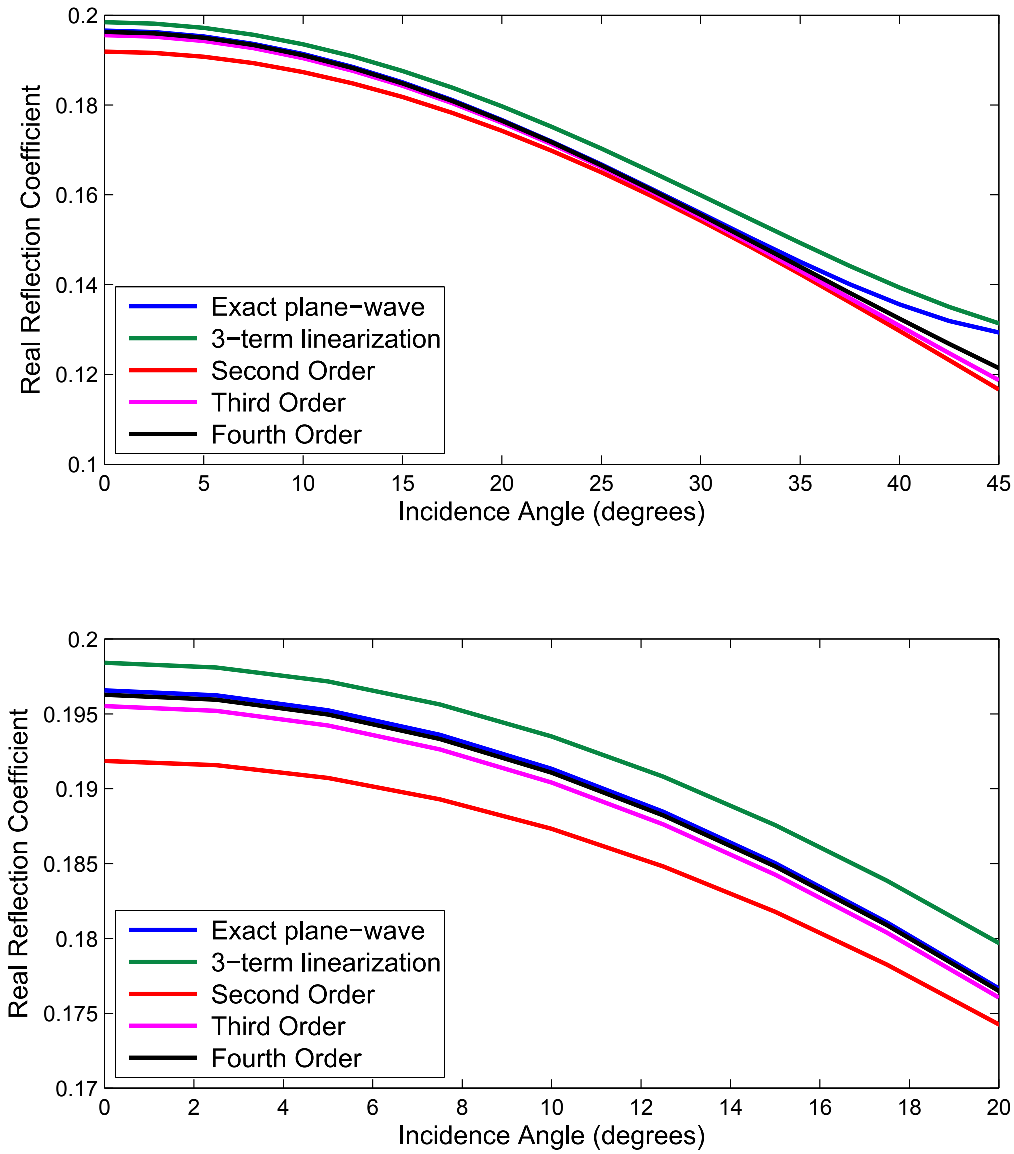

FIG. 2. Comparison of higher order terms from our series expansion to the exact plane-wave reflection coefficient as well as the 3-term linearization from Vavryčuk and Pšenčík (1998) for the slow direction (of the bottom layer) of our synthetic model with stiffnesses from Mahmoudian (2013). (top) 0 - 45 degrees angle of incidence (bottom) 0 - 20 degrees angle of incidence. The 3-term linearization using average medium properties approximates the plane-wave reflection coefficient slightly better than our fourth order (lower order when not counting theta terms) at large angles of incidence for this smaller contrast.

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