Isotropic and transversely isotropic media: absorbing bottom boundary conditions

P.F. Daley

ABSTRACT

When using pseudo – spectral methods to reduce to the spatial dimensionality of the 2.5D coupled $qP-qS_V$ wave propagation problem in an isotropic or transversely isotropic (TI) medium to that in one spatial dimension and time, the introduction of an absorbing boundary, at least, at the model bottom is useful in the removal of spurious arrivals. The top model boundary is usually wanted in the numerical calculations and reflections from the model sides may be removed by a judicious choice of model parameters, which does not significantly increase the run time. In this report, a method similar to that presented in Clayton and Engquist (1977, 1980) and is derived for the coupled $P-S_V$ wave propagation problem in a transversely isotropic medium. Finite Hankel transforms are used to remove the radial coordinate (r) in what is assumed to be a radially symmetric medium. The problem that remains is a coupled problem in depth(z), where the anisotropic parameters may arbitrarily vary, in depth and time(t).

INTRODUCTION

This method is most often referred to as the pseudo-spectral method, but due to the extensive work done in this area by B.G. Mikhailenko and A.S. Alekseev it is sometimes referred to, in seismic applications, as the Alekseev-Mikhailenko Method (AMM), (Alekseev and Mikhailenko, 1980). It falls within the genetic class of pseudo-spectral methods, but is possibly more formal and rigorous in its development. However, much of their work is relatively physically inaccessible and a considerable number of the more significant contributions are in Russian. Other works of interest in this area are Gazdag (1973), Gazdag (1981) and Kosloff and Baysal (1982).

One numerical advantage of applying finite integral transforms is that the resultant FD problem is in one spatial variable and time and there are no cross derivative terms. These are differentials of the form $\partial/\partial x_i\left[c\left(x_1,x_2,x_3\right)\partial u_k/\partial x_j\right]\quad i,j,k=1,2,3:\quad i\neq j$. Several approaches for dealing with these in a finite difference context may be found in Zahradník et al. (1993).

Apart from a number of other numerical considerations, the removal of spurious reflections from the pseudo model bottom is required. This is done here using the method described in Clayton and Engquist (1977). In that paper and other related papers, the following statement, or something similar appears:

*"Paraxial approximations for the elastic wave equation [*and elastic TI wave equation *a*s well as more complicated wave equations*] analogous to those of the scalar wave equation can also be found. We cannot, however, perform the analysis by considering expansions of the dispersion relation because the differential equations for vector fields are not uniquely specified from their dispersion relations. Instead, we use the scalar case to provide a hint as to the*

general form of the paraxial approximation and fit the coefficients by matching to the full elastic wave equation."

What is being said here is that if paraxial approximations are derived for the two coupled equations of particle motion they are a partial solution of the absorbing boundary problem. The full solution requires the integration of these scalar equations into a scalar equivalent of the two coupled equations of motion. It has been determined that using just the first part of the solution produces better than expected results.

THEORETICAL OVERVIEW

Consider the problem of coupled $P-S_V$ wave propagation in a radially symmetric (no lateral inhomogeneities), vertically inhomogeneous isotropic and transversely isotropic half space. The equations of motion are defined by the elastodynamic (Navier) equations (Martynov and Mikhailenko, 1984 Mikhailenko and Korneev, 1984, or Mikhailenko, 1985, as examples)

Isotropic:

$$\rho\frac{\partial^2 U}{\partial t^2}=\left(\lambda+2\mu\right)\left[\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}-\frac{U}{r^2}\right]+\lambda\frac{\partial^2 V}{\partial r\partial z}+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial U}{\partial z}+\frac{\partial V}{\partial r}\right)\right]+\rho F_r \tag{1}$$

$$\rho\frac{\partial^2 V}{\partial t^2}=\mu\left[\frac{\partial}{\partial r}\left(\frac{\partial U}{\partial z}+\frac{\partial V}{\partial r}\right)+\frac{1}{r}\left(\frac{\partial U}{\partial z}+\frac{\partial V}{\partial r}\right)\right]+\frac{\partial}{\partial z}\left[\lambda\left(\frac{\partial U}{\partial r}+\frac{U}{r}\right)+\left(\lambda+2\mu\right)\frac{\partial V}{\partial z}\right]+\rho F_z \tag{2}$$

Transversely isotropic:

$$\rho\frac{\partial^2 U}{\partial t^2}=c_{11}\left[\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}-\frac{U}{r^2}\right]+c_{13}\frac{\partial^2 V}{\partial r\partial z}+\frac{\partial}{\partial z}\left[c_{55}\left(\frac{\partial U}{\partial z}+\frac{\partial V}{\partial r}\right)\right]+\rho F_r \tag{3}$$

$$\rho\frac{\partial^2 V}{\partial t^2}=c_{55}\left[\frac{\partial}{\partial r}\left(\frac{\partial U}{\partial z}+\frac{\partial V}{\partial r}\right)+\frac{1}{r}\left(\frac{\partial U}{\partial z}+\frac{\partial V}{\partial r}\right)\right]+\frac{\partial}{\partial z}\left[c_{13}\left(\frac{\partial U}{\partial r}+\frac{U}{r}\right)+c_{33}\frac{\partial V}{\partial z}\right]+\rho F_z \tag{4}$$

where the particle displacement vector $\mathbf{u}$ is of the form

$$\mathbf{u}\equiv\mathbf{u}\left(r,z,t\right)=\left(U\left(r,z,t\right),V\left(r,z,t\right)\right). \tag{5}$$

Here $U\left(r,z,t\right)$ and $V\left(r,z,t\right)$ are the radial (horizontal) and vertical components of vector particle displacement, the azimuthal component of displacement being zero for the coupled $P-S_V$ problem. The coordinates r and z are the radial and vertical coordinates in a cylindrical coordinate system, respectively, t is time The quantities λ and μ are Lamés parameters for the medium and ρ is the density, all of which may be dependent on the vertical $\left(z\right)$ coordinate. The relationship between Lamé's parameters and velocities are $v_{Sv}^2=\beta^2=\mu/\rho$ and $v_P^2=\alpha^2=\lambda+2\mu/\rho$. In the TI case the stiffnesses, $c_{ij}\ \left(ij=11,33,13,55\right)$ replace the Lamé's parametersand the density normalized stiffnesses, have the dimensions of (velocity)^2 $a_{ij}=c_{ij}/\rho$.

The problem is solved subject to the initial conditions

$$\mathbf{u}\big|_{t=0} = \frac{\partial \mathbf{u}}{\partial t}\bigg|_{t=0} = 0 \tag{6}$$

and the free surface boundary conditions that are required to be satisfied are

$$\sigma_{zz}\big|_{z=0} = 0\,, \qquad \text{and} \qquad \sigma_{rz}\big|_{z=0} = 0\,. \tag{7}$$

That is, the normal stress and shear stress are zero at the free surface. In terms of $U(r,z,t)$, $V(r,z,t)$ and Lamés parameters, λ and μ, the expressions for the normal and shear stresses at the free surface are given by

Isotropic:

$$\sigma_{zz}\big|_{z=0} = \left[\lambda\left(\frac{1}{r}\frac{\partial(rU)}{\partial r}\right) + (\lambda+2u)\frac{\partial V}{\partial z}\right] = 0 \tag{8}$$

$$\sigma_{rz}\big|_{z=0} = \mu\left(\frac{\partial U}{\partial z} + \frac{\partial V}{\partial r}\right) = 0 \tag{9}$$

Transversely isotropic:

$$\sigma_{zz}\big|_{z=0} = \left[c_{13}\left(\frac{1}{r}\frac{\partial(rU)}{\partial r}\right) + c_{33}\frac{\partial V}{\partial z}\right] = 0 \tag{10}$$

$$\sigma_{rz}\big|_{z=0} = c_{55}\left(\frac{\partial U}{\partial z} + \frac{\partial V}{\partial r}\right) = 0 \tag{11}$$

Introducing the finite Hankel integral transforms and the vector designation $\mathbf{G}\left(\tilde{k}_i, k_i, z, t\right) = \left(S\left(\tilde{k}_i, z, t\right), R\left(k_i, z, t\right)\right)$ has

$$S\left(\tilde{k}_i, z, t\right) = \int_0^a U(r,z,t)\, J_1(k_i r)\, r\, dr \tag{12}$$

$$R\left(k_i, z, t\right) = \int_0^a V(r,z,t)\, J_0\left(\tilde{k}_i r\right) r\, dr \tag{13}$$

where the k_i and $\tilde{k}_i$ are the roots of the transcendental equations

$$J_0\left(\tilde{k}_i r\right) = 0 \tag{14}$$

and

$$J_1\left(k_i r\right) = 0, \tag{15}$$

respectively. Using the two formulations of the Hankel transforms, it may be shown that both of the inverse series summations may be accomplished using only the roots of one of the Bessel function transcendental equation, $J_1\left(k_i r\right) = 0$, so that the inverse transforms are defined by

$$U\left(r,z,t\right) = \frac{2}{a^2}\sum_{i=1}^{\infty}\frac{S\left(k_i,z,t\right)J_1\left(k_i r\right)}{\left[J_0\left(k_i a\right)\right]^2} \tag{16}$$

$$V\left(r,z,t\right) = \frac{2}{a^2}\sum_{i=1}^{\infty}\frac{R\left(k_i,z,t\right)J_0\left(k_i r\right)}{\left[J_0\left(k_i a\right)\right]^2} \tag{17}$$

Thus both inverse series summations may be taken over the roots of one rather than two transcendental equations and as a consequence, $\mathbf{G}\left(k_i,z,t\right) = \left(S\left(k_i,z,t\right), R\left(k_i,z,t\right)\right)$. The matter of what, numerically, constitutes an infinite number of terms in the inverse series summations is addressed in Daley (2011). It is shown there that an earlier assumption that the source wavelet be band limited is significant in this determination. As the only spatial direction in which a finite difference is used is the *z* direction an economical manner to introduce a damping conditions at the lower *z* boundary, i.e., $\gamma_1\left(z\right)\partial R/\partial t$ and $\gamma_2\left(z\right)\partial S/\partial t$. However, as the previous may help in removing spurious arrivals, the use of the Clayton – Engquist conditions offers safety in this.

Applying the appropriate Hankel transforms to equations (1) – (4) results in

Isotropic:

$$\frac{\partial^2 S}{\partial t^2} = \frac{\partial}{\partial z}\left(\mu\frac{\partial S}{\partial z} + k_i\mu R\right) - k_i\mu\frac{\partial R}{\partial z} - k_i^2\left(\lambda + 2\mu\right)S + \rho\hat{F}_r \tag{18}$$

$$\frac{\partial^2 R}{\partial t^2} = \frac{\partial}{\partial z}\left(\left(\lambda + 2\mu\right)\frac{\partial R}{\partial z} + k_i\lambda S\right) + k_i\mu\frac{\partial S}{\partial z} - k_i^2\mu R + \rho F_z \tag{19}$$

Transversely isotropic:

$$\rho\frac{\partial^2 S}{\partial t^2} = \frac{\partial}{\partial z}\left(c_{55}\frac{\partial S}{\partial z} - k_i c_{55}R\right) - k_i c_{13}\frac{\partial R}{\partial z} - k_i^2 c_{11}S + \rho\hat{F}_r \tag{20}$$

$$\rho\frac{\partial^2 R}{\partial t^2} = \frac{\partial}{\partial z}\left(c_{33}\frac{\partial R}{\partial z} + k_i c_{13}S\right) + k_i c_{55}\frac{\partial S}{\partial z} - k_i^2 c_{55}R + \rho\hat{F}_r \tag{21}$$

while the transforms of the shear and normal stresses at the free surface, which is assumed to be planar, have the form

Isotropic:

$$\left[\mu\left(\frac{\partial S}{\partial z}-k_i R\right)\right]_{z=0}=0 \tag{22}$$

$$\left[\left(\lambda+2\mu\right)\frac{\partial R}{\partial z}+k_i\lambda R\right]_{z=0}=0 \tag{23}$$

Transversely isotropic:

$$\left.\left[c_{55}\left(\frac{\partial S}{\partial z}-k_i R\right)\right]\right|_{z=0}=0 \tag{24}$$

$$\left.\left(c_{33}\frac{\partial R}{\partial z}+k_i c_{13} S\right)\right|_{z=0}=0\,. \tag{25}$$

The transformed initial conditions at $t=0$ are

$$S\left(z,k_i,t\right)\Big|_{t=0}=0;\quad R\left(z,k_i,t\right)\Big|_{t=0}=0;\quad \left.\frac{\partial S}{\partial t}\right|_{t=0}=\left.\frac{\partial S}{\partial t}\right|_{t=0}=0. \tag{26}$$

where k_i are the roots of the transcendental equation $J_1\left(k_i a\right)=0$ which requires additional boundary conditions at $r=a$ (pseudo boundary such that)

$$U\Big|_{r=a}=\left.\frac{\partial V}{\partial r}\right|_{r=a}=0 \tag{27}$$

The pseudo boundary is placed at some distance $r=a$ so that no spurious reflections from this boundary are present in the synthetic traces. The unwanted reflections may be removed using variation of the methods described in works such as Clayton and Engquist (1977), Cerjan et al. (1985) or Reynolds (1978). Care is required in choosing this distance, as the number of terms in the inverse series summation depends on it in a linear fashion (Daley, 2011).

If it is assumed that the anisotropic parameters (stiffness coefficients) are spatially independent the Hankel transformed equations take on the simplified forms given below. For convenience, it is assumed that the first two grid points in $z\left(z_0 \text{ and } z_1\right)$, at the free surface are of this form so that equations (18) to (21) may be written there as

Isotropic:

$$\frac{\partial^2 S}{\partial t^2}=\beta^2\frac{\partial^2 S}{\partial z^2}-k_i\left(\alpha^2-\beta^2\right)\frac{\partial R}{\partial z}-k_i^2\alpha^2 S \tag{28}$$

$$\frac{\partial^2 R}{\partial t^2}=\alpha^2\frac{\partial^2 R}{\partial z^2}+k_i\left(\alpha^2-\beta^2\right)\frac{\partial S}{\partial z}-k_i^2\beta^2 R \tag{29}$$

Transversely isotropic:

$$\frac{\partial^2 R}{\partial t^2}=a_{33}\frac{\partial^2 R}{\partial z^2}+k_i\left(a_{13}+a_{55}\right)\frac{\partial S}{\partial z}-k_i^2 a_{55} R \tag{30}$$

$$\frac{\partial^2 S}{\partial t^2}=a_{55}\frac{\partial^2 S}{\partial z^2}-k_i\left(a_{13}+a_{55}\right)\frac{\partial R}{\partial z}-k_i^2 a_{11} S \tag{31}$$

and the Hankel transformed shear and normal stresses required at the free surface as boundary conditions have been given in equations (22) to (25).

An explicit finite difference scheme will be introduced into the transformed equations in depth and time $\left(z \text{ and } t\right)$. Equal grid spacing of h in the z direction and δ in time so that an arbitrary depth and time point are specified by $z_k = nh$ and $t_m = m\delta$. The order of accuracy of the finite difference process is 2nd order, $O\left(h^2,\delta^2\right)$.

An explosive point $P-$wave source is most commonly used in producing synthetic seismograms using this method. The transformed radial and vertical components of the source term are given as

$$\hat{F}_r=\frac{1}{2\pi}\delta\left(z-d\right)f\left(t\right) \tag{32}$$

$$F_z=\frac{1}{2\pi}\frac{d}{dz}\left[\delta\left(z-d\right)\right]f\left(t\right) \tag{33}$$

where $\delta\left(\zeta\right)$ is the Dirac delta function and $f\left(t\right)$ is the time dependence of the source wavelet, which is assumed to be band limited.

ABSORBING BOUNDARY AT MODEL BOTTOM

Starting with equations (28) and (29) or (30) and (31), remove all occurrences of the other component of particle displacement from these equation sets. This may seem to have no basis in mathematical theory, however, it not a totally unfounded move. What remains after doing this are two sets of uncoupled equations describing the vertical and radial components of particle displacement. These may now all be considered scalar equations and treated as such. The action described above is not without precedent as variants of this have been suggested in Clayton and Engquist (1977) and Reynolds (1978). If it is assumed that the last 3 grid points in depth are homogeneous (independent of *z*), the Hankel transformed *scalar* equations of motion have the form

Isotropic:

$$\frac{\partial^2 S}{\partial z^2} - \frac{k_i^2 \alpha^2}{\beta^2} S - \frac{1}{\beta^2}\frac{\partial^2 S}{\partial t^2} \approx 0 \tag{34}$$

$$\frac{\partial^2 R}{\partial z^2} - \frac{k_i^2 \beta^2}{\alpha^2} R - \frac{1}{\alpha^2}\frac{\partial^2 R}{\partial t^2} \approx 0 \tag{35}$$

Transversely isotropic:

$$a_{55}\frac{\partial^2 S}{\partial z^2} - k_i^2 a_{11} S - \frac{\partial^2 S}{\partial t^2} = 0 \tag{36}$$

$$a_{33}\frac{\partial^2 R}{\partial z^2} - k_i^2 a_{55} R - \frac{\partial^2 R}{\partial t^2} = 0 \tag{37}$$

A plane wave solution is assumed as

$$R \propto exp\left[-i\omega t + ik_z z\right] \qquad \text{and} \qquad S \propto exp\left[-i\omega t + ik_z z\right] \tag{38}$$

Pseudo-differential operators are defined as

$$\left(-i\omega\right) \rightarrow \frac{\partial}{\partial t} \qquad \text{and} \qquad \left(ik_z\right) \rightarrow \frac{\partial}{\partial z} \tag{39}$$

Applying the plane wave solution to equations (36) and (37) has

$$a_{55}\left(ik_z\right)^2 S - k_i^2 a_{11} S - \left(-i\omega\right)^2 S = 0 \tag{40}$$

$$a_{33}\left(ik_z\right)^2 R - k_i^2 a_{55} R - \left(-i\omega\right)^2 R = 0 \tag{41}$$

Consider equation (40) and construct a parabolic approximation to it. A parabolic approximation to this equation may be used to do the same to (34), (35) and (37)

$$\begin{gathered} a_{55}\left(ik_z\right)^2 S - k_i^2 a_{11} S - \left(-i\omega\right)^2 S = 0 \quad \text{so that} \\ \frac{\left(ik_z\right)}{\left(-i\omega\right)} = \frac{1}{\sqrt{a_{55}}}\left(1 + \frac{k_i^2 a_{11}}{\left(-i\omega\right)^2}\right)^{1/2} \end{gathered} \tag{42}$$

Continuing with the derivation

$$\frac{(ik_z)}{(-i\omega)} \approx \left(\frac{1}{\sqrt{a_{55}}} + \frac{k_i^2 a_{11}}{2(-i\omega)^2 \sqrt{a_{55}}} \right)$$
$$(ik_z)(-i\omega) S - \frac{(-i\omega)^2}{\sqrt{a_{55}}} S - \frac{k_i^2 a_{11}}{2\sqrt{a_{55}}} S = 0 \tag{43}$$

Reintroducing the pseudo-differential operators results in:

$$\frac{\partial^2 S}{\partial z \partial t} - \frac{1}{\sqrt{a_{55}}} \frac{\partial^2 S}{\partial t^2} - \frac{k_i^2 a_{11}}{2\sqrt{a_{55}}} S = 0 \tag{44}$$

$$\frac{\partial^2 R}{\partial z \partial t} - \frac{1}{\sqrt{a_{33}}} \frac{\partial^2 R}{\partial t^2} - \frac{k_i^2 a_{55} R}{2\sqrt{a_{33}}} = 0 \tag{45}$$

With $\mathbf{G} = (S, R)'$and using the finite difference template introduced in Clayton and Engquist (1977) the following definition may be employed

$$D_-^z D_0^t \mathbf{G}_K^n - \left(\frac{1}{2}\right) \mathbf{X}_1 D_+^t D_-^t \left(\mathbf{G}_K^n + \mathbf{G}_{K-1}^n \right) - \left(\frac{1}{4}\right) \mathbf{X}_2 \left(\mathbf{G}_K^{n-1} + \mathbf{G}_{K-1}^n \right) = 0 \tag{46}$$

where the $\mathbf{X}_j \, (j = 1, 2)$ are defined as:

Isotropic:

$$\mathbf{X}_1 = \begin{bmatrix} 1/\beta & 0 \\ 0 & 1/\alpha \end{bmatrix}, \quad \mathbf{X}_2 = k_j^2 \mathbf{X}_2 = \frac{k_j^2}{2} \begin{bmatrix} \alpha^2/\beta & 0 \\ 0 & \beta^2/\alpha \end{bmatrix} \tag{47}$$

Transversely isotropic:

$$\mathbf{X}_1 = \mathbf{X}_1 = \begin{bmatrix} 1/\sqrt{a_{55}} & 0 \\ 0 & 1/\sqrt{a_{33}} \end{bmatrix}, \quad \mathbf{X}_2 = k_j^2 \mathbf{X}_2 = \frac{k_j^2}{2} \begin{bmatrix} a_{11}/\sqrt{a_{55}} & 0 \\ 0 & a_{55}/\sqrt{a_{33}} \end{bmatrix} \tag{48}$$

The operators D_+^q, D_-^q and D_0^q are the forward, backward and center difference finite difference analogues with respect to the variable q and are given by

$$\text{Forward}: \quad D_+^z G_k^n \approx \left(G_{k+1}^n - G_k^n \right) / \Delta z \tag{49}$$

$$\text{Backward}: \quad D_-^z G_k^n \approx \left(G_k^n - G_{k-1}^n \right) / \Delta z \tag{50}$$

$$\text{Center}: \quad D_0^t G_k^n \approx \left(G_k^{n+1} - G_k^{n-1} \right) / (2\Delta t)$$
$$\text{actually} \quad D_0^t G_k^n \approx \left(G_k^{n+1/2} - G_k^{n-1/2} \right) / \Delta t \tag{51}$$

Introducing equations (49) – (51) into (46) results in the following two sets of equations for the transformed radial and vertical components of the paraxial wave equation approximation for the absorbing boundary conditions at the model bottom for isotropic and transversely isotropic media.

Radial:

$$S_K^{n+1} = \left[S_{K-1}^{n+1} + S_K^{n-1} - S_{K-1}^{n-1} + \frac{\mathbf{X}_1^{11}\Delta z}{\Delta t}\left(-2S_K^n + S_K^{n-1} + S_{K-1}^{n+1} - 2S_{K-1}^n + S_{K-1}^{n-1}\right) + \frac{\mathbf{X}_2^{11}\Delta z\Delta t}{2}\left(S_K^{n-1} + S_{K-1}^n\right)\right] \Big/ \left(1 - \frac{\mathbf{X}_1^{11}\Delta z}{\Delta t}\right) \quad (52)$$

Vertical:

$$R_K^{n+1} = \left[R_{K-1}^{n+1} + R_K^{n-1} - R_{K-1}^{n-1} + \frac{\mathbf{X}_1^{22}\Delta z}{\Delta t}\left(-2R_K^n + R_K^{n-1} + R_{K-1}^{n+1} - 2R_{K-1}^n + R_{K-1}^{n-1}\right) + \frac{\mathbf{X}_2^{22}\Delta z\Delta t}{2}\left(R_K^{n-1} + R_{K-1}^n\right)\right] \Big/ \left(1 - \frac{\mathbf{X}_1^{22}\Delta z}{\Delta t}\right) \quad (53)$$

The above two equations may be used for either the isotropic or transversely isotropic case, if the appropriate definitions of $\mathbf{X}_1$ and a $\mathbf{X}_2$ are used.

NUMERICAL RESULTS

The models used for testing the absorbing boundary conditions at the model bottom of a plane layered isotropic and transversely isotropic media is given in Tables 1 and 2 and Figures (1) and (6). The recording geometries are vertical seismic profiles (VSP) with a surface sources and receivers down hole. For the isotropic case an offset geometry is also presented. In Figures (2) and (3) are the VSP synthetics, vertical and radial components of displacement, for the isotropic model. The upper panels in each of these figures show the uncorrected synthetic while the bottom panels are the same synthetic traces with the *acoustic* model bottom absorbing boundary conditions derived here implemented. The traces for the offset case for the same isotropic model are shown in Figures (4) and (5). Again the *acoustic* boundary conditions are used. However, for the offset case, exponential damping is also employed in this area as it was found that this acquisition geometry was more sensitive (for reasons presently unknown) than the VSP case. It should be noted that there appears to be some type of coherent noise present in the traces. This is not real, but an artifact of the plotting software. Too many traces.

For the transversely isotropic problem only the VSP acquisition geometry is investigated. The vertical and radial components of particle displacement are given in Figures (7) and (8). Again the uncorrected synthetic traces appear in the upper panels of each figure while the bottom panels show the corrected synthetics. The only the *acoustic* model bottom absorbing boundary conditions are used to remove spurious arrivals. When compared to the computationally process of setting the model bottom at a depth where no possible reflected arrivals can appear in the time window holding the synthetic traces, the only difference that could be determined was at a level below the noise.

CONCLUSIONS

Using the paper of Clayton and Engquist (1977), which treats seismic wave propagation for the coupled $P-S_V$ modes in a $2D$ elastic medium, as a template, absorbing boundary conditions are derived for the model bottom boundary in a transversely isotropic medium. A vertically inhomogeneous medium has been assumed with the radial derivatives of particle motion being transformed away using finite Hankel transforms. This produces a coupled set of equations for the vertical and radial components of displacement that are to be solved using finite difference methods in the remaining spatial coordinate depth (z) and time. This problem is at least marginally different than what appears in Clayton and Engquist (1977), but a similar solution method may be employed. What results are the so called 15 degree paraxial approximations to the equations of particle displacement. Numerical experiments have shown that what was derived provides a reasonable solution and does remove spurious reflections from the model bottom, if the synthetics are not "over scaled". A modification was introduced to account for the effects of unwanted arrivals appearing in the synthetics when various forms of scaling are introduced. This alteration is a temporary measure and other forms of damping are being considered, specifically some form of exponential damping.

REFERENCES

Alekseev, A.S. and Mikhailenko, B.G., 1980, Solution of dynamic problems of elastic wave propagation in inhomogeneous media by a combination of partial separation of variables and finite difference methods, Journal of Geophysics, 48, 161-172.

Cerjan, C., Kosloff, D., Kosloff, R. and Reshef, M., 1985, A nonreflecting boundary condition for discrete acoustic and elastic wave equations, Geophysics, 50, 705-708.

Clayton, R. and Engquist, B., 1977, Absorbing boundary conditions for acoustic and elastic wave equations, Bulletin of the Seismological Society of America, 67, 1529-1540.

Clayton, R. and Engquist, B., 1980, Absorbing boundary conditions for wave-equation migration, GEOPHYSICS, 45, 895-904.

Daley, P.F., 2011, *P-S_V* wave propagation in a radially symmetric vertically inhomogeneous TI medium: Finite difference hybrid method, CREWES Research Report, Volume 23.

Gazdag, J., 1973, Numerical convective schemes based on the accurate computation of space derivatives, Journal of Computational Physics, 13, 100-113.

Gazdag, J., 1981, Modeling of the acoustic wave equation with transform methods, Geophysics, 46, 854-859.

Kosloff, D. and Baysal, E., 1982, Forward modeling by a Fourier method, Geophysics, 47, 1402-1412.

Martynov, V.N. and Mikhailenko, B.G., 1984, Numerical modelling of propagation of elastic waves in anisotropic inhomogeneous media for the half-space and the sphere, Geophysical Journal of the Royal Astronomical Society, 76, 53-63.

Mikhailenko, B.G. and Korneev, V.I., 1984,Calculation of synthetic seismograms for complex subsurface geometries by a combination of finite integral Fourier transforms and finite difference techniques, Journal of Geophysics, 54, 195-206.

Mikhailenko, B.G., 1985, Numerical experiment in seismic investigations, Journal of Geophysics, 58, 101-124.

Reynolds, A.C., 1978, Boundary conditions for the numerical solution of wave propagation problems, Geophysics, 43, 1099-1110.

Zahradnik, J., P. Moczo, and F. Hron, 1993,Testing four elastic finite difference schemes for behavior at discontinuities, Bulletin of the Seismological Society of America, 83, 107-129.

ACKNOWLEDGEMENTS

The author wishes to thank the sponsors of CREWES and NSERC (Professor G.F. Margrave, CRDPJ 379744-08) for financial support in undertaking this work.

Thickness	Density	V_P	V_S
0.5	1.0	6.0	2.5
0.5	2.05	9.0	4.0
0.5	2.0	14.82	6.7
Hspace	2.1	28.36	6.36

Table 1: The parameters for the isotropic plane layered medium used in testing the model bottom absorbing boundary condition are given in the table. Density is in *gm/cm³*, thickness in *km* and the A_{ij} in km^2/s^2.

Thickness	Density	A_{11}	A_{33}	A_{55}	A_{13}
0.5	1.0	6.0	3.0	2.5	1.5
0.5	2.05	9.0	7.7	4.0	1.2
0.5	2.0	14.82	12.21	6.7	6.7
Hspace	2.1	28.36	28.36	6.36	8.36

Table 2: The parameters for the transversely isotropic plane layered medium used in testing the model bottom absorbing boundary condition are given in the table. Density is in gm/cm^3, thickness in *km* and the A_{ij} in km^2/s^2.

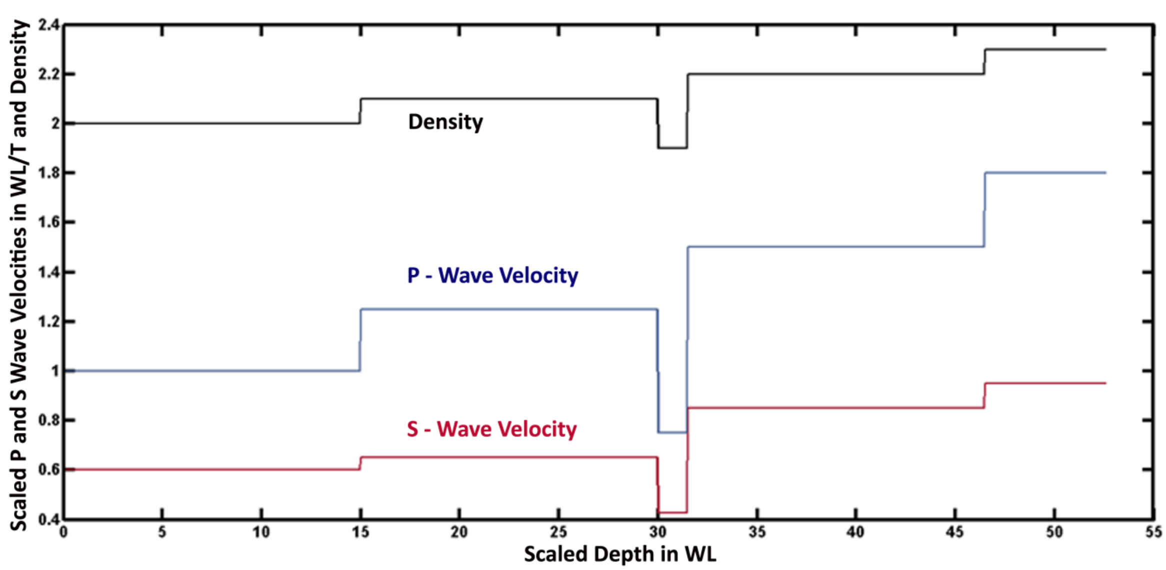
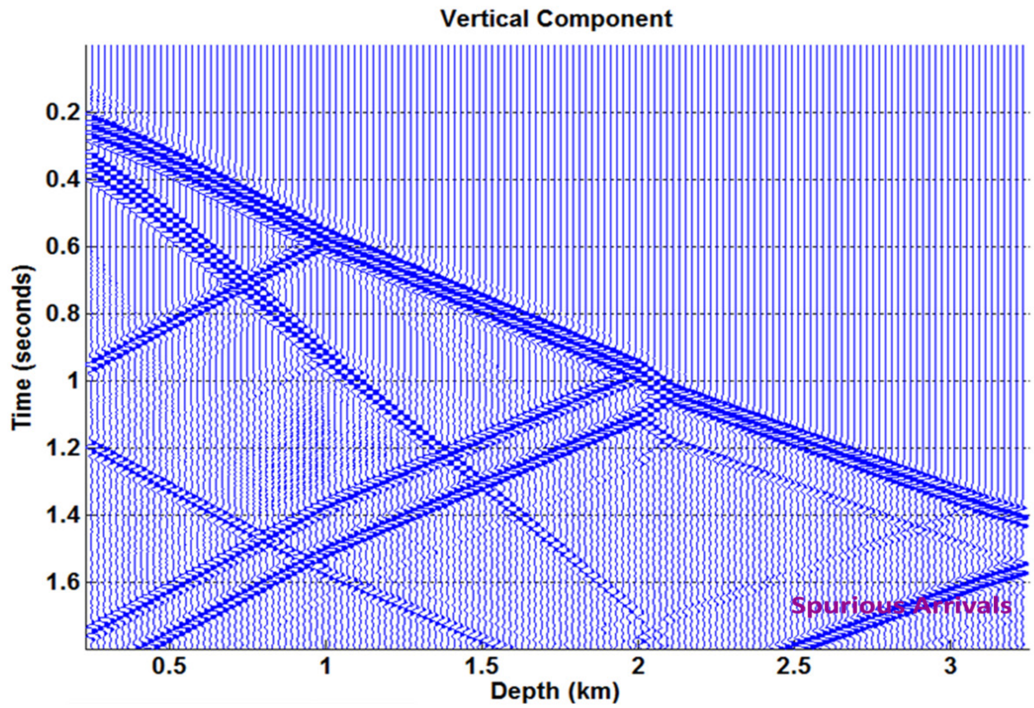


Fig. 1: The vertical *P* – wave velocity in *km/s* plotted versus depth in *km* in the upper panel. In the lower panel the scaled velocity is plotted versus the number of depth grid points.



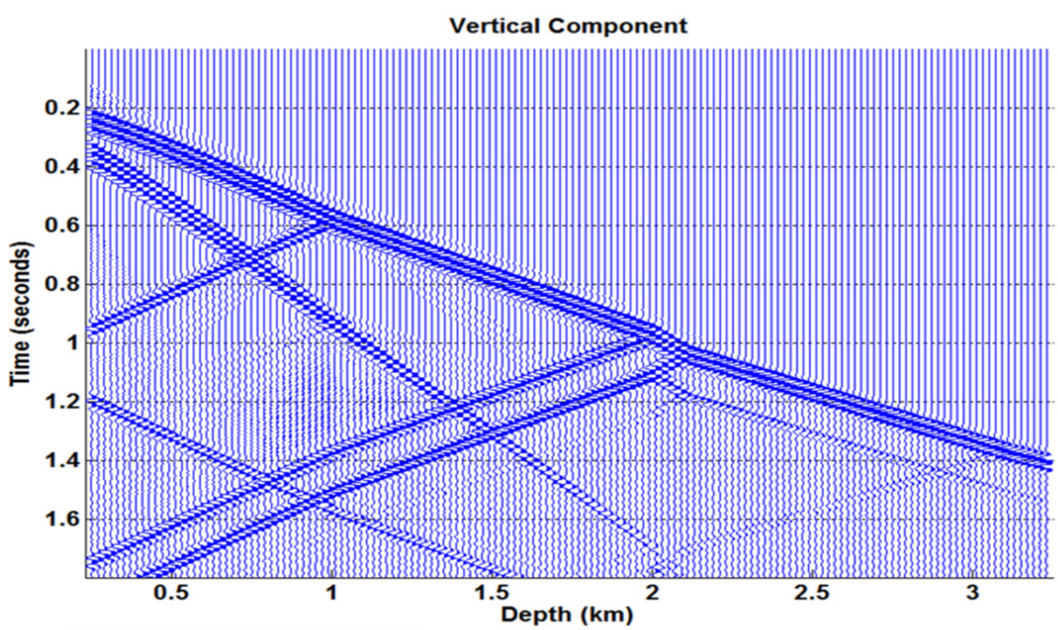
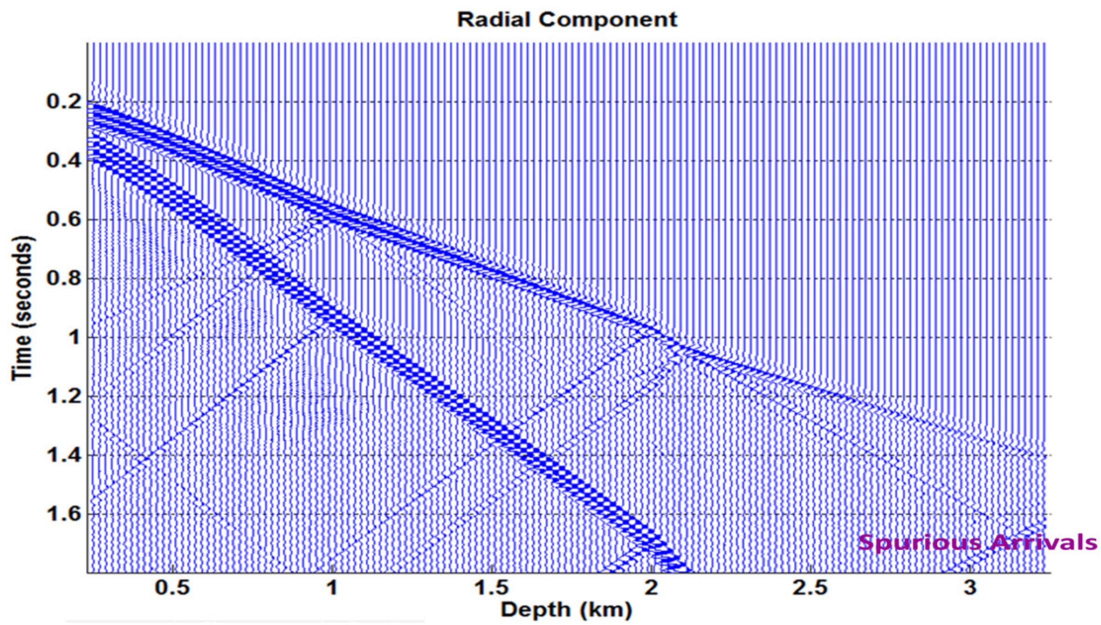


Fig. 2: The vertical component of the VSP synthetic of the model described in the text. In this panel the computations continued to the proper time indicating that no spurious reflections from the model are included in the traces.



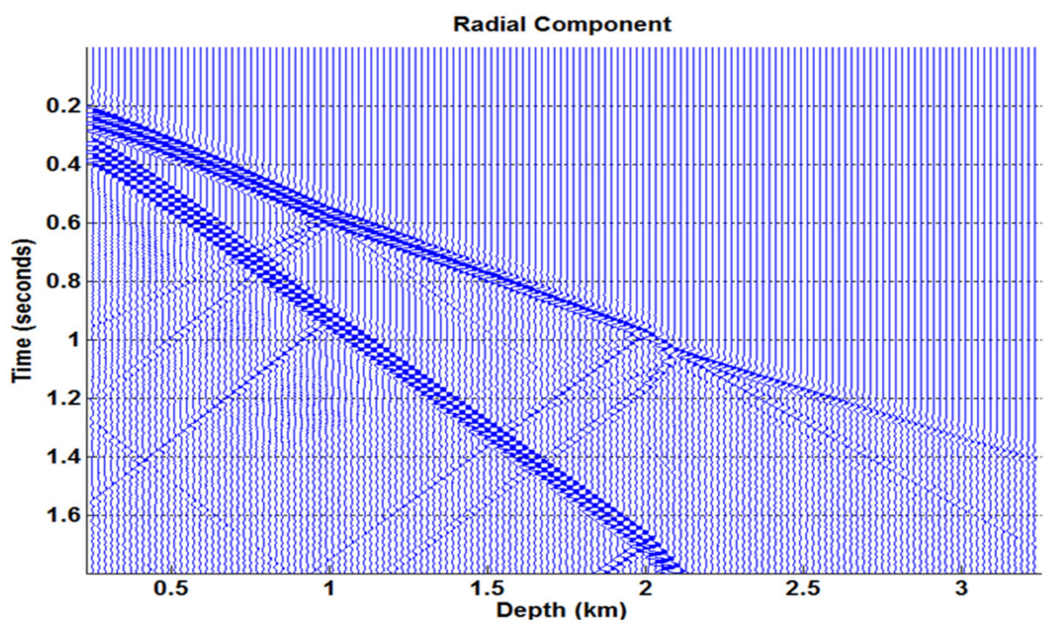


Fig. 3: The radial component of the VSP synthetic of the model described in the text. In this panel the computations continued to the proper time indicating that no spurious reflections from the model are included in the traces

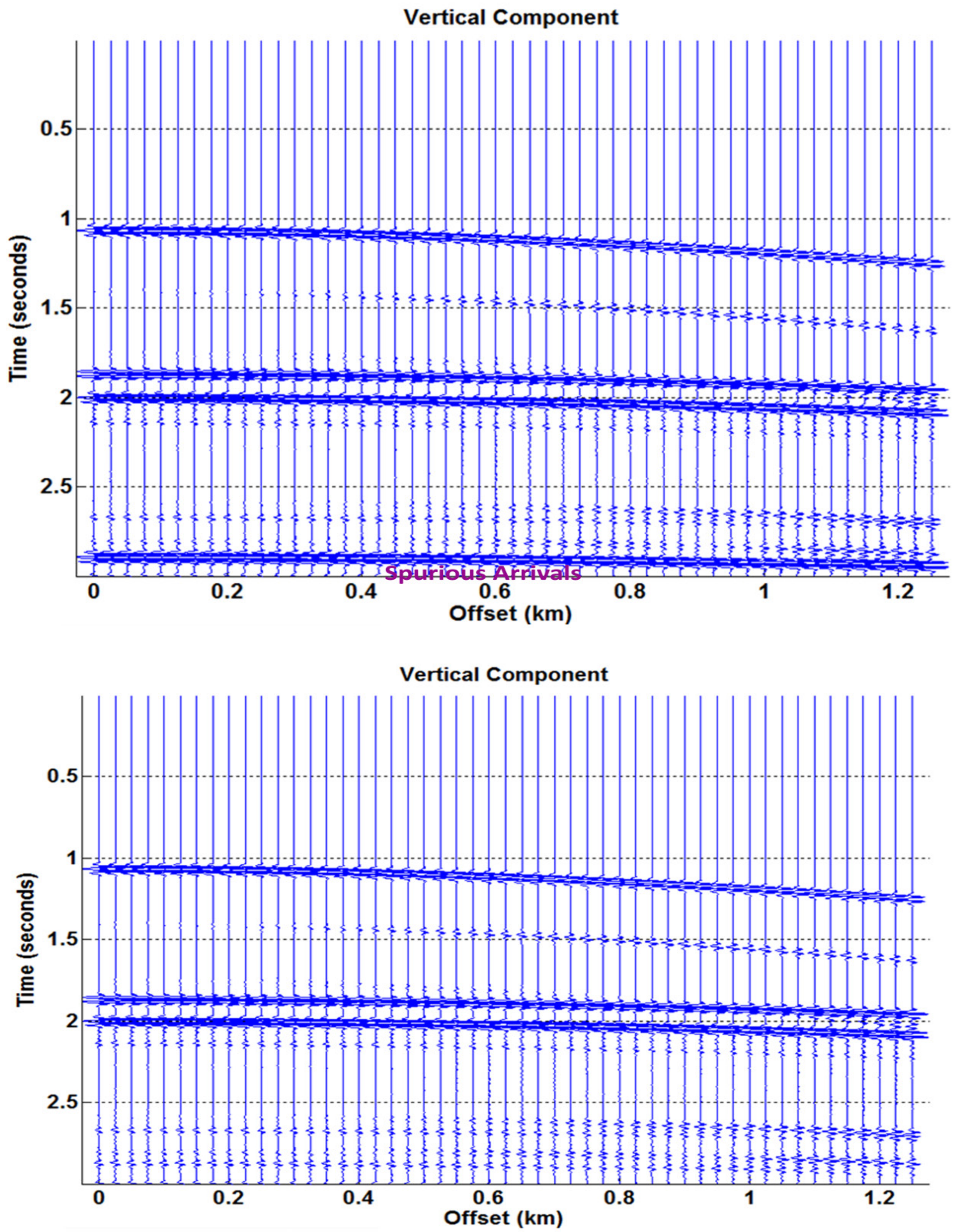


Fig. 4: The vertical component of the offset synthetic of the model described in the text. In the upper panel there are mild spurious reflections from the model bottom while in the lower panel these have been removed.

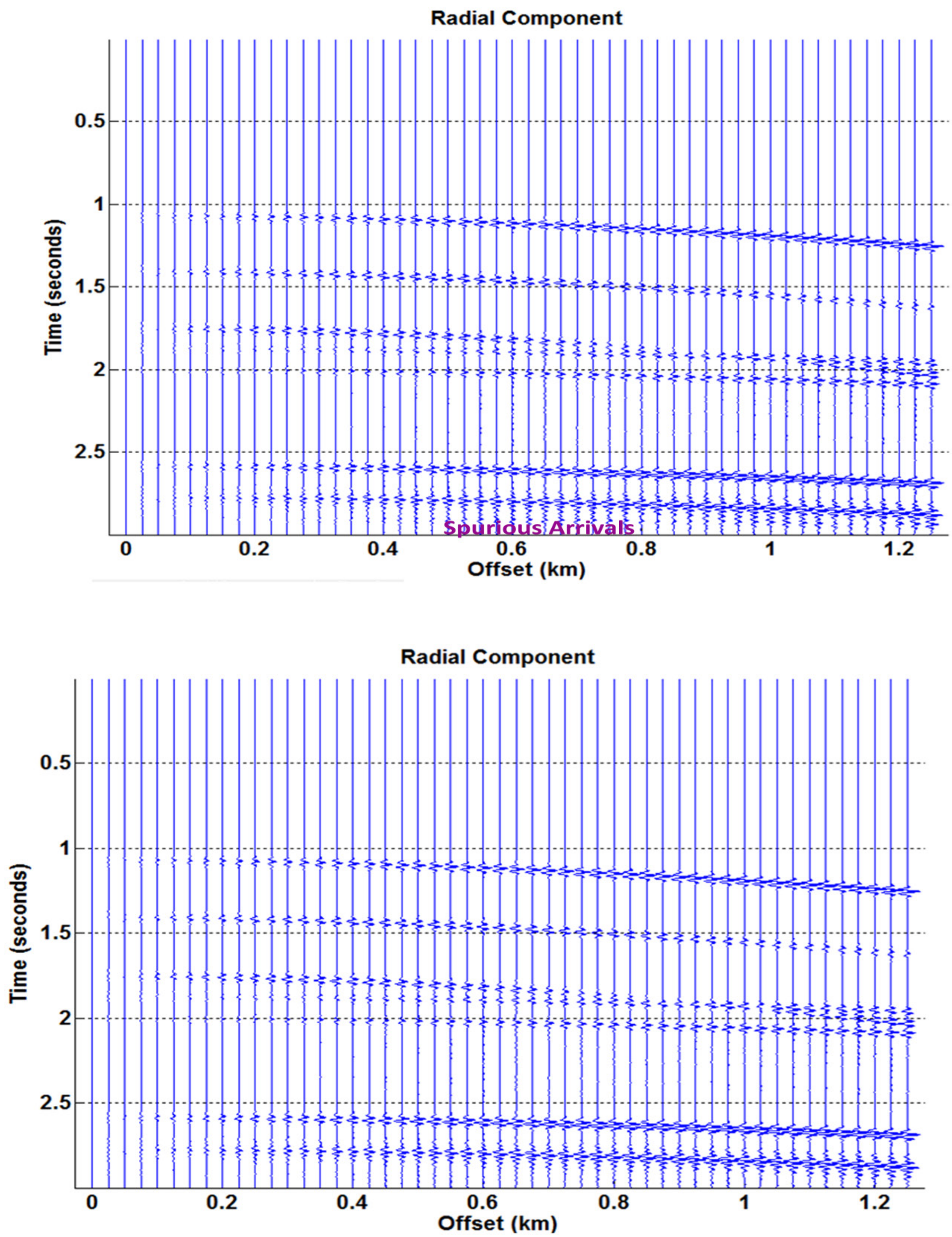


Fig. 5: The radial component of the offset synthetic of the model described in the text. In the upper panel there are mild spurious reflections from the model bottom while in the lower panel these have been removed.

.

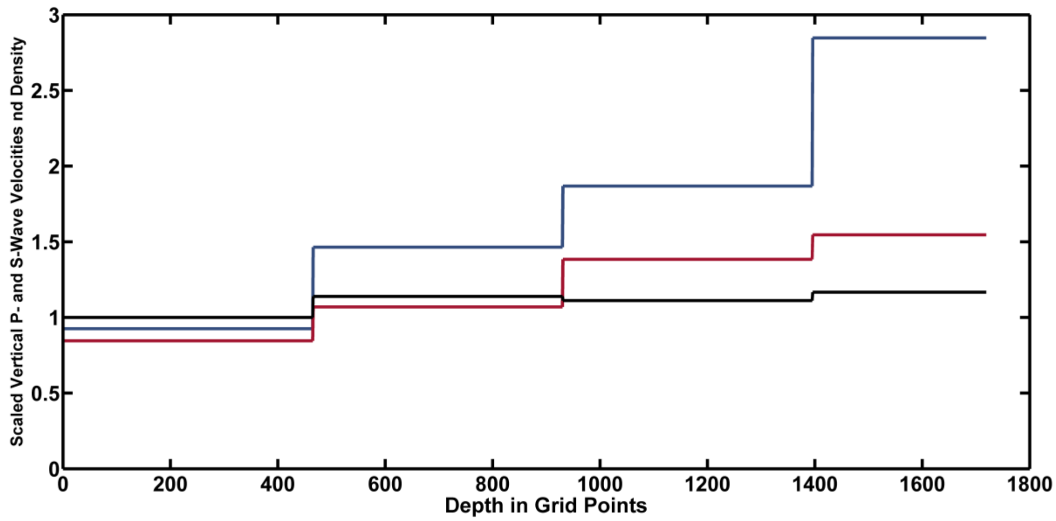


Fig. 6: The scaled vertical qP and qS_V velocities together with the density for the transversely isotropic medium used in the modeling of the next two figures.

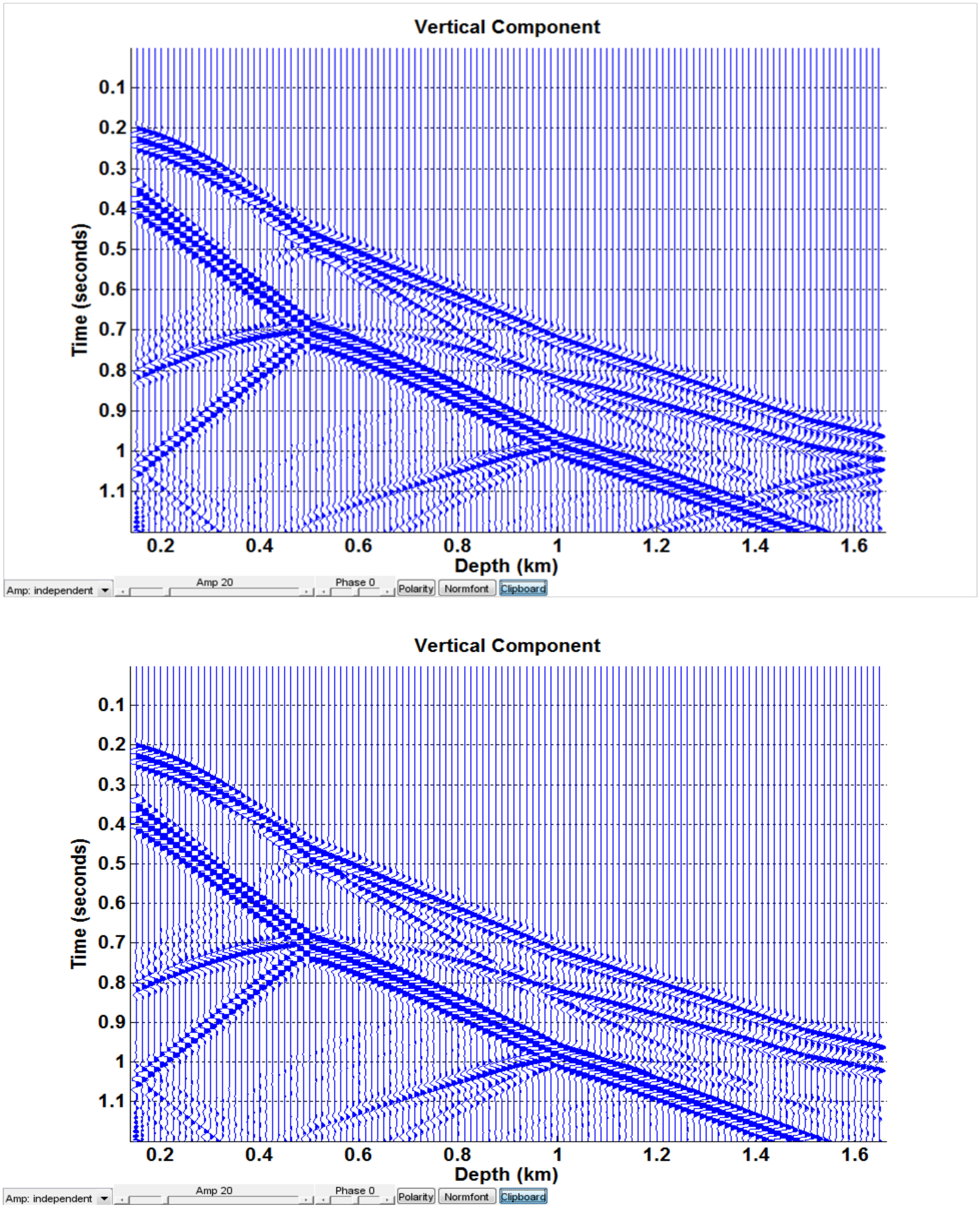


Fig. 7: The vertical component of the VSP synthetic for the model described in the text. In the upper panel the spurious reflections from the model bottom have not been removed. In the bottom panel they have. A comparison was done with a computationally intensive method and compared with almost no difference.

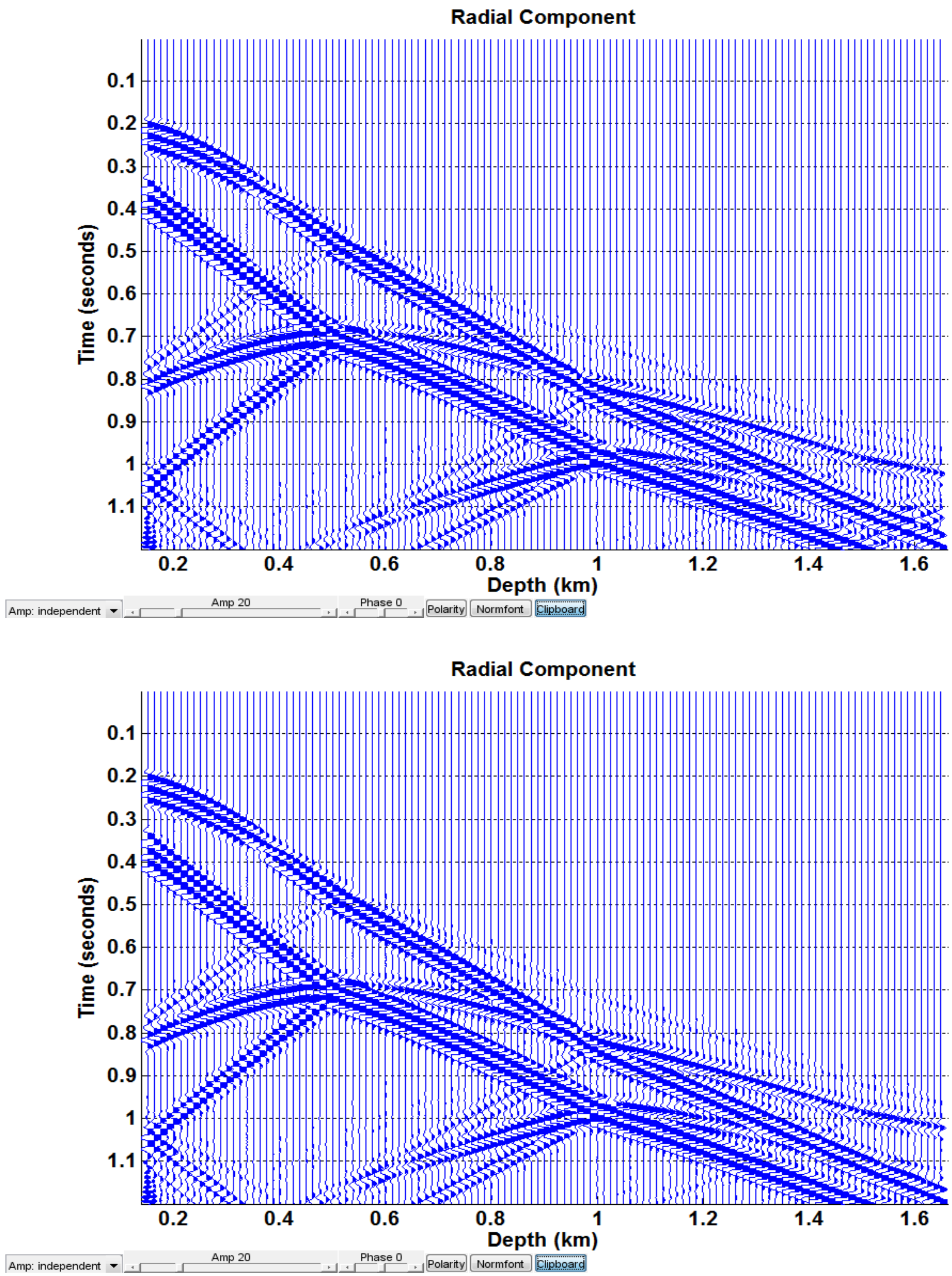


Fig. 8: The radial component of the VSP synthetic for the model described in the text. In the upper panel the spurious reflections from the model bottom have not been removed. In the bottom panel they have. A comparison was done with a computationally intensive method and compared with almost no difference.