Full waveform inversion sensitivities in anisotropic viscoelastic media

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ABSTRACT

We analyze the scattering of seismic waves from both anisotropic and viscoelastic inclusions in an attenuative isotropic background. There are mainly two methods used in investigation of seismic wave scattering, the method of so-called Aki-Richards approximation based on the linearization of the exact solutions of the Zoepprits equation, and, alternatively the Born approximation which is based on the first order perturbation theory. Solution of Zoeppritz equation even for elastic medium has a complicated form and these coefficients should be linearized with respect to the changes in elastic properties. For anisotropic viscoelastic media the situation is more complicated. Born approximation overcome this difficulty as we don't need to solve the Zoeppritz equation and linearized the the reflection coefficients. Instead by having the perturbed stiffness tensor and polarizations in the background medium we can derived the linearized reflection coefficients. The resulted scattering amplitudes are called scattering potentials which can be transformed to the weak reflection coefficients by proper transformations. We consider to the Vertical Transverse Isotropic (VTI) and orthorhombic media with low loss attenuation and weak anisotropy such that the second and higher orders of quality factors and Thompson parameters are neglected. In a viscoelastic medium we have P-wave, SI-wave and SII waves, all with complex slowness vectors. We derived the expressions for potentials of scattering of P-to-P, P-to-SI, SI-to-SI and SII-to-SII. We show that how our results cover the previously derived scattering potentials for elastic/viscoelastic media. The resulting expressions for scattering potentials are sensitivity kernels related to the Fréchet derivatives which linearly link data and parameters perturbations.

INTRODUCTION

The weak contrast linearized reflection coefficients play a major role in inversion of seismic data as they contain unique information on sensitivity of the seismic data to the changes in earth properties (Beylkin and Burridge, 1990; Tarantola, 1986). The traditional way to compute the linearized reflection coefficients is based on the solution of the Zeoppritz equation assuming that properties across the boundary are slightly change (Aki and Richards, 2002).

The exact and approximate reflection and transmission coefficients have been derived for layered viscoelastic isotropic medium taking into the changes in the viscoelastic parameters for incident homogeneous wave (Ursin and Stovas, 2002). The same problem for an inhomogeneous viscoelastic plane wave interacting with a low contrast two layered isotropic viscoelastic media wherein the jumps in the inhomogeneity angle is accommodated recently have been derived (Moradi and Innanen, 2016). It has been shown that these linearized reflection coefficients can be transformed into the viscoelastic scattering potentials as derived in the general volume scattering framework (Moradi and Innanen, 2015). Cervený & Psencík studied the homogeneous and inhomogeneous plane waves propagating in a viscoelastic anisotropic medium (Cervený and Psencík, 2005a,b, 2008). Linearized weak reflection coefficients for viscoelastic anisotropic media including the inhomogeneity angle of incident wave are derived based on the exact solutions of the Zoeppritz equations by (Behura and Tsvankin, 2009a,b).

Understanding the scattering patterns induced by perturbations in medium properties is an essential prerequisite for AVO inversion and Full Waveform Inversion (FWI) (Virieux and Operto, 2009; Fichtner, 2010; Castagna and Backus, 1993). The Born approximation method based on the perturbation theory is an efficient method to evaluate the sensitivity kernel for FWI. In this approach actual medium is regarded as a reference medium with the slightly different properties occupied with perturbations in medium properties. In the case of vertically attenuative isotropic medium perturbations are in density, vertical P- and S-wave velocities, vertical P- and S-wave quality factors, three anisotropic Thompson parameters and three Q-dependent Thompson parameters. Insight into the seismic wave propagation in an attenuative anisotropic earth is of fundamental interest. Our paper is a self contained presentation of the scattering volume from inclusions both in anisotropic and viscoelastic properties. Comparing to the isotropic elastic medium derivation of such approximations is extremely complicated and needs the assumption of both weak anisotropy and low attenuation in lower and upper media.

A summary of our paper follows. It should be noted that we work throughout with a theory of scattering of seismic waves in a vertically isotropic viscoelastic media: the extension of the scattering of seismic waves in isotropic media. In section 2 we discuss the complex stiffness tensor for viscoelastic VTI media staring with the matrix form followed by the subscript notation of the stiffness tensor. In entire paper we assume (a) the anisotropy is weak, and (b) the media is low-loss attenuative. In view of (a) and (b) we introduce the Q-dependent Thompson parameters in terms of real stiffness tensor components and Quality factor matrix components. In section 3 we describe the perturbations in stiffness tensor which are essential in deriving the approximate forms of the scattering potentials. In particular we will show how the perturbed VTI stiffness tensor decomposed into the isotropic part and the contributions from the anisotropic parameters. In section 4 we present the general form of scattering potential for scattering of P-wave to P wave, P-wave to SI-wave, SI-wave to SI-wave and SII-wave to SII-wave. We also present the polarization and slowness vectors of incident and scattered P- and SI-waves which are essential to evaluate the scattering potentials. The results obtained for scattering potentials will be discussed in more detail in section 5. It will be shown that scattering potentials can be decomposed into the isotropic elastic, anisotropic elastic, isotropic viscoelastic and anisotropic visocelastic components.

STIFFNESS TENSOR FOR VTI VISCOELASTIC MEDIUM AND COMPLEX THOMPSON PARAMETERS

One of the most common anisotropic model that have been used in exploration seismology is the VTI/HTI media (Rüger, 2001). For a VTI media with axis of symmetry along the z-direction, there are parallel planes perpendicular to the z-axis. The stiffness tensor in terms of symmetric 6×6 matrix is given by

$$c_{\mathrm{VTI}} = \begin{pmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & 0 & 0 & 0 \\ c_{11}-2c_{66} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}. \tag{1}$$

P-wave phase velocity along the vertical axis z is given by $V_{\mathrm{P}} = \sqrt{c_{33}/\rho}$, vertically S-wave velocity for shear wave polarized in z-direction is given by $V_{\mathrm{S}}^{\perp} = \sqrt{c_{55}/\rho}$ and the velocity of the vertically propagating shear wave polarized in the y direction is given by $V_{\mathrm{S}} = \sqrt{c_{44}/\rho}$. Since in VTI media $c_{55} = c_{44}$ we have $V_{\mathrm{S}}^{\perp} = V_{\mathrm{S}}$, as a result in what follows for VTI media the shear wave velocity is shown by V_{S}. However for Horizontal Transverse Isotropic (HTI) media $V_{\mathrm{S}}^{\perp} \neq V_{\mathrm{S}}$.

In this section we derive the components of the complex VTI stiffness tensor in terms of elastic, anelastic and anisotropic parameters. We start with the tensor form of the stiffness tensor in subscript notation Ikelle and Amundsen (2005)

$$\begin{aligned} \hat{c}^{\mathrm{VTI}}_{ijkl} &= (\hat{c}_{11} - 2\hat{c}_{66})\delta_{ij}\delta_{kl} + \hat{c}_{66}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \\ &+ (2\hat{c}_{66} - \hat{c}_{11} + \hat{c}_{13})(\delta_{ij}\delta_{k3}\delta_{l3} + \delta_{kl}\delta_{i3}\delta_{j3}) \\ &+ (\hat{c}_{55} - \hat{c}_{66})(\delta_{ik}\delta_{j3}\delta_{l3} + \delta_{jk}\delta_{i3}\delta_{l3} + \delta_{il}\delta_{j3}\delta_{k3} + \delta_{jl}\delta_{i3}\delta_{k3}) \\ &+ (\hat{c}_{11} - 2\hat{c}_{13} + \hat{c}_{33} - 4\hat{c}_{55})\delta_{i3}\delta_{j3}\delta_{k3}\delta_{l3}. \end{aligned} \tag{2}$$

We use the upper mark " ˆ " to distinct the complex stiffness tensor $\hat{c}_{ijkl}$ from real c_{ijkl}. The above form of $\hat{c}_{ijkl}$ in terms of Keroneker delta function is very useful to express the scattering potential in terms of inner product of the polarizations and slowness vectors. It is also useful to decompose the scattering potential into the contributions from isotropic and anisotropic terms. In what follows we will explain in details the relationship between the components of stiffness tensor and Thomsen anisotropic parameters and quality factors. In an viscoelastic media, attenuation is characterized by quality factor Q. The following definitions establishes the framework of our analysis of scattering induced by anisotropic viscoelastic inclusions. In an attenuative media the stiffness tensor is complex which the real part is related to the elastic and anisotropic properties and imaginary part is related to the quality factors. Corresponding to each independent component of stiffness tensor there is a quality factor defined by $Q_{mn} = c_{mn}/c^{\mathrm{I}}_{mn}$, where c_{mn} and c^{I}_{mn} are real and imaginary parts of the complex stiffness tensor $\hat{c}_{mn}$. We also use the Voigt notation where $m = ij$ and $n = kl$. As a result $\hat{c}_{mn}$ can be written as a function of the quality factor tensor

$$\hat{c}_{mn} = c_{mn}\left(1 + iQ^{-1}_{mn}\right). \tag{3}$$

The dependency on Q factor express the dissipative effects. To describe the effects of anisotropy of the reflectivity there is a notation proposed by Thomsen (1986) which enables us to separate the influence of anisotropy from the other properties. Complex Thomsen parameters are defined

$$\hat{\varepsilon} = \frac{\hat{c}_{11} - \hat{c}_{33}}{2\hat{c}_{33}}, \tag{4}$$

$$\hat{\gamma} = \frac{\hat{c}_{66} - c_{55}}{2\hat{c}_{55}}, \tag{5}$$

$$\hat{\delta} = \frac{(\hat{c}_{13} + \hat{c}_{55})^2 - (\hat{c}_{33} - \hat{c}_{55})^2}{2\hat{c}_{33}(\hat{c}_{33} - \hat{c}_{55})}. \tag{6}$$

In order to address the contributions of attenuation in scattering potentials we need to decomposed the Thomsen parameters into the real and imaginary parts. Incorporating (3) in (4)-(6) and assuming the low-attenuation condition $Q_{ij}^{-1} \ll 1$ we get

$$\hat{\varepsilon} = \varepsilon + \frac{i}{2} Q_{33}^{-1} \varepsilon_Q, \tag{7}$$

$$\hat{\delta} = \delta + \frac{i}{2} Q_{33}^{-1} \delta_Q, \tag{8}$$

$$\hat{\gamma} = \gamma + \frac{i}{2} Q_{55}^{-1} \gamma_Q, \tag{9}$$

where ε, δ and γ are the well known Thompson parameters.These parameters are related to the phase velocities in an VTI media. ε is the P-wave anisotropy parameter refers to the anisotropy of rock in the absence of attenuation. It is the difference between the vertical and horizontal P-wave velocities. The parameter δ called small offset NMO factor which controls the near-vertical anisotropy as it is not a function of horizontal P-wave velocity. γ is related to the SH-wave anisotropy which is the difference between the vertical and horizontal SH-wave velocities. In addition the contributions from attenuation in medium in the Thomsen parameters are given by Yaping and Tsvankin (2006)

$$\begin{aligned}
\varepsilon_Q &= \frac{Q_{33} - Q_{11}}{Q_{11}} \\
\delta_Q &= 2\frac{c_{13}(c_{13} + c_{55})}{c_{33}(c_{33} - c_{55})} \frac{Q_{33} - Q_{13}}{Q_{13}} + \frac{c_{55}(c_{13} + c_{33})^2}{c_{33}(c_{33} - c_{55})^2} \frac{Q_{33} - Q_{55}}{Q_{55}}, \\
\gamma_Q &= \frac{Q_{55} - Q_{66}}{Q_{66}}.
\end{aligned}$$

Since we are interested in understanding of dependency of the perturbations in complex stiffness tensor in terms of the changes in medium properties, we first write the components of $\hat{c}_{ij}$ in terms of Thomsen parameters and quality factors. Incorporating equations (7)-(9) leads us to express the components of the stiffness tensor in terms of Thomsen parameters

$$\begin{aligned}
\hat{c}_{33} =& c_{33}(1 + iQ_{33}^{-1}), \\
\hat{c}_{55} =& c_{55}(1 + iQ_{55}^{-1}), \\
\hat{c}_{11} =& c_{33}(1 + 2\varepsilon) + iQ_{33}^{-1} c_{33}(1 + 2\varepsilon + \varepsilon_Q), \\
\hat{c}_{66} =& c_{55}(1 + 2\gamma) + iQ_{55}^{-1} c_{55}(1 + 2\gamma + \gamma_Q), \\
\hat{c}_{13} =& c_{33}(1 + \delta) - 2c_{55} + iQ_{33}^{-1} c_{33}(1 + \delta + \delta_Q) - 2iQ_{55}^{-1} c_{55}.
\end{aligned} \tag{10}$$

Throughout the present work we assume that the weak anisotropy $|\gamma|, |\delta|, |\varepsilon| \ll 1$ and weak attenuation $Q_{33}^{-1}, Q_{55}^{-1} \ll 1$. It is clear that real part of the stiffness tensor depends to five elastic-anisotropic components; c_{33} related to the P-wave impendence, c_{55} related to the shear modulus, and Thomsen parameters $\varepsilon,\ \delta,\ \gamma$. The imaginary part including five viscoelastic-anisotropic parameters; P-wave quality factor Q_{33}^{-1}, S-wave quality factor Q_{55}^{-1} and Q-dependent Thomsen parameters $\varepsilon_Q,\ \Delta_Q,\ \gamma_Q$. We want to have a theory which describe the scattering from inclusions in viscoelastic VTI media. The natural basic tool for such a theory will be explained in next section based on the equations we have derived yet. In what follows for notational simplicity we use Q_{P} for P-wave quality factor instead of Q_{33} and Q_{S} for S-wave quality factor instead of Q_{55}.

PERTURBATIONS IN STIFFNESS TENSOR AND BORN APPROXIMATION

In this section we derive the central result of this paper, scattering potential for scattering of P-,SI and SII waves. Figure (1.a) , is a schematic description of the two layer viscoelastic VTI medium with weak anisotropy and attenuation. The properties can be divided into four parts: elastic properties including density, P- and S-wave velocities; viscoelastic properties including P- and S-wave quality factors; anisotropic parameters including three Thomsen parameters and three anisotropic viscoelastic Thomsen parameters. For a low contrast medium properties across the boundary are slightly different. As a result a small portion of the incidence wave is reflected from the boundary and the majority transmitted to the lower medium. In this case we can linearized the reflection coefficients in terms of the first order perturbations in medium properties.

On the other hand figure (1.b) illustrate the configuration of the scattering in anisotropic viscoelastic medium in the context of Born approximation. The background medium is described by three elastic parameters P-wave velocity V_{P0}, S-wave velocity V_{S0} and density ρ_0; two viscoelastic parameters P-wave quality factor Q_{P0} and S-wave quality factor Q_{S0}; three anisotropic parameters $\varepsilon_0,\ \delta_0$ and γ_0 and corresponding Q-dependent parameters $\varepsilon_{Q0}, \delta_{Q0}, \delta_{Q0}$. Wave traveling in the reference medium interacts with these scatter points that randomly distributed. Technically an incident wave undergoes a sequence of multiple scattering events from the perturbations. If we consider the whole scattering process as a series, the first term describes the single scattering of the incident wave, while the following terms describe then scattering of successively higher order. For the small perturbations, higher-order terms have negligible contributions. Thus, only the first iteration of the series is taken into account, i.e. only single scattering. Scattering from elastic inclusions is well known and studied by many authors and recently the volume scattering of the inhomogeneous and homogeneous waves in an low-loss viscoelastic media has been developed by Moradi and Innanen (2016).

For a medium with welded boundary the difference between the density in lower layer (ρ_2) and density in upper layer (ρ_1) is represented by $\Delta\rho = \rho_2 - \rho_1$. Also properties without subscripts $1(2)$ refers to the average of upper and lower properties, for example $\rho = (\rho_1 + \rho_2)/2$, is the average of the upper and lower density. Fractional changes in density across the boundary is given by $\Delta\rho/\rho$ which for low-contrast medium is much smaller than unity. The same notation is valid for the fractional changes in P- and S-wave velocities and P- and S-wave quality factors. We use the fractional changes for the five

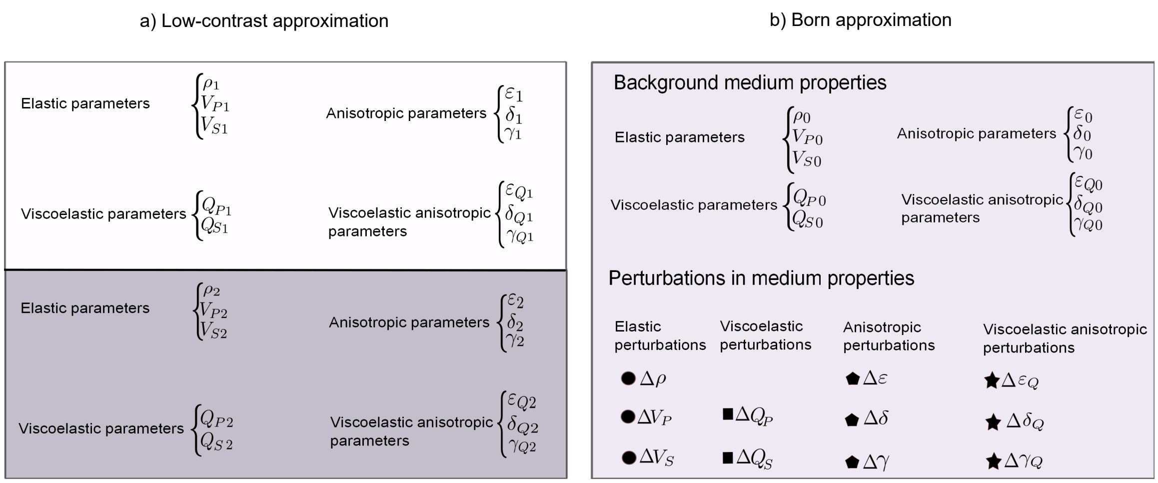

FIG. 1. a) Schematic description of the two anisotropic viscoelastic layer Medium, the upper medium is characterized by its three elastic parameters P-wave velocity V_{P1}, S-wave velocity V_{S1} and density ρ_1; two viscoelastic parameters P-wave quality factor Q_{P1} and S-wave quality factor Q_{S1}; three anisotropic parameters $\varepsilon_1,\ \delta_1$ and γ_1 and corresponding Thomsen parameters quality factor $\varepsilon_{Q1}, \delta_{Q1}, \delta_{Q1}$. is characterized by its three elastic parameters P-wave velocity V_{P2}, S-wave velocity V_{S2} and density ρ_2; two viscoelastic parameters P-wave quality factor Q_{P2} and S-wave quality factor Q_{S2}; three anisotropic parameters $\varepsilon_2,\ \delta_2$ and γ_2 and corresponding Thomsen parameters quality factor $\varepsilon_{Q1}, \delta_{Q2}, \delta_{Q2}$. b) Diagram illustrating the mathematics of Born approximation based on the perturbation theory. The background medium is anisotropic viscoelastic medium characterized by its three elastic parameters P-wave velocity V_{P0}, S-wave velocity V_{S0} and density ρ_0; two viscoelastic parameters P-wave quality factor Q_{P0} and S-wave quality factor Q_{S0}; three anisotropic parameters $\varepsilon_0,\ \delta_0$ and γ_0 and corresponding Thomsen parameters quality factor $\varepsilon_{Q0}, \delta_{Q0}, \delta_{Q0}$. Perturbations in an anisotropic viscoelastic medium are characterized by 11 components represented by $\Delta\rho,\ \Delta V_{\mathrm{P}},\ \Delta V_{\mathrm{SE}},\ \Delta\varepsilon,\ \Delta\delta,\ \Delta\gamma,\ \Delta Q_{\mathrm{P}},\ \Delta Q_{\mathrm{S}},\ \Delta\varepsilon_Q,\ \Delta\delta_Q,\ \Delta\gamma_Q$.

aforementioned parameters as the differences in these quantities are grater than unity and can not be used as the perturbation terms. However for Thompson parameters since for the weak anisotropy they are much smaller than unity we can use the differences in Thompson parameters as perturbation terms.

In the context of Born approximation $\Delta\rho = \rho - \rho_0$ denotes the difference between the density in actual medium ρ and density in the background medium ρ_0. In this case the fractional changes in properties can be represented by either $\Delta\rho/\rho_0$ or $\Delta\rho/\rho$ as both of them are much smaller than unity and can be used as the perturbation. Similar to the low contrast medium, the perturbation terms for anisotropic parameters are given by the difference between the properties instead of the fractional changes. Variation in properties also called fractional changes or inclusions. Waves propagating in the background can be scattered by the inclusions in eleven properties are shown in figure (1.b).

Let us start our discussion with perturbations in anisotropic parameters. Changes in complex Thomsen parameters can be expressed in terms of changes in real and in Q-dependent Thomsen parameters

$$\Delta\hat{\varepsilon} = \left(\varepsilon + \frac{i}{2}Q_{\mathrm{P}}^{-1}\varepsilon_Q\right) - \left(\varepsilon_0 + \frac{i}{2}Q_{\mathrm{P0}}^{-1}\varepsilon_{Q0}\right), \tag{11}$$

$$\Delta\hat{\delta} = \left(\delta + \frac{i}{2}Q_{\mathrm{P}}^{-1}\delta_Q\right) - \left(\delta_0 + \frac{i}{2}Q_{\mathrm{P0}}^{-1}\delta_{Q0}\right), \tag{12}$$

$$\Delta\hat{\gamma} = \left(\gamma + \frac{i}{2}Q_{\mathrm{S}}^{-1}\gamma_Q\right) - \left(\gamma_0 + \frac{i}{2}Q_{\mathrm{S0}}^{-1}\gamma_{Q0}\right). \tag{13}$$

As we have discussed earlier, quantities without the subscript '0' refers to the quantities in the actual medium. To simplify the above expressions we replace all quantities in actual medium in terms of their values in reference medium and perturbations. First for the Thomsen parameters we have

$$\begin{aligned} \varepsilon &= \varepsilon_0 + \Delta\varepsilon, \quad & \varepsilon_Q &= \varepsilon_{Q0} + \Delta\varepsilon_Q, \\ \delta &= \delta_0 + \Delta\delta, \quad & \delta_Q &= \delta_{Q0} + \Delta\delta_Q, \\ \gamma &= \gamma_0 + \Delta\gamma, \quad & \gamma_Q &= \gamma_{Q0} + \Delta\gamma_Q. \end{aligned} \tag{14}$$

For inverse P- and S-wave quality factors Q_{P}^{-1} and Q_{S}^{-1} we can write

$$\begin{aligned} Q_{\mathrm{P}}^{-1} &= (Q_{\mathrm{P0}} + \Delta Q_{\mathrm{P}})^{-1} \approx Q_{\mathrm{P0}}^{-1}\left(1 - \frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}}\right), \\ Q_{\mathrm{S}}^{-1} &= (Q_{\mathrm{S0}} + \Delta Q_{\mathrm{S}})^{-1} \approx Q_{\mathrm{S0}}^{-1}\left(1 - \frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}}\right). \end{aligned} \tag{15}$$

Where we take advantages of both low-loss attenuation and weak contrast assumptions respectively given by $\left(Q_{\mathrm{P0}}^{-1}, Q_{\mathrm{S0}}^{-1}\right) \ll 1$ and $(\Delta Q_{\mathrm{P}}/Q_{\mathrm{P}}, \Delta Q_{\mathrm{S}}/Q_{\mathrm{S}}) \ll 1$. Incorporating equations (14) and (15) into equations (11)-(13) and considering the first order in perturbations, we arrive at

$$\Delta\hat{\varepsilon} = \Delta\varepsilon + \frac{i}{2}Q_{\mathrm{P0}}^{-1}\Delta\varepsilon_Q, \tag{16}$$

$$\Delta\hat{\delta} = \Delta\delta + \frac{i}{2}Q_{\mathrm{P0}}^{-1}\Delta\delta_Q, \tag{17}$$

$$\Delta\hat{\gamma} = \Delta\gamma + \frac{i}{2}Q_{\mathrm{S0}}^{-1}\Delta\gamma_Q. \tag{18}$$

As we can see for a elastic background as $Q_{\mathrm{P0}}^{-1} = Q_{\mathrm{S0}}^{-1} = 0$, the Q-dependent part vanishes and has no effect on the radiation patterns. In trying to understand the dependency of the perturbations in stiffness tensor to the changes in density, velocities and quality factors, we expand the changes in isotropic components c_{33} and c_{55}, as all other components of stiffness tensor are expressed in terms of these two components. Let us first consider to the change in c_{33} component

$$\Delta\hat{c}_{33} = \hat{c}_{33} - \hat{c}_{330}^{(0)} = \rho\hat{V}_P^2 - \rho_0\hat{V}_{P0}^2. \tag{19}$$

Where the complex $\hat{V}_P$ and $\hat{V}_{P0}$ are the P-wave velocity in actual and reference background which are written in terms of elastic P-wave velocity and P-wave quality factors as

$$\begin{aligned} \hat{V}_{\mathrm{P}}^2 &= V_{\mathrm{P}}^2\left(1 + iQ_{\mathrm{P}}^{-1}/2\right)^2 \approx V_{\mathrm{P}}^2\left(1 + iQ_{\mathrm{P}}^{-1}\right), \\ \hat{V}_{\mathrm{P0}}^2 &= V_{\mathrm{P0}}^2\left(1 + iQ_{\mathrm{P0}}^{-1}/2\right)^2 \approx V_{\mathrm{P0}}^2\left(1 + iQ_{\mathrm{P0}}^{-1}\right). \end{aligned} \tag{20}$$

Inserting expressions in (20) into (19) and using $\rho = \rho_0 + \Delta\rho$ and $V_{\mathrm{P}}^2 = V_{\mathrm{P0}}^2 + 2V_{\mathrm{P0}}\Delta V_{\mathrm{P}}$ we finally arrive at

$$\frac{\Delta\hat{c}_{33}}{\hat{c}_{330}} = \left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_P}{V_P}\right) - iQ_{\mathrm{P0}}^{-1}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}}, \tag{21}$$

where $\hat{c}_{\mathrm{P0}} = \rho_0\hat{V}_{\mathrm{P0}}^2$. We can see that the fractional perturbation in $\hat{c}_{33}$ decomposed into two component. The real part is the perturbations in density and P-wave velocity and the imaginary part as a function of changes in density, P-wave velocity and changes in P-wave quality factor. In this expression we can see that even if the changes in P-wave quality factor is zero but the reference medium is viscoelastic, the contributions for anelasticity is not zero. In a similar manner we calculate the fractional perturbation in c_{55}

$$\frac{\Delta\hat{c}_{55}}{\hat{c}_{\mathrm{S0}}} = \left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_S}{V_S}\right) - iQ_{\mathrm{S0}}^{-1}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}}, \tag{22}$$

where $\hat{c}_{\mathrm{S0}} = \rho_0\hat{V}_{\mathrm{S0}}^2$. As we can see changes in c_{55} depend to the changes in density, S-wave velocity and S-wave quality factor. Changes in other components of the stiffness tensor are expressed in terms of changes in c_{33}, c_{55} and changes in Thomsen parameters.

$$\begin{aligned} \Delta\hat{c}_{11} &= \hat{c}_{11} - \hat{c}_{11}^{(0)} = \Delta\hat{c}_{33} + 2\hat{c}_{33}^{(0)}\Delta\hat{\varepsilon}, \\ \Delta\hat{c}_{13} &= \hat{c}_{13} - \hat{c}_{13}^{(0)} = \Delta\hat{c}_{33} - 2\Delta\hat{c}_{55} + c_{33}^{(0)}\Delta\hat{\delta}, \\ \Delta\hat{c}_{66} &= \hat{c}_{66} - \hat{c}_{66}^{(0)} = \Delta\hat{c}_{55} + 2\hat{c}_{55}^{(0)}\Delta\hat{\gamma}, \\ \Delta\hat{c}_{12} &= \hat{c}_{12} - \hat{c}_{12}^{(0)} = \Delta\hat{c}_{33} - 2\Delta\hat{c}_{55} + 2\hat{c}_{33}^{(0)}\Delta\hat{\varepsilon} - 4\hat{c}_{55}^{(0)}\Delta\hat{\gamma}. \end{aligned} \tag{23}$$

The perturbation in the aforementioned components of stiffness tensor are functions of changes in elastic and anelastic parameters, Thomsen parameters and Q-dependent Thomsen parameters. These are the basic elements of out of which scattering potentials are constructed by changes in different properties in media. To conclude this section let us observe how changes in $\hat{c}_{ijkl}$ can decomposed into the isotropic and anisotropic terms. We will see later that such expressions help us to compare our result with the previously driven scattering potentials for elastic, and viscoelastic media. Using (2) and (23), it follows now that the changes in complex stiffness tensor can be written in the following form

$$\Delta\hat{c}_{ijkl} = \Delta\hat{c}^{\mathrm{Iso}}_{ijkl} + \Delta\hat{c}^{\varepsilon}_{ijkl} + \Delta\hat{c}^{\delta}_{ijkl} + \Delta\hat{c}^{\gamma}_{ijkl}, \tag{24}$$

where the isotropic part of the perturbation is given by

$$\Delta\hat{c}^{\mathrm{Iso}}_{ijkl} = \Delta\hat{c}_{33}\delta_{ij}\delta_{kl} + \Delta\hat{c}_{55}(\delta_{ik}\delta_{jk} - 2\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk}), \tag{25}$$

and the perturbations related to the Thomsen parameters are

$$\begin{aligned}
\Delta\hat{c}^{\varepsilon}_{ijkl} &= 2\hat{c}^{(0)}_{33}\Delta\hat{\varepsilon}\left\{\delta_{ij}\delta_{kl}\right\}_{[1,2]}, \\
\Delta\hat{c}^{\delta}_{ijkl} &= \hat{c}^{(0)}_{33}\Delta\hat{\delta}(\delta_{ij}\delta_{k3}\delta_{l3} + \delta_{kl}\delta_{i3}\delta_{j3})_{[1,2]}, \\
\Delta\hat{c}^{\gamma}_{ijkl} &= 2\hat{c}^{(0)}_{55}\Delta\hat{\gamma}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl})_{[1,2]} - 2\hat{c}^{(0)}_{55}\Delta\hat{\gamma}(\delta_{jk}\delta_{i3}\delta_{l3} + \delta_{jl}\delta_{i3}\delta_{k3} - 2\delta_{kl}\delta_{i3}\delta_{j3}).
\end{aligned} \tag{26}$$

$[1,2]$ means the subscripts only take 1,2. The isotropic part $\Delta\hat{c}^{\mathrm{Iso}}_{ijkl}$ including the changes in density, P-wave velocity, S-wave velocity and P-wave quality factor, S-wave quality factor. $\Delta\hat{c}^{\varepsilon}_{ijkl}$ is sensitive to changes in Thomsen parameter ε and Q-dependent Thomsen parameter ε_Q. Similar dependency is valid for $\Delta\hat{c}^{\gamma}_{ijkl}$ and $\Delta\hat{c}^{\delta}_{ijkl}$. In next section based on definitions of the polarization and slowness vectors and perturbations in stiffness tensor we formulate the scattering potentials.

SCATTERING POTENTIALS

Born approximation yields an expression for scattered wave which is linear in fractional changes in medium properties. Second and higher order terms in this approximation are neglected as these terms are related to the stronger contrast between the reference and actual medium properties. Perturbations in stiffness tensor is essential to derive the scattering potentials. In the case of elastic medium we have the perturbation in density, P- and S-wave velocity. If we add attenuation and anisotropy to medium, in addition to changes in elastic parameters we have the perturbations in three Thomsen parameters and three Q-related Thomsen parameters we introduced in last section. Let us first consider to the case that background medium and perturbations are both anisotropic viscoelastic with weak anisotropy and weak attenuation. In this case the all parameters related to the background medium are labeled by "0" otherwise parameters are related to the actual medium. Based on the Born approximation, the difference between actual and background medium are characterized by the perturbations. In what follows we consider the green function in actual medium as the scattered wavefield whose incident wave is the green function in reference medium. The so called Born kernel which is related to the scattering potential is given by

(Beylkin and Burridge, 1990)

$$\begin{aligned}\mathrm{K}_{pq}(\mathbf{x},\mathbf{x}_r,\mathbf{x}_s,\omega) \equiv&\omega^2\Delta\rho(\mathbf{x})\mathrm{G}^{(0)}_{ip}(\mathbf{x}_r,\mathbf{x},\omega)\mathrm{G}^{(0)}_{iq}(\mathbf{x},\mathbf{x}_s,\omega)\\ &-\Delta c_{ijkl}(\mathbf{x})\frac{\partial \mathrm{G}^{(0)}_{ip}(\mathbf{x}_r,\mathbf{x},\omega)}{\partial x_j}\frac{\partial \mathrm{G}^{(0)}_{kq}(\mathbf{x},\mathbf{x}_s,\omega)}{\partial x_l},\end{aligned} \tag{27}$$

In the right hand side we applied the Einstein sum rule which states that there is sum over repeated indexes. In addition, $\Delta\hat{c}_{ijkl} = \hat{c}_{ijkl} - \hat{c}^{(0)}_{ijkl}$ is the difference between the non zero components of the stiffness tensor in actual and reference media. Additionally $\mathrm{G}^{(0)}_{iq}(\mathbf{x},\mathbf{x}_s,\omega)$ is the green function in the reference medium responsible for the propagation of the wave from source point $\mathbf{x}_s$ to the point $\mathbf{x}$ where perturbations in density, $\Delta\rho$, and stiffness tensor Δc_{ijkl} interact with the wavefield. After scattering, green function $\mathrm{G}^{(0)}_{ip}(\mathbf{x}_r,\mathbf{x},\omega)$ deliver the wavefield to the receiver point $\mathbf{x}_r$. We can write the green function in terms of the polarization vector in the source point $\mathbf{x}_s$ as

$$\begin{aligned}\mathrm{G}^{(0)}_{ip}(\mathbf{x}_r,\mathbf{x},\omega) &\equiv \mathcal{S}_i g_p\left(\mathbf{x}_r-\mathbf{x},\omega\right),\\ \mathrm{G}^{(0)}_{iq}(\mathbf{x},\mathbf{x}_s,\omega) &\equiv \mathcal{I}_i g_q\left(\mathbf{x}-\mathbf{x}_s,\omega\right),\end{aligned} \tag{28}$$

where $\boldsymbol{\mathcal{S}}$ and $\boldsymbol{\mathcal{I}}$ respectively are the polarization vector of the scattered and incident wave, defined at the scatter point $\mathbf{x}$. In addition $g_p\left(\mathbf{x}_r-\mathbf{x},\omega\right)$ is a vector field including the function $e^{i\omega\mathbf{k}^{\mathrm{Sc}}\cdot(\mathbf{x}_r-\mathbf{x})}$ where $\mathbf{k}^{\mathrm{Sc}}$ is the slowness vector of the scattered wave field. In the same way, $g_q\left(\mathbf{x}-\mathbf{x}_s,\omega\right)$ is a vector field defined at the scatter point $\mathbf{x}$ including the function $e^{i\omega\mathbf{k}^{\mathrm{In}}\cdot(\mathbf{x}-\mathbf{x}_s)}$ where $\mathbf{k}^{\mathrm{In}}$ is the slowness vector of the incident wavefield. Consequently, the differentiations in (27) takes the form

$$\begin{aligned}\frac{\partial \mathrm{G}^{(0)}_{ip}(\mathbf{x}_r,\mathbf{x},\omega)}{\partial x_j} &= -i\omega\mathcal{S}_i\mathrm{k}^{\mathrm{Sc}}_j g_p\left(\mathbf{x}_r-\mathbf{x},\omega\right),\\ \frac{\partial \mathrm{G}^{(0)}_{kq}(\mathbf{x},\mathbf{x}_s,\omega)}{\partial x_l} &= i\omega\mathcal{I}_k\mathrm{k}^{\mathrm{In}}_l g_q\left(\mathbf{x}-\mathbf{x}_s,\omega\right).\end{aligned} \tag{29}$$

Inserting (28) and (29) in equation (27) we get $\mathrm{K}_{pq} = \omega^2\mathrm{S}(g_p g_q)$, where the frequency independent scattering potential is given by

$$\mathrm{S} = (\boldsymbol{\mathcal{S}}\cdot\boldsymbol{\mathcal{I}})\Delta\rho - \eta_{mn}\Delta\hat{c}_{mn} = (\boldsymbol{\mathcal{S}}\cdot\boldsymbol{\mathcal{I}})\Delta\rho - (\mathcal{S}_i\mathrm{k}^{\mathrm{Sc}}_j\mathcal{I}_k\mathrm{k}^{\mathrm{In}}_l)\Delta\hat{c}_{ijkl} \tag{30}$$

where $m = ij$ and $n = kl$ refer to the Voigt notation. The scattering potential is a central concept in Full waveform inversion as it is considered as sensitivity kernels. It also describe the radiation pattern of scattered wavefield. Equation (30) is derived from the Lippmann-Schwinger equation which represent the scattered wave as a superposition of the outgoing waves scattered from the inclusions in media. In previous section we showed that changes in stiffness tensor can be decomposed into the isotropic and anisotropic parts. By incorporating (25) and (26) in (30) and rearranging the the scattering potential in terms of changes in isotropic and anisotropic parameters we have

$$\mathrm{S}_{\mathrm{VTI}} = \mathrm{S}^{\mathrm{Iso}} + \mathrm{S}^{\mathrm{Ani}}_{\mathrm{VTI}}, \tag{31}$$

with isotropic and anisotropic parts respectively given by

$$
\begin{aligned}
\mathrm{S}^{\mathrm{Iso}} &= (\boldsymbol{\mathcal{S}} \cdot \boldsymbol{\mathcal{I}})\Delta\rho \\
&- (\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{In}})\Delta\hat{c}_{33} \\
&- \left\{(\boldsymbol{\mathcal{S}} \cdot \boldsymbol{\mathcal{I}})(\mathbf{k}^{\mathrm{Sc}} \cdot \mathbf{k}^{\mathrm{In}}) + (\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{In}})(\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{Sc}}) - 2(\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{In}})\right\}\Delta\hat{c}_{55}, \\
\mathrm{S}^{\mathrm{Ani}}_{\mathrm{VTI}} &= -2\hat{c}^{(0)}_{33}\left\{(\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{In}})\right\}_{[x,y]}\Delta\hat{\varepsilon} \\
&- \hat{c}^{(0)}_{33}\left\{(\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{Sc}})_{[x,y]}\mathcal{I}_{\mathrm{z}}\mathrm{k}^{\mathrm{In}}_{\mathrm{z}} + (\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{In}})_{[x,y]}\mathcal{S}_{\mathrm{z}}\mathrm{k}^{\mathrm{Sc}}_{\mathrm{z}}\right\}\Delta\hat{\delta} \\
&- 2\hat{c}^{(0)}_{55}\left\{(\boldsymbol{\mathcal{S}} \cdot \boldsymbol{\mathcal{I}})(\mathbf{k}^{\mathrm{Sc}} \cdot \mathbf{k}^{\mathrm{In}}) + (\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{In}})(\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{Sc}}) - 2(\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{In}})\right\}_{[x,y]}\Delta\hat{\gamma}.
\end{aligned}
\tag{32}
$$

Where superscripts 'Sc' and 'In', respectively refer to the scattered and incident waves. Additionally $[x, y]$ indicates the only x and y components in the expression. For example $(\boldsymbol{\mathcal{S}} \cdot \boldsymbol{\mathcal{I}})_{[x,y]} = \mathcal{S}_x\mathcal{I}_x + \mathcal{S}_y\mathcal{I}_y$. This unique decomposition of the scattering potential into isotropic and anisotropic parts is quite significant as we will see in the course of our discussions.

We note that the isotropic part includes not only contributions from changes in elastic properties but also including the contributions from anelasticity in reference medium as well as changes in quality factors. A similar explanation is valid for the anisotropic part, it is sensitive to the changes in both real Thomsen parameters and Q-dependent Thomsen parameters. To evaluate the scattering potential we need to determine the slowness and polarization vectors for scattered and incident waves. In a viscoelastic media wave number vector is given by a complex vector $\mathbf{K} = \mathbf{P} - i\mathbf{A}$, where $\mathbf{P}$ is called the propagation vector and $\mathbf{A}$ is attenuation vector which determines the direction of maximum attenuation. If propagation and attenuation be in the same direction wave is called homogeneous. In what follows we consider to the incident homogeneous wave which results the scattered homogenous wave as well. In this case the polarization and slowness vectors for scattered and incident waves for P-wave are given by

$$
\begin{aligned}
\boldsymbol{\mathcal{I}}_{\mathrm{P}} &= \hat{V}_{\mathrm{P0}}\mathbf{k}^{\mathrm{In}}_{\mathrm{P}}, \\
\boldsymbol{\mathcal{S}}_{\mathrm{P}} &= \hat{V}_{\mathrm{P0}}\mathbf{k}^{\mathrm{Sc}}_{\mathrm{P}}, \\
\mathbf{k}^{\mathrm{In}}_{\mathrm{P}} &= \frac{\mathbf{K}^{\mathrm{In}}_{\mathrm{P}}}{\omega} = \frac{1}{\omega}\left(\mathbf{P}^{\mathrm{In}}_{\mathrm{P}} - i\mathbf{A}^{\mathrm{In}}_{\mathrm{P}}\right) \\
\mathbf{k}^{\mathrm{Sc}}_{\mathrm{P}} &= \frac{\mathbf{K}^{\mathrm{Sc}}_{\mathrm{P}}}{\omega} = \frac{1}{\omega}\left(\mathbf{P}^{\mathrm{Sc}}_{\mathrm{P}} - i\mathbf{A}^{\mathrm{Sc}}_{\mathrm{P}}\right)
\end{aligned}
\tag{33}
$$

In above equations, θ_P is the P-wave incident angle, the angle that direction of the incident P-wave makes with the z-axis. $\boldsymbol{\mathcal{I}}_{\mathrm{P}}$ and $\boldsymbol{\mathcal{S}}_{\mathrm{P}}$ respectively are the incident and scattered P-wave polarization vectors; $\mathbf{k}^{\mathrm{In}}_{\mathrm{P}}$ and $\mathbf{k}^{\mathrm{Sc}}_{\mathrm{P}}$ respectively are the the incident and scattered P-wave slowness vectors. Also incident P-wave propagation and attenuation vectors respectively defined by $\mathbf{P}^{\mathrm{In}}_{\mathrm{P}}$ and $\mathbf{A}^{\mathrm{In}}_{\mathrm{P}}$, and for scattered wave by $\mathbf{P}^{\mathrm{Sc}}_{\mathrm{P}}$ and $\mathbf{A}^{\mathrm{Sc}}_{\mathrm{P}}$ (Appendix B). Additionally complex P-wave velocity in background medium is defined by $\hat{V}_{\mathrm{P0}} = V_{\mathrm{P0}}(1 + \frac{i}{2}Q^{-1}_{\mathrm{P0}})$ with elastic P-wave velocity V_{P0} and P-wave quality factor Q_{P0} both in reference medium. Furthermore for SI-wave also we have

$$\begin{aligned}
\boldsymbol{\mathcal{I}}_{\mathrm{S}} &= \hat{V}_{\mathrm{S0}}\left(\mathbf{y}\times\mathbf{k}_{\mathrm{S}}^{\mathrm{In}}\right),\\
\boldsymbol{\mathcal{S}}_{\mathrm{S}} &= \hat{V}_{\mathrm{S0}}\left(\mathbf{y}\times\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}}\right),\\
\mathbf{k}_{\mathrm{S}}^{\mathrm{In}} &= \frac{\mathbf{K}_{\mathrm{S}}^{\mathrm{In}}}{\omega} = \frac{1}{\omega}\left(\mathbf{P}_{\mathrm{S}}^{\mathrm{In}} - i\mathbf{A}_{\mathrm{S}}^{\mathrm{In}}\right),\\
\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}} &= \frac{\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}}}{\omega} = \frac{1}{\omega}\left(\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}} - i\mathbf{A}_{\mathrm{S}}^{\mathrm{Sc}}\right).
\end{aligned} \tag{34}$$

Here we defined θ_S as the S-wave incident angle, $\boldsymbol{\mathcal{I}}_{\mathrm{S}}$ and $\boldsymbol{\mathcal{S}}_{\mathrm{S}}$ respectively are the incident and scattered P-wave polarization vectors; $\mathbf{k}_{\mathrm{S}}^{\mathrm{In}}$ and $\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}}$ respectively are the the incident and scattered S-wave slowness vectors. Additionally complex S-wave velocity in background medium is defined by $\hat{V}_{\mathrm{S0}} = V_{\mathrm{S0}}(1+\frac{i}{2}Q_{\mathrm{S0}}^{-1})$ with elastic S-wave velocity V_{S0} and S-wave quality factor Q_{S0} both in reference medium. Also incident S-wave propagation and attenuation vectors respectively defined by $\mathbf{P}_{\mathrm{S}}^{\mathrm{In}}$ and $\mathbf{A}_{\mathrm{S}}^{\mathrm{In}}$, and for scattered wave by $\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}}$ and $\mathbf{A}_{\mathrm{S}}^{\mathrm{Sc}}$ (Appendix).

We note that the perturbations in stiffness tensor, expressed in the last section, is a function of the changes in eleven anisotropic viscoelastic properties as a result the scattering potential can change with corresponding medium properties. Inserting the polarization/slowness components (33) into (32), the scattering potential for scattering of P-wave to P-wave is given by

$$\begin{aligned}
\mathrm{S}_{\mathrm{PP}} &= (\boldsymbol{\mathcal{S}}_{\mathrm{P}}\cdot\boldsymbol{\mathcal{I}}_{\mathrm{P}})\Delta\rho\\
&- (\boldsymbol{\mathcal{S}}_{\mathrm{P}}\cdot\mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}}_{\mathrm{P}}\cdot\mathbf{k}_{\mathrm{P}}^{\mathrm{In}})\Delta\hat{c}_{33}\\
&- \left[(\boldsymbol{\mathcal{S}}_{\mathrm{P}}\cdot\boldsymbol{\mathcal{I}}_{\mathrm{P}})(\mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}}\cdot\mathbf{k}_{\mathrm{P}}^{\mathrm{In}}) - 2(\boldsymbol{\mathcal{S}}_{\mathrm{P}}\cdot\mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}}_{\mathrm{P}}\cdot\mathbf{k}_{\mathrm{P}}^{\mathrm{In}}) + (\boldsymbol{\mathcal{S}}_{\mathrm{P}}\cdot\mathbf{k}_{\mathrm{P}}^{\mathrm{In}})(\boldsymbol{\mathcal{I}}_{\mathrm{P}}\cdot\mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}})\right]\Delta\hat{c}_{55}\\
&- 2\hat{c}_{33}^{(0)}(\mathcal{S}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{In}})\Delta\hat{\varepsilon}\\
&- \hat{c}_{33}^{(0)}(\mathcal{S}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Pz}}\mathrm{k}_{\mathrm{Pz}}^{\mathrm{In}} + \mathcal{S}_{\mathrm{Pz}}\mathrm{k}_{\mathrm{Pz}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{In}})\Delta\hat{\delta}.
\end{aligned}$$

Where, $\Delta\hat{c}_{33}$ given (21) contains the contributions from the fractional in changes in density, P-wave velocity, P-wave quality factor. Also $\Delta\hat{c}_{55}$ given by (22) including the fractional in changes in density, S-wave velocity, S-wave quality factor. Additionally $\Delta\hat{\varepsilon}$ and $\Delta\hat{\delta}$ are given by equations (11) and (12) include the changes in Thomsen anisotropic parameters and Q-dependent anisotropic parameters. Consequently the P-to-P scattering potential is sensitive to changes in all other properties except changes in γ and γ_Q. Furthermore for converted P-wave we have

$$\begin{aligned}
\mathrm{S}_{\mathrm{PSI}} &= (\boldsymbol{\mathcal{S}}_{\mathrm{S}}\cdot\boldsymbol{\mathcal{I}}_{\mathrm{P}})\Delta\rho\\
&- \left[(\boldsymbol{\mathcal{S}}_{\mathrm{S}}\cdot\boldsymbol{\mathcal{I}}_{\mathrm{P}})(\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}}\cdot\mathbf{k}_{\mathrm{P}}^{\mathrm{In}}) + (\boldsymbol{\mathcal{S}}_{\mathrm{S}}\cdot\mathbf{k}_{\mathrm{P}}^{\mathrm{In}})(\boldsymbol{\mathcal{I}}_{\mathrm{P}}\cdot\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}})\right]\Delta\hat{c}_{55}\\
&- 2\hat{c}_{33}^{(0)}(\mathcal{S}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{In}})\Delta\hat{\varepsilon}\\
&- \hat{c}_{33}^{(0)}(\mathcal{S}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Pz}}\mathrm{k}_{\mathrm{Pz}}^{\mathrm{In}} + \mathcal{S}_{\mathrm{Sz}}\mathrm{k}_{\mathrm{Sz}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{In}})\Delta\hat{\delta}.
\end{aligned}$$

Comparing to the P-to-P mode, the $\Delta\hat{c}_{33}$ terms does not appear here, as a result changes in P-wave velocity and P-wave quality factors have a no contributions in P-to-SI scattering potential. For scattering of SI-wave to SI-wave the scattering potential

$$\begin{aligned}
\mathrm{S}_{\mathrm{SISI}} &= (\boldsymbol{\mathcal{S}}_{\mathrm{S}} \cdot \boldsymbol{\mathcal{I}}_{\mathrm{S}})\Delta\rho \\
&- \left[(\boldsymbol{\mathcal{S}}_{\mathrm{S}} \cdot \boldsymbol{\mathcal{I}}_{\mathrm{S}})(\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}} \cdot \mathbf{k}_{\mathrm{S}}^{\mathrm{In}}) + (\boldsymbol{\mathcal{S}}_{\mathrm{S}} \cdot \mathbf{k}_{\mathrm{S}}^{\mathrm{In}})(\boldsymbol{\mathcal{I}}_{\mathrm{S}} \cdot \mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}})\right] \Delta\hat{c}_{55} \\
&- 2\hat{c}_{33}^{(0)}(\mathcal{S}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{In}})\Delta\hat{\varepsilon} \\
&- \hat{c}_{33}^{(0)}(\mathcal{S}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Sz}}\mathrm{k}_{\mathrm{Sz}}^{\mathrm{In}} + \mathcal{S}_{\mathrm{Sz}}\mathrm{k}_{\mathrm{Sz}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{In}})\Delta\hat{\delta}.
\end{aligned}$$

The dependency of SI-to-SI mode to changes in properties is the same as P-to-SI mode. For scattering of SII-wave to SII-wave the scattering potential is

$$\mathrm{S}_{\mathrm{SIISII}} = \Delta\rho - (\mathbf{k}_{\mathrm{S}}^{\mathrm{In}} \cdot \mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}})\Delta\hat{c}_{55} - 2c_{55}^{(0)}(\mathrm{k}_{\mathrm{Sx}}^{\mathrm{Sc}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{In}})\Delta\hat{\gamma}.$$

These expressions for scattering potentials demonstrate the role of changes in anisotropic parameters in the scattering process. Changes in $\hat{\varepsilon}$ and $\hat{\delta}$ affect the P-to-P, P-to-SI and SI-to-SI scattering modes meanwhile changes in $\hat{\gamma}$ occur only for SII to SII scattering mode. In next section we will show that scattering potential can be separate out to the the following components

- **Isotropic Elastic (IS)**: sensitive to the changes in density P-and S-wave velocity. This terms is the scattering potential for scattering of seismic wave in an isotropic elastic reference media.
- **Anisotropic Elastic (AE)**: sensitive to the changes in Thomsen parameters. In the case that media is isotropic this term goes to zero. (IS+AE)-term is the scattering potential for scattering of elastic wave in an anisotropic-elastic referent medium.
- **Isotropic Viscoelastic (IV)**: is sensitive to the changes in density, P-and S-wave velocities and P- and S-wave quality factors. In the case that Quality factors goes to zero this term vanishes. (IS+iIV)-term is the scattering potential for scattering of viscoelastic wave in an isotropic viscoelastic reference media.
- **Anisotropic Viscoelastic (AV)**: is sensitive to the changes in Q-dependent Thomsen parameters. In the case that media is either isotropic or elastic this term is zero.

So far we have discussed the elements of scattering potentials for various types of incident homogenous viscoelastic P, SI and SII waves. To conclude this section let us consider that the scattering potentials derived so far can describe the distinct scattering problems including, scattering of elastic waves in both isotropic and anisotropic media and scattering of viscoelastic waves in an isotropic viscoelastic media. Also the scattering potentials coincides with the previously derived amplitude variations with offset for reflection of viscoelastic waves from boundary separating the two viscoelastic media or two anisotropic viscoelastic media. In next section we discuss these problems one by one.

SCATTERING FORM ANISOTROPIC VISCOELASTIC INCLUSIONS

In what follows, we will present the scattering potentials developed in previous chapter in more detail. We assume the low attenuation in which the higher orders of inverse quality factors Q_{P}^{-1} and Q_{S}^{-1} are negligible. The scattering potential can be decomposed into the contributions in various types of medium properties in medium in Figure 2. Since we are interested in the effects of changes in any medium properties either elastic or viscoelastic/anisotropic, it is useful to decompose the scattering potential into the contribution in changes in elastic, viscoelastic, anisotropic and viscoelastic-anisotropic parameters. Let us consider the reference medium to be VTI viscoelastic media filled with the perturbations in viscoelastic and anisotropic parameters. In what follows we defined the normalized scattering potential as $\rho_0^{-1}\mathrm{S}$. In this case the scattering potential decomposed into four components

$$[\mathrm{PP}] = [\mathrm{PP}]_{\mathrm{IE}} + [\mathrm{PP}]_{\mathrm{AE}} + i\,[\mathrm{PP}]_{\mathrm{IV}} + i\,[\mathrm{PP}]_{\mathrm{AV}}\,, \tag{35}$$

with elastic, anisotropic, viscoelastic and viscoelastic anisotropic components

$$[\mathrm{PP}]_{\mathrm{IE}} = [\mathrm{PP}]_{\mathrm{IE}}^{\rho}\frac{\Delta\rho}{\rho} + [\mathrm{PP}]_{\mathrm{IE}}^{\mathrm{Vp}}\frac{\Delta V_P}{V_P} + [\mathrm{PP}]_{\mathrm{IE}}^{\mathrm{Vs}}\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}, \tag{36}$$

$$[\mathrm{PP}]_{\mathrm{AE}} = [\mathrm{PP}]_{\mathrm{AE}}^{\varepsilon}\,\Delta\varepsilon + [\mathrm{PP}]_{\mathrm{AE}}^{\delta}\,\Delta\delta, \tag{37}$$

$$[\mathrm{PP}]_{\mathrm{IV}} = [\mathrm{PP}]_{\mathrm{IV}}^{\rho}\frac{\Delta\rho}{\rho} + [\mathrm{PP}]_{\mathrm{IV}}^{\mathrm{Vs}}\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}} + [\mathrm{PP}]_{\mathrm{IV}}^{\mathrm{Qp}}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}} + [\mathrm{PP}]_{\mathrm{IV}}^{\mathrm{Qs}}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}}, \tag{38}$$

$$[\mathrm{PP}]_{\mathrm{AV}} = [\mathrm{PP}]_{\mathrm{AV}}^{\varepsilon}\,\Delta\varepsilon + [\mathrm{PP}]_{\mathrm{AV}}^{\delta}\,\Delta\delta + [\mathrm{PP}]_{\mathrm{AV}}^{\varepsilon_{\mathrm{Q}}}\,\Delta\varepsilon_{\mathrm{Q}} + [\mathrm{PP}]_{\mathrm{AV}}^{\delta_{\mathrm{Q}}}\,\Delta\delta_{\mathrm{Q}}, \tag{39}$$

where the sensitivities to each properties are

$$
\begin{aligned}
[\mathrm{PP}]_{\mathrm{IE}}^{\rho} &= -2 + 2\sin^2\theta_{\mathrm{P}} + 2V_{\mathrm{SP}}^2\sin^2 2\theta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{IE}}^{\mathrm{Vp}} &= -2,\\
[\mathrm{PP}]_{\mathrm{IE}}^{\mathrm{Vs}} &= 4V_{\mathrm{PS}}^2\sin^2 2\theta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{AE}}^{\varepsilon} &= -2\sin^4\theta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{AE}}^{\delta} &= -\frac{1}{2}\sin^2 2\theta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{IV}}^{\rho} &= 2V_{\mathrm{SP}}^2\sin^2 2\theta_{\mathrm{P}}(Q_{\mathrm{S0}}^{-1} - Q_{\mathrm{P0}}^{-1}) + Q_{\mathrm{P0}}^{-1}\left(\sin 2\theta_{\mathrm{P}} + 2V_{\mathrm{SP}}^2\sin 4\theta_{\mathrm{P}}\right)\tan\delta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{IV}}^{\mathrm{Vs}} &= 4V_{\mathrm{SP}}^2\sin^2 2\theta_{\mathrm{P}}(Q_{\mathrm{S0}}^{-1} - Q_{\mathrm{P0}}^{-1}) + 4Q_{\mathrm{P0}}^{-1}V_{\mathrm{PS}}^2\sin 4\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{IV}}^{\mathrm{Qp}} &= Q_{\mathrm{P0}}^{-1},\\
[\mathrm{PP}]_{\mathrm{IV}}^{\mathrm{Qs}} &= -2Q_{\mathrm{S0}}^{-1}V_{\mathrm{SP}}^2\sin^2 2\theta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{AV}}^{\varepsilon} &= -2Q_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{P}}\sin^2\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{AV}}^{\delta} &= -\frac{1}{2}Q_{\mathrm{P0}}^{-1}\sin 4\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{AV}}^{\varepsilon_{\mathrm{Q}}} &= -Q_{\mathrm{P0}}^{-1}\sin^4\theta_{\mathrm{P}},\\
[\mathrm{PP}]_{\mathrm{AV}}^{\delta_{\mathrm{Q}}} &= -\frac{1}{4}Q_{\mathrm{P0}}^{-1}\sin^2 2\theta_{\mathrm{P}}.
\end{aligned}
$$

Here we define $V_{\mathrm{SP0}} = \frac{V_{\mathrm{S0}}}{V_{\mathrm{P0}}}$. $[\mathrm{PP}]_{\mathrm{IE}}$ is the scattering potential illustrates the scattering of the P-wave to P-wave in an isotropic elastic media. It has three components, $[\mathrm{PP}]_{\mathrm{IE}}^{\rho}$ is the sensitivity of the scattered wavefield to density, $[\mathrm{PP}]_{\mathrm{IE}}^{\mathrm{Vp}}$ is the sensitivity to the P-wave velocity and $[\mathrm{PP}]_{\mathrm{IE}}^{\mathrm{Vs}}$ sensitivity to the S-wave velocity.

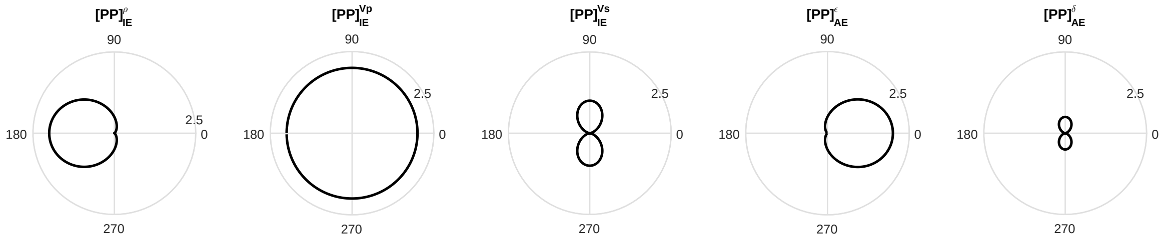

FIG. 2. Sensitivity of the elastic part of the P-to-P scattering potential to the changes in properties versus incident P-wave angle θ_{P}. From left to right, $[\mathrm{PP}]^{\rho}_{\mathrm{IE}}$, sensitivity to the density; $[\mathrm{PP}]^{\mathrm{Vp}}_{\mathrm{IE}}$, sensitivity to the P-wave velocity; $[\mathrm{PP}]^{\mathrm{Vs}}_{\mathrm{IE}}$, sensitivity to the S-wave velocity; $[\mathrm{PP}]^{\varepsilon}_{\mathrm{AE}}$ sensitivity to the Thompson parameter ε and $[\mathrm{PP}]^{\delta}_{\mathrm{AE}}$ sensitivity to the Thompson parameter δ. The S- to P-wave velocity ratio for reference medium is chosen to be $1/2$.

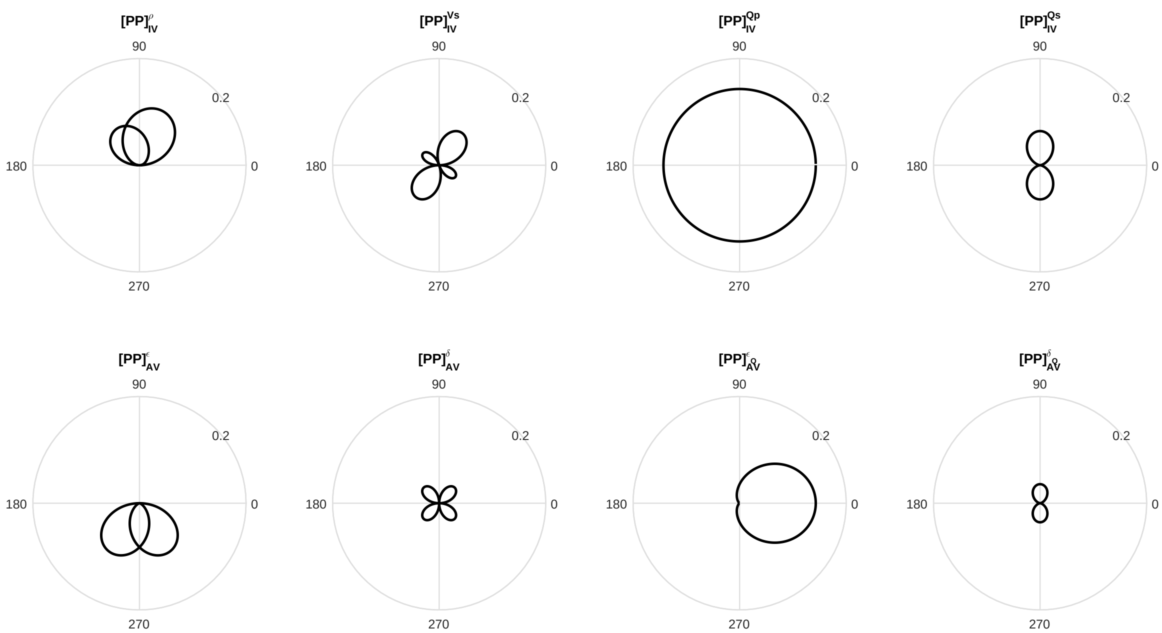

FIG. 3. Sensitivity of the viscoelastic part of the P-to-P scattering potential to the changes in properties versus incident P-wave angle θ_{P}. Top plots are the sensitivity of the isotropic viscoelastic components. From left to right, $[\mathrm{PP}]^{\rho}_{\mathrm{IV}}$, sensitivity to the density; $[\mathrm{PP}]^{\mathrm{Vs}}_{\mathrm{IV}}$, sensitivity to the S-wave velocity; $[\mathrm{PP}]^{\mathrm{Qp}}_{\mathrm{IV}}$, sensitivity to the P-wave quality factor; $[\mathrm{PP}]^{\mathrm{Qs}}_{\mathrm{IV}}$, sensitivity to the S-wave quality factor; The lower plots are the sensitivity of the anisotropic viscoelastic components. From left to right ;$[\mathrm{PP}]^{\varepsilon}_{\mathrm{AV}}$, sensitivity to the Thomsen parameter ε; $[\mathrm{PP}]^{\delta}_{\mathrm{AV}}$ sensitivity to the Thomsen parameter δ; $[\mathrm{PP}]^{\varepsilon_Q}_{\mathrm{AV}}$, sensitivity to the viscoelastic Thomsen parameter ε_Q; $[\mathrm{PP}]^{\delta_Q}_{\mathrm{AV}}$ sensitivity to the Thomsen parameter δ_Q. Quality factor of P-wave for reference medium is to be 10 and for S-wave is 7. Also the S- to P-wave velocity ratio for reference medium is chosen to be $1/2$. P-wave attenuation angle is chosen to be $\delta_{\mathrm{P}} = \pi./6$.

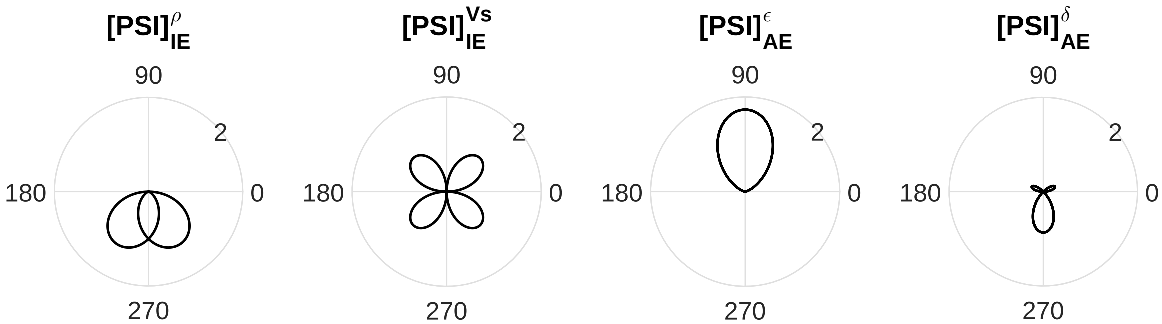

FIG. 4. Sensitivity of the elastic part of the P-to-SI scattering potential to the changes in properties versus incident P-wave angle $\theta_{\rm P}$. From left to right, $[\rm PSI]^{\rho}_{\rm IE}$, sensitivity to the density; $[\rm PSI]^{\rm Vs}_{\rm IE}$, sensitivity to the S-wave velocity; $[\rm PSI]^{\varepsilon}_{\rm AE}$ sensitivity to the Thompson parameter ε and $[\rm PSI]^{\delta}_{\rm AE}$ sensitivity to the Thompson parameter δ. The S- to P-wave velocity ratio for reference medium is chosen to be $1/2$.

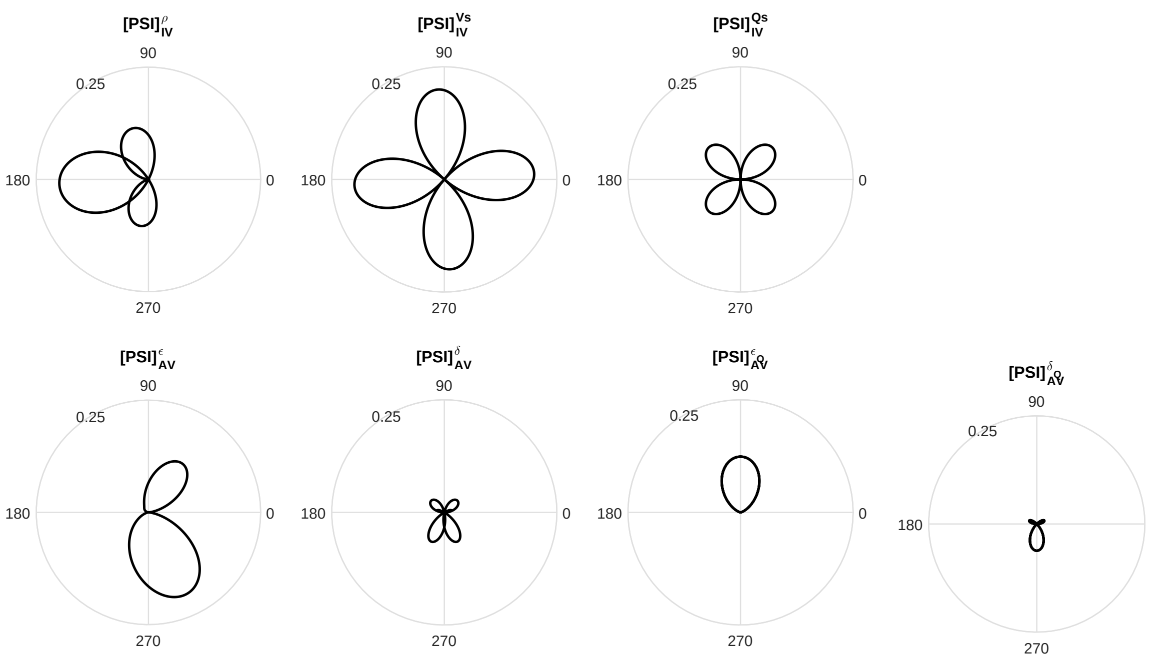

FIG. 5. Sensitivity of the viscoelastic part of the P-to-SI scattering potential to the changes in properties versus incident P-wave angle $\theta_{\rm P}$. Top plots are the sensitivity of the isotropic viscoelastic components. From left to right, $[\rm PSI]^{\rho}_{\rm IV}$, sensitivity to the density; $[\rm PSI]^{\rm Vs}_{\rm IV}$, sensitivity to the S-wave velocity; $[\rm PSI]^{\rm Qp}_{\rm IV}$, sensitivity to the P-wave quality factor; $[\rm PSI]^{\rm Qs}_{\rm IV}$, sensitivity to the S-wave quality factor; The lower plots are the sensitivity of the anisotropic viscoelastic components. From left to right ;$[\rm PSI]^{\varepsilon}_{\rm AV}$, sensitivity to the Thomsen parameter ε; $[\rm PSI]^{\delta}_{\rm AV}$ sensitivity to the Thomsen parameter δ; $[\rm PSI]^{\varepsilon_Q}_{\rm AV}$, sensitivity to the viscoelastic Thomsen parameter ε_Q; $[\rm PSI]^{\delta_Q}_{\rm AV}$ sensitivity to the Thomsen parameter δ_Q. Quality factor of P-wave for reference medium is to be 10 and for S-wave is 7. Also the S- to P-wave velocity ratio for reference medium is chosen to be $1/2$. P- and S-wave attenuation angle is chosen to be $\delta_{\rm P} = \pi./6$.

The elastic component $[\mathrm{PP}]_{\mathrm{IE}}$ is the function of fractional changes in density, P-wave and S-wave velocities. The anisotropic component $[\mathrm{PP}]_{\mathrm{AE}}$ desponds to the changes in Thomsen parameters ε and δ. The viscoelastic component variate with the fractional changes in P- and S-wave quality factors and S-wave velocity. Viscoelastic anisotropic components depends to the changes in Q-dependent Thomsen parameters $\Delta\varepsilon_Q$ and $\Delta\delta_Q$. For normal incident as $\theta_{\mathrm{P}} = 0$ the contributions from anisotropic and viscoelastic anisotropic are zero. In $[\mathrm{PP}]_{\mathrm{AE}}$ and $[\mathrm{PP}]_{\mathrm{AV}}$ components there is no influence of changes in vertical P- and S-wave velocities and corresponding quality factors. Also for small opening angles only changes in δ and δ_Q influence the scattering potential. I the case that there is no fractional changes in ε and δ, but the reference medium is anisotropic viscoelastic, scattering potential is sensitive to the change in Q-dependent Thomsen parameters.

Our results coincides with the previously driven scattering potentials for special cases like elastic and viscoelastic media. $[\mathrm{PP}]_{\mathrm{IE}}$ is the scattering potential for the case that both reference medium and inclusions are elastic (Stolt and Weglein, 2012). $([\mathrm{PP}]_{\mathrm{IE}} + i\,[\mathrm{PP}]_{\mathrm{IV}})$ is the scattering potential for the scattering of inhomogeneous P-wave to P-wave in an isotropic viscoelastic media (Moradi and Innanen, 2015). Finally in the case that both reference medium and perturbations are VTI anisotropic media, the scattering potential is given by $([\mathrm{PP}]_{\mathrm{IE}} + [\mathrm{PP}]_{\mathrm{AE}})$.

In figure 2 we plot elastic isotropic and anisotropic sensitivities for scattering of P-wave to P-wave versus the incident P-wave angle θ_{P}. Angle of incident have been considered to be in the range $(0^\circ, 360^\circ)$. The incident inhomogeneous P-wave propagates in an isotropic viscoelastic reference medium and it can be scattered to either inhomogeneous P-wave or SI-wave. The sensitivity of the elastic scattering potential potential, $[\mathrm{PP}]_{\mathrm{IE}}^{\rho}$, to the density has two lobes reaches maximum absolute values at 0° and 180°. Radiation pattern of sensitivity to P-wave velocity, $[\mathrm{PP}]_{\mathrm{IE}}^{\mathrm{V_P}}$, is circle independent of angle of incident. The similar interpretation for the radiation patterns in figure 3 for viscoelastic components of the P-to-P scattering potential. Figures 4-7 illustrates the radiation patters for elastic and viscoelastic components of the P-to-SI, SI-to-SI and SII-to-SII scattering potentials.

The scattering potential that we derived can be transformed to the low-contrast reflection coefficients corresponding to incident P-wave to reflected P-wave. The proper transformation is given by

$$\mathrm{R_{PP}} = \frac{1}{4\cos^2\theta_{\mathrm{P}}}[\mathrm{PP}] = \hat{\mathrm{A}}_{\mathrm{PP}} + \hat{\mathrm{B}}_{\mathrm{PP}}\sin^2\theta_{\mathrm{P}} + \hat{\mathrm{C}}_{\mathrm{PP}}\sin^2\theta_{\mathrm{P}}\tan^2\theta_{\mathrm{P}} \tag{40}$$

where the coefficients $\hat{\mathrm{A}}_{\mathrm{PP}}$, $\hat{\mathrm{B}}_{\mathrm{PP}}$ and $\hat{\mathrm{C}}_{\mathrm{PP}}$ are complex depending to the changes in properties as follows

$$\begin{aligned}
\hat{\mathrm{A}}_{\mathrm{PP}}^{\mathrm{VTI}} &= \frac{1}{2}\left(\frac{\Delta\rho}{\rho}+\frac{\Delta V_P}{V_P}\right)-\frac{i}{4}Q_{\mathrm{P0}}^{-1}\frac{\Delta Q_{\mathrm{P0}}}{Q_{\mathrm{P0}}}\\
\hat{\mathrm{B}}_{\mathrm{PP}} &= \frac{1}{2}\left[\frac{\Delta V_P}{V_P}-4V_{\mathrm{SP}}^2\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right)+\Delta\delta\right]-i\left[\frac{1}{4}Q_{\mathrm{P0}}^{-1}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}}-2V_{\mathrm{SP}}^2Q_{\mathrm{S0}}^{-1}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}}\right]\\
&\quad -i\left[2V_{\mathrm{SP}}^2(Q_{\mathrm{S0}}^{-1}-Q_{\mathrm{P0}}^{-1})\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right)-\frac{1}{4}Q_{\mathrm{P0}}^{-1}\Delta\delta_Q\right]\\
\hat{\mathrm{C}}_{\mathrm{PP}} &= \frac{1}{2}\left[\frac{\Delta V_P}{V_P}+\Delta\varepsilon\right]-\frac{i}{4}Q_{\mathrm{P0}}^{-1}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}}+\frac{i}{4}Q_{\mathrm{P0}}^{-1}\Delta\varepsilon_Q
\end{aligned} \tag{41}$$

Term $\hat{\mathrm{A}}_{\mathrm{PP}}$ is called zero offset or normal incident reflection coefficient. It can be seen that this term depends only to the changes in density, vertical P-wave velocity and P-wave quality factor. Anisotropic and viscoelastic anisotropic properties does not influence this term. The second coefficient $\hat{\mathrm{B}}_{\mathrm{PP}}$ is called gradient and responsible for $\sin^2\theta_P$ term. $\hat{\mathrm{B}}_{\mathrm{PP}}$ variate with changes in density, vertical P- and S-wave velocities, P- and S-wave quality factors and δ and δ_Q. However changes in ε and ε_Q have no effect in the gradient term. The last term $\hat{\mathrm{C}}_{\mathrm{PP}}$ which is called curvature is influential for large angles of incident. This term is affect by changes in P-wave related properties, vertical P-wave velocity, P-wave quality factor, ε and ε_Q. We can rearrange the reflectivity into the following form

$$\mathrm{R}_{\mathrm{PP}}^{\mathrm{VTI}}=\mathrm{R}_{\mathrm{PP}}^{\mathrm{IE}}+\mathrm{R}_{\mathrm{PP}}^{\mathrm{AE}}+i\mathrm{R}_{\mathrm{PP}}^{\mathrm{IV}}+i\mathrm{R}_{\mathrm{PP}}^{\mathrm{AV}}, \tag{42}$$

$\mathrm{R}_{\mathrm{PP}}^{\mathrm{IE}}$ is the P-to-P reflection coefficients for low contrast elastic media well known to the Aki-Richards approximation Aki and Richards (2002). $(\mathrm{R}_{\mathrm{PP}}^{\mathrm{IE}}+i\mathrm{R}_{\mathrm{PP}}^{\mathrm{IV}})$ is the approximate reflection coefficient for low contrast interfaces separating two arbitrary low-loss viscoelastic media Moradi and Innanen (2016). Approximate PP reflection coefficient for weak-contrast interfaces in weakly VTI anisotropic elastic media is given by $(\mathrm{R}_{\mathrm{PP}}^{\mathrm{IE}}+\mathrm{R}_{\mathrm{PP}}^{\mathrm{AE}})$ (Ruger 2000). Finally $\mathrm{R}_{\mathrm{PP}}^{\mathrm{VTI}}$ is the linearized homogeneous P-to-P reflection coefficients in attenuative anisotropic VTI media (Behura and Tsvankin, 2009a). The only apparent difference is that they present the P- and S-wave complex velocities as $\hat{V}_{\mathrm{P(S)}}=V_{\mathrm{P(S)}}(1+i\mathcal{A}_{\mathrm{P(S)}})$ where $\mathcal{A}_{\mathrm{P}}=Q_{33}^{-1}/2$ and $\mathcal{A}_{\mathrm{S}}=Q_{55}^{-1}/2$. As a result the changes in $\mathcal{A}_{\mathrm{P}}$ and $\mathcal{A}_{\mathrm{S}}$ can be written as

$$\begin{aligned}
\Delta\mathcal{A}_{\mathrm{P}}=\mathcal{A}_{\mathrm{P}}-\mathcal{A}_{\mathrm{P0}}=-\frac{1}{2}Q_{\mathrm{P0}}^{-1}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}},\\
\Delta\mathcal{A}_{\mathrm{S}}=\mathcal{A}_{\mathrm{S}}-\mathcal{A}_{\mathrm{S0}}=-\frac{1}{2}Q_{\mathrm{S0}}^{-1}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}},
\end{aligned}$$

Now let us consider the converted wave, the discussion for the P-to-S wave scattering can be carried out in an analogous manner that we did for PP scattering potential. The scattering potential for scattering the P-wave to SI wave is given by

$$[\mathrm{PSI}]=[\mathrm{PSI}]_{\mathrm{IE}}+[\mathrm{PSI}]_{\mathrm{AE}}+i\,[\mathrm{PSI}]_{\mathrm{IV}}+i\,[\mathrm{PSI}]_{\mathrm{AV}}\,, \tag{43}$$

with elastic, anisotropic, viscoelastic and viscoelastic anisotropic components

$$
\begin{aligned}
[\mathrm{PSI}]_{\mathrm{IE}} &= [\mathrm{PSI}]_{\mathrm{IE}}^{\rho}\,\frac{\Delta\rho}{\rho} + [\mathrm{PSI}]_{\mathrm{IE}}^{\mathrm{Vs}}\,\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}} \\
[\mathrm{PSI}]_{\mathrm{AE}} &= [\mathrm{PSI}]_{\mathrm{AE}}^{\varepsilon}\,\Delta\varepsilon + [\mathrm{PSI}]_{\mathrm{AE}}^{\delta}\,\Delta\delta \\
[\mathrm{PSI}]_{\mathrm{IV}} &= [\mathrm{PSI}]_{\mathrm{IV}}^{\rho}\,\frac{\Delta\rho}{\rho} + [\mathrm{PSI}]_{\mathrm{IV}}^{\mathrm{Vs}}\,\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}} + [\mathrm{PSI}]_{\mathrm{IV}}^{\mathrm{Qs}}\,\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}} \\
[\mathrm{PSI}]_{\mathrm{AV}} &= [\mathrm{PSI}]_{\mathrm{AV}}^{\varepsilon}\,\Delta\varepsilon + [\mathrm{PSI}]_{\mathrm{AV}}^{\delta}\,\Delta\delta + [\mathrm{PSI}]_{\mathrm{AV}}^{\varepsilon_{\mathrm{Q}}}\,\Delta\varepsilon_{\mathrm{Q}} + [\mathrm{PSI}]_{\mathrm{AV}}^{\delta_{\mathrm{Q}}}\,\Delta\delta_{\mathrm{Q}}
\end{aligned}
$$

where

$$
\begin{aligned}
[\mathrm{PSI}]_{\mathrm{IE}}^{\rho} &= -\sin(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}) - V_{\mathrm{SP0}}\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}) \\
[\mathrm{PSI}]_{\mathrm{IE}}^{\mathrm{Vs}} &= -2V_{\mathrm{SP0}}\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}) \\
[\mathrm{PSI}]_{\mathrm{AE}}^{\varepsilon} &= V_{\mathrm{PS0}}\sin 2\theta_{\mathrm{S}}\sin^2\theta_{\mathrm{P}} \\
[\mathrm{PSI}]_{\mathrm{AE}}^{\delta} &= \frac{1}{2}V_{\mathrm{PS0}}\cos 2\theta_{\mathrm{P}}\sin 2\theta_{\mathrm{S}} \\
[\mathrm{PSI}]_{\mathrm{IV}}^{\rho} &= -\frac{1}{2}V_{\mathrm{SP0}}(Q_{\mathrm{S0}}^{-1}-Q_{\mathrm{P0}}^{-1})\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}) \\
&\quad -\frac{1}{2}\left[\cos(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}) + 2V_{\mathrm{SP0}}\cos 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\right]\left(Q_{\mathrm{S0}}^{-1}\tan\delta_{\mathrm{S}} + Q_{\mathrm{P0}}^{-1}\tan\delta_{\mathrm{P}}\right) \\
[\mathrm{PSI}]_{\mathrm{IV}}^{\mathrm{Vs}} &= -V_{\mathrm{SP0}}(Q_{\mathrm{S0}}^{-1}-Q_{\mathrm{P0}}^{-1})\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}) \\
&\quad -2V_{\mathrm{SP0}}\cos 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\left(Q_{\mathrm{S0}}^{-1}\tan\delta_{\mathrm{S}} + Q_{\mathrm{P0}}^{-1}\tan\delta_{\mathrm{P}}\right) \\
[\mathrm{PSI}]_{\mathrm{IV}}^{\mathrm{Qs}} &= V_{\mathrm{SP0}}Q_{\mathrm{S0}}^{-1}\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}} \\
[\mathrm{PSI}]_{\mathrm{AV}}^{\varepsilon} &= -\frac{1}{2}V_{\mathrm{PS0}}(Q_{\mathrm{S0}}^{-1}-Q_{\mathrm{P0}}^{-1})\sin 2\theta_{\mathrm{S}}\sin^2\theta_{\mathrm{P}} \\
&\quad + V_{\mathrm{PS0}}\left(Q_{\mathrm{S0}}^{-1}\cos 2\theta_{\mathrm{S}}\sin\theta_{\mathrm{P}}\tan\delta_{\mathrm{S}} + Q_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{S}}\cos\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\right)\sin\theta_{\mathrm{P}} \\
[\mathrm{PSI}]_{\mathrm{AV}}^{\delta} &= -\frac{1}{4}V_{\mathrm{PS0}}(Q_{\mathrm{S0}}^{-1}-Q_{\mathrm{P0}}^{-1})\cos 2\theta_{\mathrm{P}}\sin 2\theta_{\mathrm{S}} \\
&\quad + \frac{1}{2}V_{\mathrm{PS}}\left(Q_{\mathrm{S0}}^{-1}\cos 2\theta_{\mathrm{S}}\cos 2\theta_{\mathrm{P}}\tan\delta_{\mathrm{S}} - Q_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{S}}\sin 2\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\right) \\
[\mathrm{PSI}]_{\mathrm{AV}}^{\varepsilon_{\mathrm{Q}}} &= \frac{1}{2}V_{\mathrm{PS0}}Q_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{S}}\sin^2\theta_{\mathrm{P}} \\
[\mathrm{PSI}]_{\mathrm{AV}}^{\delta_{\mathrm{Q}}} &= \frac{1}{4}V_{\mathrm{PS0}}Q_{\mathrm{P0}}^{-1}\cos 2\theta_{\mathrm{P}}\sin 2\theta_{\mathrm{S}}
\end{aligned}
$$

The elastic and viscoelastic component of the scattering potential $[\mathrm{PSI}]_{\mathrm{IE}}$, $[\mathrm{PSI}]_{\mathrm{IV}}$ are sensitive to the fractional changes in density and vertical S-wave velocity and S-wave quality factor. Changes in vertical P-wave velocity and P-wave quality factor doesn't have any effects on these two terms. In addiction $([\mathrm{PSI}]_{\mathrm{IE}} + i\,[\mathrm{PSI}]_{\mathrm{IV}})$ is the scattering potential describe the scattering of the inhomogeneous P-wave into the inhomogeneous SI-wave in an isotropic viscoelastic media. Regarding to the anisotropic and anisotropic viscoelastic terms, for small angle of incident they are sensitive only to the changes in δ and δ_Q. For large incident angle, changes in P-wave Thomsen parameter ε and δ would be influential.

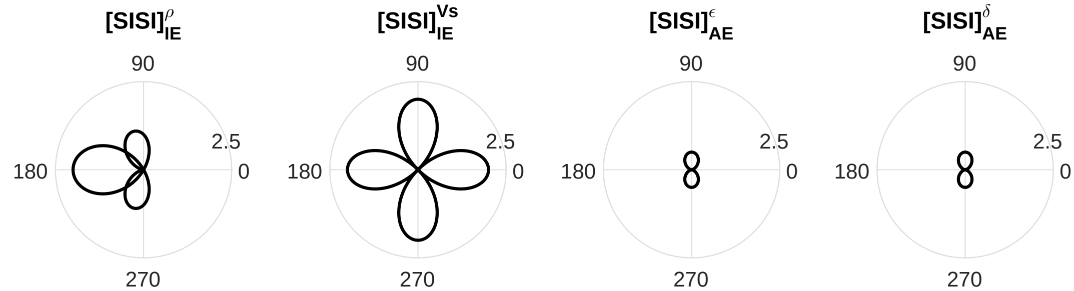

FIG. 6. Sensitivity of the elastic part of the SI-to-SI scattering potential to the changes in properties versus incident S-wave angle θ_{S}. From left to right, $[\mathrm{SISI}]^{\rho}_{\mathrm{IE}}$, sensitivity to the density; $[\mathrm{SISI}]^{\mathrm{Vs}}_{\mathrm{IE}}$, sensitivity to the S-wave velocity; $[\mathrm{SIISII}]^{\mathrm{Vs}}_{\mathrm{IE}}$, sensitivity to the S-wave velocity; $[\mathrm{SISI}]^{\gamma}_{\mathrm{AE}}$ sensitivity to the Thompson parameter γ. P-to-S velocity ratio, quality factor and attention angle same as figure 5.

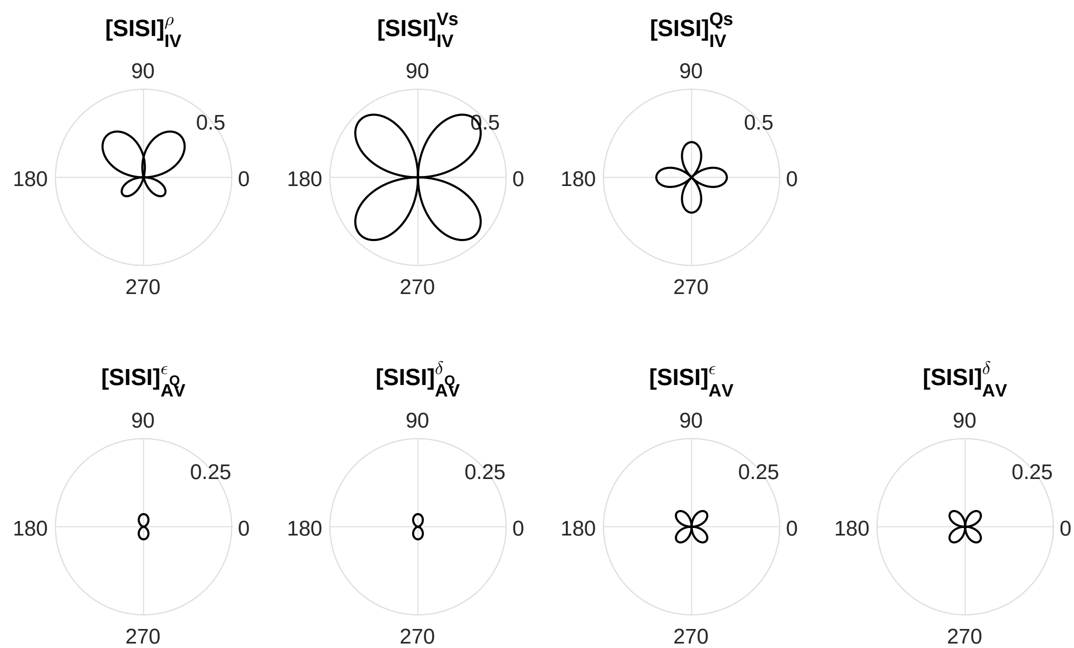

FIG. 7. Sensitivity of the viscoelastic part of the SI-to-SI scattering potential to the changes in properties versus incident S-wave angle θ_{S}. Top plots are the sensitivity of the isotropic viscoelastic components. From left to right, $[\mathrm{SISI}]^{\rho}_{\mathrm{IV}}$, sensitivity to the density; $[\mathrm{SISI}]^{\mathrm{Vs}}_{\mathrm{IV}}$, sensitivity to the S-wave velocity; $[\mathrm{SISI}]^{\mathrm{Qs}}_{\mathrm{IV}}$, sensitivity to the S-wave quality factor; The lower plots are the sensitivity of the anisotropic viscoelastic components. From left to right ;$[\mathrm{SISI}]^{\varepsilon}_{\mathrm{AV}}$, sensitivity to the Thomsen parameter ε; $[\mathrm{SISI}]^{\delta}_{\mathrm{AV}}$ sensitivity to the Thomsen parameter δ; $[\mathrm{SISI}]^{\varepsilon_Q}_{\mathrm{AV}}$, sensitivity to the viscoelastic Thomsen parameter ε_Q; $[\mathrm{SISI}]^{\delta_Q}_{\mathrm{AV}}$ sensitivity to the Thomsen parameter δ_Q. Quality factor of S-wave for reference medium is 7. P-to-S velocity ratio, quality factor and attention angle same as figure 3.

Similar to P-to-P case, for no contrast in anisotropic parameters ε and δ with anisotropic viscoelastic medium the scattering potential is sensitive to the changes in ε_Q and δ_Q.

To evaluate the reflectivity scattering potential should be multiply with

$$\mathrm{R_{PSI}} = -\frac{\sin\theta_P}{2\cos\theta_S \sin(\theta_P + \theta_S)}[\mathrm{PSI}], \tag{44}$$

In this case without loss of generality consider to the reflectivity for small incident angle. We also use the Snell's law in to express the reflected S-wave angle θ_S in terms of incident P-wave angle θ_P. Keeping up to third order of $\sin\theta_\mathrm{P}$ the reflectivity in terms of power of $\sin\theta_\mathrm{P}$ in an standard form of converted wave is given by

$$\mathrm{R}^{\mathrm{VTI}}_{\mathrm{PSI}} = (A^{\mathrm{IE}}_{\mathrm{PSI}} + A^{\mathrm{AE}}_{\mathrm{PSI}} + iA^{\mathrm{IV}}_{\mathrm{PSI}} + iA^{\mathrm{VA}}_{\mathrm{PSI}})\sin\theta_\mathrm{P} + (B^{\mathrm{IE}}_{\mathrm{PSI}} + B^{\mathrm{AE}}_{\mathrm{PSI}} + iB^{\mathrm{IV}}_{\mathrm{PSI}} + iB^{\mathrm{VA}}_{\mathrm{PSI}})\sin^3\theta_\mathrm{P} \tag{45}$$

where the complex coefficients are

$$\begin{aligned}
A^{\mathrm{IE}}_{\mathrm{PSI}} =& -\left(\frac{1}{2} + V_{\mathrm{SP}}\right)\frac{\Delta\rho}{\rho} - 2V_{\mathrm{SP}}\frac{\Delta V_\mathrm{S}}{V_\mathrm{S}} \\
B^{\mathrm{IE}}_{\mathrm{PSI}} =& \frac{1}{4}V_{\mathrm{SP}}(3V_{\mathrm{SP}} + 2)\frac{\Delta\rho}{\rho} + V_{\mathrm{SP}}(1 + 2V_{\mathrm{SP}})\frac{\Delta V_\mathrm{S}}{V_\mathrm{S}} \\
A^{\mathrm{AE}}_{\mathrm{PSI}} =& \frac{1}{1 + V_{\mathrm{SP}}}\Delta\delta \\
B^{\mathrm{AE}}_{\mathrm{PSI}} =& \frac{1}{1 + V_{\mathrm{SP}}}\Delta\varepsilon + \frac{V_{\mathrm{SP}} - 4}{2(1 + V_{\mathrm{SP}})}\Delta\delta \\
A^{\mathrm{IV}}_{\mathrm{PSI}} =& -\frac{1}{2}V_{\mathrm{SP}}\left(Q^{-1}_{\mathrm{S0}} - Q^{-1}_{\mathrm{P0}}\right)\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_\mathrm{S}}{V_\mathrm{S}}\right) + V_{\mathrm{SP}}Q^{-1}_{\mathrm{S0}}\frac{\Delta Q_\mathrm{S}}{Q_\mathrm{S}} \\
B^{\mathrm{IV}}_{\mathrm{PSI}} =& \frac{1}{4}(3V_{\mathrm{SP}} + 1)\left(Q^{-1}_{\mathrm{S0}} - Q^{-1}_{\mathrm{P0}}\right)\frac{\Delta\rho}{\rho} \\
& - \frac{1}{2}V_{\mathrm{SP}}(1 + 2V_{\mathrm{SP}})Q^{-1}_{\mathrm{S0}}\frac{\Delta Q_\mathrm{S}}{Q_\mathrm{S}} + \frac{1}{2}V_{\mathrm{SP}}(4V_{\mathrm{SP}} + 1)\left(Q^{-1}_{\mathrm{S0}} - Q^{-1}_{\mathrm{P0}}\right)\frac{\Delta V_\mathrm{S}}{V_\mathrm{S}} \\
A^{\mathrm{VA}}_{\mathrm{PSI}} =& \frac{V_{\mathrm{SP}}}{2(1 + V_{\mathrm{SP}})^2}\left(Q^{-1}_{\mathrm{S0}} - Q^{-1}_{\mathrm{P0}}\right)\Delta\delta + \frac{Q^{-1}_{\mathrm{P0}}}{2(1 + V_{\mathrm{SP}})}\Delta\delta_Q \\
B^{\mathrm{VA}}_{\mathrm{PSI}} =& -\frac{V_{\mathrm{SP}}\left(Q^{-1}_{\mathrm{S0}} - Q^{-1}_{\mathrm{P0}}\right)}{2(1 + V_{\mathrm{SP}})^2}\left\{\Delta\varepsilon - \frac{5}{2}\Delta\delta\right\} \\
& + \frac{1}{2(1 + V_{\mathrm{SP}})}Q^{-1}_{\mathrm{P0}}\Delta\varepsilon_Q + \frac{V_{\mathrm{SP}} - 4}{4(1 + V_{\mathrm{SP}})}Q^{-1}_{\mathrm{P0}}\Delta\delta_Q
\end{aligned}$$

We can see that the isotropic elastic part including the first two coefficients ${}^{\mathrm{P}}_{\mathrm{SI}}A^{\mathrm{IE}}$ and ${}^{\mathrm{P}}_{\mathrm{SI}}B^{\mathrm{IE}}$ is sensitive to the changes in density and S-wave quality factor. Anisotropic elastic part is not affected by changes in anisotropic parameter γ, however the reflectivity changes with $\Delta\varepsilon$ and $\Delta\delta$. For isotropic viscoelastic part denoted by coefficients ${}^{\mathrm{P}}_{\mathrm{SI}}A^{\mathrm{AE}}_{\mathrm{VTI}}$ and ${}^{\mathrm{P}}_{\mathrm{SI}}B^{\mathrm{AE}}_{\mathrm{VTI}}$ are sensitive to changes in density and S-wave velocity and S-wave quality factor. Finally the anisotropic viscoelastic part is sensitive to changes in both Thomsen parameters ε and δ and their corresponding Q-dependent parameters ε_Q and δ_Q.

The scattering potential for SI to SI waves

$$[\mathrm{SISI}] = [\mathrm{SISI}]_{\mathrm{IE}} + [\mathrm{SISI}]_{\mathrm{AE}} + i\,[\mathrm{SISI}]_{\mathrm{IV}} + i\,[\mathrm{SISI}]_{\mathrm{AV}}\,, \tag{46}$$

with elastic, anisotropic, viscoelastic and viscoelastic anisotropic components

$$[\mathrm{SISI}]_{\mathrm{IE}} = -(\cos 2\theta_{\mathrm{S}} + \cos 4\theta_{\mathrm{S}})\frac{\Delta\rho}{\rho} - 2\cos 4\theta_{\mathrm{S}}\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}$$

$$[\mathrm{SISI}]_{\mathrm{AE}} = \frac{1}{2}\sin^2 2\theta_{\mathrm{S}}(\Delta\delta - \Delta\varepsilon)$$

$$[\mathrm{SISI}]_{\mathrm{IV}} = \cos 4\theta_{\mathrm{S}} Q_{\mathrm{S0}}^{-1}\frac{\Delta Q_S}{Q_S} + Q_{\mathrm{S0}}^{-1}\sin 2\theta_{\mathrm{S}}\tan\delta_{\mathrm{S}}\frac{\Delta\rho}{\rho} + 2Q_S^{-1}\sin 4\theta_{\mathrm{S}}\tan\delta_{\mathrm{S}}\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_S}{V_S}\right)$$

$$[\mathrm{SISI}]_{\mathrm{AV}} = \frac{1}{4}\sin^2 2\theta_{\mathrm{S}} Q_{\mathrm{P0}}^{-1}(\Delta\delta_Q - \Delta\varepsilon_Q) - \frac{1}{2}Q_{\mathrm{S}}^{-1}\sin 4\theta_{\mathrm{S}}\tan\delta_{\mathrm{S}}(\Delta\varepsilon - \Delta\delta)$$

Here $[\mathrm{SISI}]_{\mathrm{IE}}$ is the scattering potential of scattering of SV-wave to the SV wave in an isotropic elastic background where there are no attention and anisotropy present. This term is sensitive to the changes in density and S-wave velocity only, on other words perturbation in P-wave velocity can not scatter the incident SV-wave. $([\mathrm{SISI}]_{\mathrm{IE}} + i\,[\mathrm{SISI}]_{\mathrm{IV}})$ terms refers to the scattering of the homogeneous SI-wave in an isotropic viscoelastic background. We can see that in the presence of the attenuation, incident SI-wave only influenced by the change in the S-wave quality factor Q_{55}. In total, in an viscoelastic anisotropic media changes in seven parameters can cause the scattering, $(\rho, V_{\mathrm{S}}, \Delta\delta, \Delta\varepsilon, \Delta\delta_Q, \Delta\varepsilon_Q)$.

Regarding to the reflectivity, we divide it into the real and imaginary parts

$$\mathrm{R}_{\mathrm{SISI}}^{\mathrm{VTI}} = \frac{1}{4\cos^2\theta_S}[\mathrm{SISI}] = \Re(\mathrm{R}_{\mathrm{SISI}}^{\mathrm{VTI}}) + i\Im(\mathrm{R}_{\mathrm{SISI}}^{\mathrm{VTI}}),$$

Where $\Re(\mathrm{R}_{\mathrm{SISI}}^{\mathrm{VTI}})$ is the approximate reflection coefficient for a weak contrast interface between two slightly different weakly anisotropic VTI media

$$\begin{aligned}\Re(\mathrm{R}_{\mathrm{SISI}}^{\mathrm{VTI}}) = &-\frac{1}{2}\left[\frac{\Delta\rho}{\rho} + \frac{\Delta V_S}{V_S}\right] + \left[\frac{7}{2}\frac{\Delta V_S}{V_S} + 2\frac{\Delta\rho}{\rho} - \frac{1}{2}(\Delta\delta - \Delta\varepsilon)\right]\sin^2\theta_S \\ &- \left[\frac{1}{2}\frac{\Delta V_S}{V_S}\right]\sin^2\theta_S\tan^2\theta_S\end{aligned}$$

The first term is the normal incident reflection coefficients depends to the fractional changes in density and S-wave velocity, anisotropy does not have any influence in this term. The second term called gradient or the reflection coefficient for the small angle of incident. Changes in anisotropic parameters only influence this term as there is no influence on the third term called curvature from anisotropy. The imaginary part is related to the attenuation in medium given by

$$\begin{aligned}\Im(\mathrm{R}_{\mathrm{SISI}}^{\mathrm{VTI}}) = &\frac{1}{4}Q_{\mathrm{S0}}^{-1}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}} - \frac{1}{4}\left[7Q_{\mathrm{S0}}^{-1}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}} + Q_{\mathrm{P0}}^{-1}(\Delta\delta_Q - \Delta\varepsilon_Q)\right]\sin^2\theta_S \\ &+ \frac{1}{4}\left[Q_{\mathrm{S0}}^{-1}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}}\right]\sin^2\theta_S\tan^2\theta_S\end{aligned}$$

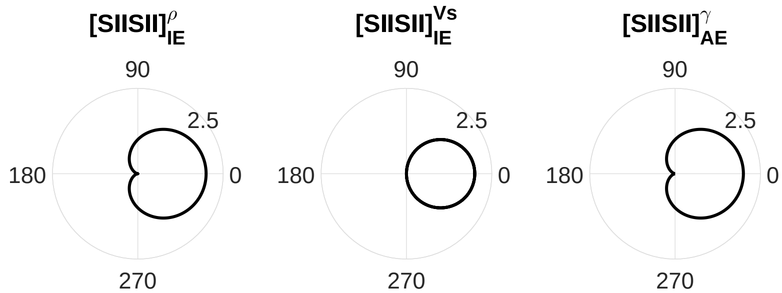

FIG. 8. Sensitivity of the elastic part of the SII-to-SII scattering potential to the changes in properties. From left to right, $[\mathrm{SIISII}]^{\rho}_{\mathrm{IE}}$, sensitivity to the density; $[\mathrm{SIISII}]^{\mathrm{Vs}}_{\mathrm{IE}}$, sensitivity to the S-wave velocity; $[\mathrm{SIISII}]^{\mathrm{Vs}}_{\mathrm{IE}}$, sensitivity to the S-wave velocity; $[\mathrm{SIISII}]^{\gamma}_{\mathrm{AE}}$ sensitivity to the Thompson parameter γ.

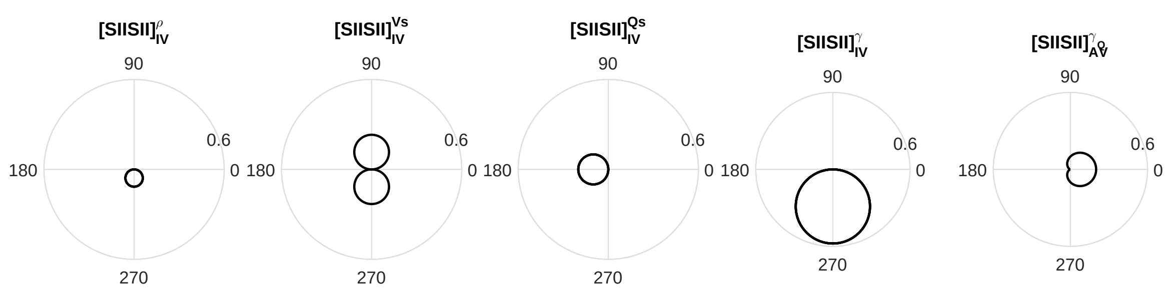

FIG. 9. Sensitivity of the viscoelastic part of the SII-to-SII scattering potential to the changes in properties versus incident S-wave angle θ_{S}. Top plots are the sensitivity of the isotropic viscoelastic components. From left to right, $[\mathrm{SIISII}]^{\rho}_{\mathrm{IV}}$, sensitivity to the density; $[\mathrm{SIISII}]^{\mathrm{Vs}}_{\mathrm{IV}}$, sensitivity to the S-wave velocity; $[\mathrm{SIISII}]^{\mathrm{Qs}}_{\mathrm{IV}}$, sensitivity to the S-wave quality factor; The lower plots are the sensitivity of the anisotropic viscoelastic components. From left to right ;$[\mathrm{SIISII}]^{\gamma}_{\mathrm{AV}}$, sensitivity to the Thomsen parameter γ; $[\mathrm{SIISII}]^{\gamma_Q}_{\mathrm{AV}}$ sensitivity to the Thomsen parameter γ_Q. Quality factor of S-wave for reference medium is 7. P-to-S velocity ratio, quality factor and attention angle same as figure 3.

Finally the scattering potential for SII-to-SII scattering potential

$$[\mathrm{SIISII}] = [\mathrm{SIISII}]_{\mathrm{IE}} + [\mathrm{SIISII}]_{\mathrm{AE}} + i\,[\mathrm{SIISII}]_{\mathrm{IV}} + i\,[\mathrm{SIISII}]_{\mathrm{AV}}\,, \tag{47}$$

with elastic, anisotropic, viscoelastic and viscoelastic anisotropic components

$$\begin{aligned}
[\mathrm{SIISII}]_{\mathrm{IE}} &= \frac{\Delta\rho}{\rho} + \cos 2\theta_{\mathrm{S}}\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_S}{V_S}\right)\\
[\mathrm{SIISII}]_{\mathrm{AE}} &= -2\sin^2\theta_{\mathrm{S}}\Delta\gamma\\
[\mathrm{SIISII}]_{\mathrm{IV}} &= -Q_{\mathrm{S0}}^{-1}\sin 2\theta_{\mathrm{S}}\tan\delta_{\mathrm{S}}\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_S}{V_S}\right) - Q_{\mathrm{S0}}^{-1}\cos 2\theta_{\mathrm{S}}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}}\\
[\mathrm{SIISII}]_{\mathrm{AV}} &= -Q_{\mathrm{S0}}^{-1}\sin^2\theta_{\mathrm{S}}\Delta\gamma_Q - Q_{\mathrm{S0}}^{-1}\sin 2\theta_S\tan\delta_{\mathrm{S}}\Delta\gamma
\end{aligned}$$

In the absence of anisotropy the above expression reduces to the scattering potential for the scattering of the inhomogeneous SII wave to the inhomogeneous SII wave. After multiplying by $-1/4\cos^2\theta_S$ the reflectivity is

$$\begin{aligned}
\mathrm{R}_{\mathrm{SIISII}}^{\mathrm{VTI}} = &-\frac{1}{2}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta V_S}{V_S}\right) + \frac{1}{2}\left(\frac{\Delta V_S}{V_S} + \Delta\gamma\right)\tan^2\theta_S\\
&+ \frac{i}{4}Q_{\mathrm{S0}}^{-1}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}} - \frac{i}{4}Q_{\mathrm{S0}}^{-1}\left[2\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}} - \Delta\gamma_Q\right]\tan^2\theta_S
\end{aligned}$$

ATTENUATIVE ORTHORHOMBIC MEDIA

A medium with orthorhombic symmetry has three mutually orthogonal mirror planes of symmetry and described by nine independent elements (Tsvankin, 1997). In each symmetry plane the the medium exhibits the transverse isotropy. The stiffness tensor for orthorhombic media given by

$$\hat{C}_{\mathrm{orth}} = \begin{pmatrix}
\hat{C}_{11} & \hat{C}_{12} & \hat{C}_{13} & 0 & 0 & 0\\
\hat{C}_{12} & \hat{C}_{22} & \hat{C}_{23} & 0 & 0 & 0\\
\hat{C}_{13} & \hat{C}_{23} & \hat{C}_{33} & 0 & 0 & 0\\
0 & 0 & 0 & \hat{C}_{44} & 0 & 0\\
0 & 0 & 0 & 0 & \hat{C}_{55} & 0\\
0 & 0 & 0 & 0 & 0 & \hat{c}_{66}
\end{pmatrix}. \tag{48}$$

The real part of the diagonal elements relate to the velocities along the coordinate axis. As we will describe later, any component of the stiffness tensor is complex whose imaginary

Model 1		Model 2		Model 3	
C_{11}	C_{11}^{I}	C_{11}	Q_{11}	V_{P}	Q_{33}
C_{12}	C_{12}^{I}	C_{12}	Q_{12}	V_{S}	Q_{55}
C_{13}	C_{13}^{I}	C_{13}	Q_{13}	$\varepsilon^{(1)}$	$\varepsilon_Q^{(1)}$
C_{23}	C_{23}^{I}	C_{23}	Q_{23}	$\varepsilon^{(2)}$	$\varepsilon_Q^{(1)}$
C_{22}	C_{22}^{I}	C_{22}	Q_{22}	$\delta^{(1)}$	$\delta_Q^{(1)}$
C_{33}	C_{33}^{I}	C_{33}	Q_{33}	$\delta^{(2)}$	$\delta_Q^{(2)}$
C_{44}	C_{44}^{I}	C_{44}	Q_{44}	$\delta^{(3)}$	$\delta_Q^{(3)}$
C_{55}	C_{55}^{I}	C_{55}	Q_{55}	$\gamma^{(1)}$	$\gamma_Q^{(1)}$
C_{66}	C_{66}^{I}	C_{66}	Q_{66}	$\gamma^{(2)}$	$\gamma_Q^{(2)}$

Table 1. Three models of parametrization to describe the attenuative orthorhombic media.

part is related to the quality factor.

$$\begin{aligned}
\hat{C}_{ijkl} &= \hat{C}_{23}\delta ij\delta_{kl} + \hat{C}_{66}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \\
&+ (\hat{C}_{11} - \hat{C}_{23} - 2\hat{C}_{66})\delta_{i1}\delta_{j1}\delta_{k1}\delta_{l1} \\
&+ (\hat{C}_{22} - \hat{C}_{23} - 2\hat{C}_{66})\delta_{i2}\delta_{j2}\delta_{k2}\delta_{l2} \\
&+ (\hat{C}_{33} - \hat{C}_{23} - 2\hat{C}_{66})\delta_{i3}\delta_{j3}\delta_{k3}\delta_{l3} \\
&+ (\hat{C}_{12} - \hat{C}_{23})(\delta_{i1}\delta_{j1}\delta_{k2}\delta_{l2} + \delta_{i2}\delta_{j2}\delta_{k1}\delta_{l1}) \\
&+ (\hat{C}_{13} - \hat{C}_{23})(\delta_{i1}\delta_{j1}\delta_{k3}\delta_{l3} + \delta_{i3}\delta_{j3}\delta_{k1}\delta_{l1}) \\
&+ (\hat{C}_{44} - \hat{C}_{66})(\delta_{i2}\delta_{j3} + \delta_{i3}\delta_{j2})(\delta_{k2}\delta_{l3} + \delta_{k3}\delta_{l2}) \\
&+ (\hat{C}_{55} - \hat{C}_{66})(\delta_{i1}\delta_{j3} + \delta_{i3}\delta_{j1})(\delta_{k1}\delta_{l3} + \delta_{k3}\delta_{l1}).
\end{aligned} \tag{49}$$

Where we use the 'hat' symbol to distinct the complex quantity from real. By introducing the quality factor as a $Q_{ij} = C_{ij}/C_{ij}^{\mathrm{I}}$ where C_{ij} and C_{ij}^{I} are real and imaginary parts of the stiffness components. We can see corresponding to each independent components of the stiffness tensor there is a component of quality factor. In what follows we will show an another set of parameters characterized the attenuation in orthorhombic media. In fact instead of above parameters we can have seven Q-dependent Thompson parameters plus two S- and P-wave quality factors. Tsvankin defined the Thomsen parameters to characterize the weak anisotropy in attenuative orthorhombic media by assuming the weak attenuation. In table 1 illustrate the different types of parametrization that can be used to describe the orthorhombic media. In model 1 medium described by real and imaginary parts of the stiffness tensor, 18 parameters in total. In model 2 instead of imaginary parts of the stiffness tensor, viscoelasticity describes by the components of the quality factor tensor. Model 3 which is the most useful parametrization is described by P- and S-wave velocities, seven Thomsen parameters and for the viscoelastic part, by P- and S-wave quality factors and seven Q-dependent anisotropic parameters. We suppose that the values of density and stiffness tensor components change slightly from their corresponding reference values $\rho^{(0)}$ and

$C_{ijkl}^{(0)}$

$$\rho = \rho_0 + \Delta\rho \tag{50}$$

$$C_{ijkl} = C_{ijkl}^{(0)} + \Delta C_{ijkl} \tag{51}$$

Where superscript '0' indicates the value in reference medium. As we discussed before these changes cause the scattering of incident wave. In fact the perturbed terms act as the source term for the scattered wave satisfy in the wave equation. First we consider to the case that the actual medium is elastic anisotropic orthorhombic, so that the changes in nine elements of stiffness tensor C_{11},C_{22},C_{33},C_{44},C_{55},C_{66},C_{12},C_{13},C_{23} involve in scattering. We showed in previous section that these components can be written in terms of density, P-wave, S-wave velocity and Thompson parameters. Now the perturbation in stiffness tensor is given by

$$\begin{aligned}
\Delta\hat{C}_{ijkl}^{\text{ort}} &= \Delta\hat{C}_{33}\mathrm{E}_{ijkl} \\
&+ \frac{1}{2}\Delta\hat{C}_{44}\left\{\mathrm{F}_{ijkl} + (\mathrm{F}_{ijkl})_{[2,3]} - (\mathrm{F}_{ijkl})_{[1,3]}\right\} \\
&+ \frac{1}{2}\Delta\hat{C}_{55}\left\{\mathrm{F}_{ijkl} + (\mathrm{F}_{ijkl})_{[1,3]} - (\mathrm{F}_{ijkl})_{[2,3]}\right\} \\
&+ \hat{C}_{33}^{(0)}\Delta\hat{\delta}^{(1)}(\delta_{i2}\delta_{j2}\delta_{k3}\delta_{l3} + \delta_{i3}\delta_{j3}\delta_{k2}\delta_{l2}) \\
&+ \hat{C}_{33}^{(0)}\Delta\hat{\delta}^{(2)}(\delta_{i1}\delta_{j1}\delta_{k3}\delta_{l3} + \delta_{i3}\delta_{j3}\delta_{k1}\delta_{l1}) \\
&+ \hat{C}_{33}^{(0)}\Delta\hat{\delta}^{(3)}(\delta_{i1}\delta_{j1}\delta_{k2}\delta_{l2} + \delta_{i2}\delta_{j2}\delta_{k1}\delta_{l1}) \\
&+ \hat{C}_{55}^{(0)}\Delta\hat{\gamma}^{(1)}\left(\mathrm{F}_{ijkl}\right)_{[1,2]} \\
&+ \hat{C}_{44}^{(0)}\Delta\hat{\gamma}^{(2)}\left(\mathrm{F}_{ijkl}\right)_{[1,2]} \\
&+ 2\hat{C}_{33}^{(0)}\Delta\hat{\varepsilon}^{(1)}\delta_{i2}\delta_{j2}\delta_{k2}\delta_{l2} \\
&+ 2\hat{C}_{33}^{(0)}\Delta\hat{\varepsilon}^{(2)}\left\{(\mathrm{E}_{ijkl})_{[1,2]} - \delta_{i2}\delta_{j2}\delta_{k2}\delta_{l2}\right\}.
\end{aligned} \tag{52}$$

Where we defined the symmetric tensors E_{ijkl} and F_{ijkl}

$$\begin{aligned}
\mathrm{E}_{ijkl} &= \delta_{ij}\delta_{kl}, \\
\mathrm{F}_{ijkl} &= \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - 2\delta ij\delta_{kl}.
\end{aligned}$$

The important feature of the above expression for perturbation in stiffness tensor is the separation of the contributions of the anisotropy in scattering. for VTI media as $\hat{C}_{55} = \hat{C}_{44}$ and $\delta^{(1)} = \delta^{(2)} = \delta$ and $\delta^{(3)} = 0$ $\varepsilon^{(1)} = \varepsilon^{(2)} = \varepsilon$ $\gamma^{(1)} = \gamma^{(2)}$ and $C_{55}^{(0)} = C_{44}^{(0)}$

$$\begin{aligned}
\Delta\hat{C}_{ijkl}^{\text{vti}} &= \Delta\hat{C}_{33}\mathrm{E}_{ijkl} + \Delta\hat{C}_{55}\mathrm{F}_{ijkl} \\
&+ \hat{C}_{33}^{(0)}\Delta\hat{\delta}(\delta_{ij}\delta_{k3}\delta_{l3} + \delta_{kl}\delta_{i3}\delta_{j3} - 2\delta_{i3}\delta_{j3}\delta_{k3}\delta_{l3}) \\
&+ 2\hat{C}_{55}^{(0)}\Delta\hat{\gamma}\left(\mathrm{F}_{ijkl}\right)_{[1,2]} \\
&+ 2\hat{C}_{33}^{(0)}\Delta\hat{\varepsilon}(\mathrm{E}_{ijkl})_{[1,2]}.
\end{aligned} \tag{53}$$

Where the first two terms are the perturbations from elastic parameters. We can summarize what we have done so far. To apply the perturbation theory to extract the scattering potentials due to the various types of inclusions in attenuative orthorhombic media, we start with the subscript notation of the stiffness tensor. Afterwards we introduced the anisotropic Thompson parameters and corresponding Q-dependent parameters. To see the effects of anisotropy on the scattering problem we derive a useful formula for the perturbation in stiffness tensor including the changes in both elastic and anisotropic parameters. In fact given the perturbed stiffness tensor we can construct the scattering potential components. From the structure of the above expression can also be seen in an intuitive manner that how changes in both elastic and anisotropic parameters affect the radiation patterns. The scattering potential for orthorombic media is given by

$$\begin{aligned}
\mathrm{S} = &(\boldsymbol{\mathcal{S}} \cdot \boldsymbol{\mathcal{I}})\Delta\rho - \Delta\hat{C}_{33}\mathcal{F} \\
&- \frac{1}{2}\Delta\hat{C}_{44}(\mathcal{E} + \mathcal{E}_{[2,3]} - \mathcal{E}_{[1,3]}) \\
&- \frac{1}{2}\Delta\hat{C}_{55}(\mathcal{E} + \mathcal{E}_{[1,3]} - \mathcal{E}_{[2,3]}) \\
&- \hat{C}_{33}^{(0)}\Delta\hat{\delta}^{(1)}(\mathcal{S}_2\mathrm{k}_2^{\mathrm{Sc}}\mathcal{I}_3\mathrm{k}_3^{\mathrm{I}} + \mathcal{S}_3\mathrm{k}_3^{\mathrm{Sc}}\mathcal{I}_2\mathrm{k}_2^{\mathrm{I}}) \\
&- \hat{C}_{33}^{(0)}\Delta\hat{\delta}^{(2)}(\mathcal{S}_1\mathrm{k}_1^{\mathrm{Sc}}\mathcal{I}_3\mathrm{k}_3^{\mathrm{I}} + \mathcal{S}_3\mathrm{k}_3^{\mathrm{Sc}}\mathcal{I}_1\mathrm{k}_1^{\mathrm{I}}) \\
&- \hat{C}_{33}^{(0)}\Delta\hat{\delta}^{(3)}(\mathcal{S}_1\mathrm{k}_1^{\mathrm{Sc}}\mathcal{I}_2\mathrm{k}_2^{\mathrm{I}} + \mathcal{S}_2\mathrm{k}_2^{\mathrm{Sc}}\mathcal{I}_1\mathrm{k}_1^{\mathrm{I}}) \\
&- \left(\hat{C}_{55}^{(0)}\Delta\hat{\gamma}^{(1)} + \hat{C}_{44}^{(0)}\Delta\hat{\gamma}^{(2)}\right)\mathcal{E}_{[1,2]} \\
&- 2\hat{C}_{33}^{(0)}\Delta\hat{\varepsilon}^{(1)}(\mathcal{S}_2\mathrm{k}_2^{\mathrm{Sc}}\mathcal{I}_2\mathrm{k}_2^{\mathrm{I}}) \\
&- 2\hat{C}_{33}^{(0)}\Delta\hat{\varepsilon}^{(2)}\left(\mathcal{F}_{[1,2]} - \mathcal{S}_2\mathrm{k}_2^{\mathrm{Sc}}\mathcal{I}_2\mathrm{k}_2^{\mathrm{I}}\right),
\end{aligned} \tag{54}$$

where we have defined

$$\begin{aligned}
\mathcal{F} &= (\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{In}}) \\
\mathcal{E} &= (\boldsymbol{\mathcal{S}} \cdot \boldsymbol{\mathcal{I}})(\mathbf{k}^{\mathrm{Sc}} \cdot \mathbf{k}^{\mathrm{In}}) + (\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{In}})(\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{Sc}}) - 2(\boldsymbol{\mathcal{S}} \cdot \mathbf{k}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}} \cdot \mathbf{k}^{\mathrm{In}})
\end{aligned}$$

In addition the we define the subscript notation $[1, 2]$ by means that the expression only includes the 1 and 2 components, similar explanation for expressions with subscript $[2, 3]$ and $[1, 3]$. We consider to the incident homogeneous P-wave (with zero attenuation angle) propagate in an isotropic viscoelastic reference medium interaction with the anisotropic-viscoelastic inclusions. From eq (54) the normalized scattering potential is given by

$$
\begin{aligned}
\mathbb{S}_{\rm PP} = \rho_0^{-1}\mathrm{S}_{\rm PP} = \frac{\Delta\rho}{\rho}(2\sin^2\theta_P - 1) - \frac{\Delta\hat{C}_{33}}{\hat{C}_{33}} \\
&+ 4\sin^2\theta_P\cos^2\theta_P(1-\cos 2\varphi_P)\hat{V}_{\rm PS}^2\frac{\Delta\hat{C}_{44}}{\hat{C}_{44}} \\
&+ 4\sin^2\theta_P\cos^2\theta_P(1+\cos 2\varphi_P)\hat{V}_{\rm PS\perp}^2\frac{\Delta\hat{C}_{55}}{\hat{C}_{55}} \\
&- 2\Delta\hat{\delta}^{(1)}\sin^2\theta_P\cos^2\theta_P\sin^2\varphi_P \\
&- 2\Delta\hat{\delta}^{(2)}\sin^2\theta_P\cos^2\theta_P\cos^2\varphi_P \\
&- 2\Delta\hat{\delta}^{(3)}\sin^4\theta_P\cos^2\varphi_P\sin^2\varphi_P \\
&- 2\Delta\hat{\varepsilon}^{(1)}\sin^4\theta_P\sin^4\varphi_P \\
&- 2\Delta\hat{\varepsilon}^{(2)}\left(1-\sin^4\varphi_P\right)\sin^4\theta_P.
\end{aligned}
$$

Where $\hat{V}_{\rm PS} = \frac{\hat{V}_{\rm S0}}{\hat{V}_{\rm P0}}$ and $\hat{V}_{\rm PS\perp} = \frac{\hat{V}_{\rm S\perp 0}}{\hat{V}_{\rm P0}}$. For symmetry plane $[x_1, x_3]$ ($\varphi_P = 0$) the above expression reduce to

$$
{}^{\rm P}_{\rm P}\mathrm{S}^{\rm orth}_{\rm xz} = {}^{\rm P}_{\rm P}\mathrm{L}^{\rm orth}_{\rm xz} + \left({}^{\rm P}_{\rm P}\mathrm{M}^{\rm orth}_{\rm xz}\right)\sin^2\theta_P + \left({}^{\rm P}_{\rm P}\mathrm{N}^{\rm orth}_{\rm xz}\right)\sin^4\theta_P, \tag{55}
$$

where the fisrt term called the intercept or normall incident term is given by

$$
{}^{\rm P}_{\rm P}\mathrm{L}^{\rm orth}_{\rm xz} = -2\frac{\Delta V_P}{V_P} + iQ_{330}^{-1}\frac{\Delta Q_{33}}{Q_{33}}
$$

The second term which controls the $\sin^2\theta_P$ term corresponds to the small angle scattering potential

$$
\begin{aligned}
{}^{\rm P}_{\rm P}\mathrm{M}^{\rm orth}_{\rm xz} = 2\frac{\Delta\rho}{\rho} + \frac{8V_{S\perp}^2}{V_P^2}\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{S\perp}}{V_{S\perp}}\right] - 2\Delta\delta^{(2)} \\
&+ i8\left(\frac{V_{S\perp}}{V_P}\right)^2\left(Q_{550}^{-1} - Q_{330}^{-1}\right)\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{S\perp}}{V_{S\perp}}\right] \\
&- i8Q_{550}^{-1}\left(\frac{V_{S\perp}}{V_P}\right)^2\frac{\Delta Q_{55}}{Q_{55}} - iQ_{330}^{-1}\Delta\delta_Q^{(2)}
\end{aligned}
$$

finally the third term would be more influential for large angles

$$
\begin{aligned}
{}^{\rm P}_{\rm P}\mathrm{N}^{\rm orth}_{\rm xz} = -\frac{8V_{S\perp}^2}{V_P^2}\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{S\perp}}{V_{S\perp}}\right] + 2\Delta\delta^{(2)} - 2\Delta\varepsilon^{(2)} \\
&- i8\left(\frac{V_{S\perp}}{V_P}\right)^2\left(Q_{550}^{-1} - Q_{330}^{-1}\right)\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{S\perp}}{V_{S\perp}}\right] \\
&+ i8Q_{550}^{-1}\left(\frac{V_{S\perp}}{V_P}\right)^2\frac{\Delta Q_{55}}{Q_{55}} + iQ_{330}^{-1}(\Delta\delta_Q^{(2)} - \Delta\varepsilon_Q^{(2)})
\end{aligned}
$$

As the attenuation and anisotropy goes to zero the above equation reduces to the PP-scattering potential for a P-wave traveling in an isotropic elastic media interacting with the perturbations in density and P- and S-wave velocities. Removing the anisotropy and keeping the anelasticity in (55) results the scattering potential for scattering of the P-wave in an low-loss viscoelastic media with perturbations in five viscoelastic parameters. Now let us analyse each term in equation (55). This equation illustrate the scattering of a homogeneous P-wave in an low-loss viscoelastic medium interacting with the perturbations in anisotropic-viscoelastic perturbations. For normal angle of incident as $\theta_{\mathrm{P}} = 0$, PP radiation pattern depends only to the changes in P-wave velocity and P-wave quality factor. Anisotropic parameters do not influence the scattered P-wave for vertically incident P-wave. The second term which related to the small angle of incident varies with the changes in five anisotropic-viscoelastic parameters $\left(\rho, V_{S\perp}, Q_{55}, \delta^{(2)}, \delta_Q^{(2)}\right)$.

CONCLUSION AND SUMMARY

Even for elastic medium exact reflection coefficients is a very complicated function of medium properties. Nevertheless, under favorable conditions, if the changes in medium properties across the boundary are small and for small angle of incident, it is possible to find a reliable approximate solutions for reflection coefficients. Scattering potentials based on the Born approximation describes the low contrast layered medium reflection coefficients for scattering of seismic waves from complex structures including the anisotropy and attenuation. The scattering of the seismic waves from a viscoelastic VTI media is well described by the scattering potential described by solutions by means of perturbation theory. The advantages of this approach is that it does not need the exact solutions of the wave equation. Our work is concerned with the scattering potential, relies on the perturbation theory, for scattering of viscoelastic waves in an anisotropic viscoelastic media. Instead of struggling with the mathematical difficulties of solutions of Zoeppritz equation and linearization, we employ the geometry of the Born approximation which provides a useful and simple language in which the amplitude variation with offset equations can be formulated effectively and clearly.

When attenuation is included (to be specific let us say by adding the imaginary part to the stiffness tensor) there arise additional terms associated with the quality factors and anelastic Thomsen parameters. These extra terms can be seen as a deviation from the anisotropic stiffness tensor. In our calculations we assumed that the medium is weak anisotropic and attenuative, as a result inverse quality factors and Thomsen parameters are much smaller than unity. Also the fractional changes in properties are such small that higher orders can be neglected. In a first step using the Thomsen notation for definition of anisotropic properties in medium, we extract the complex Thomsen parameters where the real part characterized the anisotropy in medium and imaginary part refers to the anisotrpic-viscoelastic properties in medium.

The detailed analysis has been performed how the scattering patterns depend to the fractional changes in anisotropic and viscoelastic properties in medium. In particular we decomposed the P- to P and P-to-SI scattering potentials into elastic, anisotropic, viscoelastic and anisotropic-viscoelastic components. Elastic components includes the fractional changes in density, vertical P-wave velocity and vertical S-wave velocity; anisotropic component includes the changes in anisotropic Thomsen parameters; viscoelastic components including the fractional changes in vertical P- and S-wave quality factors and fractional changes in density, vertical P-wave velocity and vertical S-wave velocity; anisotropic component including the changes in Q-dependent Thomsen parameters. The elastic and anisotropic components are the real part of the scattering potential and viscoelastic and viscoelastic-anisotropic components are the imaginary parts of the scattering potential.

HTI ANISOTROPIC MEDIA

Tensor form of the stiffness tensor for horizontally isotropic viscoelastic media can be expressed as Ikelle and Amundsen (2005)

$$\begin{aligned}\hat{c}_{ijkl}^{\mathrm{HTI}} &= (\hat{c}_{33} - 2\hat{c}_{44})\delta_{ij}\delta_{kl} + \hat{c}_{44}(\delta_{ik}\delta_{jk} + \delta_{il}\delta_{jk}) \\ &+ (2\hat{c}_{44} - \hat{c}_{33} + \hat{c}_{13})(\delta_{ij}\delta_{k3}\delta_{l3} + \delta_{kl}\delta_{i3}\delta_{j3}) \\ &+ (\hat{c}_{66} - \hat{c}_{44})(\delta_{ik}\delta_{j3}\delta_{l3} + \delta_{jk}\delta_{i3}\delta_{l3} + \delta_{il}\delta_{j3}\delta_{k3} + \delta_{jl}\delta_{i3}\delta_{k3}) \\ &+ (\hat{c}_{11} - 2\hat{c}_{13} + \hat{c}_{33} - 4\hat{c}_{66})\delta_{i3}\delta_{j3}\delta_{k3}\delta_{l3}\end{aligned} \tag{56}$$

or in a matrix form

$$c_{\mathrm{HTI}} = \begin{pmatrix} c_{11} & c_{13} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{33} & c_{33} - 2c_{44} & 0 & 0 & 0 \\ c_{13} & c_{33} - 2c_{44} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{55} \end{pmatrix}. \tag{57}$$

Here, similar to the VTI media, P-wave phase velocity along the vertical axis z is given by $V_{\mathrm{P}} = \sqrt{c_{33}/\rho}$, vertically S-wave velocity for shear wave polarized in z-direction is given by $V_{\mathrm{S}}^{\perp} = \sqrt{c_{55}/\rho}$ and the velocity of the vertically propagating shear wave polarized in the y direction is given by $V_{\mathrm{S}} = \sqrt{c_{44}/\rho}$. Also the Thompsen parameters are defined as

$$\begin{aligned}\varepsilon^{(V)} &= \frac{c_{11} - c_{33}}{2c_{33}}, \\ \delta^{(V)} &= \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{33} - c_{55})}, \\ \gamma^{(V)} &= \frac{c_{55} - c_{44}}{2c_{44}}.\end{aligned}$$

We note that for weak anisotropy condition, $V_{\mathrm{S}}^{\perp} = V_{\mathrm{S}}(1 - \gamma)$. In what follows we outline the scattering potentials for the HTI. Most of the conclusions that we disscused for the VTI media remain valid for the HTI case.

Table 2. Notation

Symbol	Explanation	Symbol	Explanation
$\mathbf{P}_{\mathrm{P}}^{\mathrm{In}}$	Incident P-wave propagation vector	$\mathbf{A}_{\mathrm{P}}^{\mathrm{In}}$	Incident P-wave attenuation vector
$\mathbf{P}_{\mathrm{S}}^{\mathrm{In}}$	Incident S-wave propagation vector	$\mathbf{A}_{\mathrm{S}}^{\mathrm{In}}$	Incident S-wave attenuation vector
$\mathbf{P}_{\mathrm{P}}^{\mathrm{Sc}}$	Scattered P-wave propagation vector	$\mathbf{A}_{\mathrm{P}}^{\mathrm{Sc}}$	Scattered P-wave attenuation vector
$\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}}$	Scattered S-wave propagation vector	$\mathbf{A}_{\mathrm{S}}^{\mathrm{Sc}}$	Scattered S-wave attenuation vector
$\mathbf{K}_{\mathrm{P}}^{\mathrm{In}}$	Incident P-wave wavnumber vector	$\mathbf{K}_{\mathrm{S}}^{\mathrm{In}}$	Incident S-wave wavnumber vector
$\mathbf{K}_{\mathrm{P}}^{\mathrm{Sc}}$	Scattered P-wave wavnumber vector	$\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}}$	Scattered S-wave wavnumber vector
$\mathbf{k}_{\mathrm{P}}^{\mathrm{In}}$	Incident P-wave slowness vector	$\mathbf{k}_{\mathrm{S}}^{\mathrm{In}}$	Incident S-wave slowness vector
$\mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}}$	Scattered P-wave slowness vector	$\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}}$	Scattered S-wave slowness vector
$\boldsymbol{\mathcal{I}}_{\mathrm{P}}$	Incident P-wave polarization vector	$\boldsymbol{\mathcal{S}}_{\mathrm{P}}$	Scattered P-wave polarization vector
$\boldsymbol{\mathcal{I}}_{\mathrm{S}}$	Incident S-wave polarization vector	$\boldsymbol{\mathcal{S}}_{\mathrm{S}}$	Scattered S-wave polarization vector
θ_{P}	Incident/Scattered P-wave phase angle	θ_{S}	Incident/Scattered S-wave phase angle
$\delta_{\mathrm{P}}^{\mathrm{In}}$	Incident P-wave attenuation angle	$\delta_{\mathrm{S}}^{\mathrm{In}}$	Incident S-wave attenuation angle
$\delta_{\mathrm{P}}^{\mathrm{Sc}}$	Scattered P-wave attenuation angle	$\delta_{\mathrm{S}}^{\mathrm{Sc}}$	Scattered S-wave attenuation angle
V_{P}	P-wave velocity	V_{S}	S-wave velocity
Q_{33}	P-wave quality factor	Q_{55}	S-wave quality factor

COMPLEX POLARIZATION-SLOWNESS VECTORS ALGEBRA

Propagation and attenuation vectors for incident P-wave are

$$\mathbf{P}_{\mathrm{P}}^{\mathrm{In}} = \frac{\omega}{V_{\mathrm{P}}}(\mathbf{z}\cos\theta_{\mathrm{P}} + \mathbf{x}\sin\theta_{\mathrm{P}}), \tag{58}$$

$$\mathbf{A}_{\mathrm{P}}^{\mathrm{In}} = \frac{\omega}{2V_{\mathrm{P}}}Q_{\mathrm{P0}}^{-1}\sec\delta_{\mathrm{P}}^{\mathrm{In}}(\mathbf{z}\cos(\theta_{\mathrm{P}} - \delta_{\mathrm{P}}^{\mathrm{In}}) + \mathbf{x}\sin(\theta_{\mathrm{P}} - \delta_{\mathrm{P}}^{\mathrm{In}})), \tag{59}$$

scattretd P-wave

$$\mathbf{P}_{\mathrm{P}}^{\mathrm{Sc}} = \frac{\omega}{V_{\mathrm{P}}}(\mathbf{x}\sin\theta_{\mathrm{P}} - \mathbf{z}\cos\theta_{\mathrm{P}}), \tag{60}$$

$$\mathbf{A}_{\mathrm{P}}^{\mathrm{Sc}} = \frac{\omega}{2V_{\mathrm{P}}}Q_{\mathrm{P0}}^{-1}\sec\delta_{\mathrm{P}}^{\mathrm{Sc}}(\mathbf{x}\sin(\theta_{\mathrm{P}} - \delta_{\mathrm{P}}^{\mathrm{Sc}}) - \mathbf{z}\cos(\theta_{\mathrm{P}} - \delta_{\mathrm{P}}^{\mathrm{Sc}})), \tag{61}$$

incident S-wave

$$\mathbf{P}_{\mathrm{S}}^{\mathrm{In}} = \frac{\omega}{V_{\mathrm{S}}}(\mathbf{z}\cos\theta_{\mathrm{S}} + \mathbf{x}\sin\theta_{\mathrm{S}}), \tag{62}$$

$$\mathbf{A}_{\mathrm{S}}^{\mathrm{In}} = \frac{\omega}{2V_{\mathrm{S}}}Q_{\mathrm{S0}}^{-1}\sec\delta_{\mathrm{S}}^{\mathrm{In}}(\mathbf{z}\cos(\theta_{\mathrm{S}} - \delta_{\mathrm{S}}^{\mathrm{In}}) + \mathbf{x}\sin(\theta_{\mathrm{S}} - \delta_{\mathrm{S}}^{\mathrm{In}})), \tag{63}$$

scattered S-wave

$$\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}} = \frac{\omega}{V_{\mathrm{S}}}(\mathbf{x}\sin\theta_{\mathrm{S}} - \mathbf{z}\cos\theta_{\mathrm{S}}), \tag{64}$$

$$\mathbf{A}_{\mathrm{S}}^{\mathrm{Sc}} = \frac{\omega}{2V_{\mathrm{S}}}Q_{\mathrm{S0}}^{-1}\sec\delta_{\mathrm{S}}^{\mathrm{Sc}}(\mathbf{x}\sin(\theta_{\mathrm{S}} - \delta_{\mathrm{S}}^{\mathrm{Sc}}) - \mathbf{z}\cos(\theta_{\mathrm{S}} - \delta_{\mathrm{S}}^{\mathrm{Sc}})). \tag{65}$$

Now the components of the wavenumber vector are given by

$$\mathrm{K}_{\mathrm{Px}}^{\mathrm{In}} = \mathrm{P}_{\mathrm{Px}}^{\mathrm{In}} - i\mathrm{A}_{\mathrm{Px}}^{\mathrm{In}} = \frac{\omega}{V_{\mathrm{P}}}\left[\sin\theta_{\mathrm{P}}\left(1-\frac{i}{2}Q_{\mathrm{P0}}^{-1}\right)+\frac{i}{2}\cos\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}^{\mathrm{In}}\right], \tag{66}$$

$$\mathrm{K}_{\mathrm{Pz}}^{\mathrm{In}} = \mathrm{P}_{\mathrm{Pz}}^{\mathrm{In}} - i\mathrm{A}_{\mathrm{Pz}}^{\mathrm{In}} = \frac{\omega}{V_{\mathrm{P}}}\left[\cos\theta_{\mathrm{P}}\left(1-\frac{i}{2}Q_{\mathrm{P0}}^{-1}\right)-\frac{i}{2}\sin\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}^{\mathrm{In}}\right], \tag{67}$$

$$\mathrm{K}_{\mathrm{Sx}}^{\mathrm{In}} = \mathrm{P}_{\mathrm{Sx}}^{\mathrm{In}} - i\mathrm{A}_{\mathrm{Sx}}^{\mathrm{In}} = \frac{\omega}{V_{\mathrm{S}}}\left[\sin\theta_{\mathrm{S}}\left(1-\frac{i}{2}Q_{\mathrm{S0}}^{-1}\right)+\frac{i}{2}\cos\theta_{\mathrm{S}}\tan\delta_{\mathrm{S}}^{\mathrm{In}}\right], \tag{68}$$

$$\mathrm{K}_{\mathrm{Sz}}^{\mathrm{In}} = \mathrm{P}_{\mathrm{Sz}}^{\mathrm{In}} - i\mathrm{A}_{\mathrm{Sz}}^{\mathrm{In}} = \frac{\omega}{V_{\mathrm{S}}}\left[\cos\theta_{\mathrm{S}}\left(1-\frac{i}{2}Q_{\mathrm{S0}}^{-1}\right)-\frac{i}{2}\sin\theta_{\mathrm{S}}\tan\delta_{\mathrm{S}}^{\mathrm{In}}\right], \tag{69}$$

$$\mathrm{K}_{\mathrm{Px}}^{\mathrm{Sc}} = \mathrm{P}_{\mathrm{Px}}^{\mathrm{Sc}} - i\mathrm{A}_{\mathrm{Px}}^{\mathrm{Sc}} = \frac{\omega}{V_{\mathrm{P}}}\left[\sin\theta_{\mathrm{P}}\left(1-\frac{i}{2}Q_{\mathrm{P0}}^{-1}\right)+\frac{i}{2}\cos\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}^{\mathrm{Sc}}\right], \tag{70}$$

$$\mathrm{K}_{\mathrm{Pz}}^{\mathrm{Sc}} = \mathrm{P}_{\mathrm{Pz}}^{\mathrm{Sc}} - i\mathrm{A}_{\mathrm{Pz}}^{\mathrm{Sc}} = -\frac{\omega}{V_{\mathrm{P}}}\left[\cos\theta_{\mathrm{P}}\left(1-\frac{i}{2}Q_{\mathrm{P0}}^{-1}\right)-\frac{i}{2}\sin\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}^{\mathrm{Sc}}\right], \tag{71}$$

$$\mathrm{K}_{\mathrm{Sx}}^{\mathrm{Sc}} = \mathrm{P}_{\mathrm{Sx}}^{\mathrm{Sc}} - i\mathrm{A}_{\mathrm{Sx}}^{\mathrm{Sc}} = \frac{\omega}{V_{\mathrm{S}}}\left[\sin\theta_{\mathrm{S}}\left(1-\frac{i}{2}Q_{\mathrm{S0}}^{-1}\right)+\frac{i}{2}\cos\theta_{\mathrm{S}}\tan\delta_{\mathrm{S}}^{\mathrm{Sc}}\right], \tag{72}$$

$$\mathrm{K}_{\mathrm{Sz}}^{\mathrm{Sc}} = \mathrm{P}_{\mathrm{Sz}}^{\mathrm{Sc}} - i\mathrm{A}_{\mathrm{Sz}}^{\mathrm{Sc}} = -\frac{\omega}{V_{\mathrm{S}}}\left[\cos\theta_{\mathrm{S}}\left(1-\frac{i}{2}Q_{\mathrm{S0}}^{-1}\right)-\frac{i}{2}\sin\theta_{\mathrm{S}}\tan\delta_{\mathrm{S}}^{\mathrm{Sc}}\right]. \tag{73}$$

Inner product of incident and reflected propagation and attenuation vectors for P-waves

$$\begin{aligned}
\mathbf{P}_{\mathrm{P}}^{\mathrm{In}}\cdot\mathbf{P}_{\mathrm{P}}^{\mathrm{Sc}} &= -\frac{\omega^2}{V_{\mathrm{P}}^2}\cos 2\theta_{\mathrm{P}},\\
\mathbf{P}_{\mathrm{P}}^{\mathrm{In}}\cdot\mathbf{A}_{\mathrm{P}}^{\mathrm{Sc}} &= -\frac{\omega^2}{2V_{\mathrm{P}}^2}Q_{\mathrm{P0}}^{-1}\sec\delta_{\mathrm{P}}^{\mathrm{Sc}}\cos(2\theta_{\mathrm{P}}-\delta_{\mathrm{P}}^{\mathrm{Sc}}),\\
\mathbf{P}_{\mathrm{P}}^{\mathrm{Sc}}\cdot\mathbf{A}_{\mathrm{P}}^{\mathrm{In}} &= -\frac{\omega^2}{2V_{\mathrm{P}}^2}Q_{\mathrm{P0}}^{-1}\sec\delta_{\mathrm{P}}^{\mathrm{In}}\cos(2\theta_{\mathrm{P}}-\delta_{\mathrm{P}}^{\mathrm{In}}),
\end{aligned} \tag{74}$$

Inner product of incident and reflected propagation and attenuation vectors for S-waves

$$\begin{aligned}
\mathbf{P}_{\mathrm{S}}^{\mathrm{In}}\cdot\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}} &= -\frac{\omega^2}{V_{\mathrm{S}}^2}\cos 2\theta_{\mathrm{S}},\\
\mathbf{P}_{\mathrm{S}}^{\mathrm{In}}\cdot\mathbf{A}_{\mathrm{S}}^{\mathrm{Sc}} &= -\frac{\omega^2}{2V_{\mathrm{S}}^2}Q_{\mathrm{S0}}^{-1}\sec\delta_{\mathrm{S}}^{\mathrm{Sc}}\cos(2\theta_{\mathrm{S}}-\delta_{\mathrm{S}}^{\mathrm{Sc}}),\\
\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}}\cdot\mathbf{A}_{\mathrm{S}}^{\mathrm{In}} &= -\frac{\omega^2}{2V_{\mathrm{S}}^2}Q_{\mathrm{S0}}^{-1}\sec\delta_{\mathrm{S}}^{\mathrm{In}}\cos(2\theta_{\mathrm{S}}-\delta_{\mathrm{S}}^{\mathrm{In}}),
\end{aligned} \tag{75}$$

Inner product of incident and reflected propagation and attenuation vectors for P-waves and S-waves

$$\begin{aligned}
\mathbf{P}_{\mathrm{P}}^{\mathrm{In}}\cdot\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}} &= -\frac{\omega^2}{V_{\mathrm{P}}V_{\mathrm{S}}}\cos(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}),\\
\mathbf{P}_{\mathrm{P}}^{\mathrm{In}}\cdot\mathbf{A}_{\mathrm{S}}^{\mathrm{Sc}} &= -\frac{\omega^2}{V_{\mathrm{P}}V_{\mathrm{S}}}Q_{S}^{-1}\sec\delta_{\mathrm{S}}^{\mathrm{Sc}}\cos(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}-\delta_{\mathrm{S}}^{\mathrm{Sc}}),\\
\mathbf{P}_{\mathrm{P}}^{\mathrm{Sc}}\cdot\mathbf{A}_{\mathrm{S}}^{\mathrm{In}} &= -\frac{\omega^2}{V_{\mathrm{P}}V_{\mathrm{S}}}Q_{S}^{-1}\sec\delta_{\mathrm{S}}^{\mathrm{In}}\cos(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}-\delta_{\mathrm{S}}^{\mathrm{In}}),
\end{aligned} \tag{76}$$

In low-loss case we have

$$
\begin{aligned}
\mathbf{K}_{\mathrm{P}}^{\mathrm{In}}\cdot\mathbf{K}_{\mathrm{P}}^{\mathrm{Sc}} &= -\frac{\omega^2}{V_{\mathrm{P}}^2}\left\{\cos 2\theta_{\mathrm{P}}(1-iQ_{\mathrm{P0}}^{-1})-\frac{i}{2}Q_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{P}}\left(\tan\delta_{\mathrm{P}}^{\mathrm{Sc}}+\tan\delta_{\mathrm{P}}^{\mathrm{In}}\right)\right\},\\
\mathbf{K}_{\mathrm{S}}^{\mathrm{In}}\cdot\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}} &= -\frac{\omega^2}{V_{\mathrm{S}}^2}\left\{\cos 2\theta_{\mathrm{S}}(1-iQ_{\mathrm{S0}}^{-1})-\frac{i}{2}Q_{\mathrm{S0}}^{-1}\sin 2\theta_{\mathrm{S}}\left(\tan\delta_{\mathrm{S}}^{\mathrm{Sc}}+\tan\delta_{\mathrm{S}}^{\mathrm{In}}\right)\right\},\\
\mathbf{K}_{\mathrm{P}}^{\mathrm{In}}\cdot\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}} &= -\frac{\omega^2}{V_{\mathrm{P}}V_{\mathrm{S}}}\left\{\cos(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\left(1-\frac{i}{2}Q_{\mathrm{S0}}^{-1}-\frac{i}{2}Q_{\mathrm{P0}}^{-1}\right)\right.\\
&\qquad\left.-\frac{i}{2}\sin(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\left(Q_{\mathrm{S0}}^{-1}\tan\delta_{\mathrm{S}}^{\mathrm{Sc}}+Q_{\mathrm{P0}}^{-1}\tan\delta_{\mathrm{P}}^{\mathrm{In}}\right)\right\}.
\end{aligned}
\tag{77}
$$

Now consider to the cross product of the vectors

$$
\begin{aligned}
\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}}\times\mathbf{P}_{\mathrm{P}}^{\mathrm{In}} &= -\mathbf{y}\frac{\omega^2}{V_{\mathrm{P}}V_{\mathrm{S}}}\sin(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}),\\
\mathbf{A}_{\mathrm{S}}^{\mathrm{Sc}}\times\mathbf{P}_{\mathrm{P}}^{\mathrm{In}} &= -\mathbf{y}\frac{\omega^2}{2V_{\mathrm{S}}V_{\mathrm{P}}}Q_{\mathrm{S0}}^{-1}\sec\delta_{\mathrm{S}}^{\mathrm{Sc}}\sin(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}-\delta_{\mathrm{S}}^{\mathrm{Sc}}),\\
\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}}\times\mathbf{A}_{\mathrm{P}}^{\mathrm{In}} &= -\mathbf{y}\frac{\omega^2}{2V_{\mathrm{S}}V_{\mathrm{P}}}Q_{\mathrm{P0}}^{-1}\sec\delta_{\mathrm{P}}^{\mathrm{In}}\sin(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}-\delta_{\mathrm{P}}^{\mathrm{In}}),\\
\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}}\times\mathbf{P}_{\mathrm{S}}^{\mathrm{In}} &= -\mathbf{y}\frac{\omega^2}{V_{\mathrm{S}}^2}\sin 2\theta_{\mathrm{S}}\\
\mathbf{A}_{\mathrm{S}}^{\mathrm{Sc}}\times\mathbf{P}_{\mathrm{S}}^{\mathrm{In}} &= -\mathbf{y}\frac{\omega^2}{2V_{\mathrm{S}}^2}Q_{S}^{-1}\sec\delta_{\mathrm{S}}^{\mathrm{Sc}}\sin(2\theta_{\mathrm{S}}-\delta_{\mathrm{S}}^{\mathrm{Sc}}),\\
\mathbf{P}_{\mathrm{S}}^{\mathrm{Sc}}\times\mathbf{A}_{\mathrm{S}}^{\mathrm{In}} &= -\mathbf{y}\frac{\omega^2}{2V_{\mathrm{S}}^2}Q_{S}^{-1}\sec\delta_{\mathrm{P}}^{\mathrm{In}}\sin(2\theta_{\mathrm{S}}-\delta_{\mathrm{S}}^{\mathrm{In}}),
\end{aligned}
\tag{78}
$$

we have

$$
\begin{aligned}
\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}}\times\mathbf{K}_{\mathrm{P}}^{\mathrm{In}} &= -\mathbf{y}\frac{\omega^2}{V_{\mathrm{P}}V_{\mathrm{S}}}\left\{\sin(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\left(1-\frac{i}{2}Q_{\mathrm{S0}}^{-1}-\frac{i}{2}Q_{\mathrm{P0}}^{-1}\right)\right.\\
&\qquad\left.+\frac{i}{2}\cos(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\left(Q_{\mathrm{S0}}^{-1}\tan\delta_{\mathrm{S}}^{\mathrm{Sc}}+Q_{\mathrm{P0}}^{-1}\tan\delta_{\mathrm{P}}^{\mathrm{In}}\right)\right\},\\
\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}}\times\mathbf{K}_{\mathrm{S}}^{\mathrm{In}} &= -\mathbf{y}\frac{\omega^2}{V_{\mathrm{S}}^2}\left\{\sin 2\theta_{\mathrm{S}}\left(1-iQ_{\mathrm{S0}}^{-1}\right)+\frac{i}{2}\cos 2\theta_{\mathrm{S}}Q_{\mathrm{S0}}^{-1}\left(\tan\delta_{\mathrm{S}}^{\mathrm{Sc}}+\tan\delta_{\mathrm{S}}^{\mathrm{In}}\right)\right\}.
\end{aligned}
\tag{79}
$$

Consider to the P-to-P scattering potential. The incident and scattered P-wave is defined by

$$
\begin{aligned}
\boldsymbol{\mathcal{S}}_{\mathrm{P}} &= \hat{V}_{\mathrm{P}}\mathbf{K}_{\mathrm{P}}^{\mathrm{Sc}},\\
\boldsymbol{\mathcal{I}}_{\mathrm{P}} &= \hat{V}_{\mathrm{P}}\mathbf{K}_{\mathrm{P}}^{\mathrm{In}}.
\end{aligned}
\tag{80}
$$

Polarization for SI-wave

$$
\begin{aligned}
\boldsymbol{\mathcal{S}}_{\mathrm{S}} &= \hat{V}_{\mathrm{S}}\mathbf{y}\times\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}} = \mathbf{y}\times(\mathbf{x}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{Sc}}+\mathbf{z}\mathrm{k}_{\mathrm{Sz}}^{\mathrm{Sc}}) = -\mathbf{z}\hat{V}_{\mathrm{S}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{Sc}}+\mathbf{x}\hat{V}_{\mathrm{S}}\mathrm{k}_{\mathrm{Sz}}^{\mathrm{Sc}}\\
\boldsymbol{\mathcal{I}}_{\mathrm{S}} &= \hat{V}_{\mathrm{S}}\mathbf{y}\times\mathbf{K}_{\mathrm{S}}^{\mathrm{In}} = \mathbf{y}\times(\mathbf{x}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{In}}+\mathbf{z}\mathrm{k}_{\mathrm{Sz}}^{\mathrm{In}}) = -\mathbf{z}\hat{V}_{\mathrm{S}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{In}}+\mathbf{x}\hat{V}_{\mathrm{S}}\mathrm{k}_{\mathrm{Sz}}^{\mathrm{In}}
\end{aligned}
\tag{81}
$$

where incident and scattered slowness P-wave vectors are defined by $\mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}}$ and $\mathbf{k}_{\mathrm{P}}^{\mathrm{In}}$.

SCATTERING POTENTIAL

P-to-P scattering potential: scattering potential is given by

$$
\begin{aligned}
[\mathrm{PP}] &= (\boldsymbol{\mathcal{S}}_{\mathrm{P}} \cdot \boldsymbol{\mathcal{I}}_{\mathrm{P}})\frac{\Delta\rho}{\rho} - \frac{\hat{c}_{33}^{(0)}}{\rho_0}(\boldsymbol{\mathcal{S}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{In}})\frac{\Delta\hat{c}_{33}}{\hat{c}_{33}^{(0)}} \\
&- \left(\frac{\hat{c}_{55}^{(0)}}{\hat{c}_{33}^{(0)}}\right)\frac{\hat{c}_{33}^{(0)}}{\rho_0}\left[(\boldsymbol{\mathcal{S}}_{\mathrm{P}} \cdot \boldsymbol{\mathcal{I}}_{\mathrm{P}})(\mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{In}}) - 2(\boldsymbol{\mathcal{S}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{In}}) + (\boldsymbol{\mathcal{S}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{In}})(\boldsymbol{\mathcal{I}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}})\right]\frac{\Delta\hat{c}_{55}}{\hat{c}_{55}^{(0)}} \\
&- 2\hat{c}_{33}^{(0)}(\mathcal{S}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{In}})\Delta\hat{\varepsilon} - \hat{c}_{33}^{(0)}(\mathcal{S}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Pz}}\mathrm{k}_{\mathrm{Pz}}^{\mathrm{In}} + \mathcal{S}_{\mathrm{Pz}}\mathrm{k}_{\mathrm{Pz}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{In}})\Delta\hat{\delta}.
\end{aligned}
$$

We consider to each term individually. In the first order approximation $\tan\delta_{\mathrm{P}}^{\mathrm{Sc}} \approx \tan\delta_{\mathrm{P}}^{\mathrm{In}} \approx \tan\delta_{\mathrm{P}}$.

$$
\begin{aligned}
&\boldsymbol{\mathcal{S}}_{\mathrm{P}} \cdot \boldsymbol{\mathcal{I}}_{\mathrm{P}} = \frac{\hat{V}_{\mathrm{P}}^2}{\omega^2}\mathbf{K}_{\mathrm{P}}^{\mathrm{In}} \cdot \mathbf{K}_{\mathrm{P}}^{\mathrm{Sc}} = -\cos 2\theta_{\mathrm{P}} + iQ_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}, \\
&\frac{\hat{c}_{33}^{(0)}}{\rho_0}(\boldsymbol{\mathcal{S}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}})(\boldsymbol{\mathcal{I}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{In}}) = \frac{\hat{V}_{\mathrm{P}}^4}{\omega^4}(\mathbf{K}_{\mathrm{P}}^{\mathrm{In}} \cdot \mathbf{K}_{\mathrm{P}}^{\mathrm{In}})(\mathbf{K}_{\mathrm{P}}^{\mathrm{Sc}} \cdot \mathbf{K}_{\mathrm{P}}^{\mathrm{Sc}}) = 1, \\
&\frac{\hat{c}_{33}^{(0)}}{\rho_0}(\boldsymbol{\mathcal{S}}_{\mathrm{P}} \cdot \boldsymbol{\mathcal{I}}_{\mathrm{P}})(\mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{In}}) = \frac{\hat{c}_{33}^{(0)}}{\rho_0}(\boldsymbol{\mathcal{S}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{In}})(\boldsymbol{\mathcal{I}}_{\mathrm{P}} \cdot \mathbf{k}_{\mathrm{P}}^{\mathrm{Sc}}) = \\
&\frac{\hat{V}_{\mathrm{P}}^4}{\omega^4}\left(\mathbf{K}_{\mathrm{P}}^{\mathrm{Sc}} \cdot \mathbf{K}_{\mathrm{P}}^{\mathrm{In}}\right)^2 = \cos^2 2\theta_{\mathrm{P}} - iQ_{\mathrm{P0}}^{-1}\sin 4\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}} \\
&\frac{\hat{c}_{33}^{(0)}}{\rho_0}\mathcal{S}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{In}} = \frac{\hat{V}_{\mathrm{P}}^4}{\omega^4}\left(\mathrm{K}_{\mathrm{Px}}^{\mathrm{In}}\mathrm{K}_{\mathrm{Px}}^{\mathrm{Sc}}\right)^2 = \sin^4\theta_{\mathrm{P}} + iQ_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{P}}\sin^2\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}, \\
&\frac{\hat{c}_{33}^{(0)}}{\rho_0}\left(\mathcal{S}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Pz}}\mathrm{k}_{\mathrm{Pz}}^{\mathrm{In}} + \mathcal{S}_{\mathrm{Pz}}\mathrm{k}_{\mathrm{Pz}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Px}}\mathrm{k}_{\mathrm{Px}}^{\mathrm{In}}\right) = \frac{1}{2}\left(\sin^2 2\theta_{\mathrm{P}} + iQ_{\mathrm{P0}}^{-1}\sin 4\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\right).
\end{aligned}
\tag{82}
$$

As a result

$$
\begin{aligned}
[\mathrm{PP}]_{\mathrm{IE}} &= -(1 + \cos 2\theta_{\mathrm{P}} - 2V_{\mathrm{SP}}^2\sin^2 2\theta_{\mathrm{P}})\frac{\Delta\rho}{\rho} - 2\frac{\Delta V_P}{V_P} + 4V_{\mathrm{SP}}^2\sin^2 2\theta_{\mathrm{P}}\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}} \\
[\mathrm{PP}]_{\mathrm{AE}} &= -2\sin^4\theta_{\mathrm{P}}\Delta\varepsilon - \frac{1}{2}\sin^2 2\theta_{\mathrm{P}}\Delta\delta \\
[\mathrm{PP}]_{\mathrm{IV}} &= 2V_{\mathrm{SP}}^2\sin^2 2\theta_{\mathrm{P}}\left\{(Q_{\mathrm{S0}}^{-1} - Q_{\mathrm{P0}}^{-1})\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right) - Q_{\mathrm{S0}}^{-1}\frac{\Delta Q_S}{Q_S}\right\} + Q_{\mathrm{P0}}^{-1}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}} \\
&\quad + Q_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\frac{\Delta\rho}{\rho} + 2Q_{\mathrm{P0}}^{-1}V_{\mathrm{SP}}^2\sin 4\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right) \\
[\mathrm{PP}]_{\mathrm{AV}} &= -Q_{\mathrm{P0}}^{-1}\sin^4\theta_{\mathrm{P}}\Delta\varepsilon_Q - \frac{1}{4}Q_{\mathrm{P0}}^{-1}\sin^2 2\theta_{\mathrm{P}}\Delta\delta_Q \\
&\quad - 2Q_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{P}}\sin^2\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\Delta\varepsilon - \frac{1}{2}Q_{\mathrm{P0}}^{-1}\sin 4\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\Delta\delta
\end{aligned}
$$

Scattering of P-to-SI

The scattering of P-to-SI wave is given by

$$[\mathrm{PSI}] = (\boldsymbol{\mathcal{S}}_\mathrm{S}\cdot\boldsymbol{\mathcal{I}}_\mathrm{P})\frac{\Delta\rho}{\rho} - \frac{c_{55}^{(0)}}{\rho_0}\left[(\boldsymbol{\mathcal{S}}_\mathrm{S}\cdot\boldsymbol{\mathcal{I}}_\mathrm{P})(\mathbf{k}_\mathrm{S}^\mathrm{Sc}\cdot\mathbf{k}_\mathrm{P}^\mathrm{In}) + (\boldsymbol{\mathcal{S}}_\mathrm{S}\cdot\mathbf{k}_\mathrm{P}^\mathrm{In})(\boldsymbol{\mathcal{I}}_\mathrm{P}\cdot\mathbf{k}_\mathrm{S}^\mathrm{Sc})\right]\frac{\Delta\hat{c}_{55}}{c_{55}^{(0)}}$$
$$- 2\frac{\hat{c}_{33}^{(0)}}{\rho_0}(\mathcal{S}_\mathrm{Sx}\mathrm{k}_\mathrm{Sx}^\mathrm{Sc}\mathcal{I}_\mathrm{Px}\mathrm{k}_\mathrm{Px}^\mathrm{In})\Delta\hat{\varepsilon} - \frac{\hat{c}_{33}^{(0)}}{\rho_0}(\mathcal{S}_\mathrm{Sx}\mathrm{k}_\mathrm{Sx}^\mathrm{Sc}\mathcal{I}_\mathrm{Pz}\mathrm{k}_\mathrm{Pz}^\mathrm{In} + \mathcal{S}_\mathrm{Sz}\mathrm{k}_\mathrm{Sz}^\mathrm{Sc}\mathcal{I}_\mathrm{Px}\mathrm{k}_\mathrm{Px}^\mathrm{In})\Delta\hat{\delta}$$

We have

$$\boldsymbol{\mathcal{S}}_\mathrm{S}\cdot\boldsymbol{\mathcal{I}}_\mathrm{P} = \frac{\hat{V}_\mathrm{P}\hat{V}_\mathrm{S}}{\omega^2}\left(\mathbf{y}\times\mathbf{K}_\mathrm{S}^\mathrm{Sc}\right)\cdot\mathbf{K}_\mathrm{P}^\mathrm{In} = \frac{\hat{V}_\mathrm{P}\hat{V}_\mathrm{S}}{\omega^2}\mathbf{y}\cdot(\mathbf{K}_\mathrm{S}^\mathrm{Sc}\times\mathbf{K}_\mathrm{P}^\mathrm{In})$$
$$\boldsymbol{\mathcal{S}}_\mathrm{S}\cdot\mathbf{k}_\mathrm{P}^\mathrm{In} = \frac{\hat{V}_\mathrm{S}}{\omega^2}\left(\mathbf{y}\times\mathbf{K}_\mathrm{S}^\mathrm{Sc}\right)\cdot\mathbf{K}_\mathrm{P}^\mathrm{In} = \frac{\hat{V}_\mathrm{S}}{\omega^2}\mathbf{y}\cdot(\mathbf{K}_\mathrm{S}^\mathrm{Sc}\times\mathbf{K}_\mathrm{P}^\mathrm{In})$$
$$\boldsymbol{\mathcal{I}}_\mathrm{P}\cdot\mathbf{k}_\mathrm{S}^\mathrm{Sc} = \frac{\hat{V}_\mathrm{P}}{\omega^2}(\mathbf{K}_\mathrm{P}^\mathrm{In}\cdot\mathbf{K}_\mathrm{S}^\mathrm{Sc})$$
$$\frac{\hat{c}_{55}^{(0)}}{\rho_0}\left[(\boldsymbol{\mathcal{S}}_\mathrm{S}\cdot\boldsymbol{\mathcal{I}}_\mathrm{P})(\mathbf{k}_\mathrm{S}^\mathrm{Sc}\cdot\mathbf{k}_\mathrm{P}^\mathrm{In}) + (\boldsymbol{\mathcal{S}}_\mathrm{S}\cdot\mathbf{k}_\mathrm{P}^\mathrm{In})(\boldsymbol{\mathcal{I}}_\mathrm{P}\cdot\mathbf{k}_\mathrm{S}^\mathrm{Sc})\right] = 2\frac{\hat{c}_{55}^{(0)}}{\rho_0}\frac{\hat{V}_\mathrm{P}\hat{V}_\mathrm{S}}{\omega^4}\mathbf{y}\cdot(\mathbf{K}_\mathrm{S}^\mathrm{Sc}\times\mathbf{K}_\mathrm{P}^\mathrm{In})(\mathbf{K}_\mathrm{P}^\mathrm{In}\cdot\mathbf{K}_\mathrm{S}^\mathrm{Sc}) =$$
$$\frac{V_\mathrm{S}}{V_\mathrm{P}}\left\{\sin 2(\theta_\mathrm{P}+\theta_\mathrm{S})\left(1+\frac{i}{2}(Q_\mathrm{S0}^{-1}-Q_\mathrm{P0}^{-1})\right) + i\cos 2(\theta_\mathrm{P}+\theta_\mathrm{S})\left(Q_\mathrm{S0}^{-1}\tan\delta_\mathrm{S} + Q_\mathrm{P0}^{-1}\tan\delta_\mathrm{P}\right)\right\} \tag{83}$$

$$2\frac{\hat{c}_{33}^{(0)}}{\rho_0}\mathcal{S}_\mathrm{Sx}\mathrm{k}_\mathrm{Sx}^\mathrm{Sc}\mathcal{I}_\mathrm{Px}\mathrm{k}_\mathrm{Px}^\mathrm{In} = -2\frac{\hat{V}_\mathrm{P}^3\hat{V}_\mathrm{S}}{\omega^4}\mathrm{K}_\mathrm{Sx}^\mathrm{Sc}\mathrm{K}_\mathrm{Sz}^\mathrm{Sc}\left(\mathrm{K}_\mathrm{Px}^\mathrm{In}\right)^2$$
$$= -\frac{V_\mathrm{P}}{V_\mathrm{S}}\left\{\sin 2\theta_\mathrm{S}\sin\theta_\mathrm{P}\left(1-\frac{i}{2}(Q_\mathrm{S0}^{-1}-Q_\mathrm{P0}^{-1})\right) + iQ_\mathrm{S0}^{-1}\cos 2\theta_\mathrm{S}\sin\theta_\mathrm{P}\tan\delta_\mathrm{S}\right.$$
$$\left. + iQ_\mathrm{P0}^{-1}\sin 2\theta_\mathrm{S}\cos\theta_\mathrm{P}\tan\delta_\mathrm{P}\right\}\sin\theta_\mathrm{P}$$

$$\frac{\hat{c}_{33}^{(0)}}{\rho_0}\left(\mathcal{S}_\mathrm{Sx}\mathrm{k}_\mathrm{Sx}^\mathrm{Sc}\mathcal{I}_\mathrm{Pz}\mathrm{k}_\mathrm{Pz}^\mathrm{In} + \mathcal{S}_\mathrm{Sz}\mathrm{k}_\mathrm{Sz}^\mathrm{Sc}\mathcal{I}_\mathrm{Px}\mathrm{k}_\mathrm{Px}^\mathrm{In}\right) = \hat{V}_\mathrm{P}^2\left(\mathcal{S}_\mathrm{Sx}\mathrm{k}_\mathrm{Sx}^\mathrm{Sc}\mathcal{I}_\mathrm{Pz}\mathrm{k}_\mathrm{Pz}^\mathrm{In} + \mathcal{S}_\mathrm{Sz}\mathrm{k}_\mathrm{Sz}^\mathrm{Sc}\mathcal{I}_\mathrm{Px}\mathrm{k}_\mathrm{Px}^\mathrm{In}\right) =$$
$$\frac{\hat{V}_\mathrm{P}^3\hat{V}_\mathrm{S}}{\omega^4}\left\{\left(\mathrm{K}_\mathrm{Pz}^\mathrm{In}\right)^2 - \left(\mathrm{K}_\mathrm{Px}^\mathrm{In}\right)^2\right\}\mathrm{K}_\mathrm{Sz}^\mathrm{Sc}\mathrm{K}_\mathrm{Sx}^\mathrm{Sc} =$$
$$-\frac{1}{2}\frac{V_\mathrm{P}}{V_\mathrm{S}}\left\{\cos 2\theta_\mathrm{P}\sin 2\theta_\mathrm{S}\left(1-\frac{i}{2}(Q_\mathrm{S0}^{-1}-Q_\mathrm{P0}^{-1})\right) + iQ_\mathrm{S0}^{-1}\cos 2\theta_\mathrm{S}\cos 2\theta_\mathrm{P}\tan\delta_\mathrm{S}\right.$$
$$\left. - iQ_\mathrm{P0}^{-1}\sin 2\theta_\mathrm{S}\sin 2\theta_\mathrm{P}\tan\delta_\mathrm{P}\right\}$$

Finally

$$
\begin{aligned}
[\mathrm{PSI}]_{\mathrm{IE}} &= -\left\{\sin(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}) + V_{\mathrm{SP}}\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\right\}\frac{\Delta\rho}{\rho} - 2V_{\mathrm{SP}}\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\frac{\Delta V_S}{V_S} \\
[\mathrm{PSI}]_{\mathrm{IV}} &= -\frac{1}{2}V_{\mathrm{SP}}\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})(Q_{\mathrm{S0}}^{-1}-Q_{\mathrm{P0}}^{-1})\frac{\Delta\rho}{\rho} \\
&\quad - V_{\mathrm{SP}}\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})(Q_{\mathrm{S0}}^{-1}-Q_{\mathrm{P0}}^{-1})\frac{\Delta V_S}{V_S} + Q_{\mathrm{S0}}^{-1}V_{\mathrm{SP}}\sin 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\frac{\Delta Q_S}{Q_S} \\
&\quad - \left\{\frac{1}{2}\cos(\theta_{\mathrm{P}}+\theta_{\mathrm{S}}) + V_{\mathrm{SP}}\cos 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\right\}\left(Q_{\mathrm{S0}}^{-1}\tan\delta_{\mathrm{S}} + Q_{\mathrm{P0}}^{-1}\tan\delta_{\mathrm{P}}\right)\frac{\Delta\rho}{\rho} \\
&\quad - 2V_{\mathrm{SP}}\cos 2(\theta_{\mathrm{P}}+\theta_{\mathrm{S}})\left(Q_{\mathrm{S0}}^{-1}\tan\delta_{\mathrm{S}} + Q_{\mathrm{P0}}^{-1}\tan\delta_{\mathrm{P}}\right)\frac{\Delta V_S}{V_S} \\
[\mathrm{PSI}]_{\mathrm{AE}} &= V_{\mathrm{PS}}\sin 2\theta_{\mathrm{S}}\sin^2\theta_{\mathrm{P}}\Delta\varepsilon + \frac{1}{2}V_{\mathrm{PS}}\cos 2\theta_{\mathrm{P}}\sin 2\theta_{\mathrm{S}}\Delta\delta
\end{aligned}
$$

$$
\begin{aligned}
[\mathrm{PSI}]_{\mathrm{AV}} &= \frac{1}{2}Q_{\mathrm{P0}}^{-1}V_{\mathrm{PS}}\sin 2\theta_{\mathrm{S}}\sin\theta_{\mathrm{P}}\Delta\varepsilon_Q - \frac{1}{2}V_{\mathrm{PS}}\sin 2\theta_{\mathrm{S}}\sin^2\theta_{\mathrm{P}}\left(Q_S^{-1}-Q_{\mathrm{P0}}^{-1}\right)\Delta\varepsilon \\
&\quad - \frac{1}{4}V_{\mathrm{PS}}\cos 2\theta_{\mathrm{P}}\sin 2\theta_{\mathrm{S}}\left(Q_S^{-1}-Q_{\mathrm{P0}}^{-1}\right)\Delta\delta + \frac{1}{4}Q_{\mathrm{P0}}^{-1}V_{\mathrm{PS}}\cos 2\theta_{\mathrm{P}}\sin 2\theta_{\mathrm{S}}\Delta\delta_Q \\
&\quad + V_{\mathrm{PS}}\left(Q_{\mathrm{S0}}^{-1}\cos 2\theta_{\mathrm{S}}\sin\theta_{\mathrm{P}}\tan\delta_{\mathrm{S}} + Q_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{S}}\cos\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\right)\sin\theta_{\mathrm{P}}\Delta\varepsilon \\
&\quad + \frac{1}{2}V_{\mathrm{PS}}\left(Q_{\mathrm{S0}}^{-1}\cos 2\theta_{\mathrm{S}}\cos 2\theta_{\mathrm{P}}\tan\delta_{\mathrm{S}} - Q_{\mathrm{P0}}^{-1}\sin 2\theta_{\mathrm{S}}\sin 2\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\right)\Delta\delta
\end{aligned}
$$

Scattering of SI-to-SI

$$
\begin{aligned}
[\mathrm{SISI}] &= (\boldsymbol{\mathcal{S}}_{\mathrm{S}}\cdot\boldsymbol{\mathcal{I}}_{\mathrm{S}})\frac{\Delta\rho}{\rho} - \frac{c_{55}^{(0)}}{\rho}\left[(\boldsymbol{\mathcal{S}}_{\mathrm{S}}\cdot\boldsymbol{\mathcal{I}}_{\mathrm{S}})(\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}}\cdot\mathbf{k}_{\mathrm{S}}^{\mathrm{In}}) + (\boldsymbol{\mathcal{S}}_{\mathrm{S}}\cdot\mathbf{k}_{\mathrm{S}}^{\mathrm{In}})(\boldsymbol{\mathcal{I}}_{\mathrm{S}}\cdot\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}})\right]\frac{\Delta\hat{c}_{55}}{c_{55}^{(0)}} \\
&\quad - 2\frac{\hat{c}_{33}^{(0)}}{\rho_0}(\mathcal{S}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{In}})\Delta\hat{\varepsilon} - \frac{\hat{c}_{33}^{(0)}}{\rho_0}(\mathcal{S}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Sz}}\mathrm{k}_{\mathrm{Sz}}^{\mathrm{In}} + \mathcal{S}_{\mathrm{Sz}}\mathrm{k}_{\mathrm{Sz}}^{\mathrm{Sc}}\mathcal{I}_{\mathrm{Sx}}\mathrm{k}_{\mathrm{Sx}}^{\mathrm{In}})\Delta\hat{\delta}
\end{aligned}
$$

we have

$$
\begin{aligned}
\boldsymbol{\mathcal{S}}_{\mathrm{S}}\cdot\boldsymbol{\mathcal{I}}_{\mathrm{S}} &= \frac{\hat{V}_{\mathrm{S}}^2}{\omega^2}\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}}\cdot\mathbf{K}_{\mathrm{S}}^{\mathrm{In}} = -\cos 2\theta_{\mathrm{S}} + iQ_{\mathrm{S0}}^{-1}\sin 2\theta_{\mathrm{S}}\tan\delta_{\mathrm{S}} \\
\boldsymbol{\mathcal{I}}_{\mathrm{S}}\cdot\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}} &= \hat{V}_{\mathrm{S}}\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}}\cdot(\mathbf{y}\times\mathbf{k}_{\mathrm{S}}^{\mathrm{In}}) = \hat{V}_{\mathrm{S}}\mathbf{y}\cdot(\mathbf{k}_{\mathrm{S}}^{\mathrm{In}}\times\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}}) = \frac{V_{\mathrm{S}}}{\omega^2}\mathbf{y}\cdot(\mathbf{K}_{\mathrm{S}}^{\mathrm{In}}\times\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}}) \\
\boldsymbol{\mathcal{S}}_{\mathrm{S}}\cdot\mathbf{k}_{\mathrm{S}}^{\mathrm{In}} &= \hat{V}_{\mathrm{S}}\mathbf{k}_{\mathrm{S}}^{\mathrm{In}}\cdot(\mathbf{y}\times\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}}) = \hat{V}_{\mathrm{S}}\mathbf{y}\cdot(\mathbf{k}_{\mathrm{S}}^{\mathrm{Sc}}\times\mathbf{k}_{\mathrm{S}}^{\mathrm{In}}) = -\frac{V_{\mathrm{S}}}{\omega^2}\mathbf{y}\cdot(\mathbf{K}_{\mathrm{S}}^{\mathrm{In}}\times\mathbf{K}_{\mathrm{S}}^{\mathrm{Sc}})
\end{aligned}
\tag{84}
$$

as a result

$$\frac{c_S^{(0)}}{\rho_0}\left[(\boldsymbol{\mathcal{S}}_{\rm S}\cdot\boldsymbol{\mathcal{I}}_{\rm S})(\mathbf{k}_{\rm S}^{\rm Sc}\cdot\mathbf{k}_{\rm S}^{\rm In})+(\boldsymbol{\mathcal{S}}_{\rm S}\cdot\mathbf{k}_{\rm P}^{\rm In})(\boldsymbol{\mathcal{I}}_{\rm S}\cdot\mathbf{k}_{\rm S}^{\rm Sc})\right]=$$
$$\frac{\hat{V}_{\rm S}^4}{\omega^4}\left[(\mathbf{K}_{\rm S}^{\rm Sc}\cdot\mathbf{K}_{\rm S}^{\rm In})^2-(\mathbf{K}_{\rm S}^{\rm In}\times\mathbf{K}_{\rm S}^{\rm Sc})\cdot(\mathbf{K}_{\rm S}^{\rm In}\times\mathbf{K}_{\rm S}^{\rm Sc})\right]=\cos 4\theta_{\rm S}-2iQ_S^{-1}\sin 4\theta_{\rm S}\tan\delta_{\rm S}$$
$$-2\frac{\hat{c}_{33}^{(0)}}{\rho_0}(\mathcal{S}_{\rm Sx}{\rm k}_{\rm Sx}^{\rm Sc}\mathcal{I}_{\rm Sx}{\rm k}_{\rm Sx}^{\rm In})\Delta\hat{\varepsilon}-\frac{\hat{c}_{33}^{(0)}}{\rho_0}(\mathcal{S}_{\rm Sx}{\rm k}_{\rm Sx}^{\rm Sc}\mathcal{I}_{\rm Sz}{\rm k}_{\rm Sz}^{\rm In}+\mathcal{S}_{\rm Sz}{\rm k}_{\rm Sz}^{\rm Sc}\mathcal{I}_{\rm Sx}{\rm k}_{\rm Sx}^{\rm In})\Delta\hat{\delta}=$$
$$-2\frac{\hat{V}_{\rm S}^4}{\omega^4}({\rm K}_{\rm Sz}^{\rm Sc}{\rm K}_{\rm Sx}^{\rm Sc}{\rm K}_{\rm Sz}^{\rm In}{\rm K}_{\rm Sx}^{\rm In})(\Delta\hat{\varepsilon}-\Delta\hat{\delta})=\frac{1}{2}\left(\sin^2 2\theta_{\rm S}-iQ_{\rm S0}^{-1}\sin 4\theta_{\rm S}\tan\delta_{\rm S}\right)$$

and finally

$$[{\rm SISI}]_{\rm IE}=-(\cos 2\theta_{\rm S}+\cos 4\theta_{\rm S})\frac{\Delta\rho}{\rho}-2\cos 4\theta_{\rm S}\frac{\Delta V_{\rm S}}{V_{\rm S}}$$
$$[{\rm SISI}]_{\rm AE}=\frac{1}{2}\sin^2 2\theta_{\rm S}(\Delta\delta-\Delta\varepsilon)$$
$$[{\rm SISI}]_{\rm IV}=\cos 4\theta_{\rm S}Q_{\rm S0}^{-1}\frac{\Delta Q_S}{Q_S}+Q_{\rm S0}^{-1}\sin 2\theta_{\rm S}\tan\delta_{\rm S}\frac{\Delta\rho}{\rho}+2Q_S^{-1}\sin 4\theta_{\rm S}\tan\delta_{\rm S}\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right)$$
$$[{\rm SISI}]_{\rm AV}=\frac{1}{4}\sin^2 2\theta_{\rm S}Q_{\rm P0}^{-1}(\Delta\delta_Q-\Delta\varepsilon_Q)-\frac{1}{2}Q_{\rm S}^{-1}\sin 4\theta_{\rm S}\tan\delta_{\rm S}(\Delta\varepsilon-\Delta\delta)$$

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