Seismic reflections from space-, time- and mixed boundaries

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ABSTRACT

It has been pointed out that if a homogeneous medium that is supporting a propagating wave were to suddenly undergo a change in medium properties, the propagating wave would immediately partition into reflected and transmitted components–exactly as if it had impinged on a spatial boundary. Bacot *et al.* in 2016 in Nature Physics published laboratory examples of this, referring to the change causing the reflection as a "time mirror". Here we model this phenomenon numerically, speculate on a practical usage of it in monitoring reservoirs undergoing rapid pressure changes, and offer a modest extension of the theoretical description. In the extension we point out that the time-mirror reflections are essentially *non-oblique*, and then, motivated by the explorationists' tendency to think about obliquely incident waves, we ask the question of how we could force a wave to impinge on a time-boundary at an angle. The answer requires the introduction of boundaries with both space- and time features. Upon setting up such a problem basic rules for reflection angles and transmission angles are derivable by appeal to Huygens' principle.

INTRODUCTION

Suppose a one-way wave is propagating along in a homogeneous medium. Suppose further that we could arrange circumstances such that at some instant t_0 during its propagation, the entire medium has its elastic properties changed. That is, after t_0, the medium remains homogeneous, with no spatial variation in its properties, but those properties are everywhere different from the ones prior to t_0. What would happen?

Because the roles played by space and time in the wave equation are very similar, there is a fascinating answer to this question. The wave described above can be thought of as impinging on a discontinuity in medium properties, but a discontinuity along the time axis rather than along a space axis. Waves impinging on discontinuities reflect from those discontinuities, so, the answer is that after the homogeneous medium changes, the wave partitions into components going forwards and backwards. The backscattered part of this new waveform is said to be a reflection, but a reflection from a time-boundary rather than a space-boundary. As strange as it sounds, this phenomenon is real, and it works in real life.

The key papers describing this phenomenon are those of Mendonca and Shukla (2002) followed by Bacot et al. (2016) and Fink and Fort (2017). In the latter papers, an experiment involving a wave (created by simulating the splash of a souvenir statue of the Eiffel tower) on the surface of a suspended container of water is described. After the wave has propagated for a period of time, the container is dropped, allowing the water to enter a state of free-fall, which instantaneously alters the velocity of the water wave. A backwards-propagating waveform is induced which, after the same amount of time, produces an image of the Eiffel tower at the instant it hit the water.

The purpose of this report is to review and numerically reproduce the main features of this phenomenon, and add some new elements. We zoom in on the special case of the reflections of *plane waves* from time boundaries. One finds that, no matter what angle a plane wave may be propagating in space, after the medium properties change (or, one learns to say, after the wave has hit the time-boundary), the reflection propagates directly backwards. In other words, all time-boundary reflections occur at the equivalent of normal incidence. This allows us to ask whether or not there exists a meaningful extension in the sphere of time-boundary reflections of the concept of oblique-incidence. Finally some speculation on applications of this idea – and the type of information about it available to a geophysicist who is sensing it remotely – in the area of seismic monitoring and exploration technology.

What is a time boundary?

To introduce the idea of reflections from a time-boundary, which requires no special equations to model, let us simulate one synthetically. Consider a 1D scalar acoustic wave equation solved with finite differences. Let a disturbance of amplitude 1.091 (for reasons to be made clear shortly) be injected into a homogeneous 1D computational domain at the left boundary, such that it propagates to the right at a velocity of c_0=2.0km/s. In Figure 1 snapshots of this waveform as it propagates are illustrated.

At time t=0.75s, let the velocity everywhere jump suddenly from 2.0km/s to 2.4km/s, as if some external process had managed to change the elastic properties of the medium on a spatial scale comparable to that of our computational volume. What happens to the wave– does it reflect? Evidently (Figure 1) it does: the waveform propagating in one direction is partitioned into a wave with left- and right-going components, whose amplitudes have changed. Notice that the amplitude of the new right-going pulse has dropped from 1.091 to 1. We will return to these numbers to better understand them in the next section.

Can we use them?

If they could be created in practice, it would not be hard to use them. Suppose the initial waveform was set up by a geophysicist constrained to sit at the left boundary in Figure 1. The phenomenon generates for her an observable result, the weak left-going waveform. This reflection contains intelligible information about the dynamic process that caused it. In fact, the amplitude of the reflection is proportional to the jump in the properties of the medium from its initial state to its final state, and it would not be difficult to infer the jump from the amplitude.

Sounds like I am saying that we can do *AVO without an interface*. Really? The pedantic answer is no, because there *is* an interface, just not a spatial interface. On the other hand, a medium that remains at all times spatially homogeneous creates a reflection, and, as we will see, the amplitude of the reflection can be easily analyzed, so answering yes is not outlandish. A more interesting criticism is not of the A in AVO, but of the O, the offset. All time-boundary reflections described in the literature occur *normally*, i.e., without obliquity. For offset to be introduced, one would have to figure out how to aim a plane wave at a time-boundary at an angle.

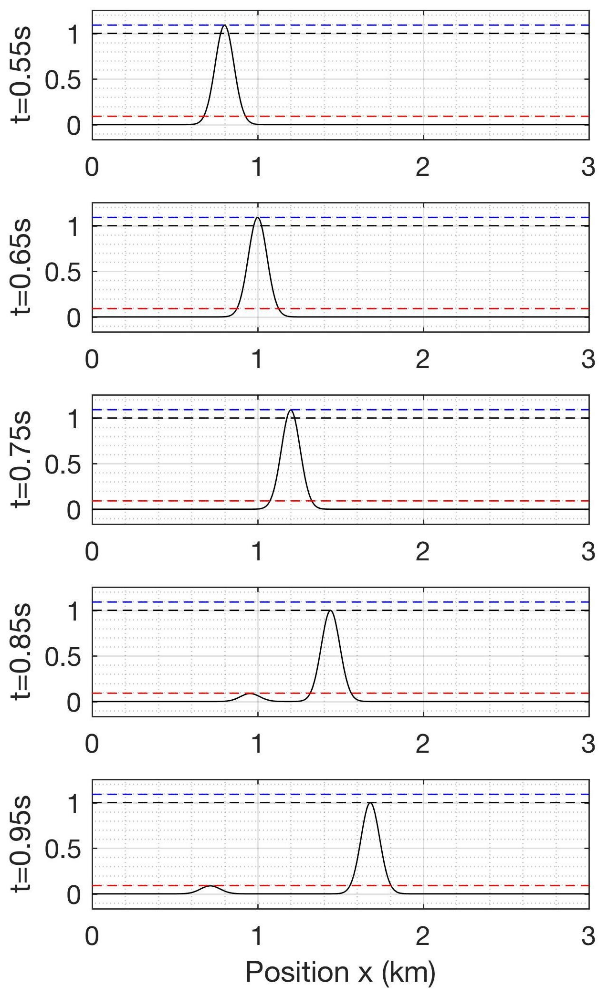

FIG. 1. Five snapshots of a waveform propagating through a 1D scalar acoustic medium, as computed with a simple finite difference solver. At t=0.75s, the velocity of the homogeneous medium is rapidly altered from 2.0km/s to 2.4km/s. The reference amplitude values of 1.091, 1, and 0.091 are labelled with blue, black and red dashed lines respectively.

DETAILED FEATURES

Time-boundary reflection and transmission coefficients

The possibility of measuring time-boundary reflections appears to have been initially guessed at because of the similarity of the roles played by time and space in wave motion. However, inspection of Figure 1, wherein the “incident” wave and the “transmitted” wave seem to have exchanged amplitudes, is suggestive that the roles are not exactly comparable. The symmetry between time- and space-boundary reflections is imperfect because the symmetry of the wave equation in a heterogeneous medium (though clearly present in some form) is imperfect. The wave equation in a perfectly homogeneous medium can be expressed in units such that the velocity is 1, which means it is symmetric under an exchange of x and t:

$$\partial_{xx}p = \partial_{tt}p \;\rightarrow\; \partial_{tt}p = \partial_{xx}p \tag{1}$$

However, if the medium properties change at some point in space, units cannot be found which preserve the symmetry and we have

$$\partial_{xx}p = c^{-2}(x)\partial_{tt}p \;\rightarrow\; \partial_{tt}p = c^{-2}(t)\partial_{xx}p. \tag{2}$$

As a consequence, when standard scalar boundary conditions (i.e., p and its derivative being continuous across the interface) are applied in the time-boundary setting, we obtain recognizable but slightly altered rules for reflection and transmission. Assuming a single right-propagating waveform for $t < 0$, before and after the medium change we have

$$p(x,t) = \begin{cases} p^{-}(x,t), & t<0 \\ p^{+}(x,t), & t>0 \end{cases} = \begin{cases} Af(x-c_0t), & t<0 \\ Bf(x-c_1t)+Cf(x+c_1t), & t>0 \end{cases}. \tag{3}$$

Continuity of field and derivative require us to impose the conditions

$$\begin{aligned} p^{+}(x,t)|_{t=0} &= p^{-}(x,t)|_{t=0} \\ \partial_t p^{+}(x,t)|_{t=0} &= \partial_t p^{-}(x,t)|_{t=0}, \end{aligned} \tag{4}$$

from which we obtain

$$\begin{aligned} A &= B + C, \\ -c_0A &= -c_1B + c_1C, \end{aligned} \tag{5}$$

or, normalizing by B,

$$T = \frac{A}{B}, \quad R = \frac{C}{B} \tag{6}$$

where

$$T = \frac{2c_1}{c_1+c_0}, \quad R = \frac{c_1-c_0}{c_1+c_0}. \tag{7}$$

This looks more or less as expected, but with one important difference. The amplitude B used to normalize A and C was not that of the incident wave. It was the amplitude of the “transmitted” wave component, i.e., the part of the wave which continues to propagate to the right after the boundary has been encountered. If we assign to the initial wave an amplitude T based on the upcoming change from c_0 to c_1, we find that the “transmitted” waveform takes on a unit amplitude.

Plane waves and normal incidence time-boundary reflections

Time-boundary reflections can be categorized as normal incidence phenomena. This emerges empirically, as we can see from the 2D numerical experiment summarized in Figure 2. A plane wave propagating at an angle oblique to the x and z coordinate axes through a homogeneous medium impinges on a pure time boundary at 0.7s. The up- and down-going waves emerging from the reflection process propagate along the same axis as the incident wave, as they would do in the case of a space-boundary reflection at normal incidence. Hence, we will refer to such as wave as having been normally-incident upon the time-boundary. Later in this document we will repeat this claim again, basing it on other arguments.

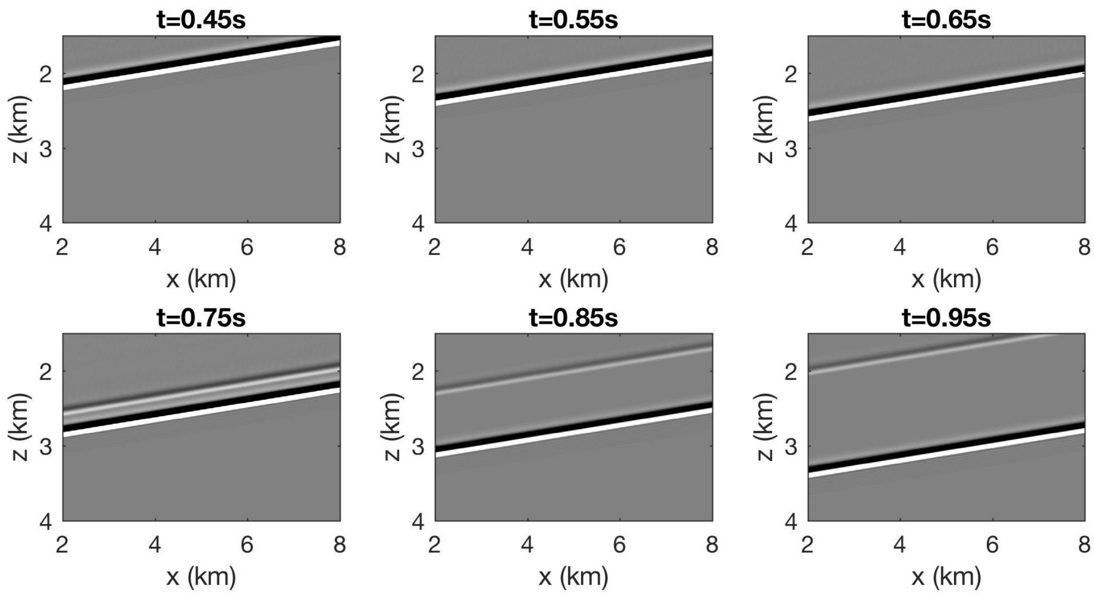

FIG. 2. A plane wave before and after its interaction with a time-boundary (occurring at 0.7s). Because the two emerging waveforms propagate forwards and backwards along the same propagation axis

Time-mirror "Bacot images", Canadian-style

Before moving on to some extensions to the time-boundary idea, let us reproduce synthetically the wave control experiment Bacot *et al.* carried out. In Figure 3, the waveform a few instants after a maple leaf hits a pond surface is illustrated propagating outward. At 0.7s, a sudden process occurs that affects the wave velocity everywhere, and a time-boundary reflection is excited. By 0.9s the collapsing waveform is discernible, and at exactly 1.18s the waves focus on the original source.

OBLIQUE INCIDENCE ON SPACE-, TIME-, AND MIXED BOUNDARIES

In seismic problems it is common to consider both normal-incidence and oblique-incidence reflections. A reflection occurring obliquely is generated by setting up a plane wave such that it impinges upon a spatial boundary at an angle. But we are considering time boundaries now. Is the idea of obliquity applicable? What does it mean for a wave to

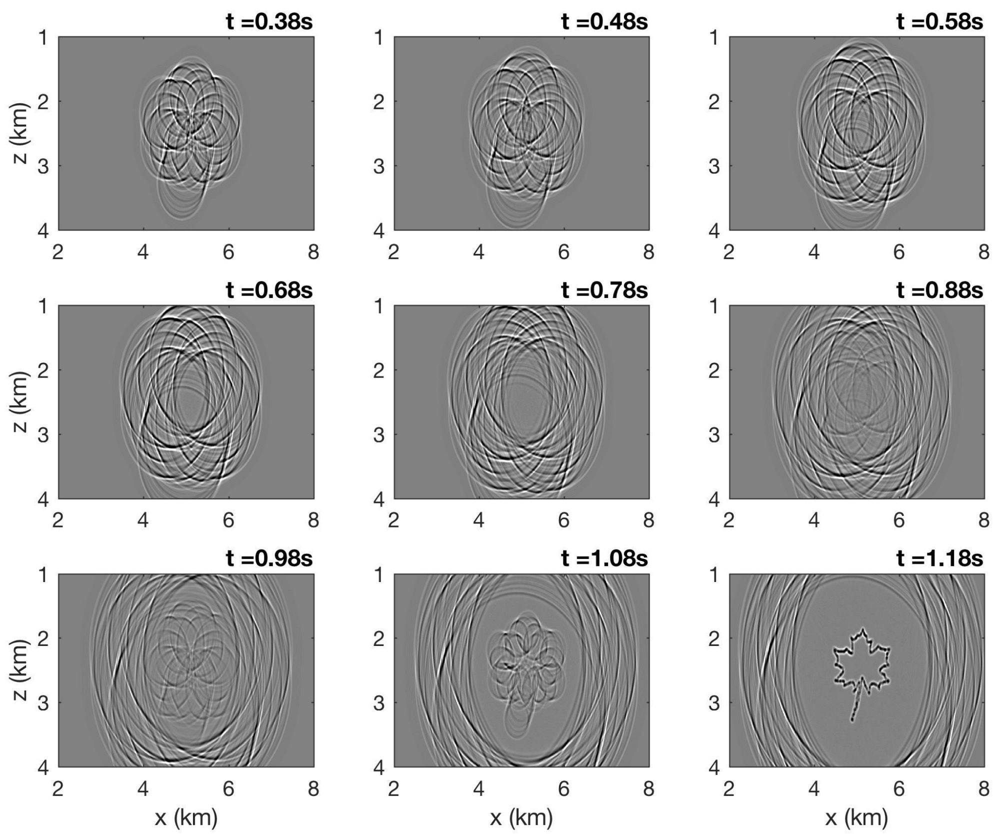

FIG. 3. Snapshots of a waveform on a (say) pond surface excited by a falling maple leaf. At 0.7s, caused by some unknown process, the surface wave velocity suddenly changes. The time-boundary reflected waveform re-focuses on the original source at 1.18s.

impinge on a time boundary at an angle? To analyze this we appeal to Huygens' principle and draw an analogy with wave reflection processes from standard space-boundaries.

Huygens' principle is useful for analyzing time-boundaries because it transforms space boundaries into time boundaries implicitly anyway. First let us agree that from a Huygens' principle point of view, reflections from time boundaries as developed by Bacot et al. are normal-incidence reflections. A plane wavefront is normally-incident on a space-boundary if it is set up so that the entire wavefront experiences the medium property change at the same instant. At that instant each point on the wavefront becomes a Huygens' source, and all of these sources which ignite simultaneously. The envelope of these sources is then seen to be a plane wavefront scattered directly away from the wavefront as it was at the moment it hit the boundary. Because this is precisely the set of phenomena that give rise to the time-boundary reflection, both of these reflection processes are classifiable as being normal-incidence.

A plane wavefront impinges on a space-boundary at *oblique incidence*, in contrast, if regularly-spaced points along the wavefront are set up to experience the medium property change non-simultaneously, at regularly-delayed instants. Thus any wavefront experiencing a time-boundary of the type introduced by Mendonca and Shukla (2002), Bacot et al. (2016), and Fink and Fort (2017), but so arranged that sequential points on the wavefront experience the property change at different, and regularly delayed, times, could be said to have been approaching the time-boundary obliquely.

To produce such a change requires, at some particular instant, one part of an incident wavefront to experience the time-boundary property change and another part not. Logically this is possible only if a space-boundary of some kind, separating the two intervals, is also present at that instant. So, time-obliquity evidently requires a boundary to be introduced whose description involves both space and time coordinates. This fits with our normal experience of a wavefront impinging at an angle on a space-boundary, which requires at least two coordinates in its description (i.e., for there to be a slope there must be a 'rise' coordinate and a 'run' coordinate).

Consider two wave experiments (Figure 4), both beginning with a plane wavefront propagating vertically downward at velocity c_0. In the first experiment (Figure 4a), it encounters a space-boundary, with dip angle γ from the horizontal, below which the wave velocity is c_1. This induces a reflection at an angle 2γ from the vertical, in agreement with standard rules for reflection. In the second experiment (Figure 4b), the medium is entirely homogeneous with wave velocity c_0 everywhere, but at some time t_0, a time-boundary is encountered and the entire medium jumps such that it has properties c_1 everywhere. In accordance with the time-boundary phenomena discussed previously, a reflection is induced which propagates vertically upward.

Suppose we created a boundary with both of these space and time aspects. The plane horizontal wavefront again propagates downward in a homogenous (c_0) medium (Figure 4c). At time t_0, let a space-boundary with dip angle γ from the horizontal appear suddenly, such that part of the incident wavefront remains in the c_0 medium, and part finds itself having crossed the time-boundary into a medium with velocity c_1. This experiment exposes

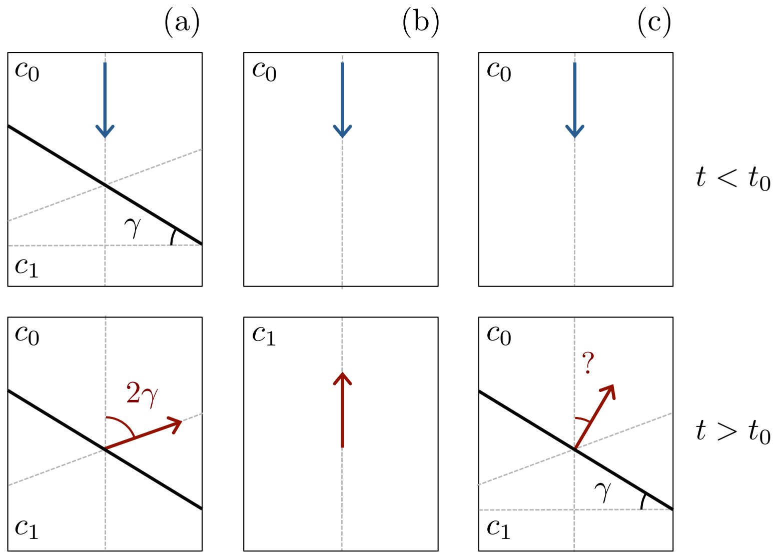

FIG. 4. Three experiments. (a) Experiment 1: a plane wave (blue arrow, top panel) is incident on a space-boundary (angle γ) which causes a reflection to take off at t_0 at angle 2γ (bottom panel). (b) Experiment 2: a plane wave encounters a time-boundary at time t_0 (top panel), after which it reflects at angle 0 (bottom panel). (c) Experiment 3: a plane wave is incident (top panel) on a space-boundary like that in (a), but one which only appears at time t_0 like the time-boundary in (b). Question: what are the characteristics (angle etc.), as picture in the bottom panel, of the reflection induced by this mixed space/time boundary?

the wavefront to a boundary that in some sense mixes both space and time components. Does the reflected wavefront induced by this mixed boundary have a direction that is a mixture of the time-boundary direction (directly upward) and the space-boundary direction (angle 2γ)? If so, what are the rules governing this direction?

Numerical simulation of the three experiments

Let us develop answers to these questions by simulating the first two different reflections, the first a standard oblique-incidence space-boundary reflection, and the second a "normal incidence" time-boundary reflection. In Figures 5a-i time snapshots of the first case are illustrated at various points as a normal incidence plane wavefront impinges on a right-dipping interface. In the last diagram wavefront lines constructed using Snell's law confirm that the transmitted (blue) and reflected (yellow) wavefronts are behaving as expected and as per Figure 4a.

In Figures 6a-i a comparable time-boundary experiment is simulated, and, as in the earlier plane-wave experiment (Figure 2), we find that the experiment sketched in Figure 4b is also accurate in essence.

In Figures 7a-i, the mixed, or space-time boundary interaction is modelled. In Figures 7a-c the plane wavefront propagates vertically downward in a homogeneous medium. At a moment between panels (c) and (d) a mixed boundary is encountered, at at times from (d)-(i) the medium now has a region with the original velocity c_0 and a region with velocity c_1.

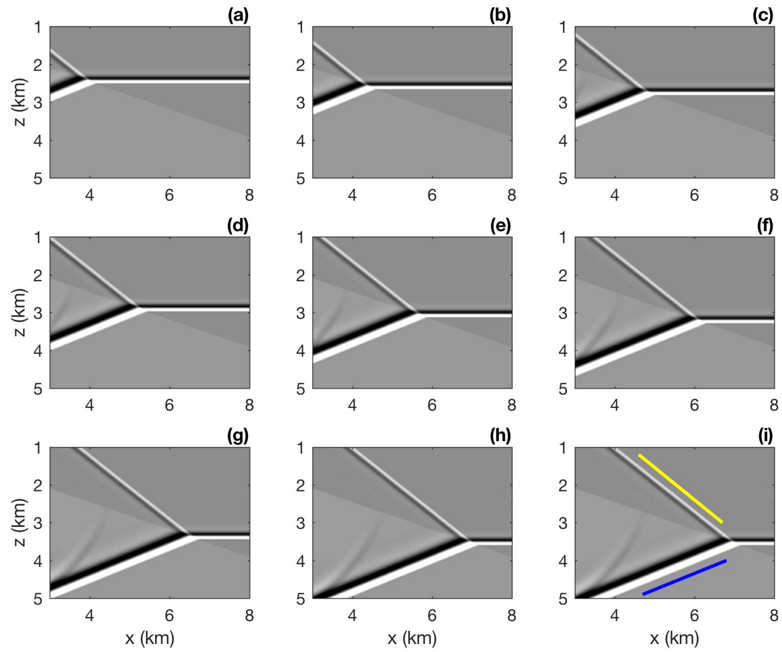

FIG. 5. Snapshots of wavefront interacting with a space-boundary at oblique incidence.

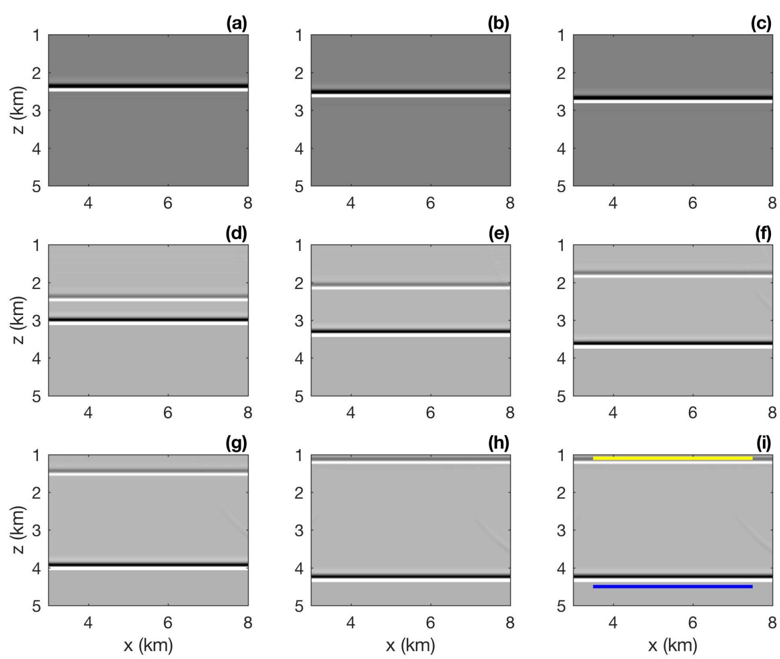

FIG. 6. Snapshots of wavefront interacting with a time-boundary at normal incidence.

The waveform which expands outward in panels (d)-(i) has six identifiable wavefronts. On the left (low x) and deeper end is a horizontal and downward propagating wavefront. On the right the original horizontal and downward propagating wavefront that has not yet encountered the space boundary is visible. Between these there is a set of four conjoined wavefronts making a tilted kite-shape. A zoomed in snapshot of this shape is illustrated in Figure 7.

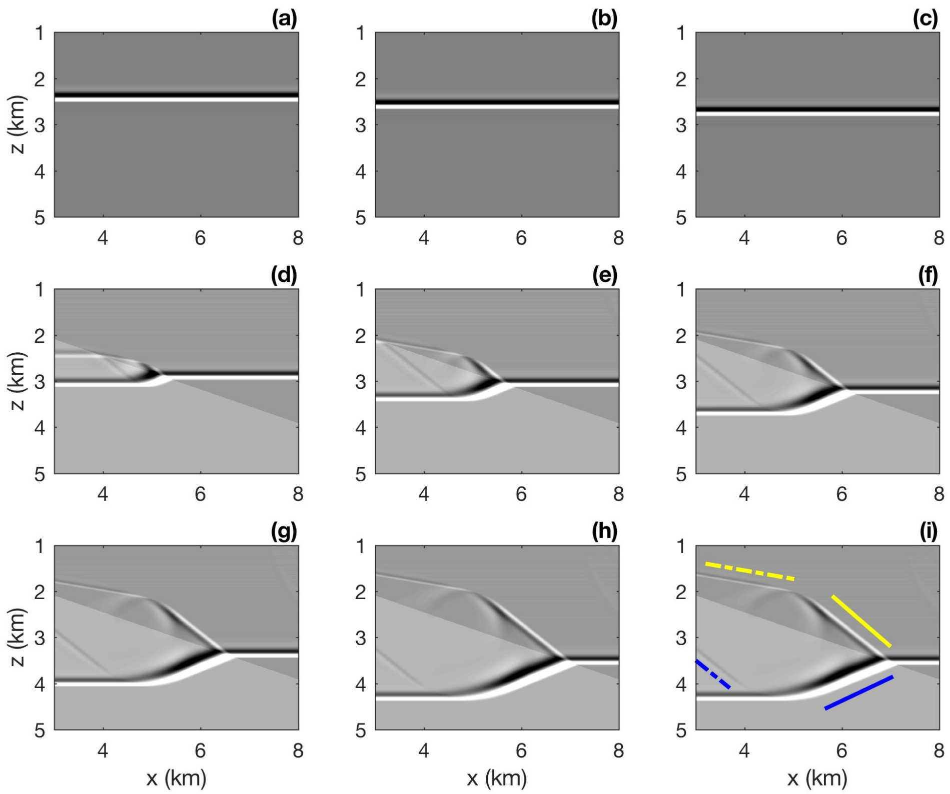

FIG. 7. Snapshots of wavefront interacting with a boundary involving both time- and space-boundary components.

Two of these wavefronts are largely the responsibility of the space-boundary, and appear to correspond to the expected reflection and transsmission responses the plane wave would have had if the space-boundary had been there alone. These are marked with solid blue and solid yellow lines in Figure 7i. The third, which is marked with a blue dashed line, is precisely parallel to the yellow solid wavefront and propagating away from the boundary. The fourth, marked with a dashed yellow line, is not parallel with any of the other three wavefronts and is not consistent with any of the wavefronts generated by either the pure time- or pure space-boundary. It propagates upward with a ray angle somewhere between the angle of pure space-boundary and the pure time-boundary reflections.

Reflection and transmission angles of the oblique wavefronts

To analyze the upgoing wave model generated by the of space- and time-boundaries acting together, labelled with the yellow-dashed wavefront in Figure 7i, we consider the

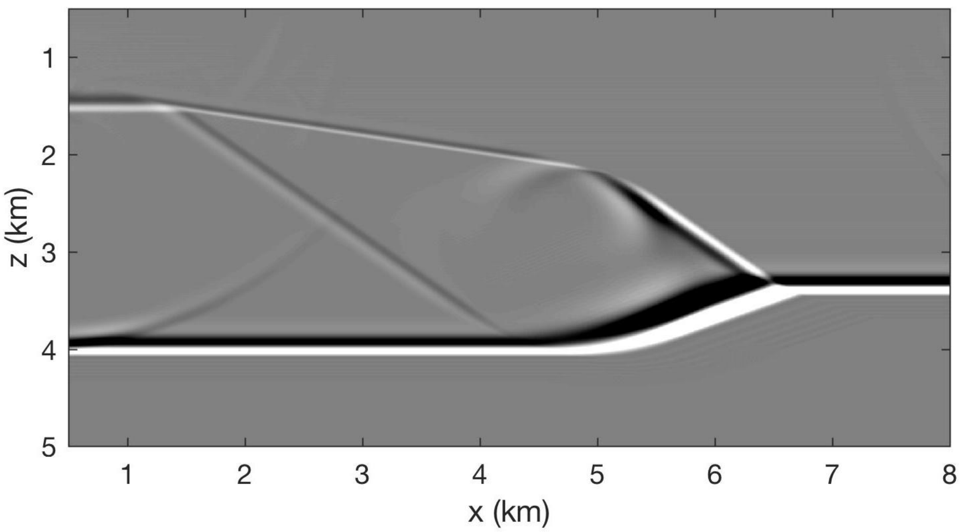

FIG. 8. Zoom in of the response a few instants after a putative mixed boundary has been encountered.

angles of the four oblique wavefronts in Figure 8. In Figure 9 these wavefronts are labelled AF, AB, ED and CD, and the angles their rays make with the vertical are labelled ϕ, θ, and θ'. Suppose a downward propagating horizontal plane wavefront passes the point O at t_0, the moment the medium jumps from being homogeneous with velocity c_0, to having a single interface separating regions with velocity c_0 and c_1 as illustrated. Allow a further time interval δt to elapse. The various new wavefronts at $t_0 + \delta t$ are in red. The component of the wavefront to the right of O at t_0 continues to propagate downward at speed c_0. It gives rise to reflected and transmitted wavefronts ED and CD respectively. The component of the initial wavefront which was to the left of O at t_0 experiences the time-boundary, and partitions into two wavefronts, one continuing downward and one propagating upward, both at speed c_1. The upward-propagating wavefront gives rise to reflected and transmitted wavefronts AB and AF respectively. The four wavefronts ED, CD, AB, and AF are the modes of interest.

The upgoing horizontal wavefront intersects O at t_0 and A at $t_0 + \delta t$. This wave transmits through the boundary forming the refracted upgoing wavefront AF. From Huygens' principle the point F (a distance $c_0\delta t$ away from O) and the point A must both lie on this refracted wavefront such that $\angle$AFO is 90°. Thus AF forms right triangle $\triangle$AOF, AB likewise forms $\triangle$AOB, CD forms $\triangle$DOC, and ED forms $\triangle$DOE. Because by again invoking Huygens' principle we know lengths AO, OD, OB, OF, OE, and OC, the directions of the four oblique rays (OF, OE, OB, and OC) in terms of θ, θ', and ϕ are fixed and can be shown to be

$$\begin{aligned} \phi &= 2\gamma \\ \theta' &= \mu - \gamma \\ \theta &= \nu - \gamma, \end{aligned} \tag{8}$$

where μ and ν satisfy

$$\frac{\sin\gamma}{c_0} = \frac{\sin\mu}{c_1}, \quad \text{and} \quad \frac{\sin\gamma}{c_1} = \frac{\sin\nu}{c_0}. \tag{9}$$

OB and OE are antiparallel and are both given by ϕ, which is the reflection angle from vertical experienced by the wavefront incident on the interface as a pure space-boundary. From equation (8) we observe that the angle μ is the refraction angle of a ray incident on the c_0/c_1 boundary at angle γ. The angle θ' is, then, simply this angle less the dip of the interface, meaning that the ray OC corresponds to the expected transmission of the horizontal incident wavefront through the space-boundary of angle γ.

The angle θ is unique to the mixed space- and time-boundary.the reflection angle produced when a time-boundary component is added to the space-boundary:

$$\theta = \sin^{-1}\left(\frac{c_0}{c_1}\sin\gamma\right) - \gamma. \tag{10}$$

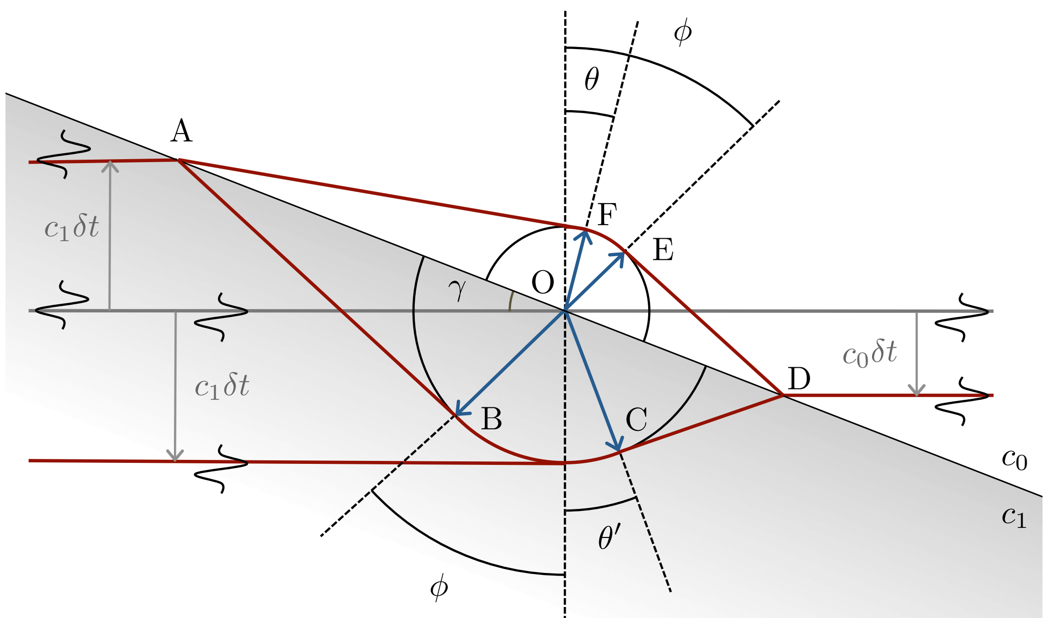

FIG. 9. Schematic representation of the wavefronts generated in the moments after a downward, vertically propagating wavefront has encountered a boundary with both time- and space- aspects.

Summarizing, the upgoing planewave induced by the time-boundary component of the mixed boundary refracts through the newly-appeared space-boundary component, creating an upgoing wavefront with an angle lying between those of the reflections from the pure time- and space-boundary components. This may be interpreted as an oblique reflection from a mixed space and time boundary.

AVO WITHOUT AN INTERFACE: THE INFORMATION CONTENT OF A TIME-BOUNDARY REFLECTION

Consider the reflection coefficient associated with a wavefront that is incident on a pure time-boundary, as determined in equation (7. We observe that this coefficient has the same form as the coefficient which would have been involved if the wavefront was incident on a spatial interface separating one medium with velocity c_0 from another with velocity c_1. In this case there is no spatial interface at all, and yet there is a reflection, and that reflection is information-carrying. The reflection coefficient directly measures the jump from one property value to another:

$$R = \frac{c_1 - c_0}{c_1 + c_0} = \frac{1}{2}\frac{\Delta c}{\tilde{c}}, \tag{11}$$

where Δc is the change in velocity and $\tilde{c}$ is the average of the two. Analyzing this reflection information, given knowledge of the incidence medium properties, in principle inferences can be

Speculative remarks on the creation of time-boundaries in reservoir settings

Dynamic reservoirs

Suppose a reservoir undergoing rapid change, due perhaps to a production process like fluid injection, was at the same time illuminated by a seismic wave (Figure 10). If conditions could be arranged such that the frequency content of an illuminating wave and the time-scales of the pressuring up were comparable, the part of the field propagating through the region nearby the well-bore would in principle experience a time-boundary and fluctuate accordingly.

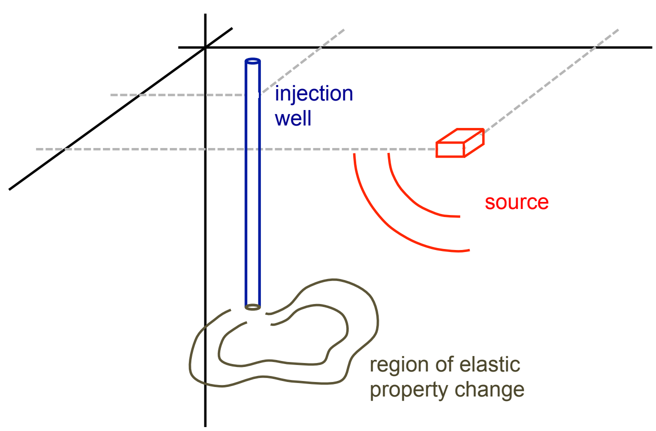

FIG. 10. Scheme: a reservoir setting in which (perhaps by a rapid pressuring-up) medium properties vary at the same time that they are illuminated by a seismic wave.

Controlled reservoirs

Research is currently underway to make the next generation of storage or hydrocarbon production reservoirs “smart”, and fully controlled, in part using nanoparticles suspended in injection fluids. Research projects in which these nanoparticles are so designed to allow electrical and seismic geophysics to determine the fraction and locations of propped versus un-propped fractures. In a [distant? ...not too distant?] future reservoir that contains significant “smart” fluids, choosing nanoparticles which react to, and align with, say, a mild magnetic field, causing locally rigid regions to appear almost instantaneously. Again, if this coincided with the passing of a seismic waveform, conditions for a time-boundary will be in place.

CONCLUSIONS

Time-boundaries are unusual, but perfectly real, types of reflector which seismic waves could in principle be arranged to impinge on. Simple time-boundaries cause essentially non-oblique reflections, but boundaries with time- and space-components can be designed which create angles of reflection that are unique, but predictable by appeal to Huygens' principle. We can speculate on practical uses for time- and mixed time- and space boundaries within enhanced reservoir monitoring and imaging – it would take quite a bit of doing to arrange for a seismic wave to hit a time-boundary in the field, but if it could be done, the information available to an observing geophysicist would be very valuable.

ACKNOWLEDGEMENTS

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