Simultaneous estimation and correction of nonstationary time-shifts and phase rotations

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SUMMARY

Constant phase rotations and constant time shifts are the constant and slope of a polynomial approximation to the seismic wavelet phase. Errors in the estimation of either one cause a bias in the subsequent estimation of the other. It follows that estimations of time-shifts followed by subsequent phase estimates, as is commonly done in well tying, is subject to this bias meaning that alignment errors cause compensating phase errors and a very questionable solution. A strategy is presented to overcome this bias whereby the alignment is estimated though correlation of trace envelopes and it is demonstrate that this is much more accurate. This strategy is then extended to the nonstationary case where, in a series of numerical experiments, it is demonstrated that nonstationary phase rotations and time delays can be reliably measured with good quality data.

INTRODUCTION

It is well understood that a time-shift can be accomplished in the frequency domain as a linear (with frequency) phase-shift. Also true is that the common practice of phase rotating seismic data to match well control is a constant phase shift. Together, phase rotation and time shift can be regarded as the intercept and slope of a linear approximation to the phase of the crosscorrelation between a seismic trace and a synthetic seismogram constructed from well-logs. For a better understanding, assume the convolutional model for the seismic trace as

$$s(t) = w(t) \cdot r(t) + n(t), \tag{1}$$

where $w(t)$ is the wavelet embedded in the trace, $r(t)$ is the reflectivity, and $n(t)$ is random noise and the bullet $(\cdot)$ denotes convolution. Next let the synthetic seismogram, aka the reference trace, constructed from the well reflectivity be

$$u(t) = w_0(t) \cdot r_0(t), \tag{2}$$

where $w_0(t)$ and $r_0(t)$ are again wavelet and reflectivity but are generally different from those in equation 1. The crosscorrelation $c_{us}(t) = u(t) \otimes s(t)$ is then

$$\begin{aligned} c_{us}(t) &= u(-t) \cdot s(t) = [w_0(-t) \cdot r_0(-t)] \cdot [w(t) \cdot r(t) + n(t)] \\ c_{us}(t) &= w_0(-t) \cdot w(t) \cdot r_0(-t) \cdot r(t) = c_{w_0 w}(t) \cdot c_{r_0 r}(t) \end{aligned} \tag{3}$$

in which use has been made of the fact that $a(t) \otimes b(t) = a(-t) \cdot b(t)$ (meaning that crosscorrelation is equivalent to time-reversal and convolution) and where $c_{r_0 n}$ was assumed to be zero because it is the crosscorrelation of two random sequences.

Now, make two simplifying assumptions about the relationship between $w(t)$ and $w_0(t)$ and between $r(t)$ and $r_0(t)$. First assume that $w(t) = R_\theta[w_0(t)]$ where R_θ is a constant-phase rotation operator. Then assume that $r(t) = T_{\Delta \mathrm{t}}(r_0(t))$ in which $T_{\Delta t}$ is a constant-time-shift operator. Thus we assume that the wavelet used in modelling and the true wavelet are different only by a constant-phase rotation and the reflectivity used in modelling and the true reflectivity are different only by a constant time-shift. These are common assumptions used in well tying and the simplest reasonable ones to make. In the frequency domain, the wavelet assumption reduces to

$$\widehat{w}(f) = \widehat{w_0}(f)e^{i\theta}, f \geq 0 \tag{4}$$

and the reflectivity assumption is

$$\hat{r}(f) = \widehat{r_0}(f)e^{i2\pi f\Delta t}, f \geq 0 \tag{5}$$

where, for convenience, both expressions have been written for non-negative frequencies only.

Now consider $c_{w_0w}(t)$ in the frequency domain and recall that time-reversal becomes phase-negation (i.e. complex conjugation). Therefore

$$\widehat{c_{w_0w}}(f) = \widehat{w_0}^*(f)\widehat{w}(f) = \widehat{w_0}^*(f)\widehat{w_0}(f)e^{i\theta} = \widehat{c_{w_0w_0}}(f)e^{i\theta} \tag{6}$$

where the superscript * indicates the complex conjugate and $c_{w_0w_0}$ is the autocorrelation of w_0 which is zero phase. Similarly, we deduce

$$\widehat{c_{r_0r}}(f) = \widehat{r_0}^*(f)\hat{r}(f) = \widehat{r_0}^*(f)\widehat{r_0}(f)e^{i2\pi f\Delta t} = \widehat{c_{r_0r_0}}(f)e^{i2\pi f\Delta t}. \tag{7}$$

Therefore, the Fourier transform of equation 3 is

$$\widehat{c_{us}} = \widehat{c_{w_0w}}(f)\widehat{c_{r_0r}}(f) = \widehat{c_{w_0w_0}}(f)\widehat{c_{r_0r_0}}(f)e^{i(\theta+2\pi f\Delta t)}. \tag{8}$$

In equation 8 $\widehat{c_{w_0w_0}}(f)$ and $\widehat{c_{r_0r_0}}(f)$ are Fourier-transformed autocorrelations and hence have no phase. Therefore the prediction for the phase of the crosscorrelation $c_{us}(t) = u(t) \otimes s(t)$ is that it is

$$\phi_{u\otimes s} = \theta + 2\pi f\Delta t \tag{9}$$

which is a linear phase with intercept θ and slope $2\pi\Delta t$.

Equation 9 shows that, with a fairly simple model, the two quantities of interest, θ and Δt, are predicted to form the phase spectrum of the fundamental measurable $c_{us}(t)$. This is very significant because it is well known that the slope and intercept of a linear data model are not statistically independent. This means that an error in the estimation of either one biases a subsequent estimation of the other. Figure 1 illustrates this situation with a simple least-squares fitting of a first-order polynomial (e.g. a straight line) to noisy data from a linear model. The model slope and intercept are 0.5 and 0.1 respectively and the simultaneous least-squares solution give excellent answers of 0.498 and 0.099. Now suppose that some independent process estimates the slope to be the incorrect value of 0.4 and a subsequent least-square solution for the intercept only is done. This results in an incorrect intercept of 0.148, which directly illustrates that an error in the slope causes a subsequent error in intercept. This also works the other way around in that if the intercept is estimated first, with some error, then a subsequent slope estimate will be correspondingly incorrect. The best way to solve this problem is a simultaneous estimation of both slope and intercept.

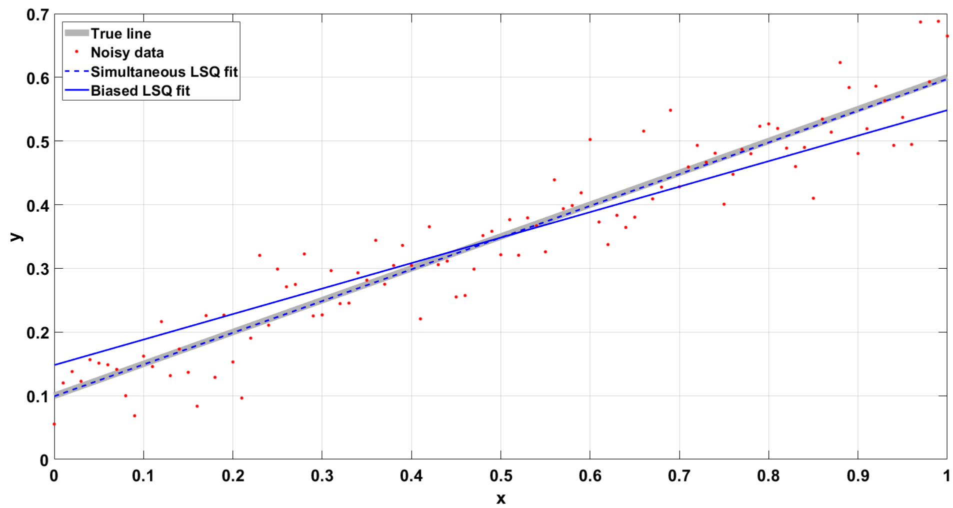

Figure 1. An example of straight-line fitting to noisy data with both a simultaneous and biased solution. The slope and intercept of the true line are 0.5 and 0.1 while the simultaneous least-squares solution estimates 0.498 and 0.099. In the biased case, the slope is constrained to be 0.4 and the resulting least-squares estimate for the intercept is then 0.148.

In the context of matching seismic data to well control, this problem of fitting a linear model to the phase of the crosscorrelation between seismic and synthetic arises. It is not practical to directly fit a linear model to the phase because the time shift can be quite large and this causes a vexing problem known as phase-wrapping. Given a linear phase relation like equation 9, even very small time shifts can easily generate phases greater than 180^o, or π radians, in absolute value. Yet for unavoidable mathematical reasons, phase estimates are "wrapped" to the interval $[-\pi,\pi]$. This means that any direct attempt to fit a linear model to the phase will fail unless the phase can be un-wrapped, which is a very complex problem. Instead, what usually happens is that the time shift is estimated by a cross-correlation between the synthetic and the seismic data, then the synthetic is shifted and the relative phase rotation is then measured. This is exactly the biased procedure discussed previously. The phase will cause errors in the time-shift estimation and the time-shift will cause errors in the phase.

Fortunately, seismic data is more complex and rich than the simple situation of Figure 1. Consequently, there may be ways to address this problem that escape this circumstance. In this paper I argue that crosscorrelation of the seismic trace and the synthetic directly is subject to the biasing problem. However, crosscorrelation of the Hilbert envelopes of trace and synthetic escapes the bias. This is because the Hilbert envelope is an estimate of the trace-amplitude that is mathematically unaffected by a constant-phase rotation. Thus for a stationary trace, the time-shift and phase can be successfully measured by first crosscorrelating the envelopes to estimate the time-shift and then estimating the phase after removing this shift.

A further complication to this problem is that both the time-shift and the phase rotation are generally functions of time. This is predicted from theory to be a consequence of anelastic attenuation and is widely observed. This problem can be addressed by performing the stationary analysis repeatedly with small trace segments created by applying a moving Gaussian window to both trace and synthetic.

In this paper, I will demonstrate this process and describe the computations involved.

THE STATIONARY CASE

Measurement of phase rotation

Given two signals, $s(t)$ and $u(t)$, we seek to estimate the constant-phase rotation that produces the best match between them. This can be formulated as an inverse problem by seeking the phase angle θ that minimizes $\|s(t) - R_\theta[u(t)]\|$ where $\| \quad \|$ denotes the L_2 norm or sum-of-squares. So we seek the phase angle minimizes the sum-squared difference between the phase rotated synthetic and the seismic trace. (It does not matter if we choose to rotate the synthetic or the trace because the phase angles determined will simply be the negatives of each other.) Another way to phase-rotate a signal, equivalent to that in equation 4, is

$$u_\theta(t) = u(t)\cos\theta + u_\perp(t)\sin\theta, \tag{10}$$

where $u_\perp(t)$ is a 90^{o} phase rotation, also called the Hilbert-transformed trace. This says that an arbitrary phase rotation can be formed as a linear combination of the signal and its 90^{o} phase rotation, with the cosine and sine weights as shown. Letting $\sin\theta = x$ and $\cos\theta = \sqrt{1-x^2}$, then

$$\|s(t) - R_\theta[u(t)]\|^2 = \sum_t \left(s(t) - u(t)\sqrt{1-x^2} - u_\perp(t)x\right)^2. \tag{11}$$

The optimal phase angle can then be found by minimizing this with respect to x and then finding the inverse sine. This is a tedious and complicated operation that involves differentiating equation 11 with respect to x, setting the result equal to zero, and solving for x. The result is a 4$^{\text{th}}$ order polynomial in x whose roots can be found numerically (two of the roots are typically complex and can be discarded). Each root must then be tested to determine if it corresponds to a maximum or a minimum. The details of this computation are not presented here but there is a function in the CREWES MATLAB toolbox that implements this method. This function is *constphase.m* and the method will be called the analytic method.

Fortunately, there is a much simpler way to find the best rotation angle and that is by a direct numerical search. Experience shows that phase-angle difference of less than 5^{o} are usually irrelevant and therefore extreme precision is not required. Furthermore, the periodicity of the sine and cosine functions means that the phase estimate will always lie between -180^{o} and 180^{o}. Therefore, it is quite sufficient to simply evaluate $\|s(t) - R_\theta u(t)\|^2$ for all integer values between -180 and 179 and the search for the numerical minimum. This method is implemented by *constphase2.m* also in the CREWES toolbox.

Figure 2 is an almost trivial example of this phase estimation. Here a 30 Hz Ricker wavelet is phase rotated by 119.2^{o} and then both the analytic method and the direct search are used to deduce this phase rotation. The calculation requires both wavelets and the analytic method yields a single number that is exactly the applied rotation while the direct search gives the entire objective function, $\|s(t) - R_\theta u(t)\|^2$, sampled at integer phase angles. Both methods get the correct result.

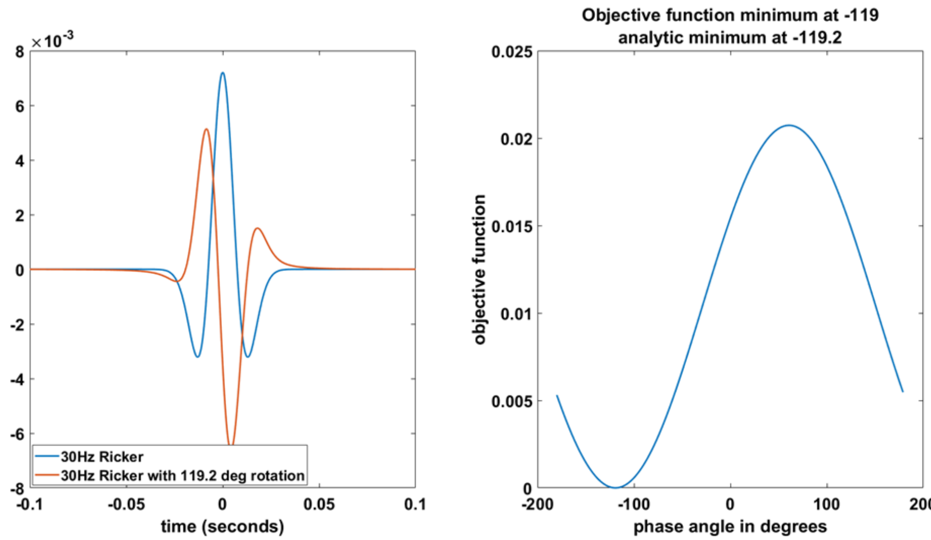

Figure 2: (Left) A 30 Hz Ricker wavelet and the same wavelet after a phase rotation of 119.2^{o}. (Right) The curve shows the objective function $\|s(t) - R_\theta u(t)\|^2$ as mapped out by direct calculation at all integer phase angles between -180 and 179. The curve has a minimum at 119^{o} while the analytic solutions gets exactly 119.2^{o}.

It is not possible to include all possible tests with these functions so instead I just summarize a few things I've learned:

1. A phase calculation always requires a reference trace and the rotated trace. The phase is always relative to the reference trace, which need not be a zero phase wavelet.
2. The method works best if the traces being compared have similar amplitude spectra. Therefore the toolbox functions provide the option to equalize the amplitude spectra before measurement.
3. The phase calculation always gives an answer even when that answer may be nearly meaningless due to conflict with time shift.

Measurement of time shift

A crosscorrelation is the classic way to measure a time shift. If $s(t)$ and $u(t)$ differ by only a constant time-shift, then computation of $c_{us} = u \otimes s$ for a suitable range of lags should detect the shift. In more detail, the time shift is the lag at which c_{us} finds it's maximum. Of course, c_{us} must be evaluated over a large enough range of lags to find the maximum. Less well known, is that the shift can also be detected by crosscorrelating the trace envelopes rather than the traces themselves, or by simply examining the envelope of the crosscorrelation of the traces. The envelope of any signal $s(t)$, denoted $e[s](t)$ is defined as

$$e[s](t) = \sqrt{s^2(t) + s_\perp^2(t)} \tag{12}$$

and it has the remarkable property that it is insensitive to phase rotations. That is, $s_\theta(t)$, which is a constant-phase rotation of $s(t)$, has the same envelope as $s(t)$. This can be easily proven by direct calculation of both envelopes and using appropriate trigonometric identities. This is very important because it means that we have an alternate method to get the time shift that is insensitive to constant phase rotations. Also, it follows directly from equation 12 that $|s(t)| \leq e[s](t)$ which means the trace is contained in the envelope. Since all constant phase rotations have the same envelope, then they are all so contained. This is illustrated in Figure 3.

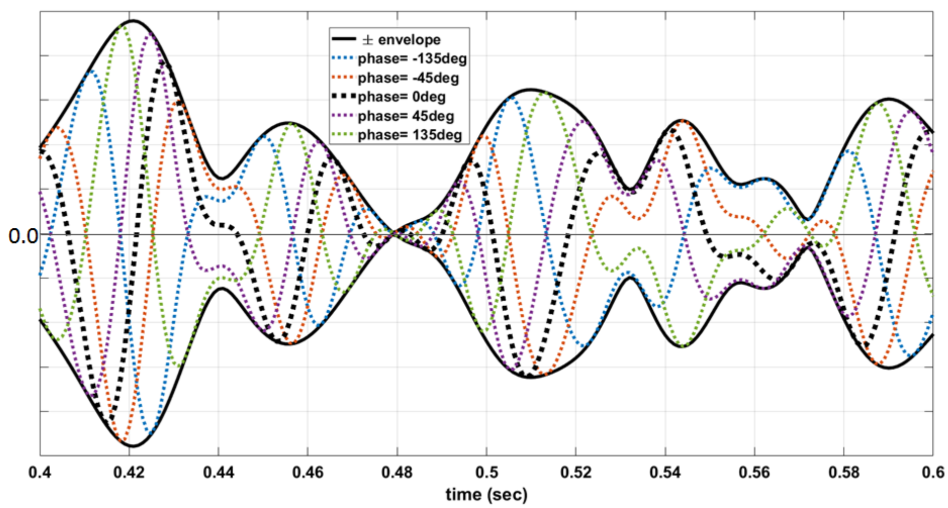

Figure 3. A portion of a seismic trace is shown together with its envelope (positive and negative) and a number of phase rotations. The envelope contains all of the phase rotations.

The argument in the introduction suggests that the phase of c_{us}, the crosscorrelation of the reference trace $u(t)$ with a seismic trace $s(t)$, is dominated by a constant phase rotation and a linear term. So, it follows that the envelope of the crosscorrelation, $e[c_{us}]$, should be insensitive to a constant phase rotation. So, either $e[u] \otimes e[s]$ or $e[u \otimes s]$ should be insensitive to phase rotations. To test these ideas, Figure 4 shows the situation where s differs from u only by a time shift and no phase rotation. On the right-hand side of this figure, the three correlation functions are shown, $u \otimes s$, $e[u \otimes s]$, and $e[u] \otimes e[s]$, and all three have their maximum at the correct position which is a lag time of -0.075 sec. Compare this to the case shown in Figure 5 where s differs from u by both a time shift and a phase rotation. When a phase shift is present both of the correlations involving envelopes still work but the standard crosscorrelation ($u \otimes s$) fails.

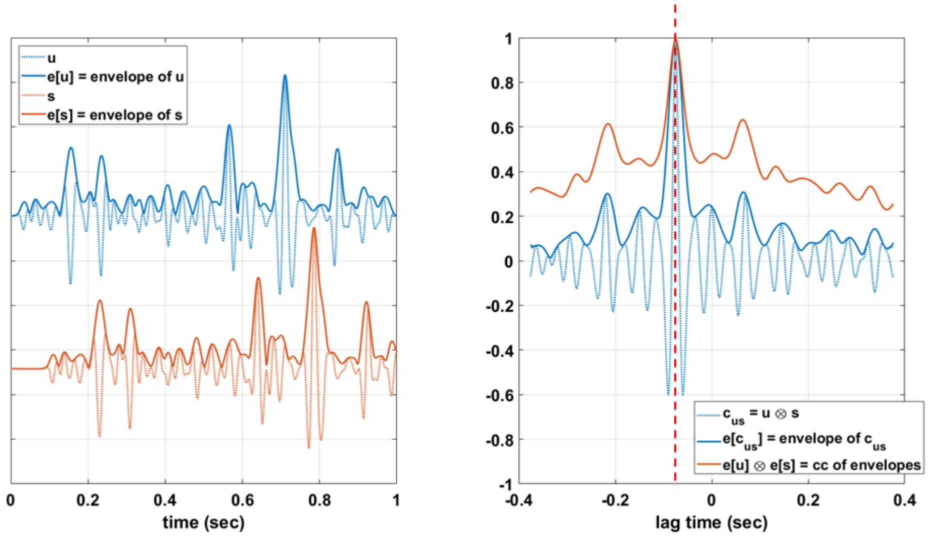

Figure 4. At left are the reference trace $u(t)$, the seismic trace $s(t)$, and their envelopes. By construction $s(t)$ is identical to $u(t)$ except for a 0.075 sec time shift. At right are the three possible correlations and all three succeed at detecting the shift because there is no phase rotation. A red dashed line indicates the correct lag.

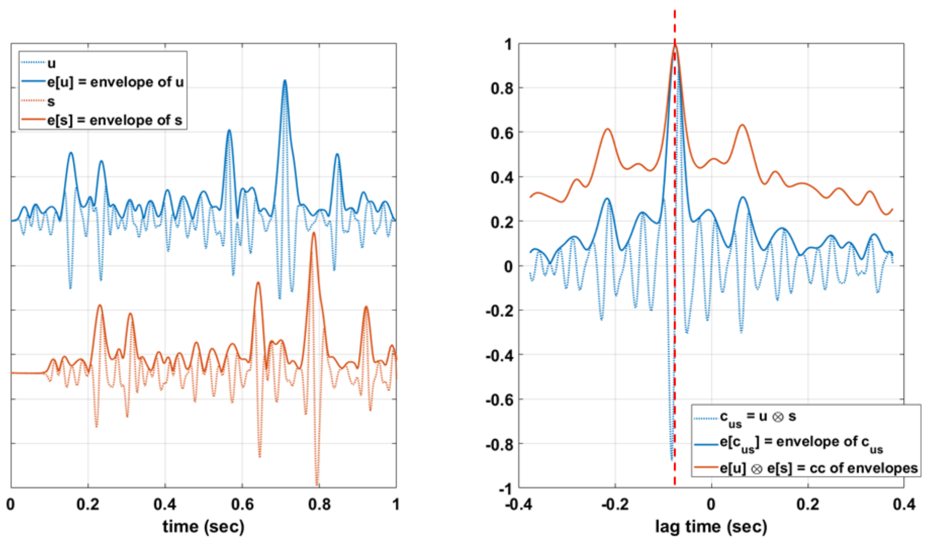

Figure 5. Similar to Figure 4 except that the seismic trace $s(t)$ differs from $u(t)$ by both a 0.075 sec time shift and a 90^{o} phase rotation. On the right, the maximum of the standard correlation $u \otimes s$ fails to pick the correct time shift but the other two correlations do correctly find the shift.

Measurement of both time shift and phase rotation

So either $e[u \otimes s]$ or $e[u] \otimes e[s]$ are crosscorrelation techniques which are insensitive to constant phase rotations while $u \otimes s$ lacks this property. The lack of a phase-rotation estimation technique that is insensitive to time shift means that the time shift must be estimated first and then the phase rotation. (For the forward problem of applying the time shift and phase rotation, the order is not important in the stationary case.) Using the same synthetic traces as before, where $s(t)$ has a 0.075 sec time shift and a 90^{o} phase rotation with respect to $u(t)$, Figure 6 compares the performance of the 3 crosscorrelation possibilities to first estimate the shift and then subsequently the phase rotation. Here $s(t)$

is shown at the top of each column and $u(t)$ is at the bottom. In between, the red trace is $s(t)$ with the estimated shift applied and the yellow trace also has the estimated phase rotation applied. Thus success is measured by the similarity of the yellow trace to $u(t)$. Above the red traces is shown the magnitude of the estimated shift while above the yellow traces is the estimated phase rotation (this is the phase required to rotate s into u so -90 is the correct result). In the first column, the ordinary crosscorrelation gets an erroneous shift of -0.0676 sec which is 7 milliseconds too small. This small shift error has a strong effect of the phase estimate which is only -9 instead of -90. The final corrected trace in the first column has a maximum correlation with u of 0.9. In contrast, both of the envelope correlation methods do much better getting both shift and phase almost exactly and having a final maximum correlation of 0.99. The sensitivity of the phase estimation to the time shift is quite strong and these results required that the initial crosscorrelation to estimate the lag be interpolated to 1/10 of the sample interval. If the crosscorrelation is evaluated without interpolation, meaning that the lag of the maximum is evaluated only to the nearest sample, the results are significantly degraded as is illustrated in Figure 7. While the final correlations are still very good, the phase rotation from the two envelope techniques is now in error by 10^{o}.

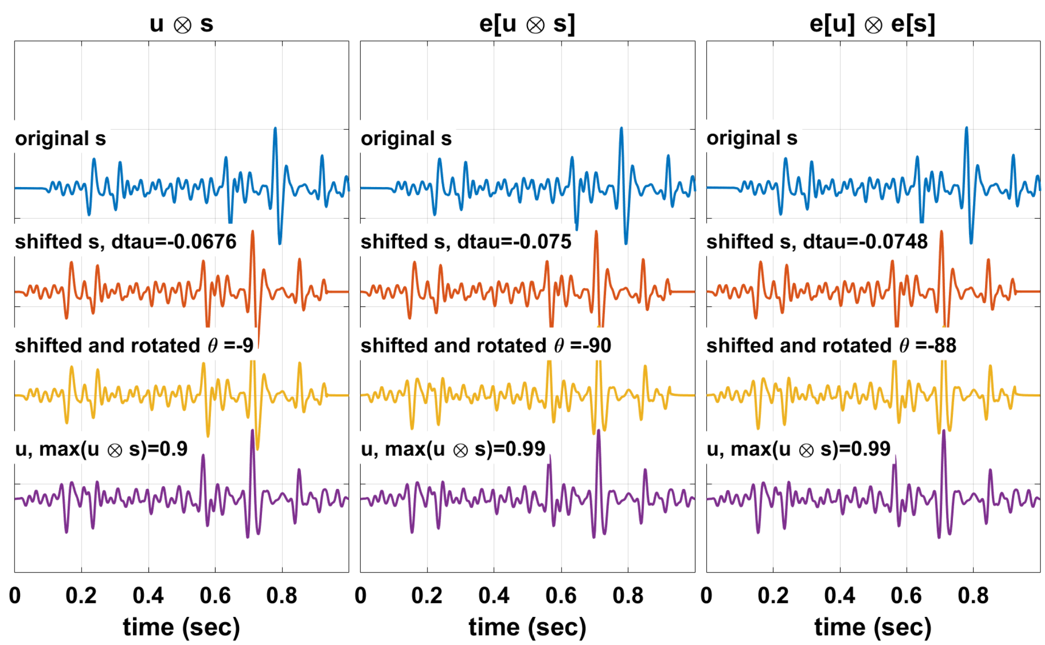

Figure 6. The three different crosscorrelation procedures are used together with the constant phase estimation to determine both shift and phase for a test trace s shown at the top of each column. The reference trace u is at the column bottoms. In this case the time shifts were determined by finding and interpolated maximum for the crosscorrelations. See the text for more discussion.

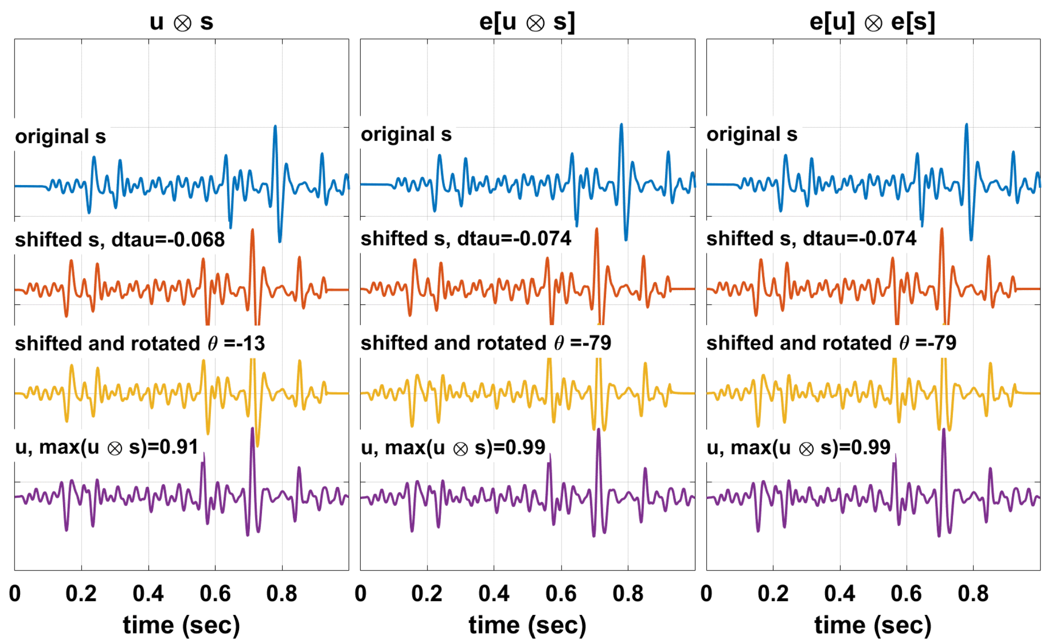

Figure 7. Similar to Figure 6 except that the time shifts were determined by finding the crosscorrelation maxima to the nearest sample only without interpolation.

THE NONSTATIONARY CASE

All seismic data is really nonstationary even though most of our data processing algorithms ignore this fact. Here "nonstationary" refers to the wavelet spectrum as it varies with both time and frequency. It is the temporal variation of the spectrum which merits the label nonstationary. This is caused by at least two effects: (1) anelastic attenuation and (2) short-path interbed multiples. As was first shown by O'Doherty and Anstey (1971) these two very different physical processes both cause the wavelet to evolve in a very similar fashion. This evolution is characterized by an amplitude spectrum that decays progressively with increasing time and a phase spectrum that also changes with time as can be inferred by the minimum-phase condition. The nonstationary evolution of wavelet phase can be characterized, to first order, by a monotonically increasing delay and a time-variant phase rotation. Application of stationary deconvolution will change the pattern of phase nonstationarity but will not remove it. This becomes evident in well tying when fitting a synthetic seismogram to processed data requires "stretching and squeezing" and time-dependent phase rotations. "Stretching and squeezing" is the common jargon for the time-dependent time-shifts that are applied to the synthetic seismogram while attempting to align it with the data. While time-dependent time-shifts are commonly used, usually only a stationary constant-phase rotation is estimated and applied. This may be due to difficulties in measurement related to short well logs, or it just may be due to tradition. Whatever the reason, the feasibility of estimating both nonstationary time-shifts and phase-rotations will be demonstrated here.

In extending the stationary results to the nonstationary setting, I will assume "local stationarity" meaning that the stationary techniques will simply be applied repeatedly in each of a set of temporal windows that span the available times. The property of the envelope being unaffected by constant phase rotations is only strictly true in the purely stationary case. Still, I will show empirically that using envelope crosscorrelations in nonstationary analyses is much better than using ordinary crosscorrelations. The main innovation in extending these methods to the nonstationary case is to use the Gabor windowing idea. Here a set of windows, usually Gaussians, are defined that break up the traces into small segments that can be compared to one another. In this study, the windows are all identical except for their center times which move regularly down the traces. For the estimation of time shifts, the Gaussian width, characterized by the standard deviation for convenience, should be chosen larger than any anticipated shift. The increment between adjacent windows is typically chose to be much smaller than the width. For example, a window width of 0.05 or 0.1 seconds and an increment of 0.01 or 0.02 are robust choices.

As with the stationary case, I begin with examples of phase rotation without delay and then delay without phase rotation before closing with an example of both together. The simplest way to apply a time-variant phase rotation is with a straight-forward generalization of equation 10

$$u_\theta(t) = u(t)\cos\theta(t) + u_\perp(t)\sin\theta(t), \qquad (13)$$

where the only change has been to replace the constant θ with the time-variant $\theta(t)$. While a stationary delay is just a static shift, a time time-variant delay is a common operation known as a *stretch*. A stretch can be written

$$u_\delta(t) = u(t + \delta(t)), \qquad (14)$$

where $\delta(t)$ describes the time-variant delay. Typically, the computation of a stretched trace (or the removal of an estimated $\delta(t)$) requires a careful interpolation of new samples from the existing trace.

Figure 8 demonstrates the successful estimation and removal of a time-variant phase rotation in the absence of any delay. An initial trace, $u(t)$, was constructed from the same reflectivity used previously and a minimum-phase wavelet. A time-variant phase function given by $\theta(t) = 90\cos 2\pi t$ was then applied using equation 13 to produce trace $s(t)$. This phase variation begins at 90^{o} at time zero, progresses to -90^{o} in the middle and back to 90^{o} at the end of the trace, passing through 0^{o} twice. The time-variant estimates of delay and phase were then made by comparing $s(t)$ with $u(t)$. Even though there is no delay in this case, a time-variant delay was estimated using both ordinary and envelope crosscorrelations and the results are seen in the middle panel. It is clear the ordinary crosscorrelations estimate an incorrect delay that is induced by the phase rotations while the envelope crosscorrelations give the correct zero result. The phase rotation estimates are essentially correct and when removed from $s(t)$ produce the purple trace in the top panel which is visually identical to $u(t)$. Both the delay estimates and the phase estimates were made using a Gabor technique using Gaussian windows of standard deviation 0.05 sec and increment 0.01 second. Annotated in the upper panel next to the yellow and purple traces are the maximum crosscorrelation values and their lag (in samples) with the optimal values of 1 and 0 being obtained with the removal of the estimated phase.

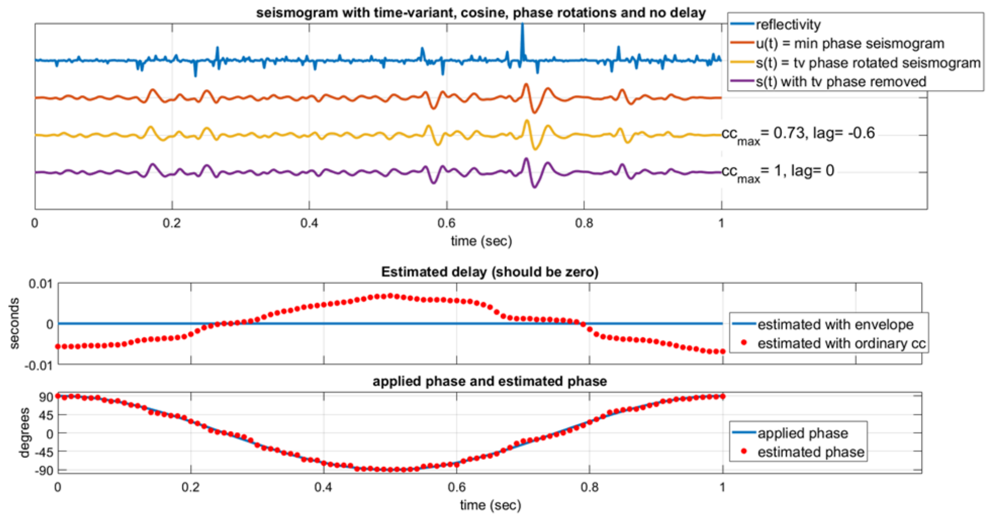

Figure 8. An initial minimum-phase seismogram, $u(t)$, (top panel) is subjected to a time-variant phase rotation where the phase is given by $90\cos 2\pi t$, and no time-variant delay, to produce trace $s(t)$. Both time–variant delay analysis and time-variant constant phase analysis were conducted. The middle panel shows that only the envelope crosscorrelation gets the correct zero delay. In the bottom panel, the time-variant phase estimates are seen to be very accurate. Crosscorrelation values (top panel) are with respect to $u(t)$.

Figure 9 is a similar experiment to that shown in Figure 8 except that here the time-variant phase rotation is progressively increasing from 0^{o} at the beginning of the trace to 180^{o} at the end. The Gabor windowing parameters were the same as before. Again the ordinary crosscorrelation sees a delay where there is none and this delay is clearly related to the applied phase. The envelope crosscorrelations give the correct null delay and the time-variant phase estimations are very close. Again the corrected $s(t)$ correlates essentially perfectly with $u(t)$.

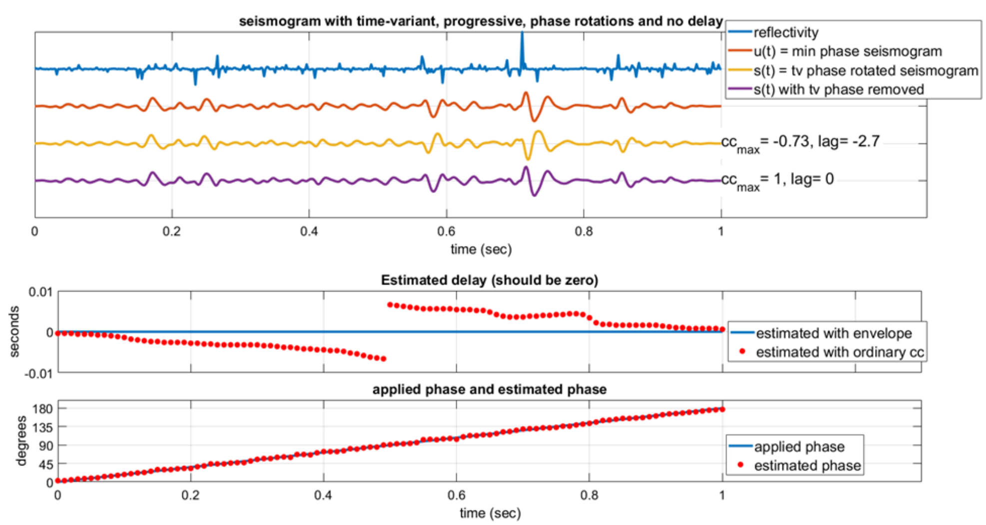

Figure 9. Similar to Figure 8 except that the time-variant phase is progressively increasing instead of oscillating in a cosine fashion. Again the ordinary crosscorrelations see a delay where there is none while the envelope crosscorrelations get the correct result. The time-variant phase estimates are very good and, when removed, give and excellent result.

Figure 10 shows a case with progressive time-delay but no phase rotations. The progressive delay increases from 0 at the trace beginning to 0.02 seconds at the end. The Gabor windowing parameters were the same as before. Since there are no phase rotations involved, both normal and envelope crosscorrelations give the same result and estimate the delay function correctly. In the bottom panel, a phase estimate is shown as measured correctly after removing the estimated delay, and then again without removing the delay. Only the first case gives the correct null result while the second case gets a completely erroneous phase that is clearly related to the applied delay. In the top panel, the purple trace is $s(t)$ after removal of the estimated delay and it correlates almost perfectly with $u(t)$. The green trace is what results if $s(t)$ is "corrected" not for delay but for the erroneous phase estimated without first removing the delay. The result is an improvement in correlation with $u(t)$ but the result is clearly incorrect and inferior to the delay-removed result.

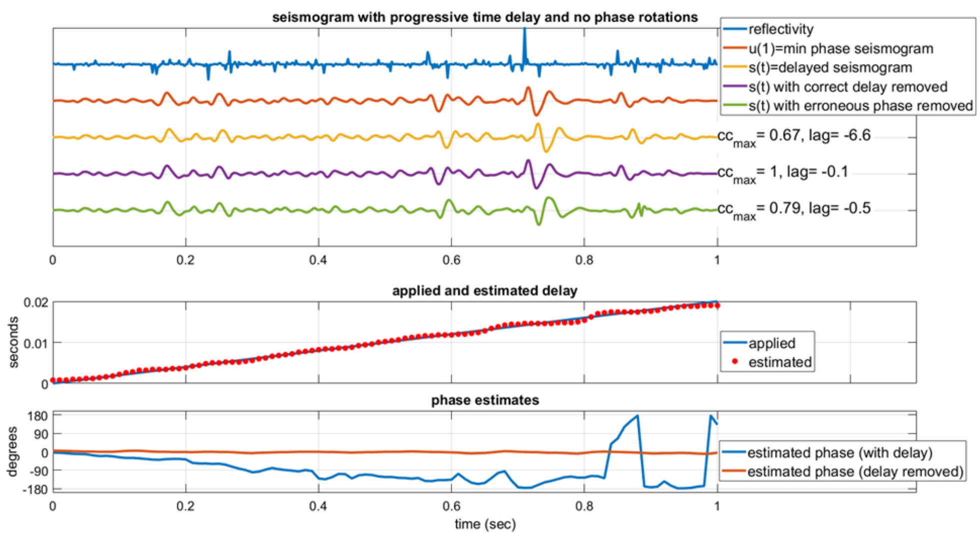

Figure 10. Here is a case of estimating time-variant delay when there are no phase rotations. Either normal or envelope crosscorrelations estimate the delay correctly. Phase estimates are shown as made after delay removal and without delay removal and only the former are correct. Versions of $s(t)$ are shown corrected for the estimated delay (purple) and corrected for the erroneous phase estimated without delay removal. Only the former correlates well with $u(t)$.

Figure 11 shows a case where $s(t)$ has both time-variant delay and time-variant phase and both are progressive. In the cases with phase only and no delay, the phase-induced delay (the delay measured without phase correction) was roughly opposite in trend to the imposed phase. Similarly, with delay only and no phase, the delay-induced phase was roughly opposite in trend to the delay. Here the imposed delay and phase have similar trends and the phase measured without delay removal is very nearly zero. The Gabor windowing parameters were the same as before (standard deviation of 0.05 sec and increment of 0.01 sec.). Delay estimates from both correlation methods are shown and only the envelope method comes close to the correct answer. The phase measured after removal of the estimated delay is very close to correct and the final corrected $s(t)$ (green trace top panel) correlates very well with $u(t)$. Although this result is very good, there is in interesting wobble in the estimated delay near 0.8 seconds. This is an example of an artefact caused by a too small Gabor window width. In Figure 12, is a repeat of this experiment with the Gabor standard deviation doubled to 0.10 sec. The delay estimate is seen to be improved and a correlated wobble in the phase curve is also reduced.

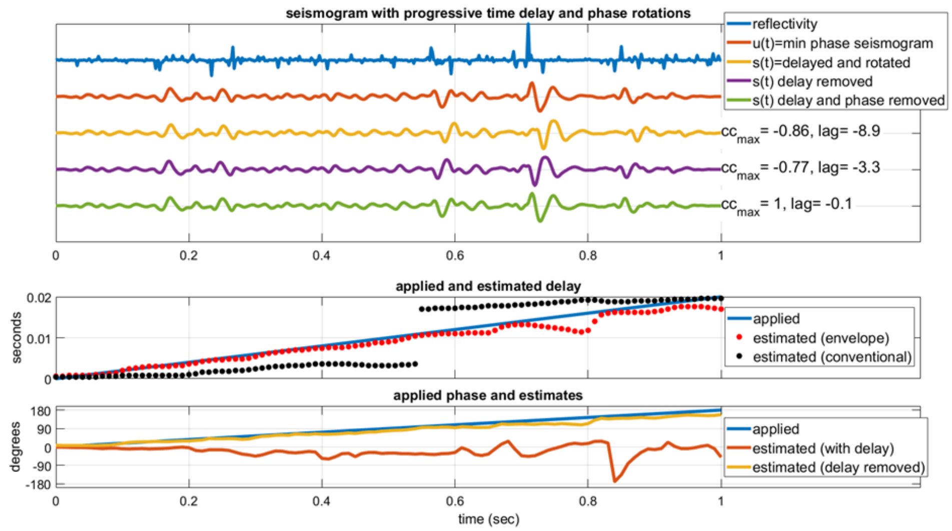

Figure 11. In this case, both a time-variant delay and a time-variant phase have been applied to the reference trace. The envelope correlation method has successfully estimate the delay while conventional correlation has not. When this delay is removed, the phase is estimated with good accuracy. When the phase is estimated without first removing the delay, the result is nearly zero. The slight error in delay estimate near 0.8 seconds is caused by a Gabor window width that was too small.

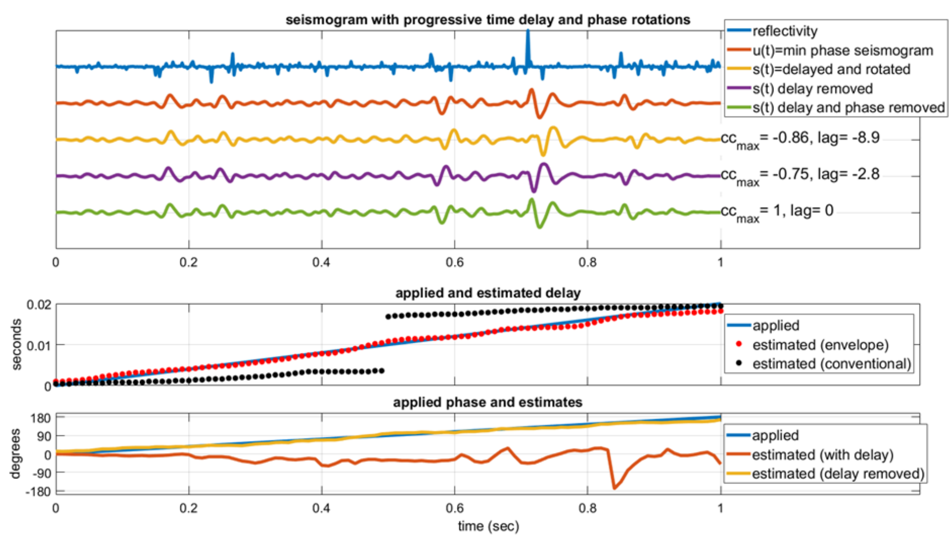

Figure 12. A repeat of the experiment of Figure 11, identical in all aspects except that the Gabor window width for delay estimation n has been doubled. This improves the delay estimates near 0.8 seconds and removes a slight corresponding phase error.

It is the phase insensitivity of the trace envelope that lead to the success of these measurements. Even though this property is only strictly true for the stationary case, the experiments shown here demonstrate that it is still a very useful approximation in reasonable nonstationary cases. Of course, it will always be possible to invent very extreme cases where this breaks down but it seems likely that there is a large class of quasistationary settings where it works very well. Figure 13 is a final attempt to bolster this claim by showing the traces $u(t)$ and $s(t)$ from the previous two figures together with their envelopes. If this figure is studied carefully, it is possible to appreciate the difficulty in guessing the relative delay between any two events from the traces themselves due to the rotating phase. When the envelopes are compared the task becomes much simpler.

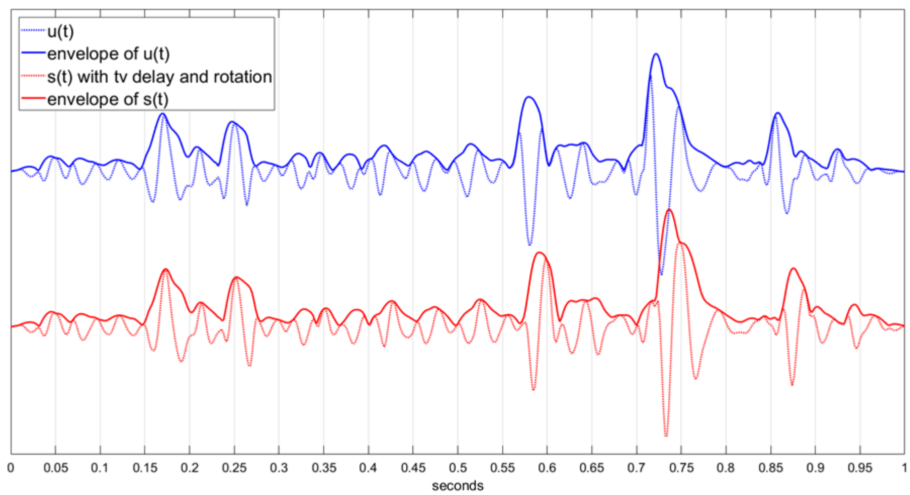

Figure 13. The traces $u(t)$ and $s(t)$ of Figures 11 and 12 are shown together with their envelopes. It is difficult to guess the time-variant delay from visual comparison of the traces themselves because of the phase rotations. However, the task is much simpler when comparing the envelopes. In fact, the two envelopes are seen to be very similar except for the temporal shifts.

SUMMARY AND CONCLUSIONS

Measurement of apparent phase rotation and time delay from the comparison of two seismic traces is complicated by the fact that the two unknowns are correlated with one another such that measurement errors in one affect the other. Relative time delay estimates, which are commonly measured by crosscorrelation, are therefore subject to considerable error if there are also phase rotation differences between the traces. The trace envelope, which is insensitive to phase rotations, can be exploited to remove this bias. Either the crosscorrelation of envelopes or the envelope of the crosscorrelation can be used to reliably measure the delay regardless of any phase rotations. In a series of controlled experiments, I have demonstrated these concepts on stationary synthetics. The extension to the nonstationary, or time-variant, case is accomplished by simply applying the stationary concepts in a series of moving Gaussian windows that are chosen to properly sample the traces. This is essentially a Gabor technique and it allows a direct extension of stationary methods to a time-variant setting. In a second series of experiments, I have demonstrated that these methods allow a very detailed estimation of both time-variant delay and time-variant phase.

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APPENDIX

Software modules used in this study are shown below. All listed functions are Matlab code found in the CREWES Matlab distribution.

phsrot … applies a constant (stationary) phase rotation.
constphase … estimates a constant (stationary) phase rotation.
env … compute the Hilbert envelope of a signal.
ccorr … crosscorrelation.
stat … stationary time shift (static shift).
maxcorr … find the maximum of a correlation function and its delay.
tvmaxcorr … time variant version of maxcorr using Gaussian windows.
tvconstphase … time variant version of constphase using Gaussian windows.
tvphaserot … time variant version of phsrot using equation 13.
stretcht … applies a time-variant delay or stretch.

If you are employed by a CREWES sponsor, then I will gladly provide a script that recreates the figures in this paper and demonstrates these codes.