Least-squares RTM of a seismic-while-drilling dataset

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ABSTRACT

Least-squares migration can, in theory, reduce the acquisition footprint and improve the illumination of the subsurface structures. It can also recover the amplitudes of the events to some extent. However, the migration operator is not complete. In other words, the operator does not span the full range of the model and the portion of the model that is in the null space of the operator will not be recovered even by posing imaging as an inverse problem. In geophysical terminology, in complex subsurface structures, rays or the wave energy will penetrate poorly in some regions, e.g., subsalt region, and that region will be a shadow zone to our acquisition system. The shadow zone is in the null space of the migration operator and the subsurface information in that region will not be recovered. Accordingly, in this research, we aim at using another set of data whose ray paths are different from the surface seismic. Seismic-while-drilling (SWD) dataset are complementary to surface data, and it brings an opportunity to address seismic illumination issue by adding new measurements into the imaging problem. Provided that we understand the correlative and non-impulsive nature of the SWD source signature, the prestack least-squares depth migration of the SWD dataset can be achieved. We study the feasibility of the least-squares reverse time migration of the SWD dataset and its potential in imaging the parts of the model which are in the shadow zone of the surface seismic acquisition.

INTRODUCTION

In conventional surface seismic acquisition, back propagating the recorded wave fields at the surface through a background medium can result in imaging the subsurface structures. This process, called seismic imaging, suffers from non-uniform illuminations. The imaging algorithms can usually address the kinematics of the subsurface structures adequately. However, they fail to properly provide the amplitude information of the events. To remedy this shortcoming authors pose the seismic imaging as an inverse problem. Least-squares migration can be used for this purpose. Least-squares migration with Kirchhoff operator was one of the early attempts (Tarantola, 1984; Nemeth et al., 1999; Trad, 2015). The algorithm is also implemented with one-way wave equation operators (Kuhl and Sacchi, 2003; Clapp et al., 2005; Kazemi and Sacchi, 2015). To fully account for all kind of dips and complexities of the subsurface structure, authors introduce two-way propagators such as reverse time migration (Baysal et al., 1983; Loewenthal and Mufti, 1983; Levin, 1984) into the least-squares imaging (Ji, 2009; Wong et al., 2011; Dai and Schuster, 2013; Zhang et al., 2014; Xue et al., 2015; Xu and Sacchi, 2017; Chen and Sacchi, 2017). Least-squares reverse time migration can, in theory, reduce the acquisition footprint and improve the illumination of the subsurface structures. It can also recover the amplitudes of the events to some extent. However, in the complex subsurface structure such as subsalt regions, waves penetrate weekly, and the reflected wave fields do not contain information from those regions. In other words, these regions will be in the shadow zone of the typical aperture limited surface seismic acquisition. Accordingly, the amplitude information in the shadow zones will not be recovered even by posing imaging as an inverse problem.

By contrast, in seismic-while-drilling (SWD) acquisition, drill bit-rock interaction can radiate significant elastic energy into the medium of interest which are strong enough to reach the surface and recorded by active receivers. Hence, the drill bit-rock interaction can be used as a seismic source. Moreover, having access to new source positions close to the target region will generate unique ray paths that are different from surface seismic. Provided that we understand the radiation pattern of the SWD sources, the SWD data contains information from within the medium of interest, at points normally not available to seismic sources. This information can be used to mitigate the subsurface illumination problem. In this study, we explore this possibility by implementing least-squares reverse time migration on a synthetic SWD dataset.

THE BORN APPROXIMATION: FORWARD/DE-MIGRATION OPERATOR AND ITS ADJOINT

The wave equation in a two dimensional constant density acoustic and isotropic medium is

$$(\omega^2\mathbf{s}^2 + \nabla^2)P = f\ \delta(\mathbf{x}-\mathbf{x}_s), \tag{1}$$

where P is pressure wave field, $\mathbf{s}$ is slowness (reciprocal of velocity), ω is temporal frequency, $\mathbf{f}$ is the source signature, $\mathbf{x}_s$ is source location and ∇^2 is Laplacian operator. To start the analysis we can assume that background smooth velocity (slowness) field is known. We can represent the squared slowness and the scalar field in terms of perturbations and backgrounds as

$$\mathbf{s}^2 = \mathbf{s}_0^2 + \mathbf{m}, \quad and \quad P = P_0 + \Delta P, \tag{2}$$

where $\mathbf{s}_0$ and P_0 are background slowness and wave field, respectively. Similarly, ΔP is the perturbation in the wave field due to $\mathbf{m}$. The parameter $\mathbf{m}$ is the perturbation in slowness-squared and is proportional to the subsurface reflectivity (Clayton and Stolt, 1981). Now, by using the Green's function G_0 satisfying the wave equation corresponding to the background medium

$$(\omega^2\mathbf{s}_0^2 + \nabla^2)\ G_0 = \delta(\mathbf{x}-\mathbf{x}_s), \tag{3}$$

the perturbed wave field can be calculated via

$$\Delta P(\omega,\mathbf{x}) \approx -\sum_{\mathbf{x}'} G_0(\mathbf{x},\omega;\mathbf{x}')\ \omega^2\mathbf{m}(\mathbf{x}')\ P_0(\omega,\mathbf{x}'). \tag{4}$$

In general, if the source is at position $\mathbf{x}_s$ and the receivers are at spatial coordinates $\mathbf{x}_r$, equation 4 can be written as

$$d(\omega,\mathbf{x}_r,\mathbf{x}_s) = \Delta P(\omega,\mathbf{x}_r,\mathbf{x}_s) \approx -\sum_{\mathbf{x}'} G_0(\mathbf{x}_r,\mathbf{x}_s,\omega;\mathbf{x}')\ \omega^2\mathbf{m}(\mathbf{x}')\ P_0(\omega,\mathbf{x}'), \tag{5}$$

which is the forward modelling operator. In matrix- vector notation, we have

$$\mathbf{d} = \mathbf{L}\ \mathbf{m}, \tag{6}$$

where $\mathbf{d}$ denotes the seismic measurements represented by a vector, the vector $\mathbf{m}$ stands for the acoustic potential, and $\mathbf{L}$ is the forward modelling operator. Now, by defining the adjoint of forward operator, we can migrate the measured data

$$\mathbf{m}_{adj}(\mathbf{x}) = -\sum_{\omega}\sum_{\mathbf{x}_s'}\ \sum_{\mathbf{x}_r'}(\omega^2 P_0^*(\mathbf{x},\omega;\mathbf{x}_s')\ G_0^*(\mathbf{x}_s',\mathbf{x}_r',\omega;\mathbf{x})\ d(\mathbf{x}_r',\mathbf{x}_s',\omega)), \tag{7}$$

where $\mathbf{m}_{adj}$ is the migrated image of the subsurface. Note that the $G_0^*(\mathbf{x}_s^{'}, \mathbf{x}_r^{'}, \omega; \mathbf{x})$ operator back propagates the receiver side wave field, i.e., recorded data at the surface, and the crosscorrelation imaging condition is applied by multiplying the back propagated receiver side wave field with the forward propagated source side wave field $P_0^*(\mathbf{x}, \omega; \mathbf{x}_s^{'})$ and finally summation over the frequencies. Equation 7 can also be written in time domain as

$$\mathbf{m}_{adj}(\mathbf{x}) = \sum_{\mathbf{x}_s} \sum_{t} p^S(\mathbf{x}; t; \mathbf{x}_s)\; p^R(\mathbf{x}; t; \mathbf{x}_s), \tag{8}$$

where p^S is the source side wave field, p^R is the receiver side wave field, and t is time. For more details, interested readers are referred to Zhang et al. (2014). Equation 7 in matrix-vector notation is

$$\mathbf{m}_{adj} = \mathbf{L}^T\, \mathbf{d}, \tag{9}$$

where $\mathbf{L}^T$ is the adjoint or migration operator. Unfortunately, $\mathbf{m}_{adj}$ is the approximated version of $\mathbf{m}$. This can be inferred by combining equations 6 and 9

$$\mathbf{m}_{adj} = \mathbf{L}^T\, \mathbf{d} = \mathbf{L}^T\mathbf{L}\, \mathbf{m}, \tag{10}$$

where $\mathbf{L}^T\mathbf{L} \neq \mathbf{I}$. Equation 10 shows that the image obtained via the adjoint operator is not equal to the true image $\mathbf{m}$ unless $\mathbf{L}^T\mathbf{L} = \mathbf{I}$. The latter is not true because $\mathbf{L}^T$ is not the inverse of $\mathbf{L}$. However, in classical migration, one considers that the adjoint operator is a good approximation to the true imaging operator and that $\mathbf{L}^T\mathbf{L}$ is diagonally dominant. Therefore, within a scale factor, one can also assume that $\mathbf{m}_{adj} \approx \mathbf{m}$. In other words, $\mathbf{m}_{adj}$ is a blurred version of $\mathbf{m}$ where the blurring operator is given by the Hessian operator $\mathbf{H} = \mathbf{L}^T\mathbf{L}$. Accordingly, if the goal is to get $\mathbf{m}$ instead of $\mathbf{m}_{adj}$, then one needs to invert the Hessian operator and solve

$$\mathbf{m} = \mathbf{H}^{-1}\mathbf{m}_{adj}\,. \tag{11}$$

Inverting the Hessian operator can be cast as an inverse problem. For more detailed derivations, interested readers are referred to Kazemi Nojadeh (2017). In the next section, we formulate inverting the Hessian as a least-squares minimization problem.

LEAST-SQUARES REVERSE TIME MIGRATION

Assuming a shot independent reflectivity model of the subsurface $\mathbf{m}$, the born forward modelling can be expressed as

$$\mathbf{d} = \mathbf{L}\mathbf{m}, \tag{12}$$

where $\mathbf{d}$ is the forward modelled data, and $\mathbf{L}$ is the forward modelling or de-migration operator. Application of the adjoint operator on the forward modelled data results in the conventional migration

$$\mathbf{m}_{adj} = \mathbf{L}^T\mathbf{d}, \tag{13}$$

where $\mathbf{L}^T$ is the reverse time migration operator. The adjoint migrated image has poor illumination and suffers from amplitude bias. To improve illumination and amplitudes, one can pose imaging as an inverse problem. Posing imaging as an inverse problem not only improves the quality of the images but also provides an opportunity to include regularization and prior information about the subsurface. By doing so, one can emphasize good features in the final image. In this report, we pose imaging as a least-squares minimization

$$\mathbf{m}_{LS} = \underset{\mathbf{m}}{\operatorname{argmin}} \; \|\mathbf{Lm} - \mathbf{d}\|_2^2 + \mu \mathcal{R}(\mathbf{m}), \tag{14}$$

where $\mathcal{R}(\mathbf{m})$ is a regularization term that enhances desired features in the model, and μ is a regularization parameter that balances the importance of data fidelity versus the regularization term. We assume that the exact subsurface model has continuous and smooth features in spatial directions and we solve

$$\mathbf{m}_{LS} = \underset{\mathbf{m}}{\operatorname{argmin}} \; \|\mathbf{Lm} - \mathbf{d}\|_2^2 + \mu \|\mathbf{Dm}\|_2^2, \tag{15}$$

where $\mathbf{D}$ is a second order derivative operator. Equation 15 has a closed form solution

$$\mathbf{m}_{LS} = (\mathbf{L}^T\mathbf{L} + \mu\mathbf{D}^T\mathbf{D})^{-1}\mathbf{L}^T\mathbf{d}. \tag{16}$$

Equation 15 can also be written as

$$\mathbf{y}_{LS} = \underset{\mathbf{y}}{\operatorname{argmin}} \; \| \begin{bmatrix} \mathbf{LD}^{-1} \\ \sqrt{\mu}\,\mathbf{I} \end{bmatrix} \mathbf{y} - \begin{bmatrix} \mathbf{d} \\ \mathbf{0} \end{bmatrix} \|_2^2 = \|\underset{\sim}{\mathbf{A}}\mathbf{y} - \mathbf{b}\|_2^2, \tag{17}$$

where $\underset{\sim}{\mathbf{A}} = \begin{bmatrix} \mathbf{LD}^{-1} \\ \sqrt{\mu}\,\mathbf{I} \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} \mathbf{d} \\ \mathbf{0} \end{bmatrix}$. Note that $\mathbf{m}_{LS} = \mathbf{D}^{-1}\mathbf{y}_{LS}$. In real-world applications, least-squares migration is considered as a medium to a large-scale problem and requires the application of iterative algorithms. We use Conjugate Gradient algorithm to solve equation 17. The Conjugate Gradient algorithm can iteratively invert the matrix without requiring any direct inversion of it. The pseudocode of the Conjugate Gradient algorithm for solving equation 17 is represented in Algorithm 1. To build the forward and adjoint operators for prestack depth migration, we need to estimate the source signature from the data. The wavelet estimation is done by using Sparse Multichannel Blind Deconvolution (SMBD) algorithm (Kazemi and Sacchi, 2014). We briefly explain the SWD wavelet estimation workflow in the next section.

SWD SOURCE SIGNATURE ESTIMATION

Seismic-while-drilling data can be modelled as

$$\mathbf{d}_j = \mathbf{W}\,\mathbf{r}_j + \mathbf{n}_j \qquad j = 1, \ldots J, \tag{18}$$

where $\mathbf{W}$ is the convolution matrix of SWD source signature, $\mathbf{r}$ is reflectivity series and $\mathbf{n}$ is the noise term. After some algebraic manipulations, it is easy to show that,

$$\mathbf{D}_p\mathbf{r}_q - \mathbf{D}_q\mathbf{r}_p = \mathbf{N}_p\mathbf{r}_q - \mathbf{N}_q\mathbf{r}_p\,, \tag{19}$$

where $\mathbf{D}_p$ and $\mathbf{D}_q$ in equation (19) represent the convolution matrices of channels p and q, respectively. $\mathbf{N}_p$ and $\mathbf{N}_q$ are convolution matrices of noise components. The combination of all possible equations leads to the following inhomogeneous system of equations

$$\mathbf{A}\,\mathbf{x} = \mathbf{e}, \tag{20}$$

Algorithm 1 Conjugate Gradient algorithm with regularization

choose $\mathbf{y}_0$,
$\mathbf{s}_0 = \mathbf{b} - \underset{\sim}{\mathbf{A}}\mathbf{y}_0$,
$\mathbf{r}_0 = \mathbf{p}_0 = \underset{\sim}{\mathbf{A}}^T(\mathbf{b} - \underset{\sim}{\mathbf{A}}\mathbf{y}_0)$,
$\mathbf{q}_0 = \underset{\sim}{\mathbf{A}}\mathbf{p}_0$,
Initialize iteration $k = 0$,
while $(||\underset{\sim}{\mathbf{A}}\mathbf{y}_k - \mathbf{b}||_2^2 > tol)$ **do**,
    $\alpha_{k+1} =< \mathbf{r}_k, \mathbf{r}_k > / < \mathbf{q}_k, \mathbf{q}_k >$,
    $\mathbf{y}_{k+1} = \mathbf{y}_k + \alpha_{k+1}\,\mathbf{p}_k$,
    $\mathbf{s}_{k+1} = \mathbf{s}_k - \alpha_{k+1}\,\mathbf{q}_k$,
    $\mathbf{r}_{k+1} = \underset{\sim}{\mathbf{A}}^T\mathbf{s}_{k+1}$,
    $\beta_{k+1} =< \mathbf{r}_{k+1}, \mathbf{r}_{k+1} > / < \mathbf{r}_k, \mathbf{r}_k >$,
    $\mathbf{p}_{k+1} = \mathbf{r}_{k+1} + \beta_{k+1}\mathbf{p}_k$,
    $\mathbf{q}_{k+1} = \underset{\sim}{\mathbf{A}}\mathbf{p}_{k+1}$,
    $k \leftarrow k + 1$,
end while
$\mathbf{y} \leftarrow \mathbf{y}_k$,
Calculate $\mathbf{m} = \mathbf{D}^{-1}\mathbf{y}$.

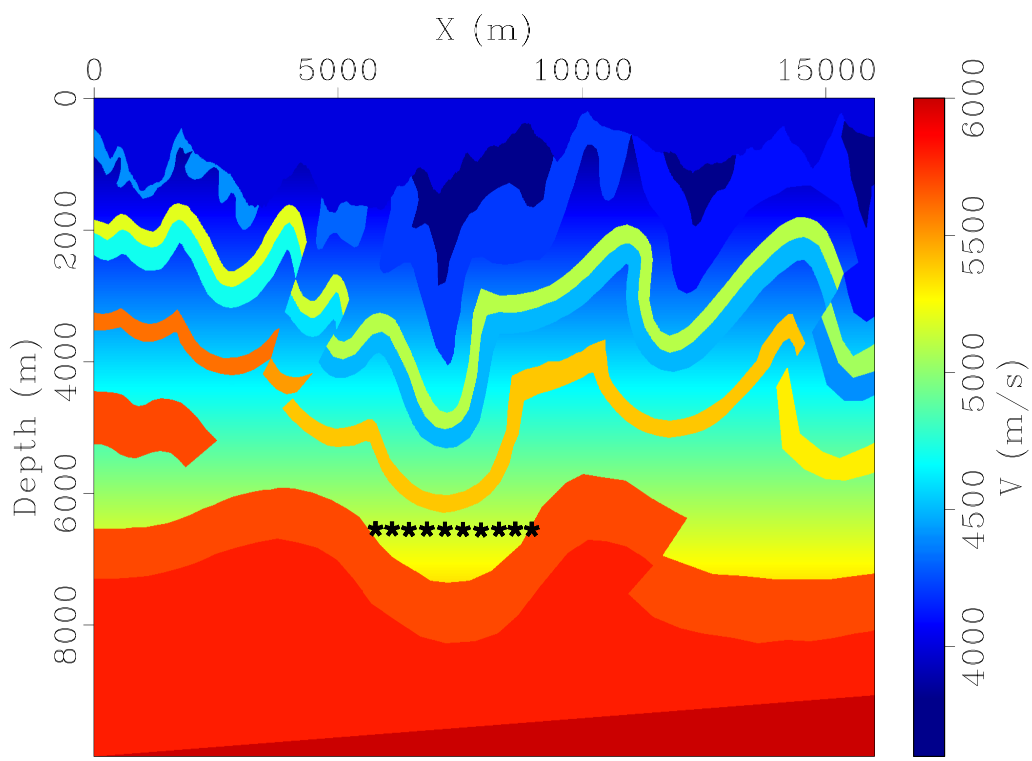

FIG. 1. SWD acquisition geometry over BP velocity model. Black stars in the middle of the model around $6.4km$ are SWD sources.

where

$$\mathbf{A} = \begin{pmatrix} \mathbf{D}_2 & -\mathbf{D}_1 & & & & & \\ \mathbf{D}_3 & & -\mathbf{D}_1 & & & & \\ \mathbf{D}_4 & & & -\mathbf{D}_1 & & & \\ \vdots & & & & \ddots & & \\ & \mathbf{D}_3 & -\mathbf{D}_2 & & & & \\ & \mathbf{D}_4 & & -\mathbf{D}_2 & & & \\ & \vdots & & & \ddots & & \\ & & & & \mathbf{D}_J & & -\mathbf{D}_{J-2} \\ & & & & & \mathbf{D}_J & -\mathbf{D}_{J-1} \end{pmatrix} \tag{21}$$

$$\text{and} \qquad \mathbf{x} = [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \ldots, \mathbf{r}_J]^T \,. \tag{22}$$

To find the reflectivity, SMBD minimizes the following cost function

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \quad \frac{1}{2}||\mathbf{A}\,\mathbf{x}||_2^2 + \lambda \sum_i (\sqrt{x_i^2 + \epsilon^2} - \epsilon), \quad \text{subject to} \quad \mathbf{x}^T\mathbf{x} = 1 \tag{23}$$

where ϵ is a small number to mimic the ℓ_1 norm behaviour and λ is a regularization parameter. As a by-product, after solving for the reflectivity, SWD source signature can be estimated using the frequency-domain least squares estimator. In the next section, we test the performance of the least-squares reverse time migration algorithm on a synthetic SWD dataset of BP model.

EXAMPLES

To evaluate the performance of the least-squares reverse time migration of a SWD dataset, we use a BP model represented in Figure 1. The drill bit- rock interaction is used as a seismic source in the deeper part of the model. The SWD acquisition geometry consists of 10 SWD sources around $6.4km$, and the receivers are listening at the surface. There are 1500 receivers at the surface with $10m$ intervals, and the source spacing is $300m$. To simulate the data, we use acoustic finite difference modelling and then convolve the data with a drill bit source signature. The drill bit-rock signature is similar to the work of Kazemi et al. (2018) and follows the assumptions of Poletto (2005). Figure 2a shows the true drill bit-rock interaction source and Figure 2b is the estimated source signature using the SMBD algorithm. The middle SWD shot gather represented in Figure 3.

The SWD data is different from the surface seismic data. The SWD source signature is correlative and non-impulsive, and this makes it difficult to track the events. Hence, pre-processing the data and estimating the SWD source signature is an important step in our least-squares migration algorithm. After feeding the raw SWD shot gathers as receiver side wave field and the estimated SWD source signature for building the source side wave field, we achieve the least-squares migrated image (Figure 4). We use 10 iterations of the Conjugate Gradient represented in Algorithm 1. The result shows that the least-squares reverse time migration of the SWD dataset successfully imaged the subsurface structures. Moreover, we show that the least-squares method fits the recorded SWD data. Finding a model that can predict the recorded data is one of the advantages of the least-squares algorithm over the conventional migration techniques. Figure 5 shows the predicted SWD data after applying the forward modelling engine $\mathbf{L}$ on the least-squares migrated image $\mathbf{m}$ shown in Figure 4.

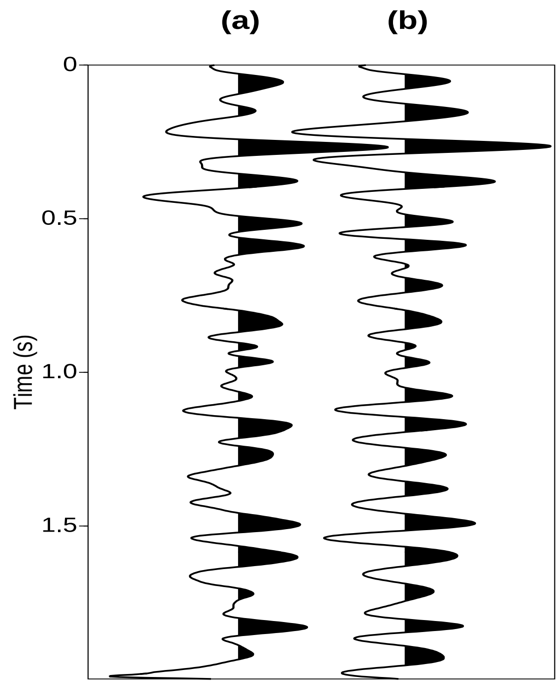

FIG. 2. Drill bit source signature. a) True and b) Estimated source signatures.

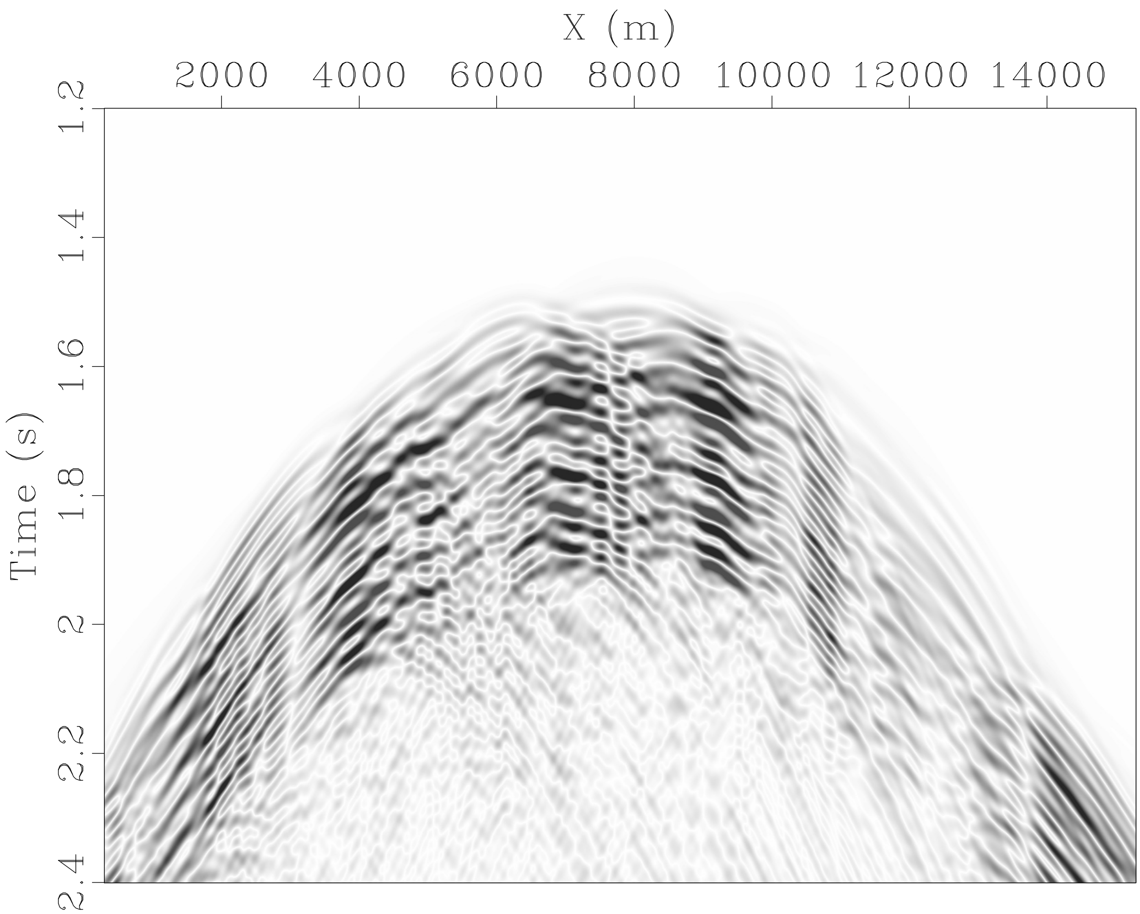

FIG. 3. SWD shot gather corresponding to the fifth SWD source.

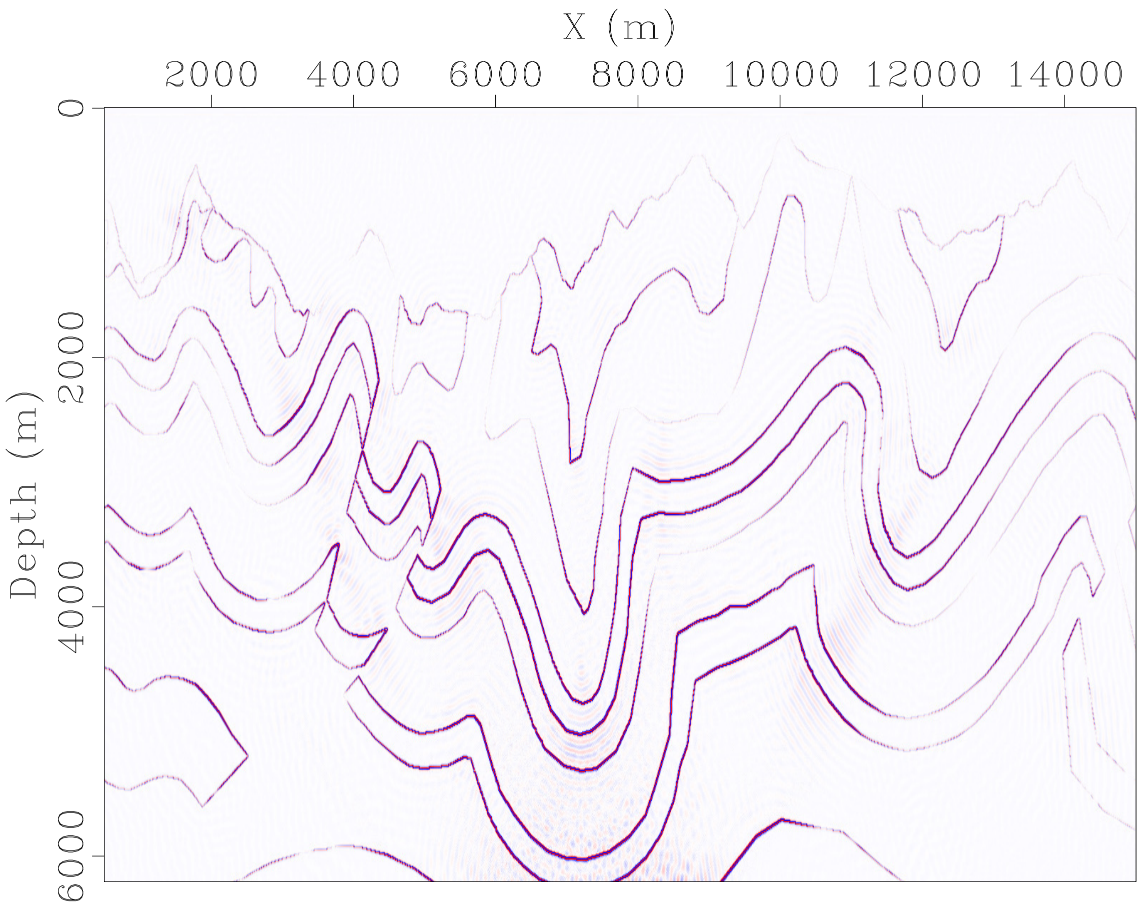

FIG. 4. Least-squares reverse time migration of the SWD dataset after 10 iterations.

CONCLUSIONS

We explored the possibility of using a drill bit-rock interaction as a seismic source. Moreover, we showed that SWD data contain valuable information from the subsurface, and, thanks to the unique ray paths of SWD acquisition, the SWD data bring an opportunity to mitigate non-uniform subsurface illumination. We formulated the least-squares reverse time migration of a SWD dataset and successfully imaged the subsurface structures. The crucial step in imaging the subsurface by using a SWD dataset is that we understand the radiation patterns of drill bit-rock interactions and be able to estimate the correlative and non-impulsive SWD source from the dataset. SWD source signature is one of the main inputs of our current workflow. To estimate the SWD source signature, we implemented a sparse multichannel blind deconvolution (SMBD) algorithm. Our preliminary results on the BP model showed that SWD data have the potential of imaging the subsurface structures. The next step is to combine surface seismic data with SWD data and apply a joint least-squares migration of combined dataset.

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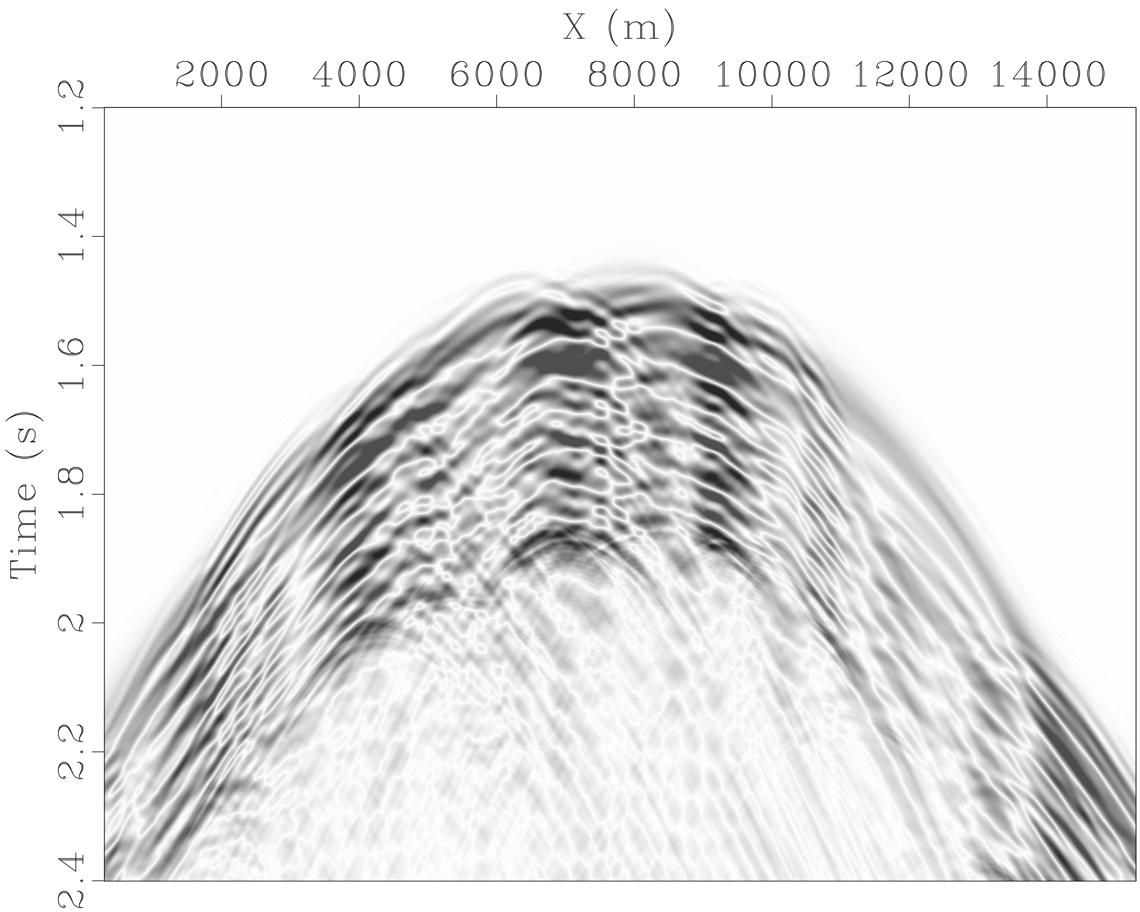

FIG. 5. Predicted fifth SWD shot gather using the inverted image shown in Figure 4.

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