A comparison of two reflection-based waveform inversion strategies

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ABSTRACT

Reflection-based waveform inversion is a set of strategies for updating the long-wavelength part of a velocity model through the use of reflection data in a full waveform inversion approach. Two analytical formulations of this type are proposed in this paper. A migration-based version uses a migration as the model of seismic reflectivity and directly calculates the effect of velocity model changes on this reflectivity. This approach is computationally intensive, and may estimate reflectivity poorly. The second approach considers only vertical shifts to a fixed reflectivity model. This reduces cost and allows for a better initial reflectivity model, but has the drawback of simplifying the effects of velocity model changes on the reflectivity. Neither approach uses demigration, instead using a long-wavelength model parameterization to ensure that reflectivities are not directly modified in the inversion.

INTRODUCTION

Full waveform inversion is a strategy for recovering a model of the subsurface which best describes the full information content of a seismic experiment (Tarantola, 1984). This approach has been highly successful in a number of applications, but often relies on measurements of diving waves to recover long wavelength features of the model (Virieux and Operto, 2009). Below the penetration depth of the diving waves, FWI often struggles to recover these longer wavelengths. While reflection events do carry information about the long wavelength structure of the subsurface, they are much more strongly influenced by the short wavelength structure of the earth. Because of this dominance of shorter wavelengths, conventional FWI struggles to recover accurate long wavelength models deep in the subsurface. A number of approaches have been developed to make better use seismic reflections in a waveform inversion approach. These are generally referred to as reflection-based waveform inversion (RWI)(Xu et al., 2012).

The strategies used in RWI usually hinge on using moveout information to recover long wavelengths. While a velocity-depth ambiguity exists when considering one dimensional seismic data, in two or three dimensions the depth of seismic reflectors is constrained. If the long wavelength velocity model is incorrect, a reflection will not be able to match both the short and long offset arrival times simultaneously. In RWI, these errors are used to drive the long wavelengths of the model toward the correct values. If the reflectors are not modified, changing only the long wavelengths of the model will generally not improve the data fitting, as the reflectors will not be in the correct position in the modified model. In order to successfully implement RWI then, two conditions need to be fulfilled. Firstly, the changes applied in an RWI update should not introduce new reflectors, only alter the background velocity. This is usually achieved by using a demigration process for forward modeling the data in the inversion and treating the migration velocity model as the inverted property (Almuteri, 2017). Secondly, the long wavelength changes in the model should cause the reflectors in the model to change position. This is usually achieved by treating the reflectors in one of two ways: either the reflectivity is a fixed model which is shifted based on the background velocity (Xu et al., 2012), or it is the result of a migration, which changes as the migration velocity is changed in the inversion (Brossier et al., 2015).

In this report, we provide analytic formulations for two different RWI strategies. One approach models reflectivity by migration at each iteration, the second applies spatial shifts to a starting reflectivity model. In both strategies we prevent the creation of reflectors through variable restriction. This allows for finite difference forward modeling without demigration assumptions.

THEORY

The FWI problem is typically posed as an optimization problem, wherein a scalar objective function measuring the mismatch between measured and numerically modeled data is minimized with respect to the model considered (Virieux and Operto, 2009). In the RWI strategies we suggest here, the objective function is defined as the square of the L_2 norm of the difference between measured and modeled data, given by

$$\frac{1}{2}||Ru - D||^2, \tag{1}$$

where D is the measured data, u is the modeled pressure field, and R is a matrix representing the receiver sampling. The modeled data are calculated by frequency domain finite differences, with the forward modeling satisfying the equation

$$S(m + m_r)u = f, \tag{2}$$

where f is a source term, S is the Helmoltz operator containing the finite difference approximation of the frequency domain wave equation, m is the model considered in the inversion, and m_r is a model of short-scale features. In effect, the forward modeling uses a subsurface model which is the sum of a long wavelength model m that is the goal of the inversion and a short wavelength model m_r. The two strategies discussed differ in how m_r is calculated.

RWI with iteratively recalculated reflectivity

In the first approach, the short scale model m_r is calculated using one iteration of the LSM-type approach described in Keating and Innanen (2018a), and is given by

$$m_r = -\alpha P \sum_{x_s,\omega} \omega^2 v^* \lambda, \tag{3}$$

where $alpha$ is a chosen scalar, P is a matrix applying a high-pass filter, x_s are the source locations, ω is angular frequency, u_mig is the pressure field satisfying

$$S(m, \omega_i)v_i = f, \tag{4}$$

and λ_mig is the pressure field satisfying

$$S(m, \omega_j)^\dagger \lambda_j = R^T(Rv_j - D). \tag{5}$$

The short scale model m_r is closely related to a migrated image, it is calculated in equation 3 by frequency domain multiplication of a forward propagated wavefield v with a back propagated λ, followed by application of a high-pass filter. With this choice for m_r, the inversion problem considered becomes

$$\min_{m} \sum_{x_s,\omega} \frac{1}{2}||Ru - D||_2^2, \tag{6}$$

subject to the constraints in equations 2, 4 and 5.

To calculate the gradient of the objective function for this problem, we consider the adjoint state method (e.g Metivier et al., 2013). The Lagrangian of this problem is

$$\begin{aligned} L = \frac{1}{2}||Ru - D||_2^2 + < S(m + m_r, \omega_n)u - f, a > \\ + \sum_{i} < S(m, \omega_i)v_i - f, b_i > \\ + \sum_{j} < S(m, \omega_j)^{\dagger}\lambda_j - R^T(Rv_j - D), c_j >, \end{aligned} \tag{7}$$

where a, b_i, and c_j are Lagrange multipliers, and $<, >$ denotes an inner product. If this function is evaluated at $\bar{u}$, $\bar{v}$, and $\bar{\lambda}$, defined as the u, v and λ_{mig} which satisfy equations 2, 4 and 5, it is equal to the objective in equation 1. The gradient of the objective function is then equal to

$$\begin{aligned} \frac{d\phi(\omega_n)}{dm} = \frac{dL(\bar{u}, \bar{v}, \bar{\lambda}_{mig})}{dm} = \frac{d\bar{u}}{dm}\frac{\partial \bar{L}}{\partial \bar{u}} + \frac{d\bar{v^*}}{dm}\frac{\partial \bar{L}}{\partial \bar{v^*}} + \frac{d\bar{\lambda}}{dm}\frac{\partial \bar{L}}{\partial \bar{\lambda}} \\ + < \partial_m S(m + m_r, \omega_n)\bar{u}, a > \\ + \sum_{i} < \partial_m S(m, \omega_i)^* \bar{v_i^*}, b_i > \\ + \sum_{j} < \partial S(m, \omega_j)^{\dagger}\bar{\lambda_j}, c_j >, \end{aligned} \tag{8}$$

where $\bar{L} = L(\bar{u}, \bar{v}, \bar{\lambda}_{mig})$. Because the derivatives of the pressure fields relative to the model parameters are computationally expensive to calculate, we choose the Lagrange multipliers such that the first three terms on the right-hand side are zero. For the first term,

$$\frac{\partial \bar{L}}{\partial \bar{u}} = R^T(R\bar{u} - D) + S(m + m_r, \omega_n)^{\dagger}a = 0, \tag{9}$$

so a can be solved for by back-propagating the data residuals. Setting the second and third terms to zero yields the equations

$$S(m, \omega_j)c_j = \alpha\omega_j^2(P < \partial_{m_r} S(m + m_r, \omega_n)\bar{u}, a >)\bar{v}_j \tag{10}$$

and

$$S(m, \omega_i)^T b_i = R^T R c_i + \alpha\omega_i^2(P < \partial_{m_r} S(m + m_r, \omega_n)\bar{u}, a >)\bar{\lambda_i}. \tag{11}$$

Equations 9, 10, and 11 can be solved for the Lagrange multipliers by performing wave propagations. When the Lagrange multipliers satisfy these equations, the gradient reduces to

$$\begin{aligned} \frac{d\phi(\omega_n)}{dm} =& < \partial_m S(m+m_r,\omega_n)\bar{u}, a > \\ &+ \sum_i < \partial_m S(m,\omega_i)^* \bar{v_i^*}, b_i > \\ &+ \sum_j < \partial S(m,\omega_j)^\dagger \bar{\lambda_j}, c_j > . \end{aligned} \tag{12}$$

This approach faces several challenges for the RWI problem. To begin with, it is difficult to choose a value for the short-scale amplitude term α. The α which gives the best result with the starting model will typically not be the best α for the true long-scale model. A poor choice of α may lead to false or local minima in the optimization precedure, harming the inversion result. Another concern may be the accuracy of the short scale model estimate, which is estimated with just one iteration of a LSM-type FWI approach at each FWI iteration.

Another concern is the large computational cost of gradient evaluation. A conventional, frequency domain FWI requires two forward modeling calculations per frequency per source. Because the Helmholtz matrix does not change for different sources, the forward modeling for all source terms at a given frequency can be calculated simultaneously at relatively small cost. This means that gradient calculation for the FWI problem has a cost of approximately two wave propagations per frequency. The RWI approach discussed here requires the calculation of one wave propagation per frequency per source for the calculations of each u, v, λ, and a. It requires a further cost of one wave propagation per frequency per frequency per source for the calculation of the b and c terms. This causes the total cost of this approach to be substantially larger than that of the FWI problem.

RWI with spatial shifting of reflectivity

The high cost of gradient calculation in the approach discussed above motivates a less expensive strategy. A large cost in that approach was the calculation of the reflectivity model at each iteration. A much less expensive approach is to modify a starting reflectivity model at each iteration based on the long-scale model changes. In an approach based on vertical shifts, the short-scale model could be defined as

$$m_r(z) = m_{r_0}(z'(z,m)), \tag{13}$$

where z represents the depth, and z' is the equivalent depth in the initial short-scale model m_{r_0}. Several functions could be considered for z' but we consider here

$$z' = \int_0^{\tau_{max}} dt v_0(\tau), \tag{14}$$

where v_0 represents the initial model,

$$\tau_{max} = \int_0^z \frac{dz}{v(m)}, \tag{15}$$

and $v(m)$ is the P-wave velocity given by the current model. The derivative of the objective function in equation 1 with this definition for m_r can again be calculated by the adjoint state method. The optimization problem becomes

$$\min_{m} \sum_{x_s,\omega} \frac{1}{2}||Ru - D||_2^2, \tag{16}$$

subject to the constraints in equations 2, and 13. The derivative of this function can be expressed as

$$\frac{d\phi}{dm} = \frac{d\phi}{dm_r}\frac{dm_r}{dm}. \tag{17}$$

The derivative $\frac{d\phi}{dm_r}$ can be shown to be equivalent to the conventional FWI gradient. The other term can be expressed as

$$\frac{dm_r}{dm} = \sum_{n} \frac{dm_{r0}}{dz'}\frac{dz'}{dv_n}\frac{dv_n}{dm}, \tag{18}$$

where v_n describes the P-wave velocity of the model over a range Δz_n. The value of $\frac{dm_{r0}}{dz'}$ can be calculated directly from m_{r_0}, and $\frac{dv_n}{dm}$ is dependent on the model parameterization used, discussed in the next section. The remaining term can be reduced to

$$\frac{dz'}{dv_n} = \frac{dt}{dv_n}\frac{dz'}{dt} = \frac{v_0(\tau_{max})H(\tau_{max} - \tau(\Delta z_n))}{v_n^2}, \tag{19}$$

where H is the Heaviside step function.

This approach has several advantages. The gradient calculation requires no extra forward modeling problems to be solved, so the computational cost of this algorithm is much less than that of the approach discussed in the previous section. Because the starting short-scale model m_{r_0} is fixed instead of being recalculated at each iteration, it can be estimated more accurately at the start of the inversion. As in the previous approach, however, the amplitude of the short scale model may be difficult to choose at the start of the inversion. The major drawback of this approach is the simple relation assumed between the long and short-scale models. In reality, the relation between the two is more complicated than a simple vertical shift, and this may prove an obstacle to the approach in complex geologic environments.

Parameterization

Demigration is often used as the forward modeling process in RWI (Almuteri, 2017). This process involves assumptions about the reflectivities and frequencies considered, and is not strictly based on a finite-difference approximation of wave propagation. The RWI strategy proposed here does not use a demigration approach. Instead of using a reflectivity estimate, the short scale model m_r is assumed to introduce most of the observed reflections. In RWI, the goal is to recover the long-scale features of the model. The gradient of the objective function in equation 1 with respect to m will typically include short-wavelength features if conventional FWI variables are used. For reflection data, these features will

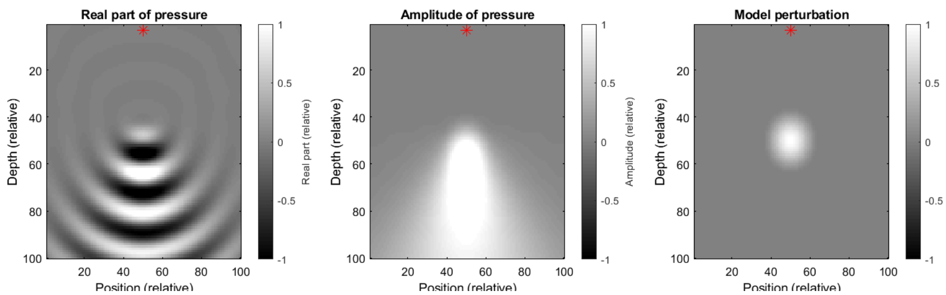

FIG. 1: Left: Real part of frequency domain radiation pattern. Center: Amplitude of frequency domain radiation pattern. Right: Variables considered in inversion. The red stars mark the source location. Changes in variables like this introduce limited reflections, allowing for the RWI strategy to be used without demigration. This should also allow for the natural treatment of diving waves simultaneously.

dominate over the desired long-wavelength features. To prevent this problem, we consider an alternate parameterization of the problem, in which the model m consists of the coefficients of long wavelength perturbations of $\frac{1}{v^2}$. If the variable a describes the squared slowness at each point of the finite difference grid,

$$a = Pm, \tag{20}$$

where P is a matrix whose columns are the long wavelength variables. The FWI problem with variables of this type is described at length in Keating and Innanen (2018a,c,b). An example of the variables used is shown in figure 1 (left). This choice of variable is made because long-scale changes in the model have the capacity to change the travel times of the modeled reflections, but do not have the capacity to introduce reflctions. This behaviour can be observed by studying radiation patterns, the derivative of the wave field with respect to a model variable. A numerically calculated radiation pattern for a 15 Hz source is shown in figure 1. The red star in this figure denotes a source position. Significant changes in can be observed at transmissive scattering angles, but there is almost no change at reflection-type scattering angles, making these parameters ideal for the RWI problem.

Another support for this variable choice can be observed in the form of the gradient. As shown by Keating and Innanen (2018b), the derivative of the objective function with respect to model variables of this type, expressed in the finite difference grid space defining a, is given by

$$g = PP^T g_a, \tag{21}$$

where g_a is the gradient with respect to the variables a. This expression shows that the gradient with respect to m is effectively equal to applying the low-pass filter defined by PP^T to the conventional FWI gradient g_a. Consequently, any short-scale features in g_a are effectively suppressed in g through appropriate choice of P. This allows for the derivative of the long-scale model to be calculated without the use of demigration.

CONCLUSIONS

Reflection-based waveform inversion is a set of waveform inversion strategies that use measured seismic reflections to recover information about the long wavelength structure of the subsurface

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