Elastic wave-equation migration for laterally varying isotropic and HTI media

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Outline

- Introduction
- Theory
  - Elastic wavefield extrapolation
  - Extension to laterally heterogeneous media
  - Migration imaging condition
- Examples
  - Elastic HTI modeled data
  - Marmousi 2: elastic OBC modeled data
- Conclusions

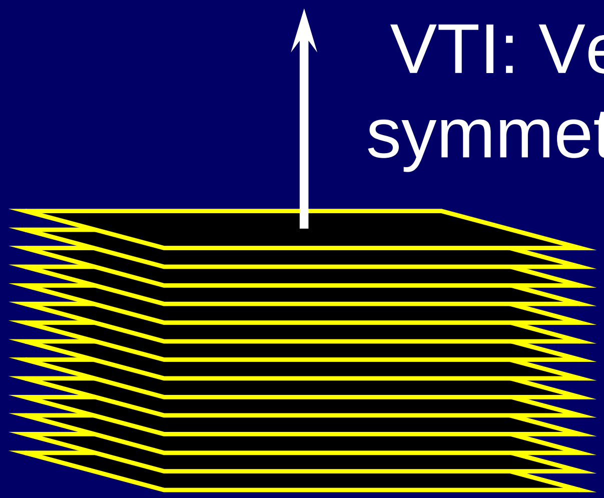
Introduction

Drawbacks to scalar extrapolation for elastic migration:

- Neglects mode conversions
- Fails to keep track of polarization changes
- Difficult to fully account for anisotropic effects, in particular shear wave splitting (birefringence) for HTI media

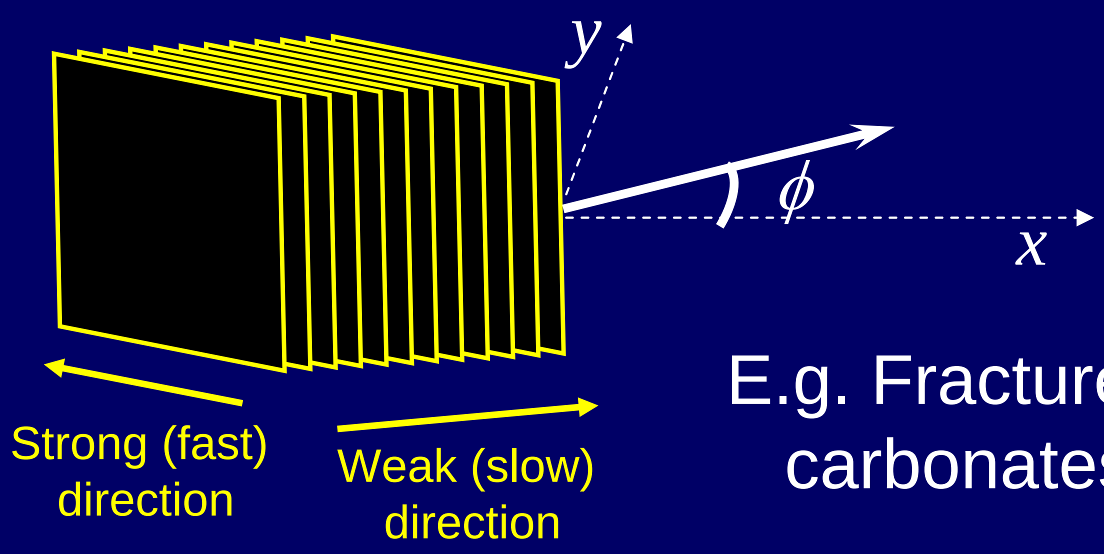
VTI and HTI: decks of Cards

VTI: Vertical symmetry axis



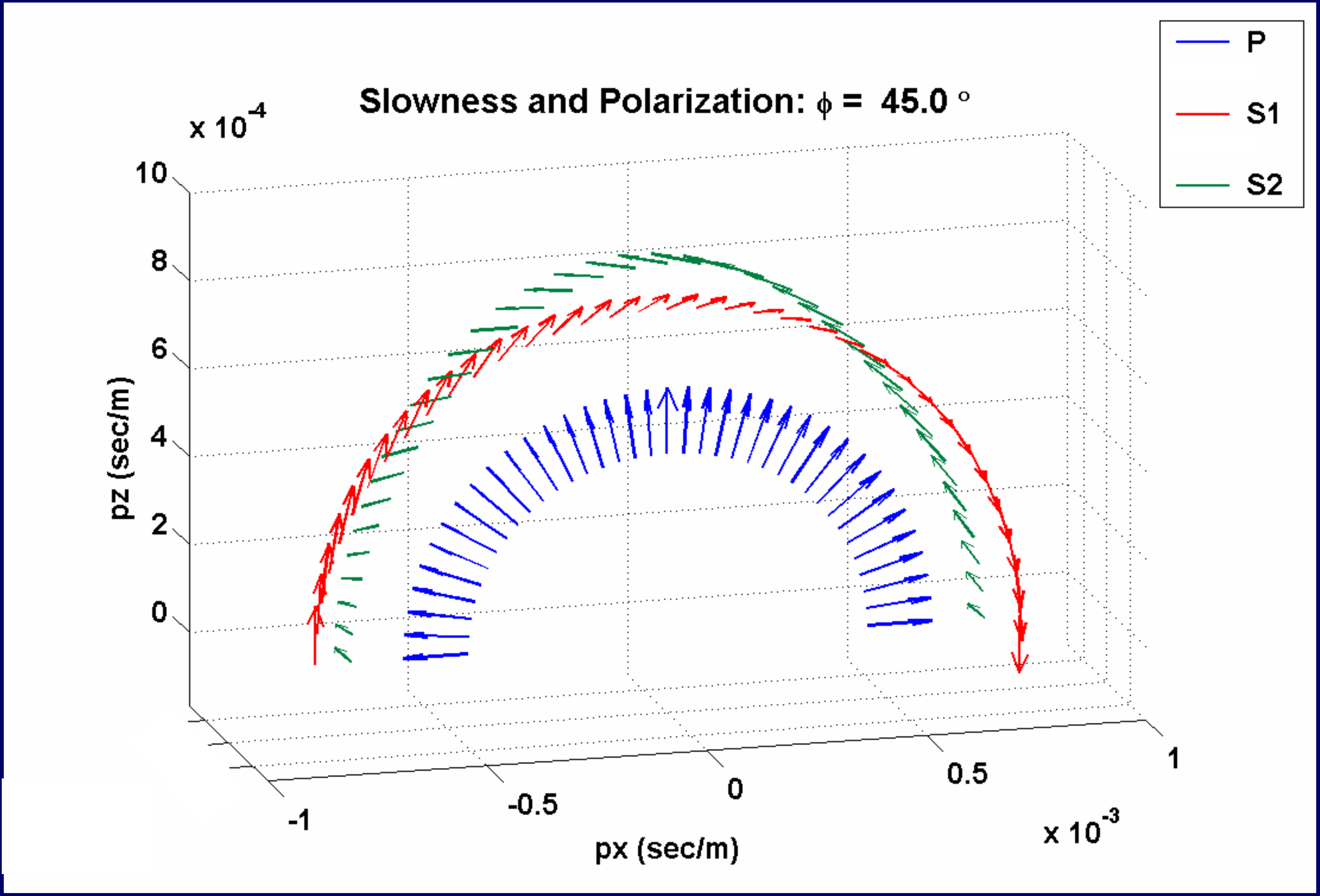
E.g. Shales

HTI: Horizontal symmetry axis



E.g. Fractured carbonates

Variation of Polarization with Slowness: HTI



Introduction

Standard processing of birefringent shear waves:

- Assumes vertical incidence waves
- Neglects variation of shear wave polarization with propagation angle
- Neglects changes in velocity, (and time delay) with propagation angle
- Often neglect variations of symmetry axis with depth

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Theory

First order, 6x6 form of elastic wave-equation:

$$\frac{\partial \mathbf{b}}{\partial z} = i\omega \mathbf{A}\mathbf{b} \qquad \mathbf{b} = \begin{pmatrix} \mathbf{u} \\ \boldsymbol{\tau} \end{pmatrix}$$

$\mathbf{A}$ = 6x6 fundamental elasticity matrix

depends on horizontal slowness p, frequency ω and elastic constants

$\mathbf{u}$ = displacement vector

τ = vertical traction vector

Theory

Diagonalize:

$$\frac{\partial \mathbf{v}}{\partial z} = i\omega \mathbf{\Lambda} \mathbf{v} \qquad \mathbf{b} = \mathbf{D}\mathbf{v} \qquad \mathbf{v} = \begin{pmatrix} \mathbf{v}_U \\ \mathbf{v}_D \end{pmatrix}$$

$\mathbf{v}_U$ = up-going wave-mode vector

$\mathbf{v}_D$ = down-going wave-mode vector

Λ = diagonal matrix of eigenvalues (vert. slowness)

$\mathbf{D}$ = eigenvector matrix (from polarizations)

Solution: $$\mathbf{v}_D(z) = e^{i\omega \mathbf{\Lambda}_D (z - z_0)} \mathbf{v}_D(z_0)$$

$V(z)$ Extrapolation

$$\mathbf{v}\left(p, z_{n+1}, \omega\right) = e^{i\omega\boldsymbol{\Lambda}_n\left(z_{n+1}-z_n\right)}\mathbf{v}\left(p, z_n, \omega\right)$$

p : horizontal slowness

z_n : n[th] depth level

$\mathbf{v}$: wave-mode vector in k-ω domain ($k = p\omega$)

Λ_n : diagonal matrix of eigenvalues (vert. slowness)

$V(z)$ Extrapolation

recomposition extrapolation decomposition

$$\mathbf{b}\left(p, z_{n+1}, \omega\right) = \mathbf{D}_n e^{i\omega\Lambda_n\left(z_{n+1}-z_n\right)}\mathbf{D}_n^{-1}\mathbf{b}\left(p, z_n, \omega\right)$$

p : horizontal slowness

z_n : n[th] depth level

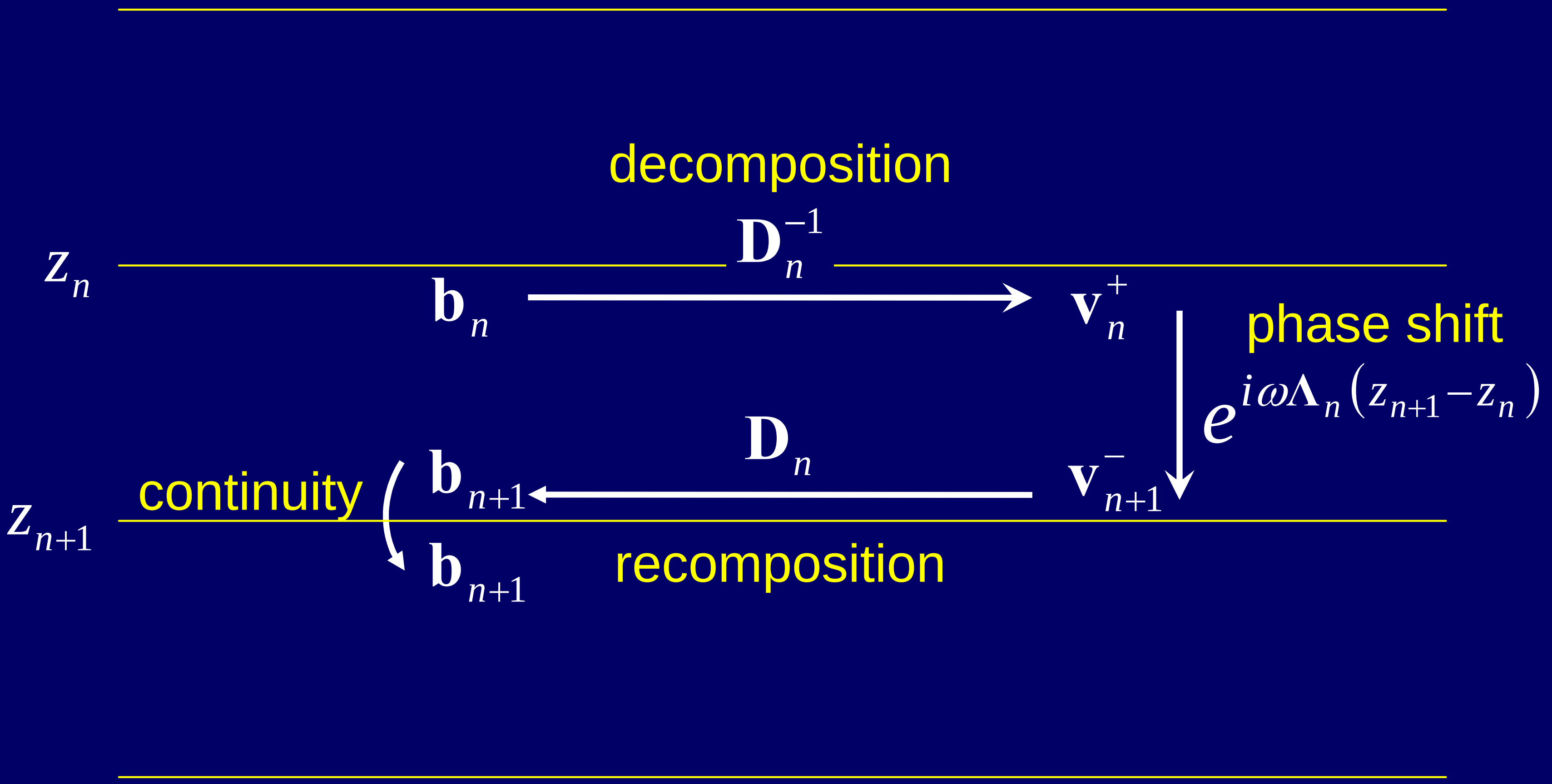
$\mathbf{v}$: wave-mode vector in k-ω domain ($k = p\omega$)

Λ_n **:** diagonal matrix of vertical slowness (P, S1, S2)

$\mathbf{b}$: displacement-stress vector in k-ω domain

$\mathbf{D}_n$: eigenvector matrix (from polarizations)

$V(z)$ Extrapolation



V(x,z) Extrapolation Operator

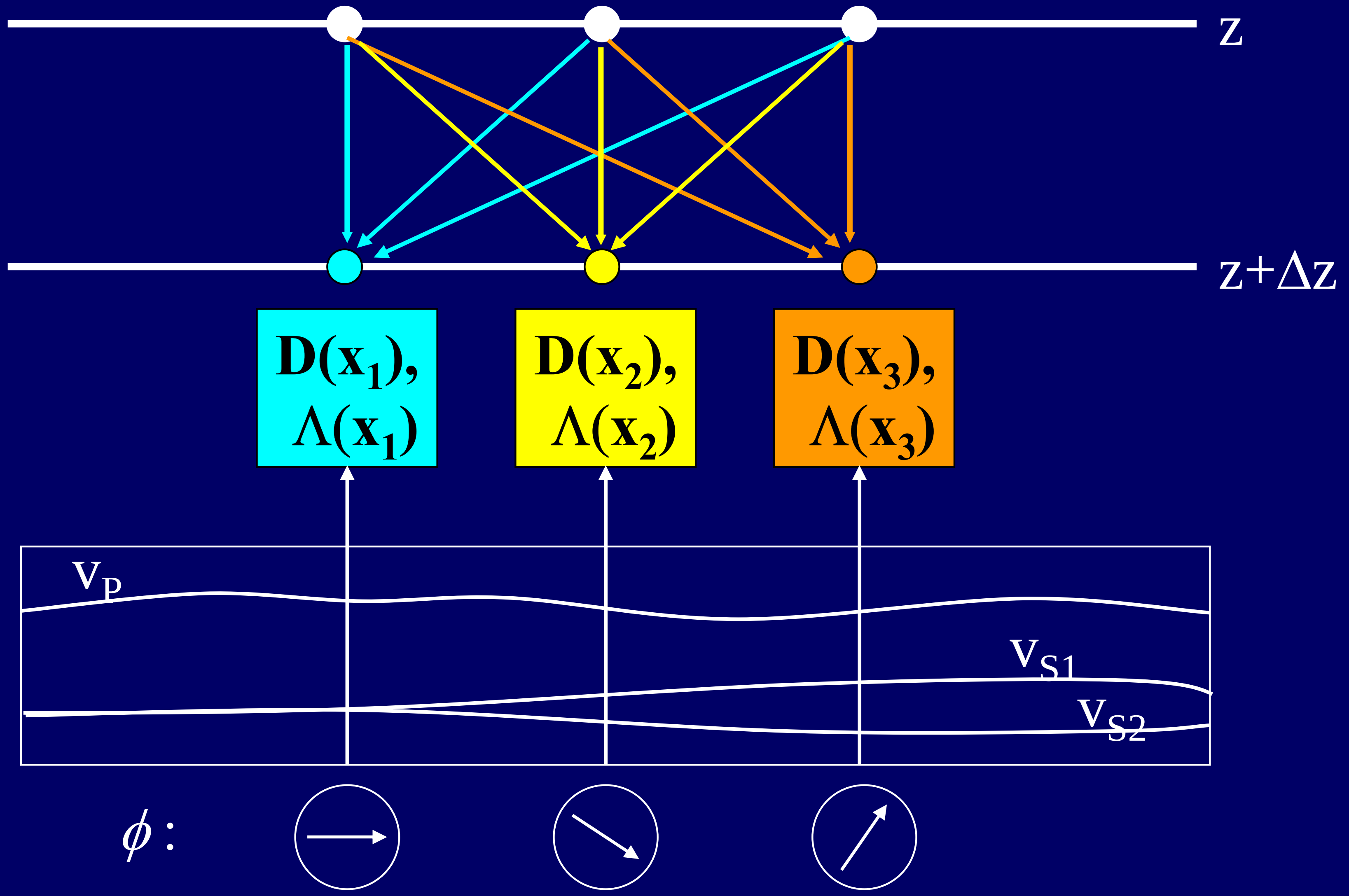
Lateral Dependence

$$\mathbf{b}_{PSPI}\left(x, z_{n+1}, \omega\right) = \frac{\omega}{2\pi}\int_{-\infty}^{\infty}\mathbf{D}_n\left(x, p\right)\mathbf{E}_n\left(x, p, \omega\right) \times \mathbf{D}_n^{-1}\left(x, p\right)\mathbf{b}\left(p, z_n, \omega\right)e^{-i\omega px}dp$$

Fourier Transform: $$\mathbf{b}\left(p, z_n, \omega\right) = \int_{-\infty}^{\infty}\mathbf{b}\left(x, z_n, \omega\right)e^{i\omega px}dx$$

Extrapolation: $$\mathbf{E}_n\left(x, p, \omega\right) = \exp\left[i\omega\mathbf{\Lambda}_n\left(x, p\right)\left(z_{n+1} - z_n\right)\right]$$

PSPI Elastic Extrapolation



Spatial Interpolation: PSPI

Standard PSPI

- Extrapolate with N reference velocities
- Interpolate based on actual velocity at each spatial position

- Isotropic elastic case: dependence on V_P and V_S is (almost) separable

  $\Rightarrow$ cost $\propto N_{Vp} + N_{Vs}$ OK

- HTI elastic case: non-separable dependence on 6 parameters

  $\Rightarrow$ cost $\propto (N_{Vp}N_{Vs})(N_\varepsilon N_\delta N_\gamma)N_\phi$ BAD!

Spatial Interpolation: PSPAW

"Phase shift plus adaptive windowing"

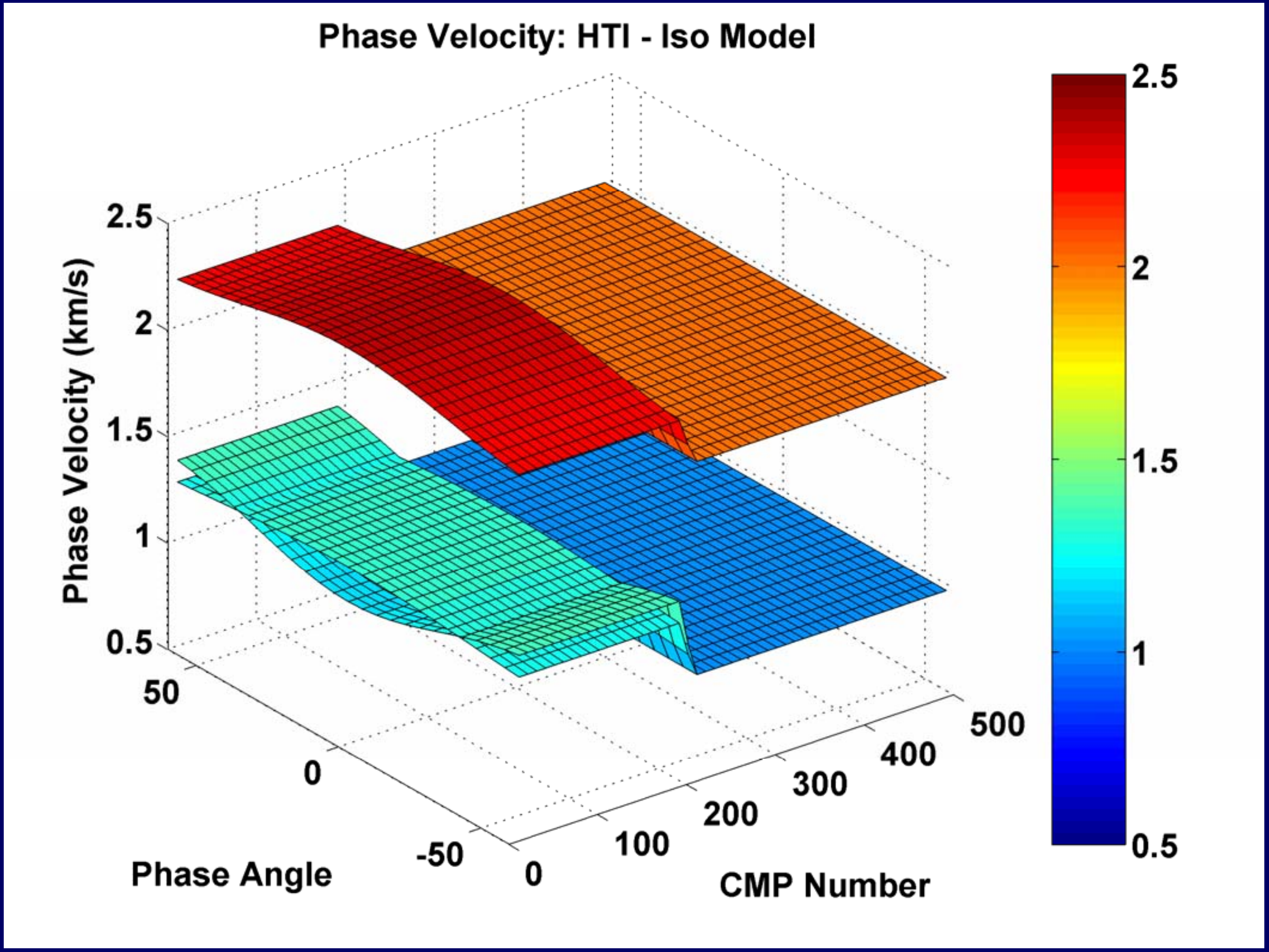
- Windows ("molecules") constructed from elementary small windows ("atoms")

  c.f. Scalar adaptive method (Grossman et al., 2002)

1. Compute phase slowness for P, S1, S2 modes as a function of lateral position and phase angle
2. For each molecule, atom acceptance based on:
   - Maximum phase error over slownesses
   - Maximum variation of HTI symmetry axis
3. Begin new molecule if either criteria are violated

$\Rightarrow$ Cost $\propto$ # Windows (usually OK)

Adaptive Windowing:



Phase slowness for HTI to Isotropic transition model

Imaging Condition

Forward extrapolated source wavefield:

$$\mathbf{w}_D = \left(w_P^D \quad w_{S1}^D \quad w_{S2}^D \right)^T$$

Backward extrapolated receiver wavefield:

$$\mathbf{v}_U = \left(v_P^U \quad v_{S1}^U \quad v_{S2}^U \right)^T$$

P-wave S1-wave S2-wave

P-P Image

$$I_{P-P} = \int \frac{\overline{w_P^D} v_P^U}{\left| w_P^D \right|^2} d\omega$$

P-S1 Image

$$I_{P-S1} = \int \frac{\overline{w_P^D} v_{S1}^U}{\left| w_P^D \right|^2} d\omega$$

etc. …

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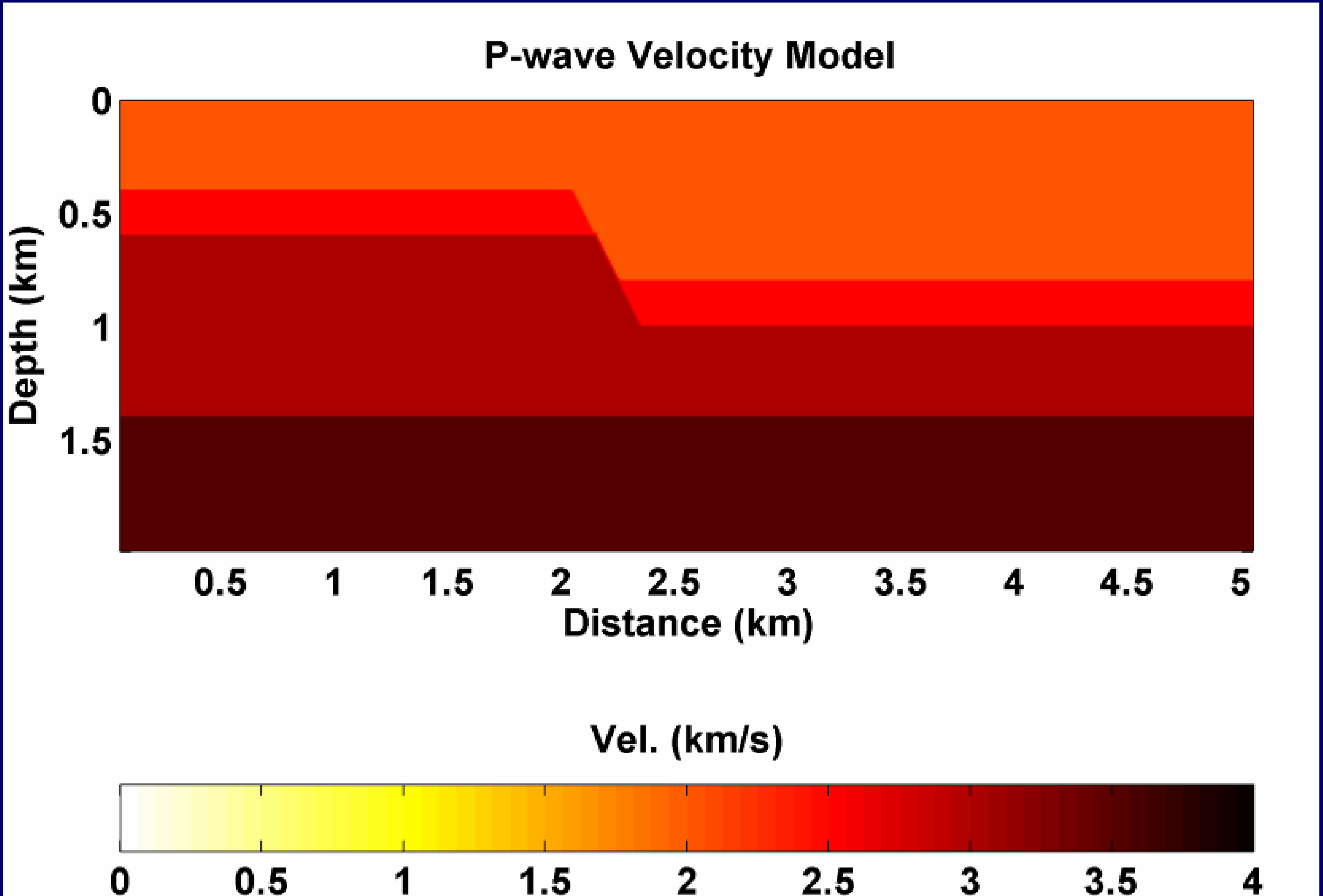
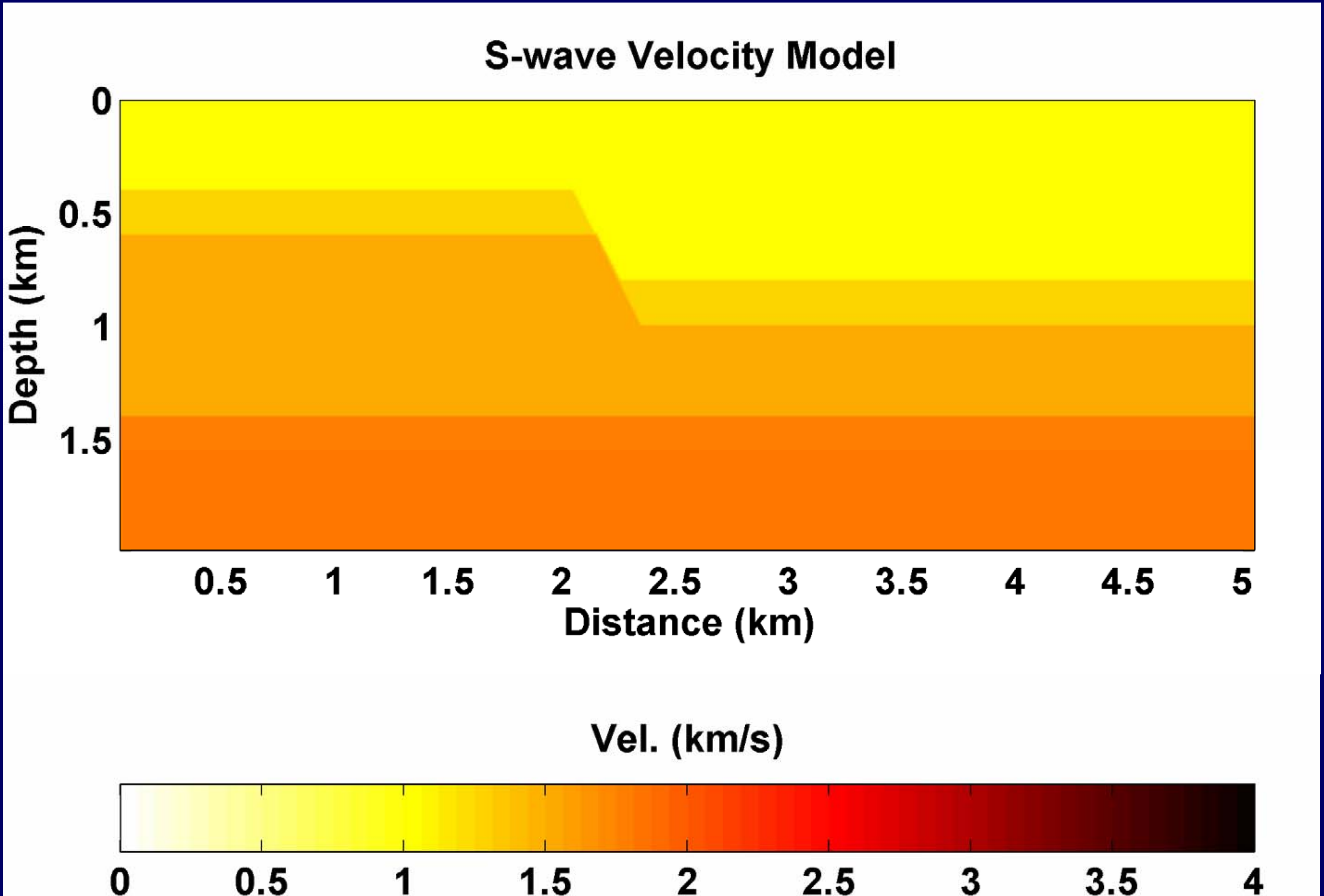
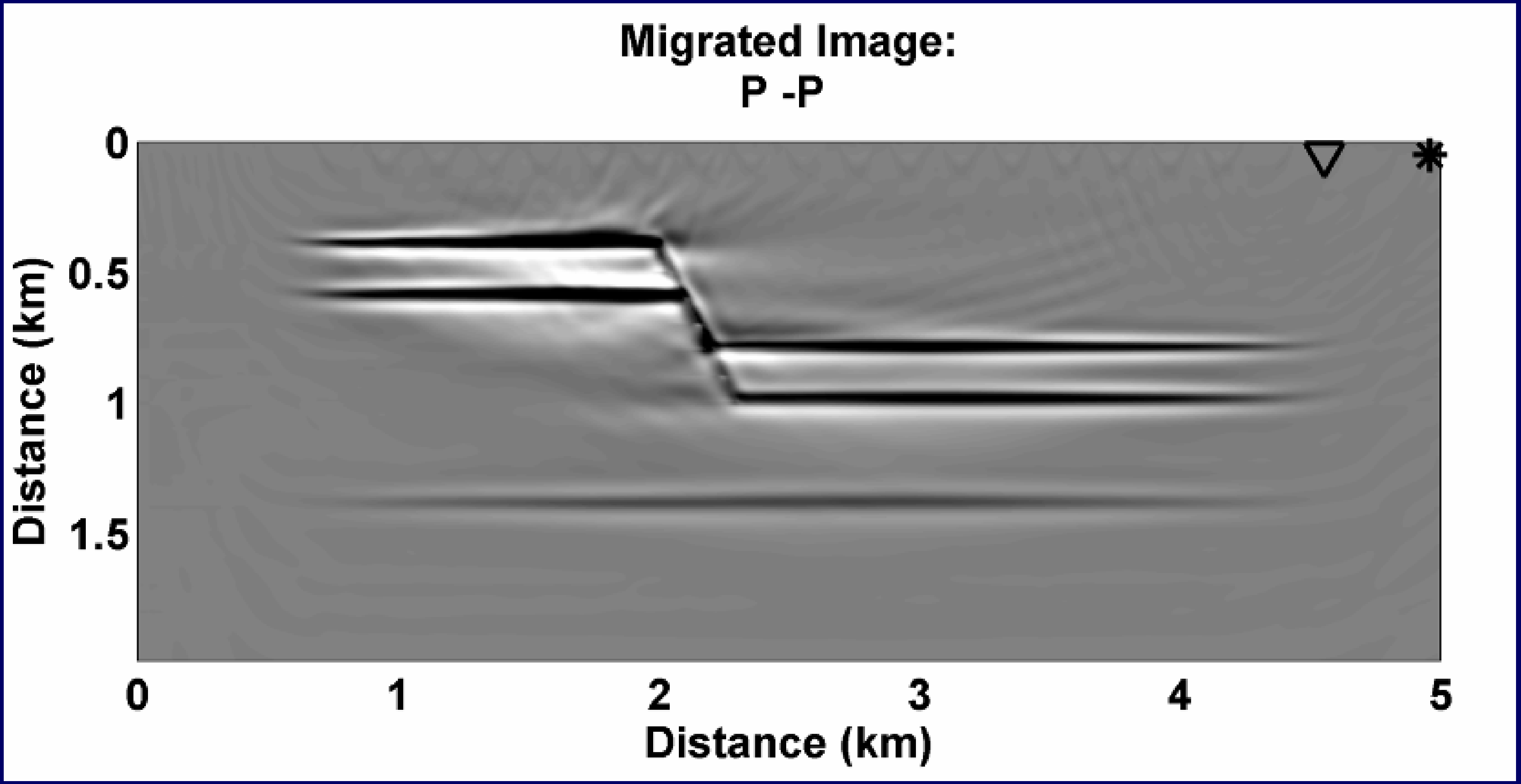
Isotropic Model



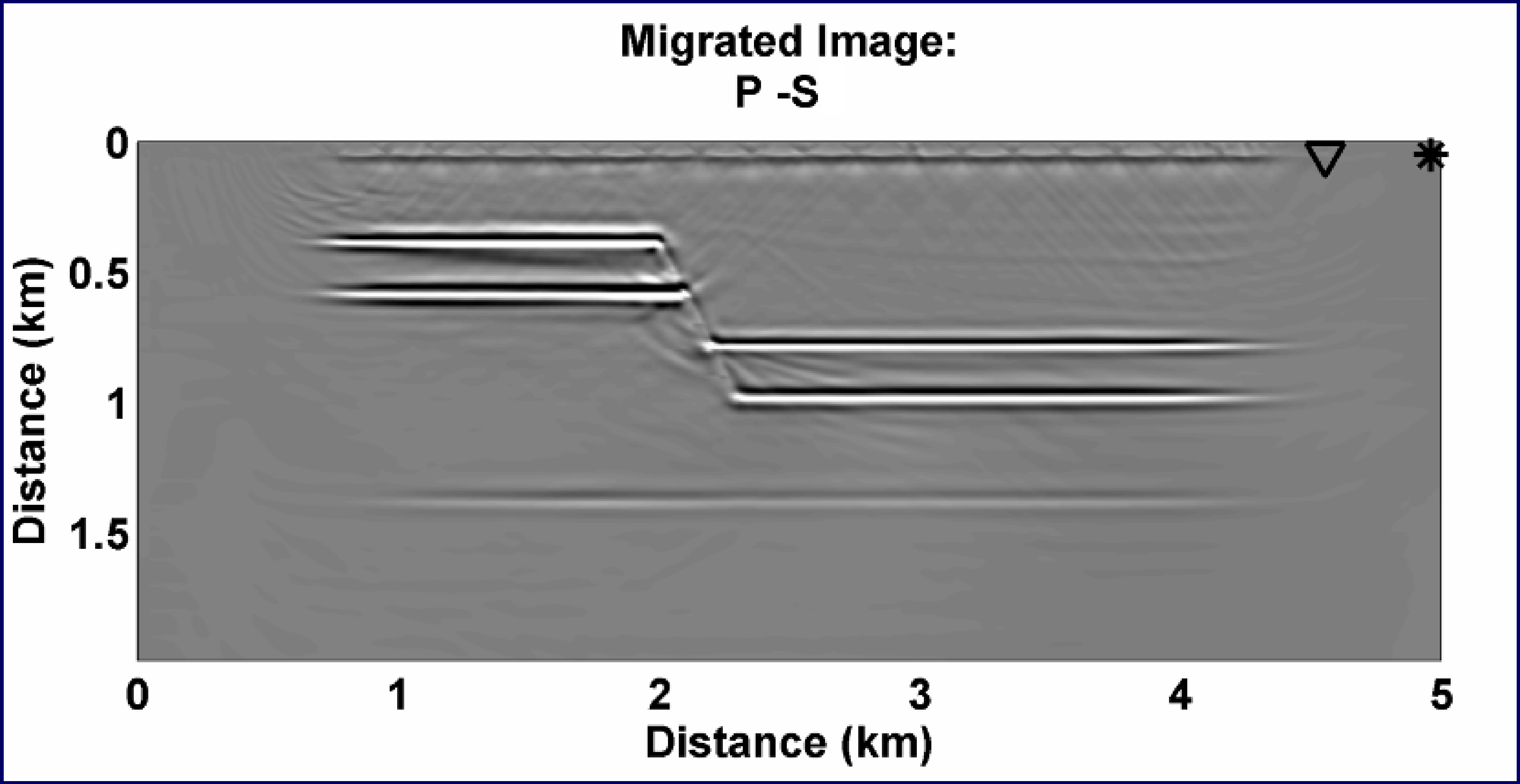
Isotropic Model



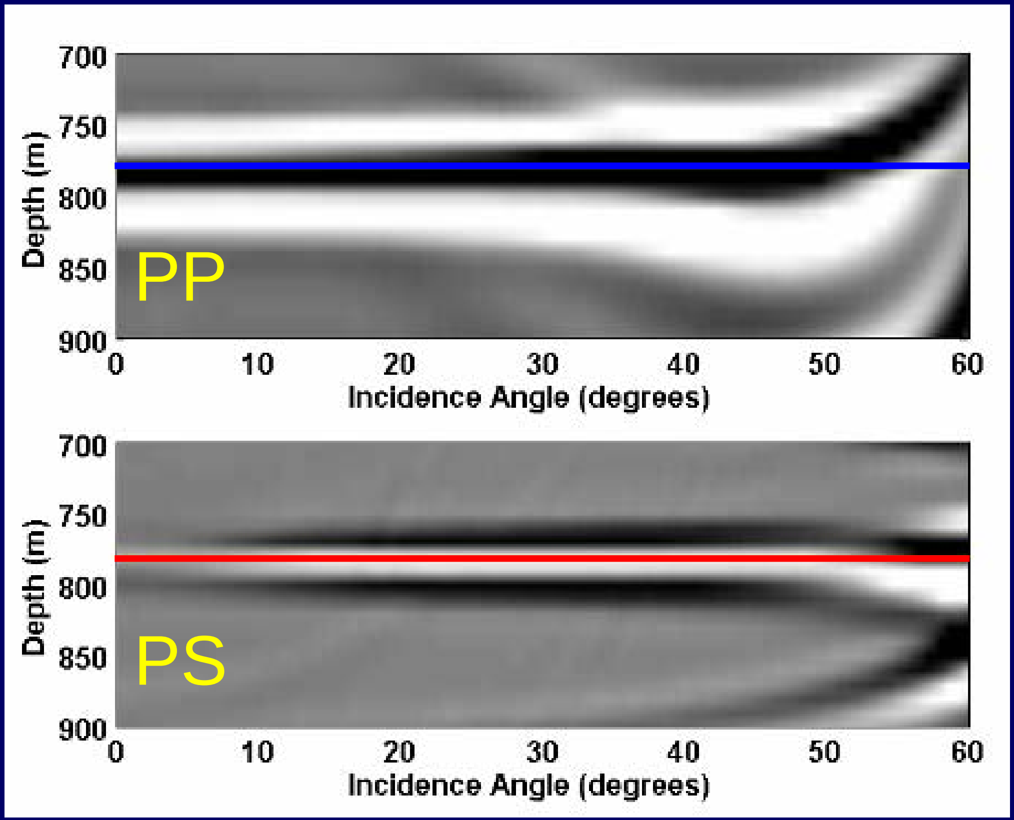
Isotropic Data P-P Image (PSPAW)

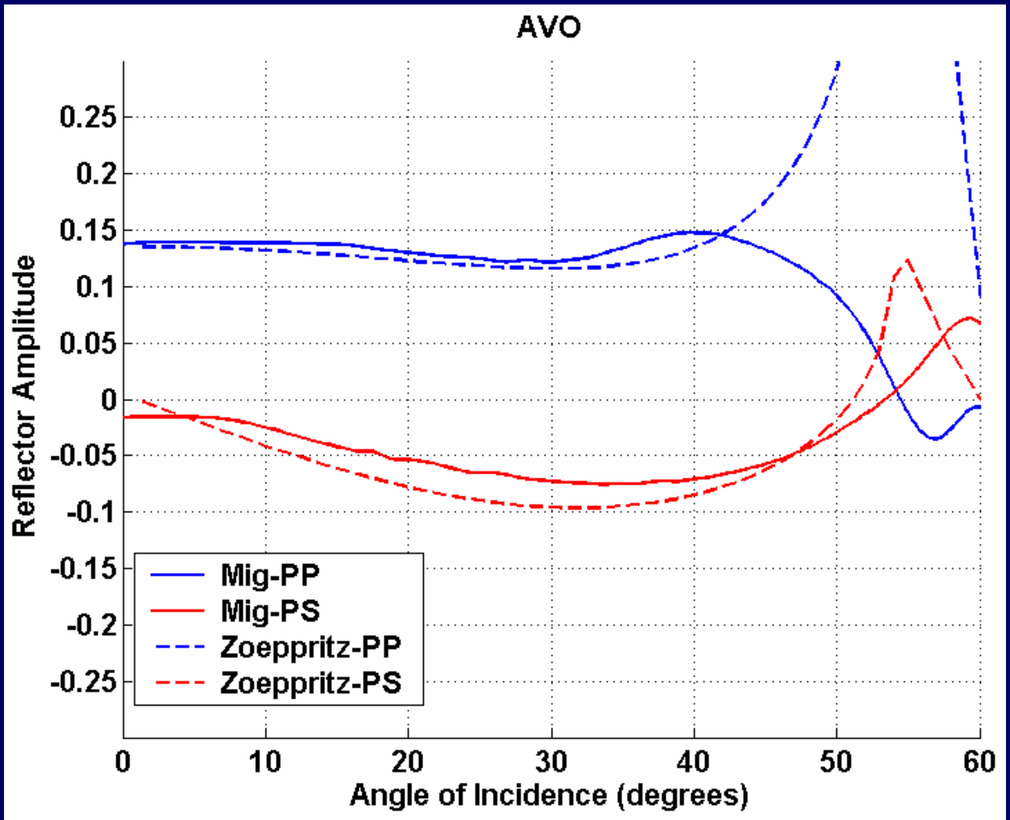


Isotropic Data P-S Image (PSPAW)

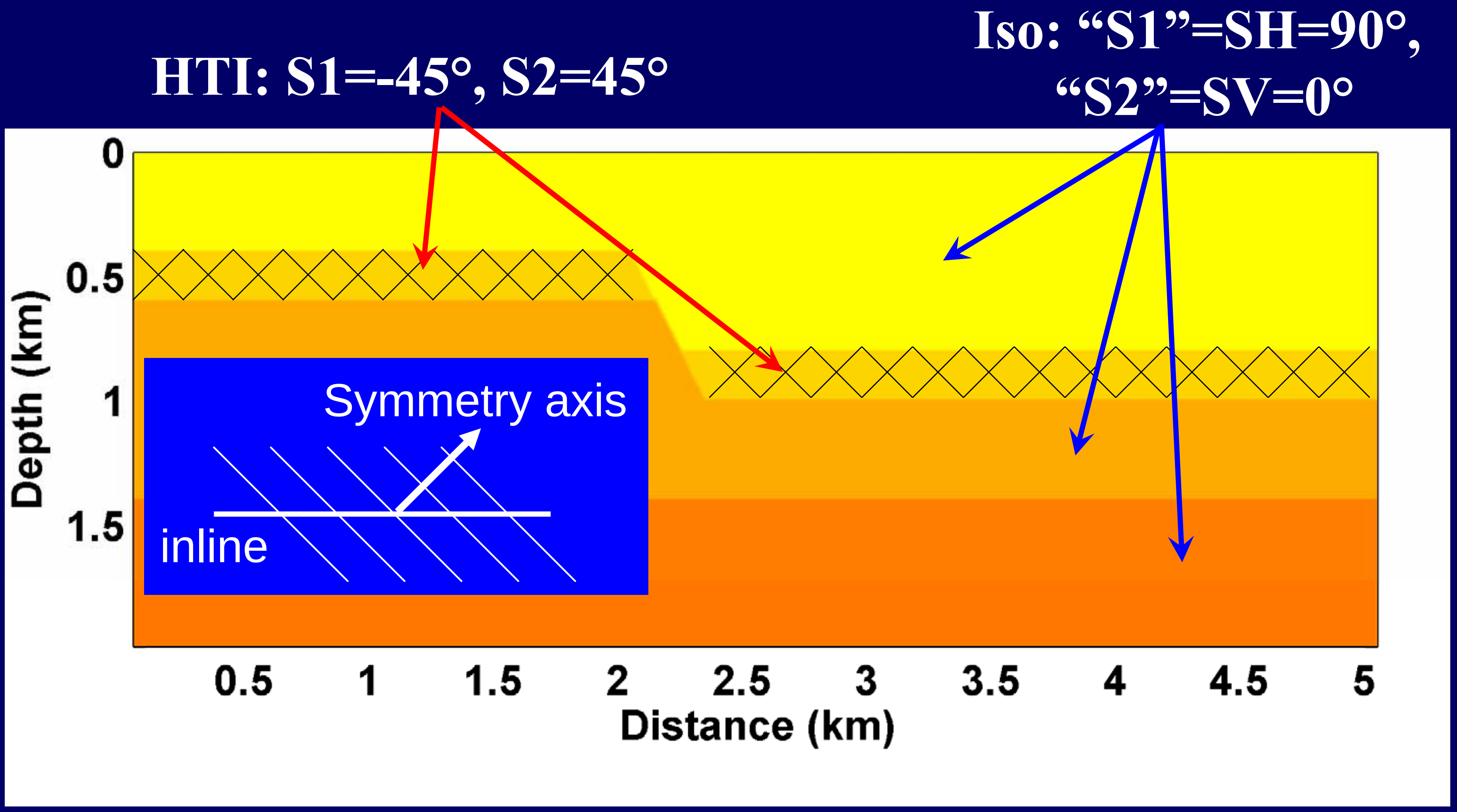


AVO on Flat Reflector from Migration of Single Shot



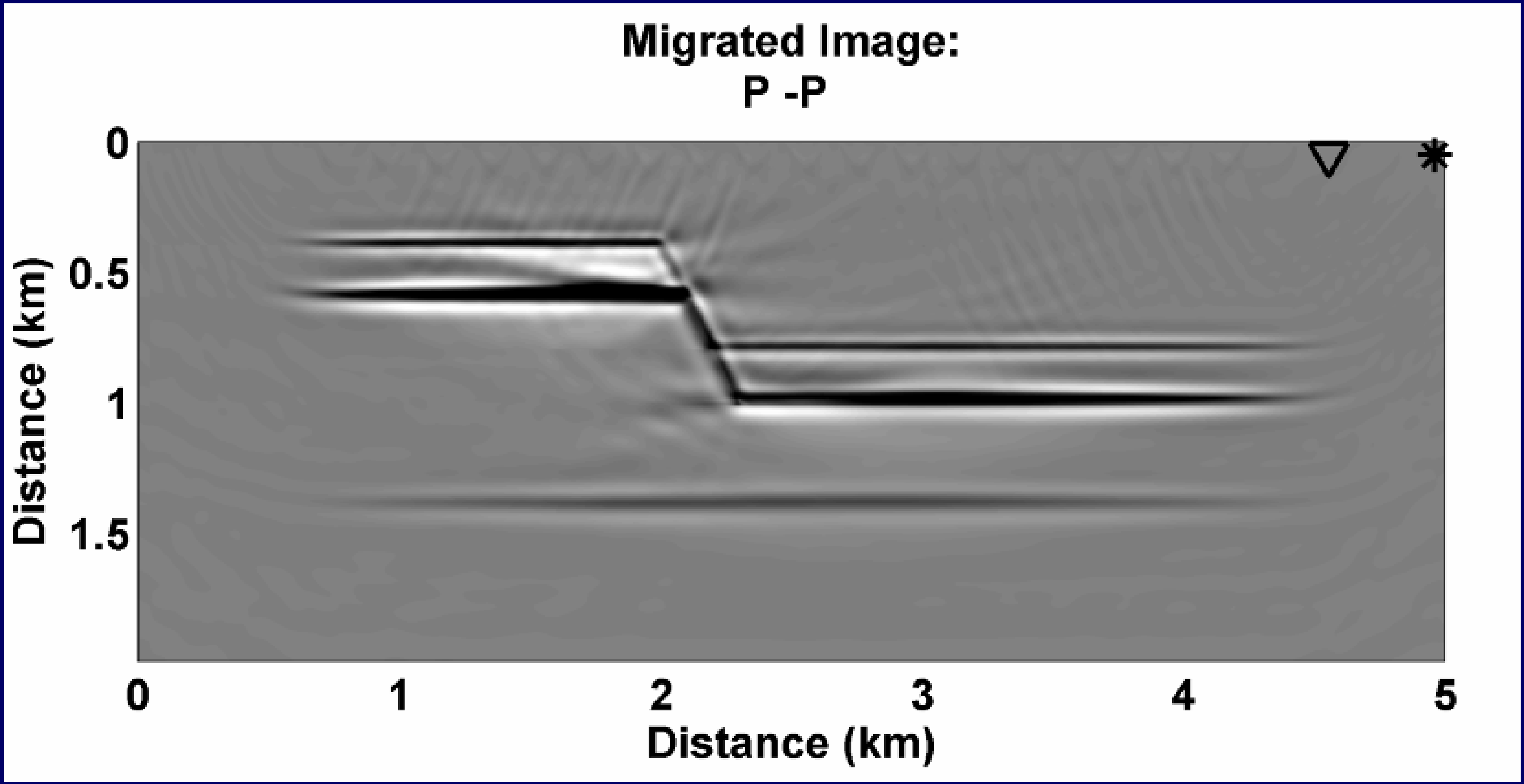


HTI Model

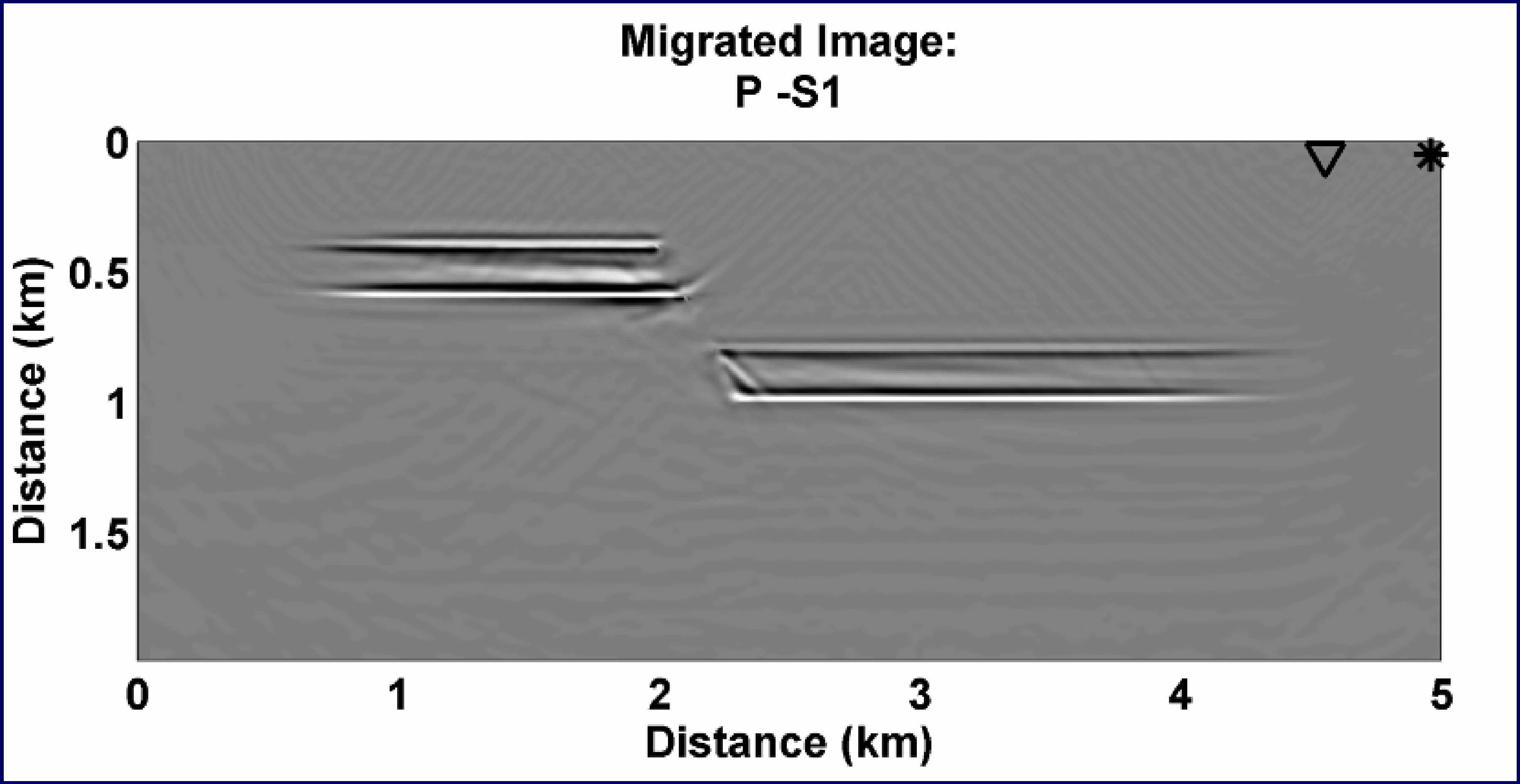


NOTE: In following images, we (arbitrarily) assign SH mode to S1, and SV to S2, for isotropic layers.

HTI Data P-P Image (PSPAW)

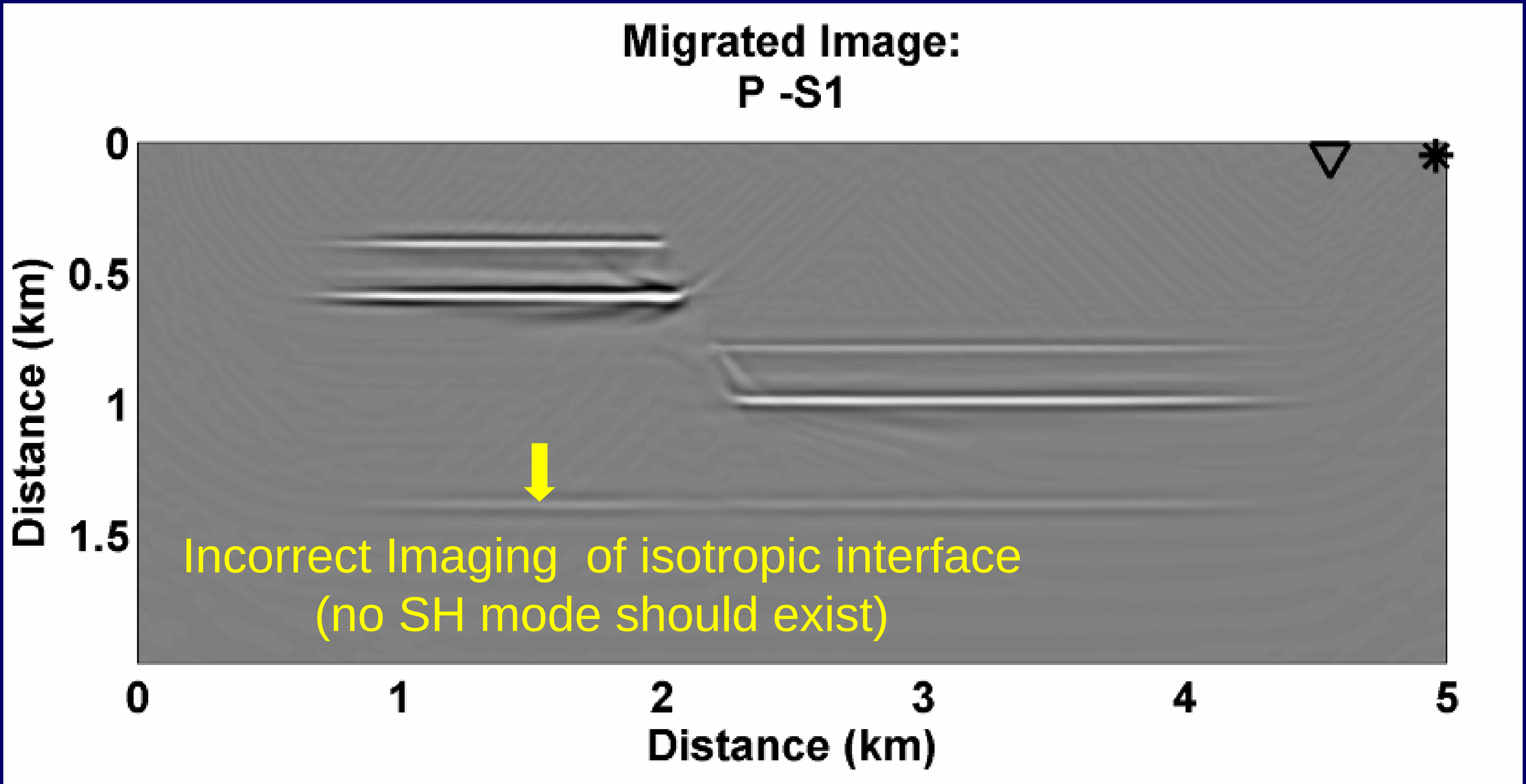


HTI Data P-S1 Image (PSPAW)



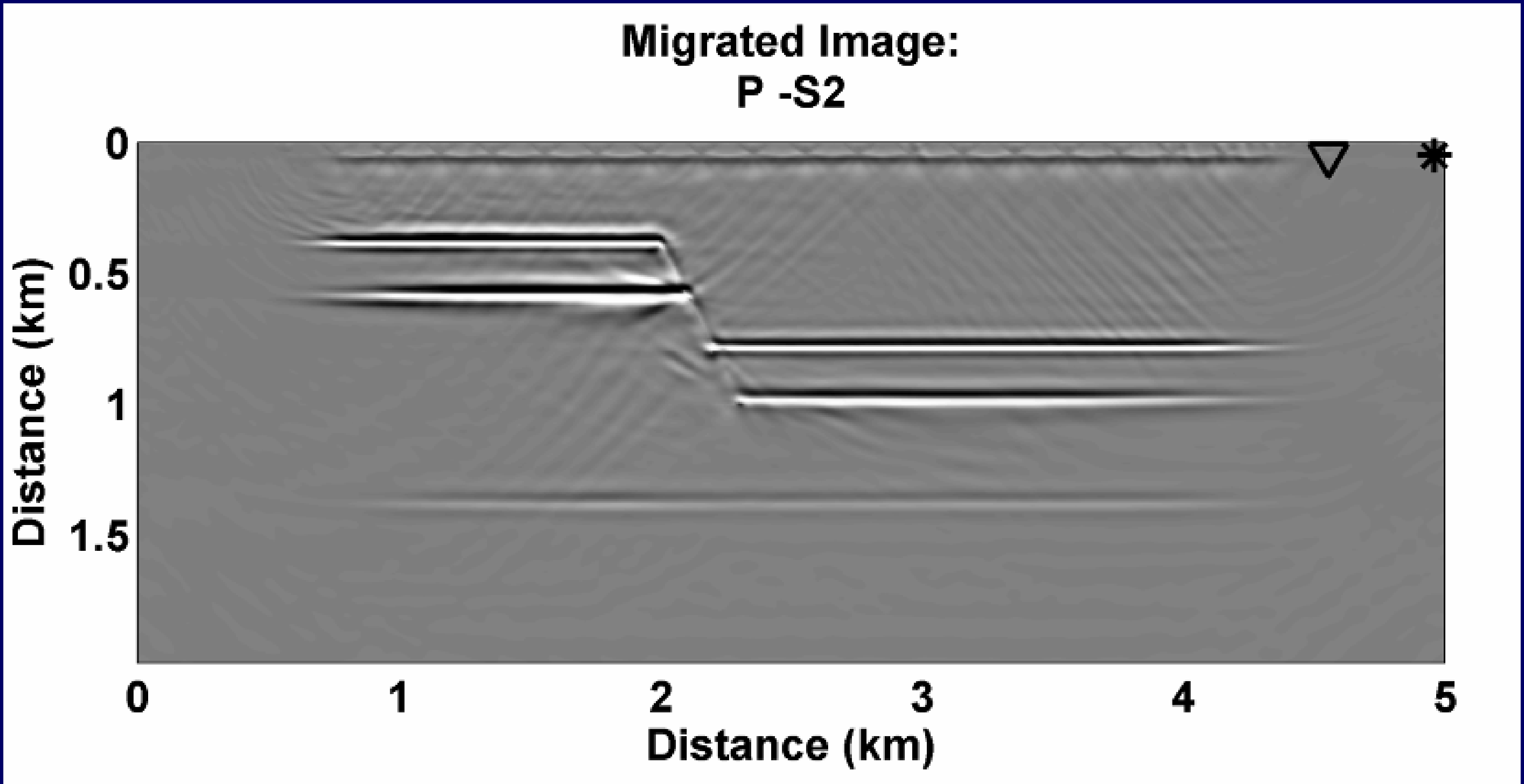
Migrated with true HTI model

HTI Data P-S1 Image (PSPAW)



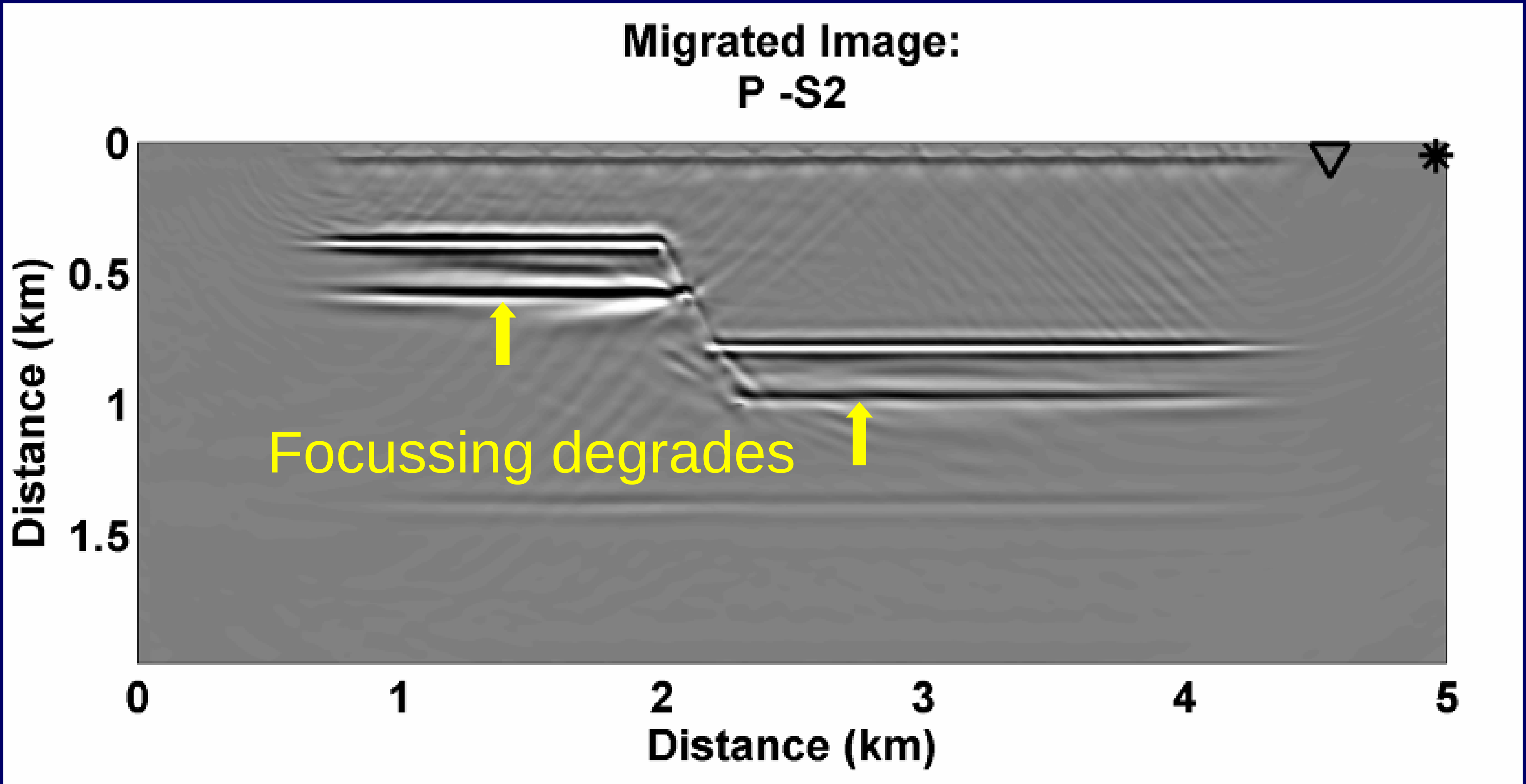
Migrated with isotropic model

HTI Model P-S2 Image (PSPAW)



Migrated with true HTI model

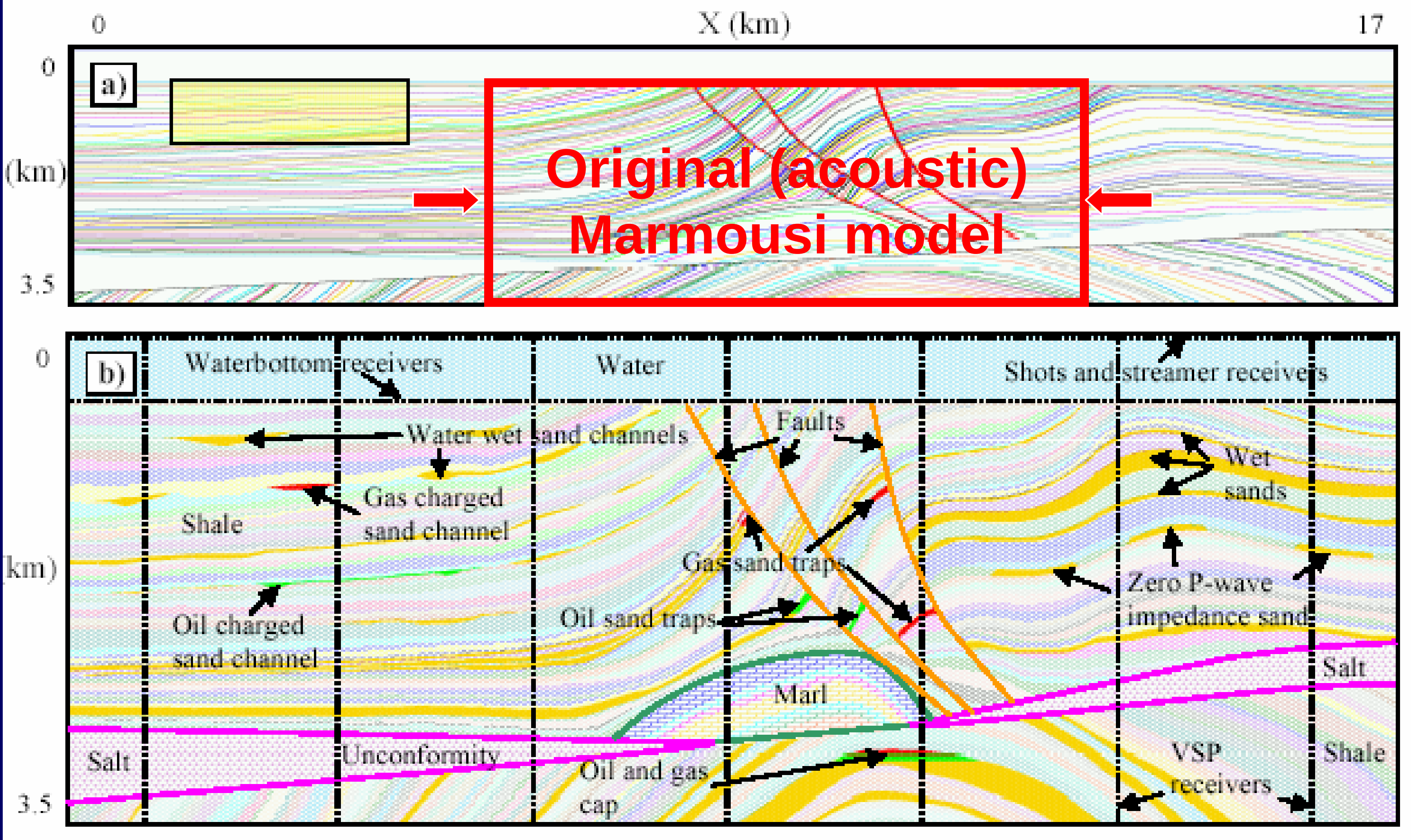
HTI Data P-S2 Image (PSPAW)



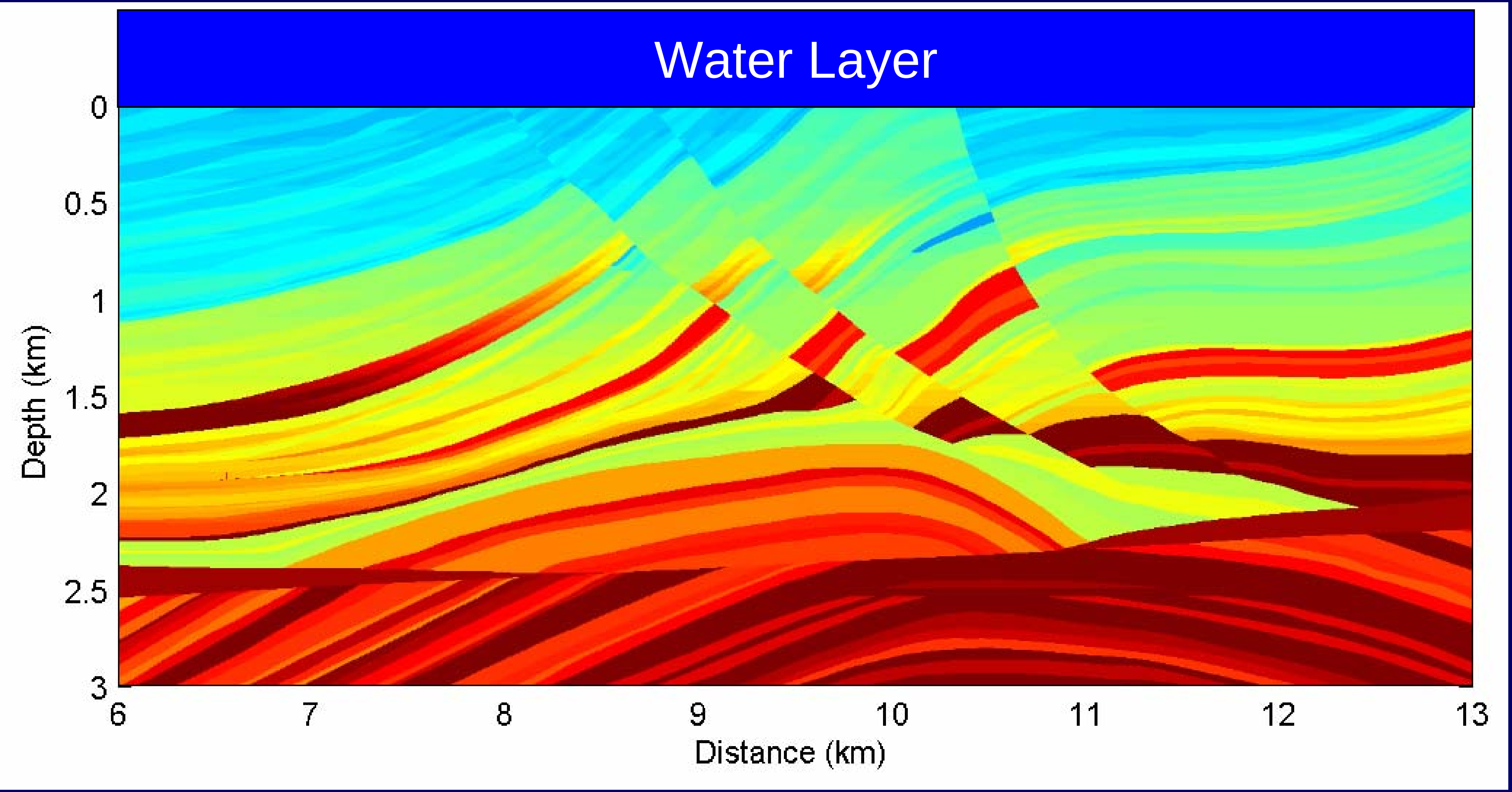
Migrated with isotropic model

The Marmousi-2 Elastic OBC Model

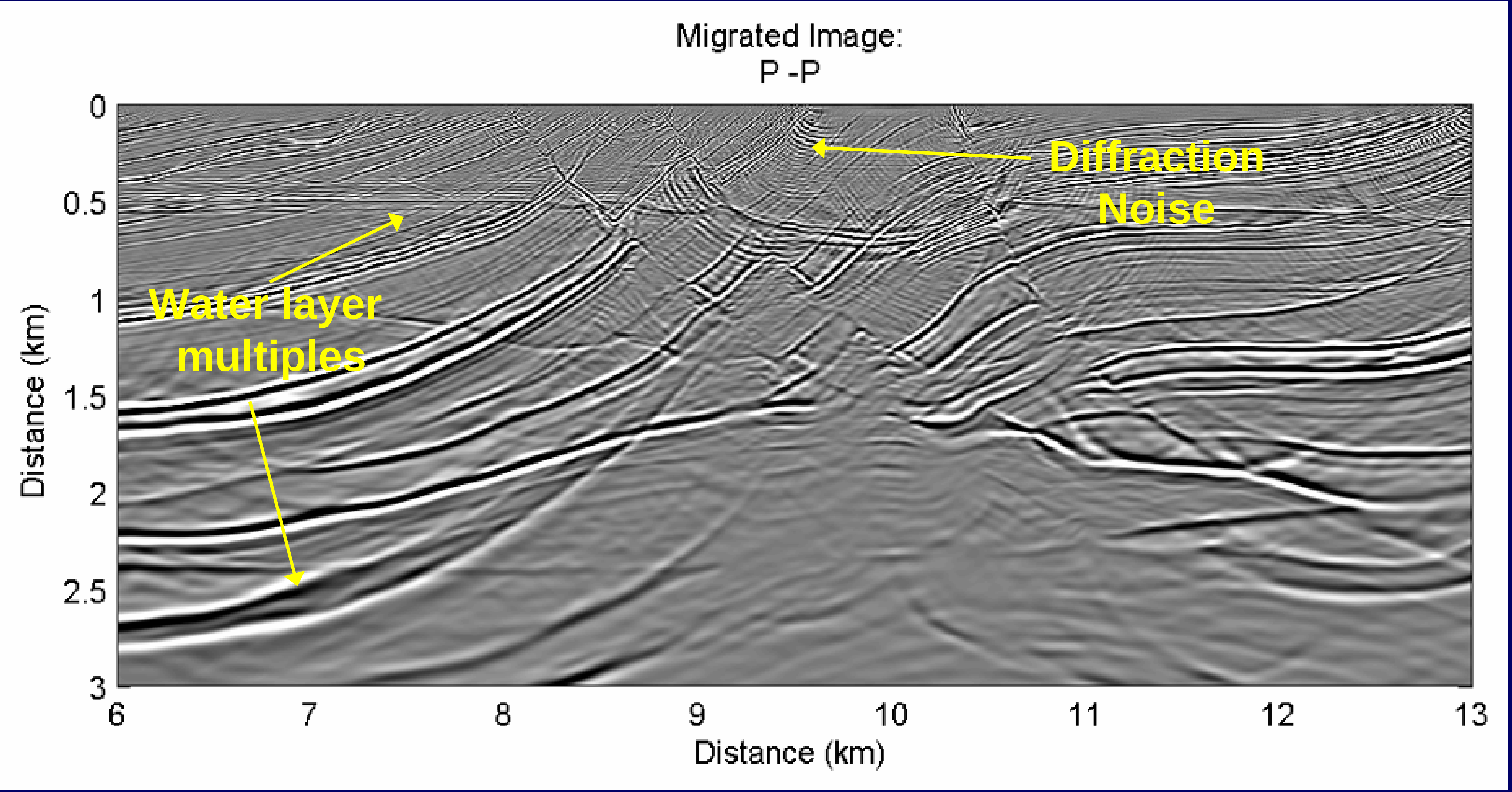
From Martin, Marfurt and Larsen, "Marmousi-2: an updated model for the investigation of AVO in structurally complex areas", SEG 2002



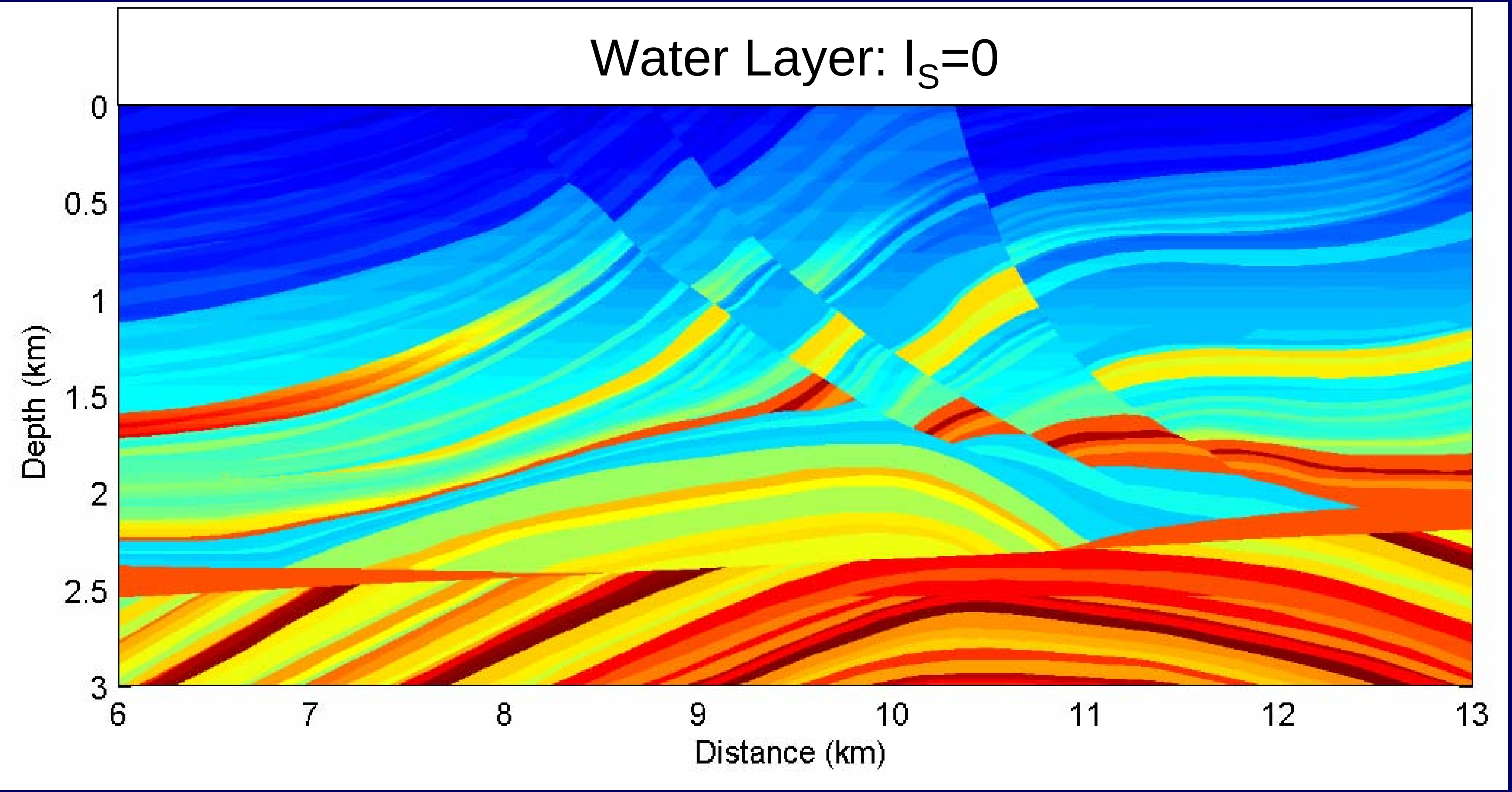
Marmousi-2 Mid-section: P-Impedance



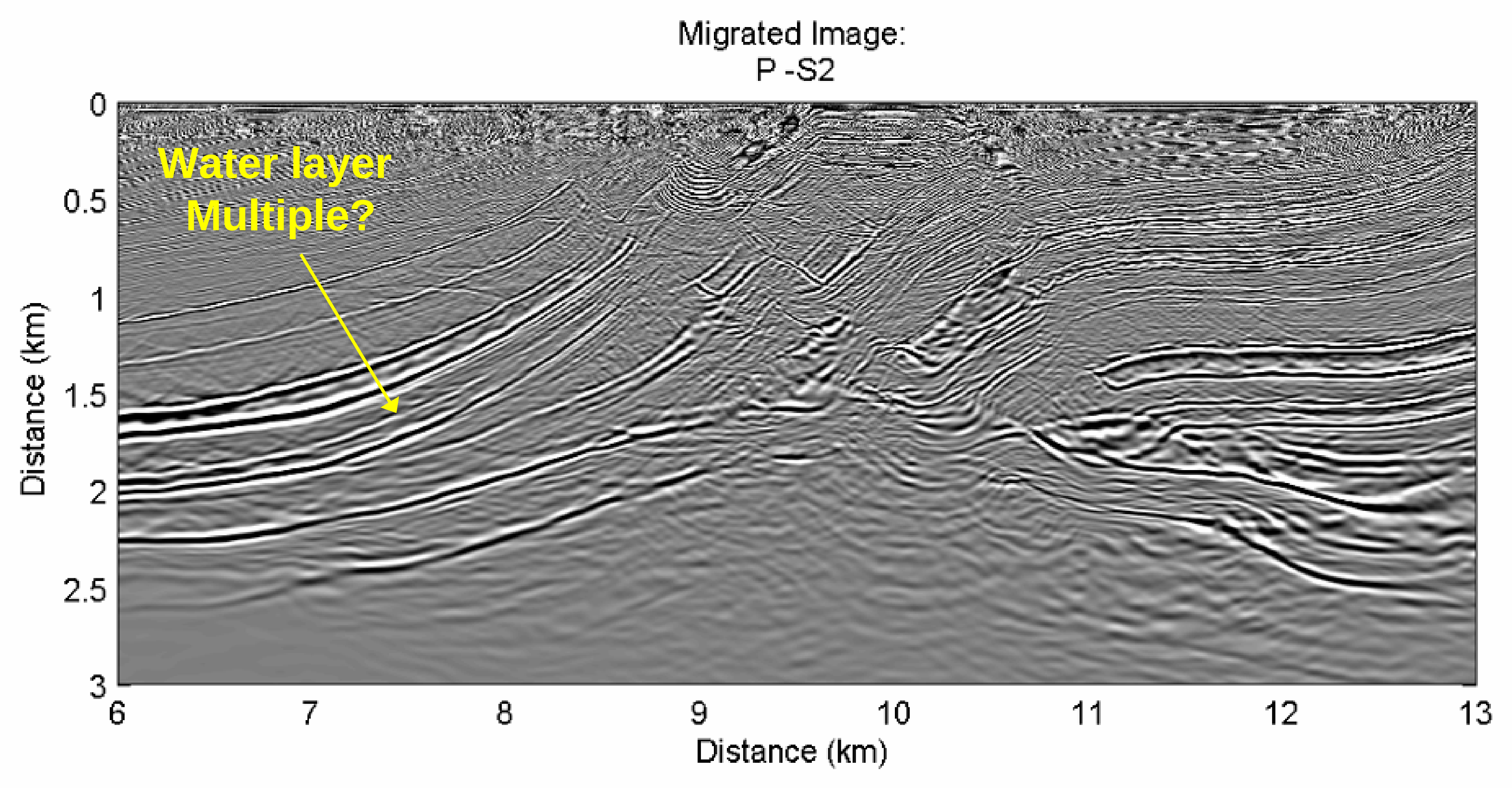
Marmousi -2 Mid-Section: PP Image (PSPI)



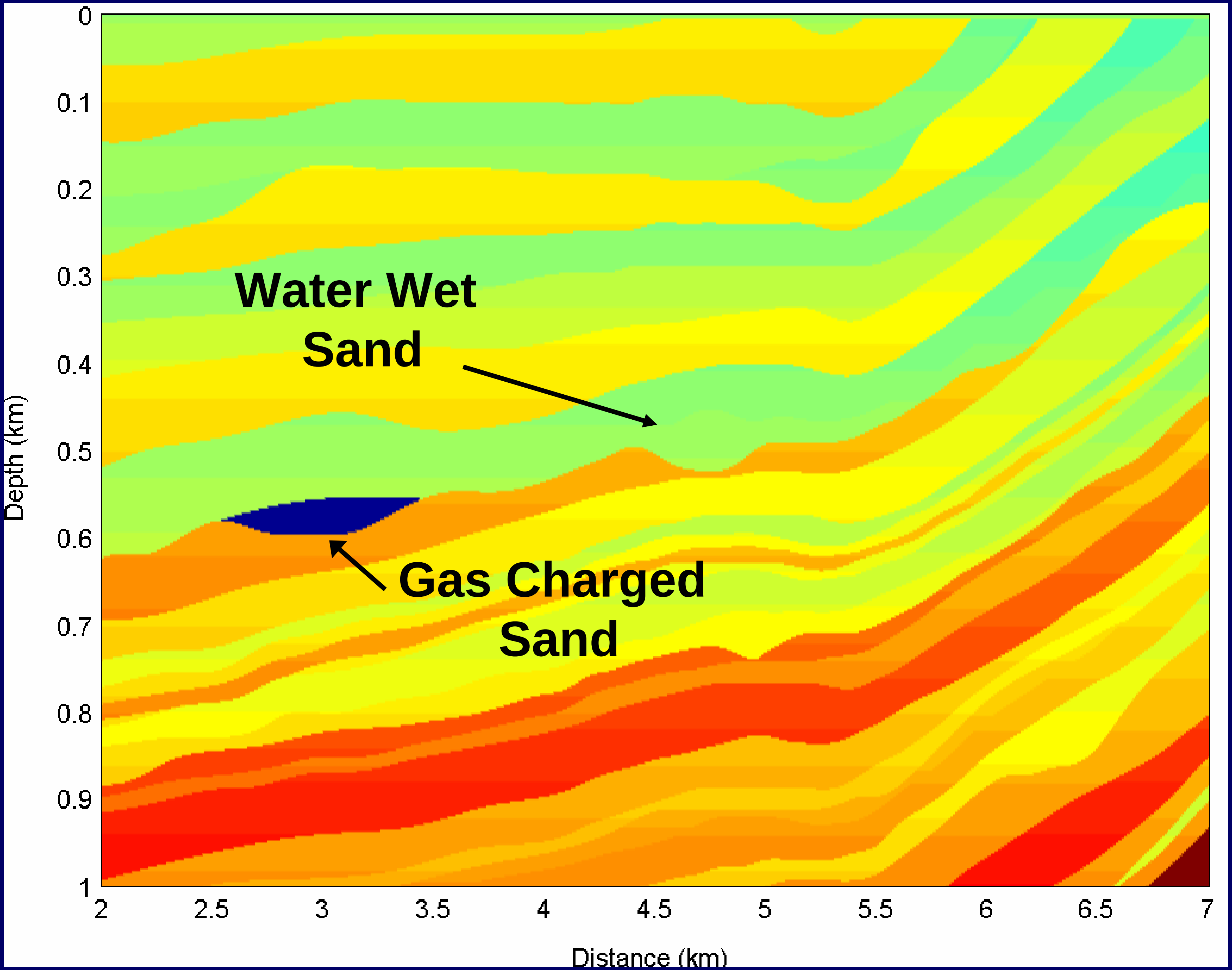
Marmousi-2 Mid-section: S-Impedance



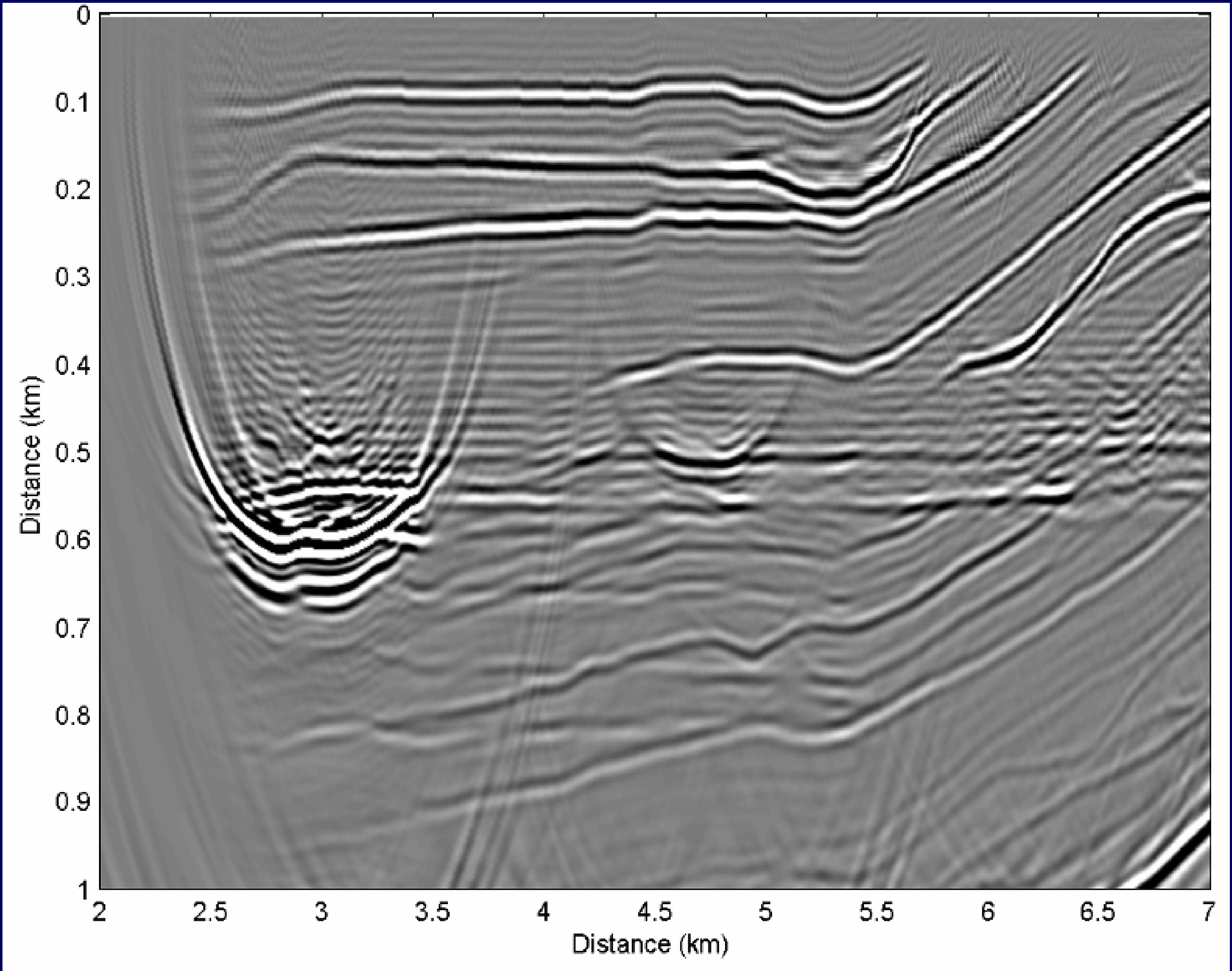
Marmousi -2 Mid-Section: PS Image (PSPI)



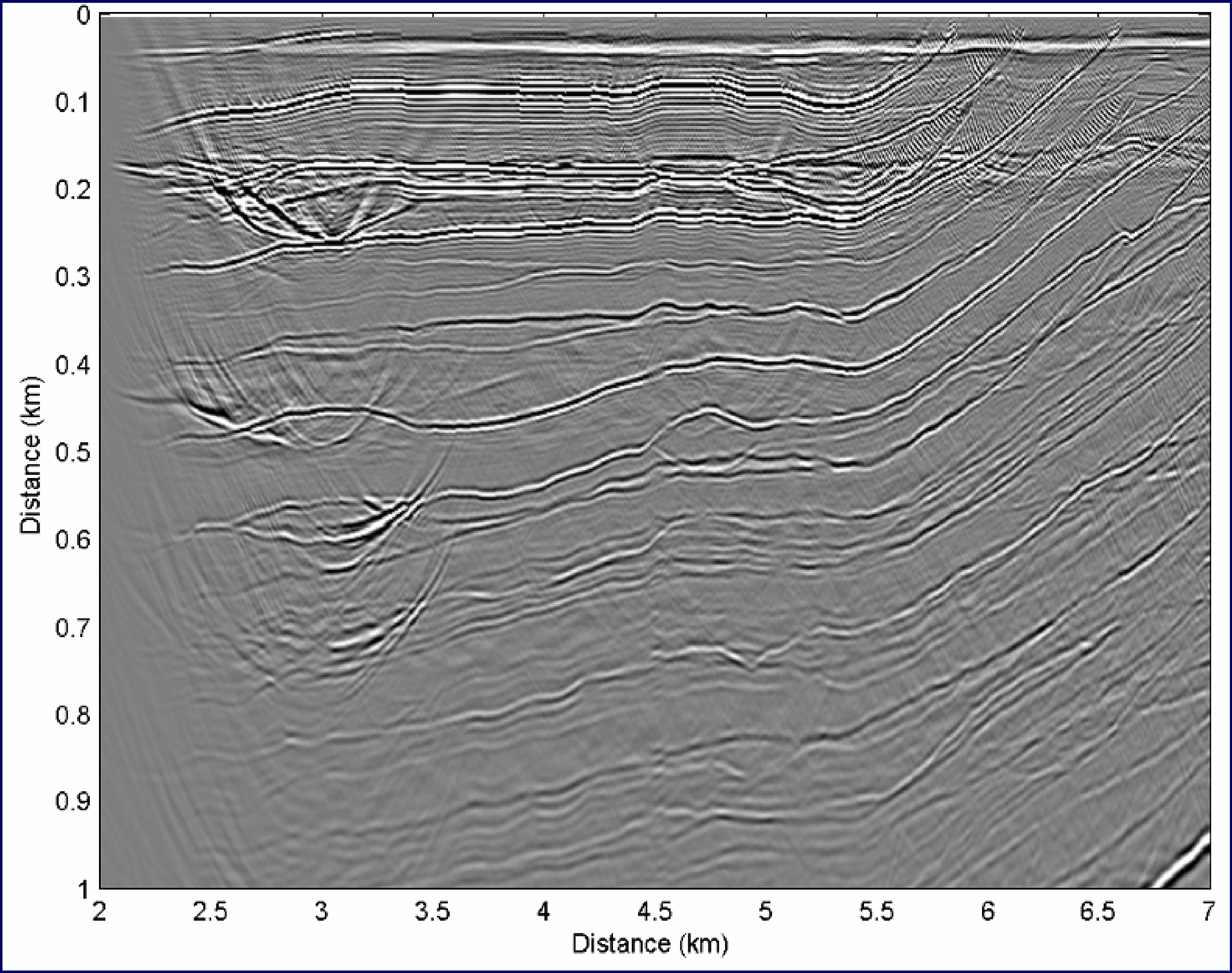
Marmousi-2 Shallow: I_P



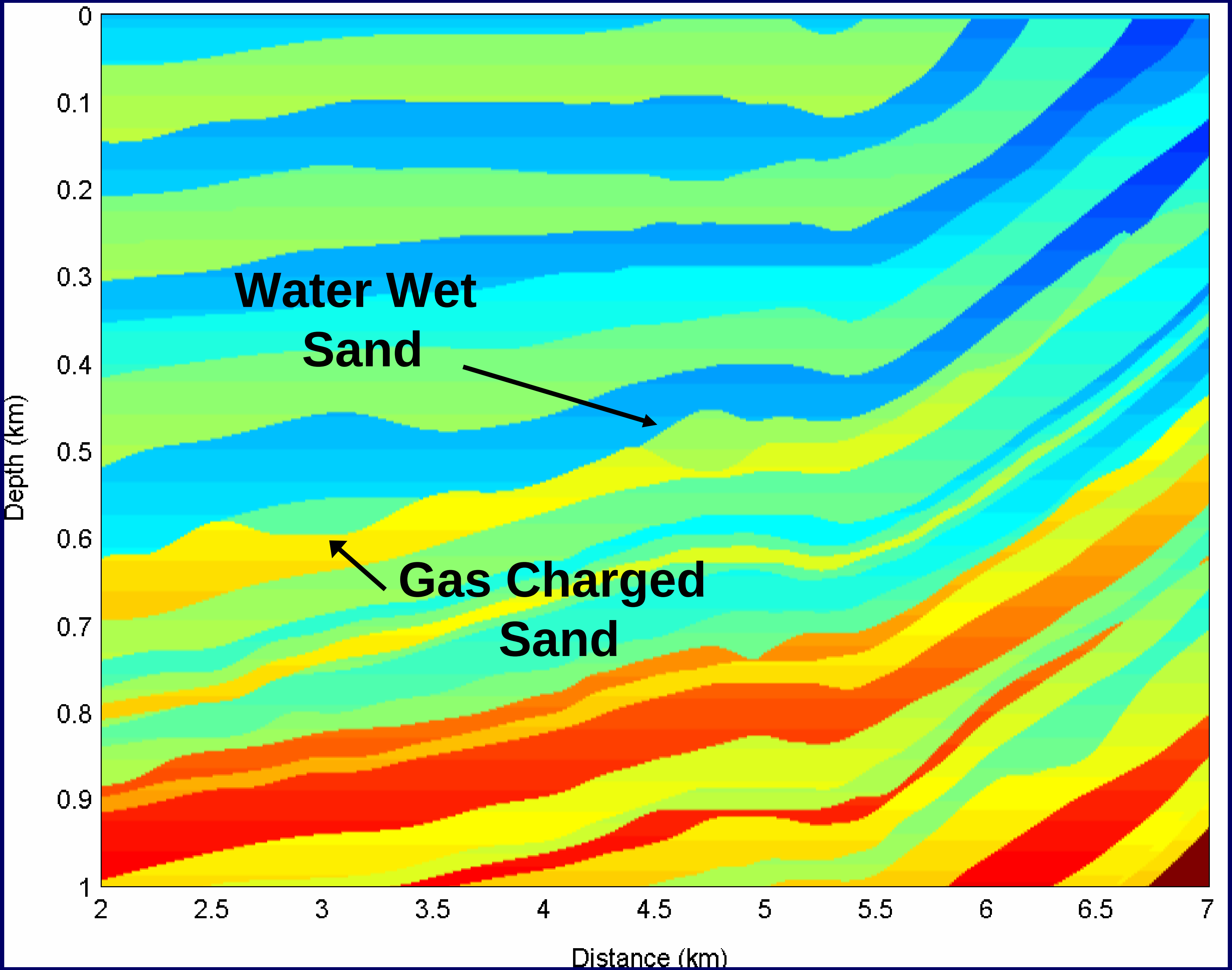
Marmousi -2 Shallow: PP Image



Marmousi -2 Shallow: PS Image



Marmousi-2 Shallow: I_S



Conclusions

- Developed elastic wave-equation migration applicable to HTI anisotropy
- AVO response compares well to Zoeppritz for flat reflector under isotropic layer
- Two PSPI-type algorithms for spatial variations
  - “Standard” PSPI for isotropic cases
  - PSPAW for HTI
- HTI migration focuses S1 and S2 images - isotropic migration fails to
- Marmousi tests demonstrate:
  - Multiples and aliased noise are problematic
  - Imaging in structural area: PP better than PS
  - Shallow resolution of PS better than PP
  - Fluid lithology discrimination

Acknowledgements

- Sponsors of CREWES
- Sponsors of POTSI: Pseudodifferential Operator Theory and Seismic Imaging
  - MITACS: Mathematics of Information Technology and Complex Systems
  - PIMS: Pacific Institute of the Mathematical Sciences
  - NSERC: Natural Sciences and Engineering Research Council of Canada
- Allied Geophysical Laboratory, University of Houston for permission to use the Marmousi II data
  - Prof. Robert Wiley
  - Gary Martin (GX Technology)
- Kevin Hall, CREWES