Gabor deconvolution, Hilbert transform and Phase corrections

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CREWES

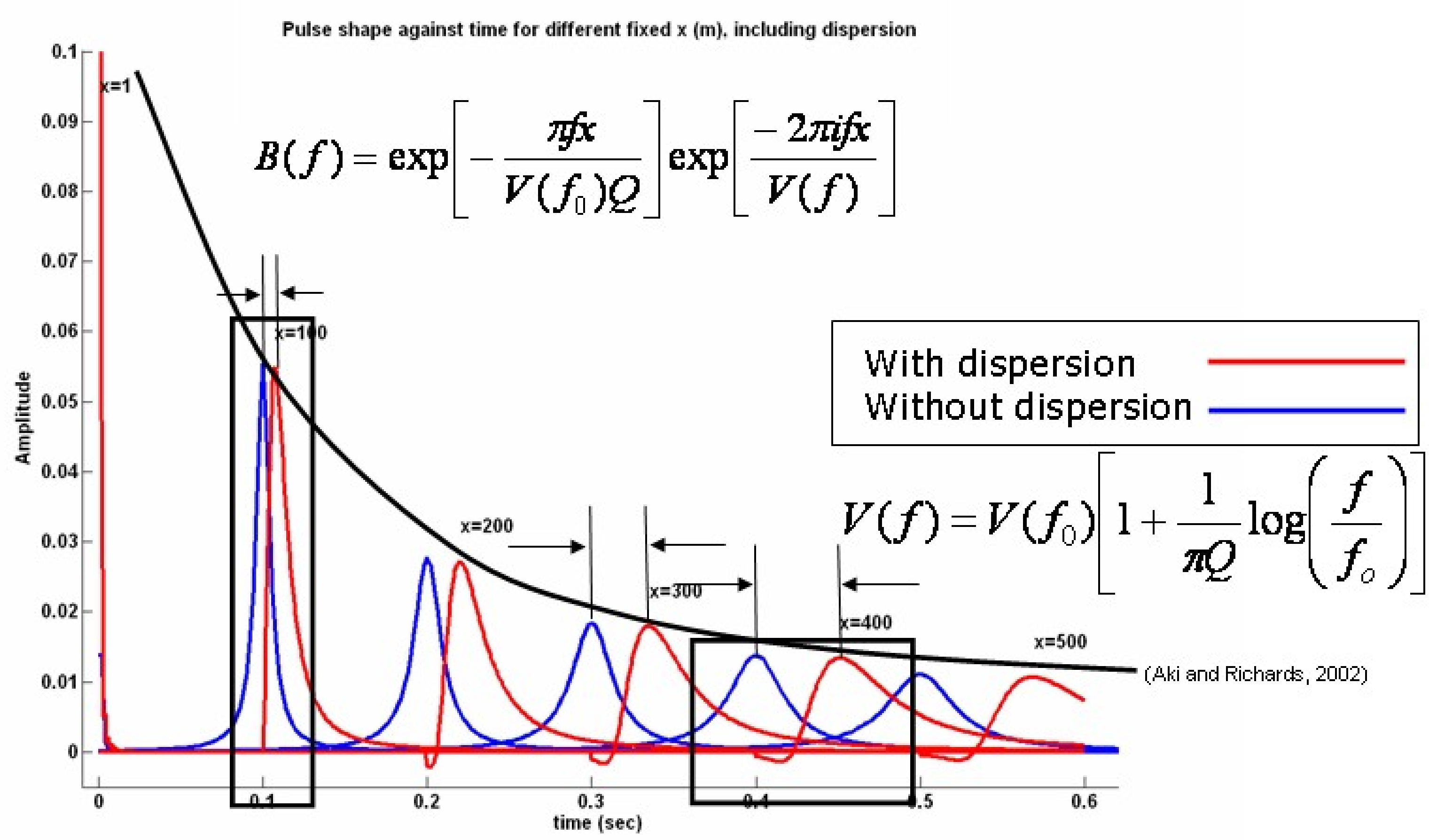
Overview

- Gabor deconvolution is a nonstationary extension of Wiener deconvolution.
- Minimum phase and Hilbert transform are fundamental concepts in the Gabor method.
- The estimation of the phase through the digital Hilbert transform can be improved by
  - Adding up a correction term to the digital Hilbert
  - Downsampling.

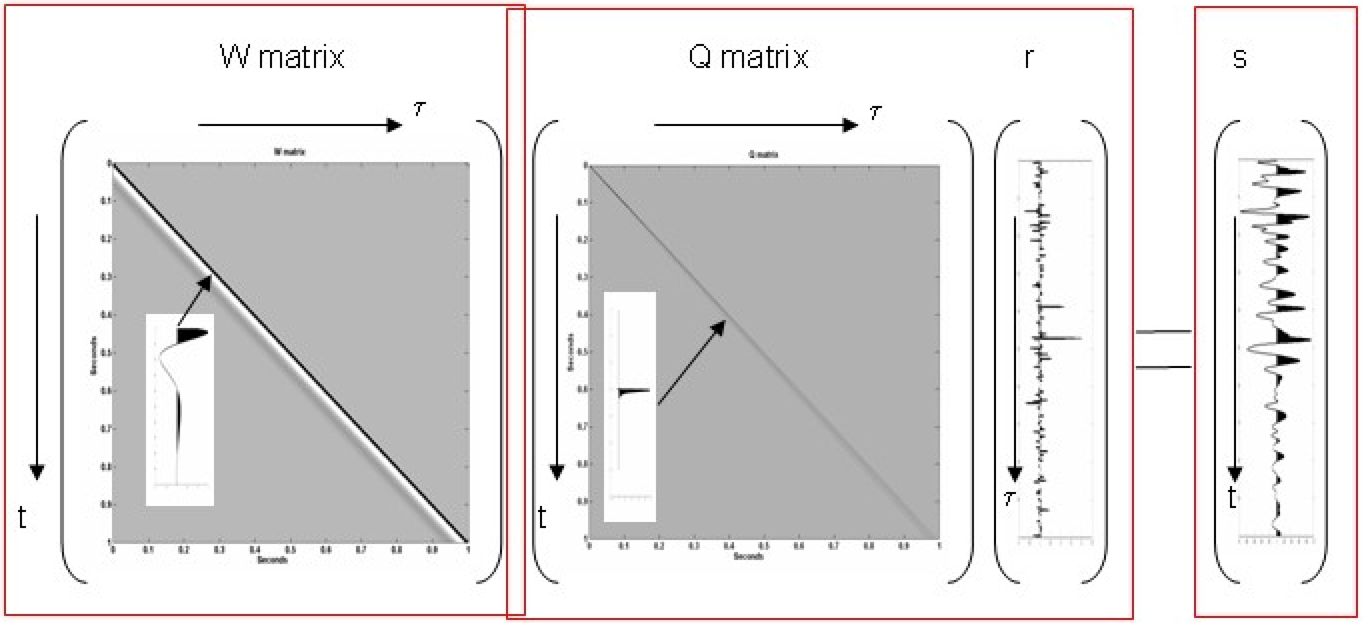
Outline

- Introduction: Gabor deconvolution
  - Constant Q theory
  - Nonstationary convolutional model
  - Gabor transform
- Minimum phase and Hilbert transform
- Phase correction in Gabor deconvolution
- Conclusions

Constant Q theory for attenuation



Nonstationary convolutional model



Fourier transform of trace

$$\hat{s}(f) = \hat{w}(f)\int_{-\infty}^{\infty} \alpha_Q(f,\tau)\, r(\tau)\, e^{-2\pi i f\tau}\, d\tau$$

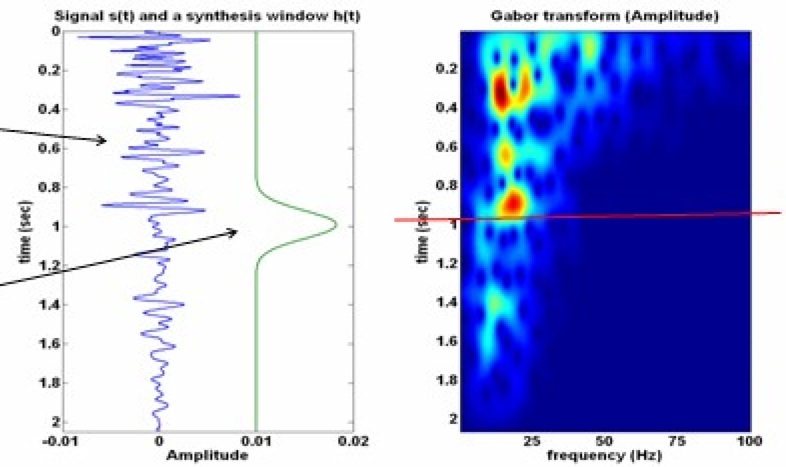
$$\alpha_Q(\omega,\tau) = \exp\left(-\omega\tau/2Q + iH(\omega\tau/2Q)\right)$$

Fourier transform of the minimum phase wavelet

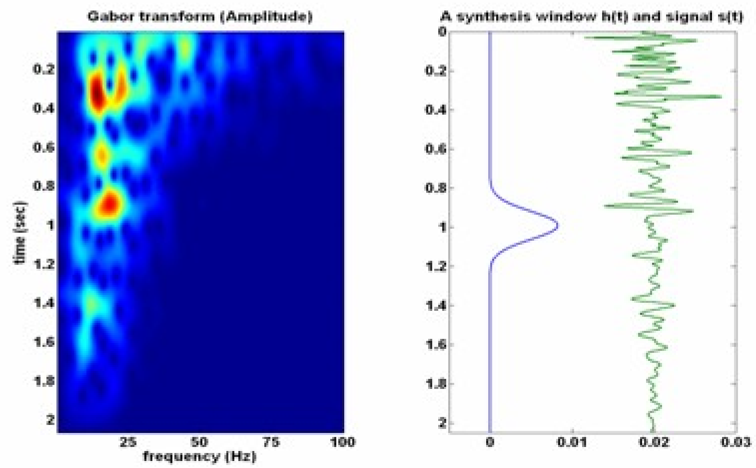
Pseudo differential operator

Gabor transform

$$G[s](\tau, f) = \int_{-\infty}^{\infty} s(t) g(t-\tau) e^{-2\pi i f t} dt$$



$$s(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G[s](\tau, f) \gamma(t-\tau) e^{2\pi i f t} df d\tau$$



Gabor deconvolution

- Factorization of the nonstationary convolutional model

$$\hat{s}(f) = \hat{w}(f)\int_{-\infty}^{\infty} \alpha_Q(f,\tau)\, r(\tau)\, e^{-2\pi i f \tau}\, d\tau$$

Fourier of the source wavelet

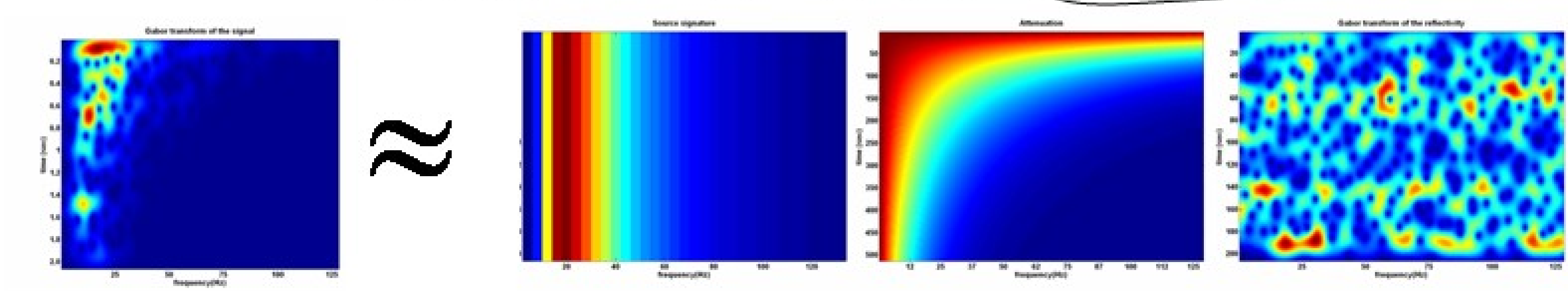
Attenuation function

Approximated factorization

Gabor of the trace

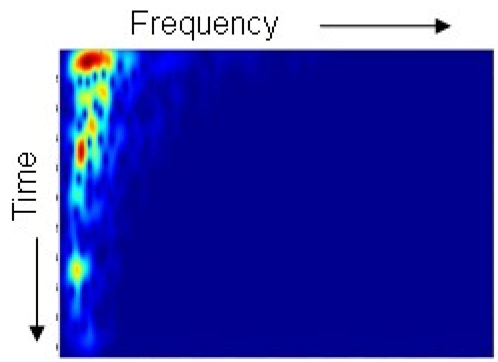
Gabor of the reflectivity

$$G[s](\tau,f) \approx \hat{w}(f)\,\alpha_Q(\tau,f)\,G[r](\tau,f)$$

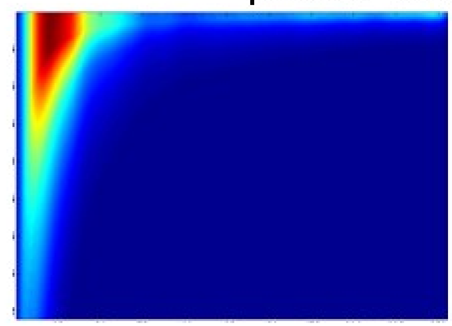


Gabor deconvolution

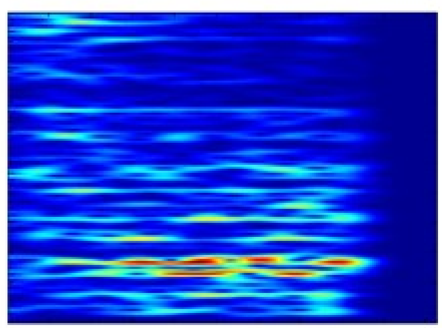
Gabor transform of the seismic trace



Deconvolutional operator:
- smoothing Gabor of trace
- phase through Hilbert transform

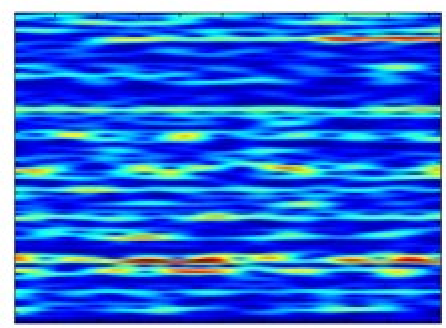


Source wavelet removal and compensation for attenuation are simultaneous



Estimate of Gabor of reflectivity:

$$\frac{\text{Gabor transform of the trace}}{\text{deconvolutional operator}}$$

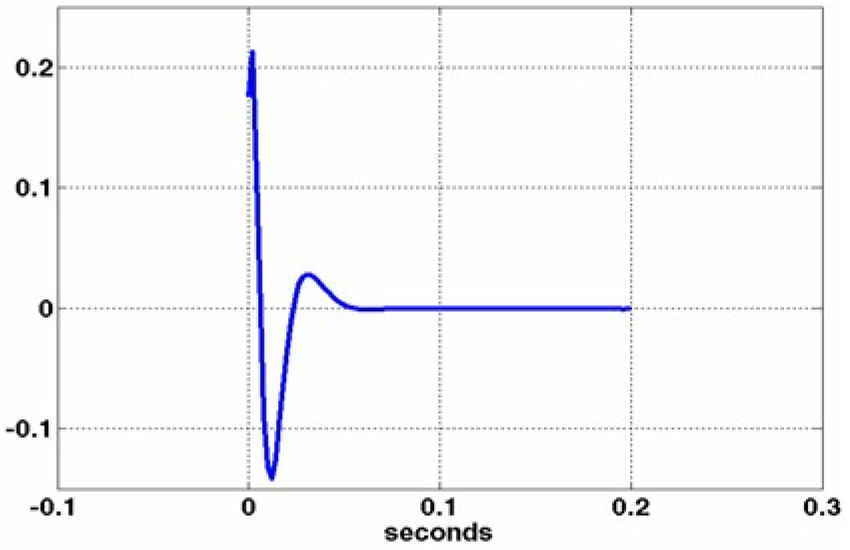


Expected Gabor of the trace

Minimum phase, linearity, causality and Hilbert transform.

- Minimum phase wavelet:
  - The wavelet with the minimum phase delay of all possible causal, invertible wavelets with the same amplitude spectrum
  - A wavelet whose Fourier phase spectrum is the Hilbert transform of the logarithm of its amplitude spectrum

Futterman (1962) showed that wave attenuation in a causal, linear theory is always minimum phase.



Analog Hilbert transform

$$\varphi_A(f) = \mathrm{H}[\alpha(f)] = \frac{f}{V(f)} - \frac{f}{V_\infty}$$

$$V(f) = V_\infty\left[1 + \frac{1}{\pi Q}\log\left(\frac{f}{f_\infty}\right)\right]$$

- Depends explicitly on the attenuation parameter Q

and the velocity *V∞* at a reference frequency *f∞*

Digital Hilbert transform

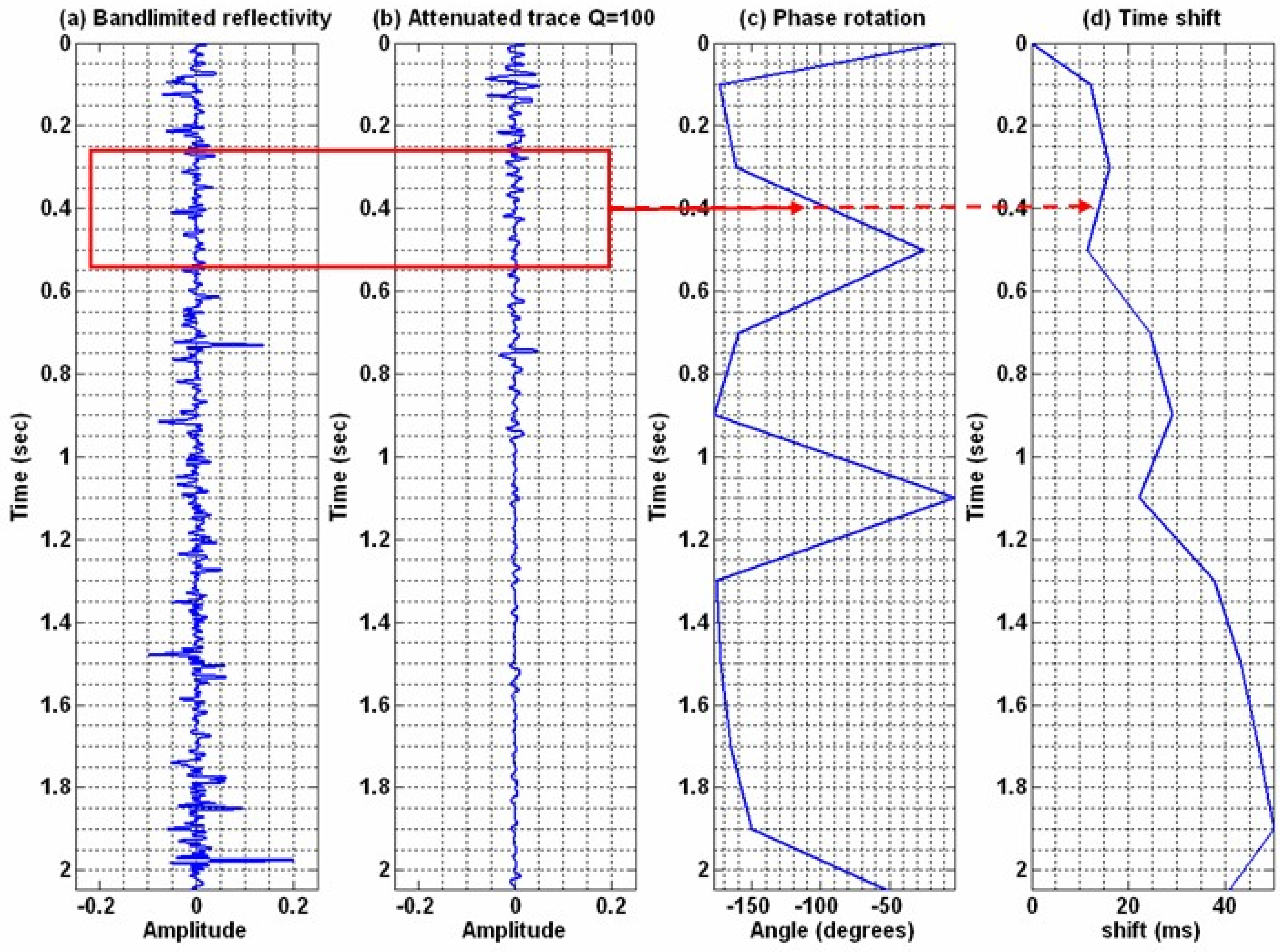
Amplitude component of the Gabor decon operator

$$\varphi_D(f) = \int_{-\infty}^{\infty} \frac{\ln|\sigma(f')|}{f - f'} df' \approx \sum_{f'=-Nyq}^{Nyq} \frac{\ln|\sigma(f')|}{f - f'} \Delta f$$

- Q does not appear explicitly in this formula

- Phase estimated from this expression is data-driven

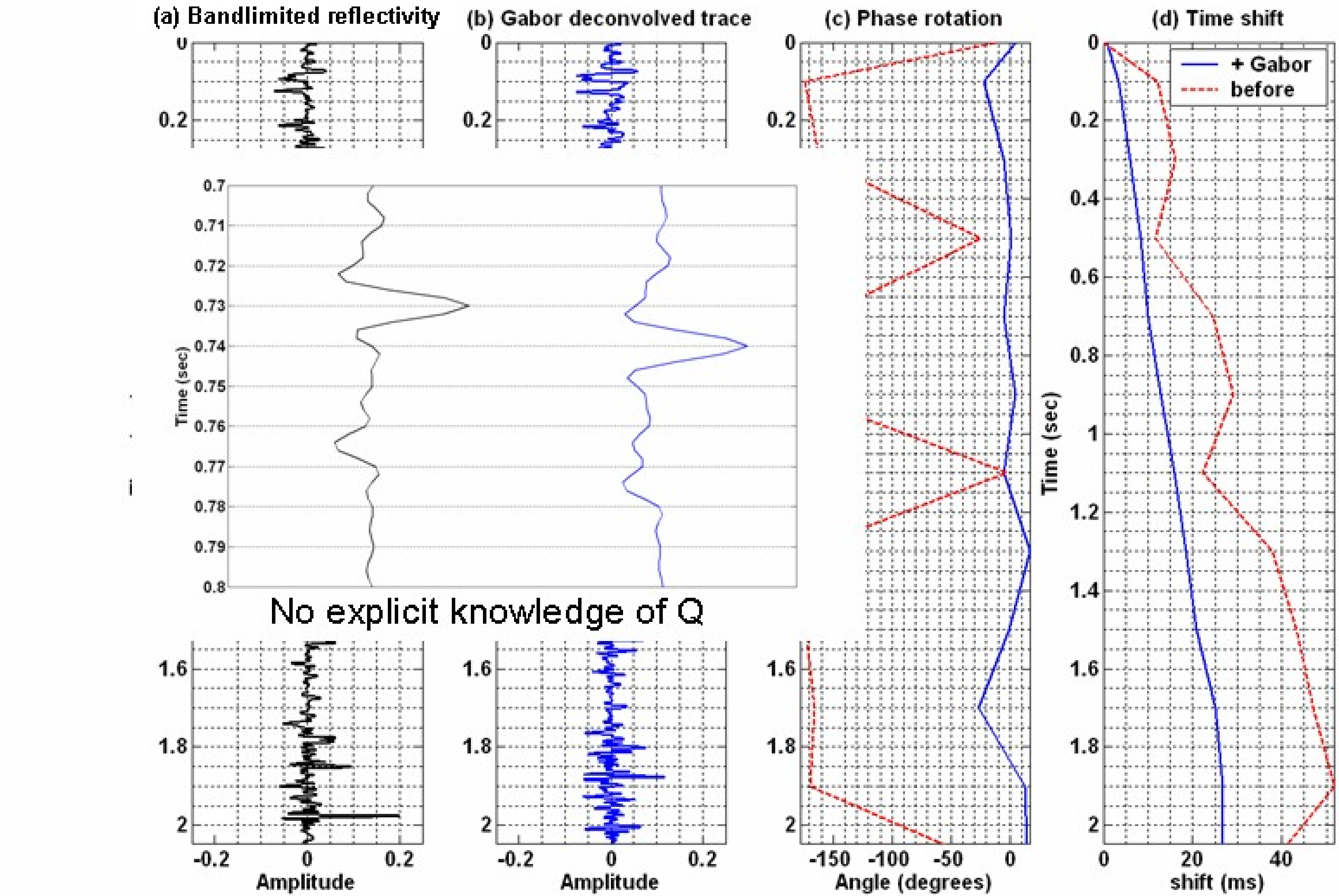
Reference traces, Q=100



After standard Gabor

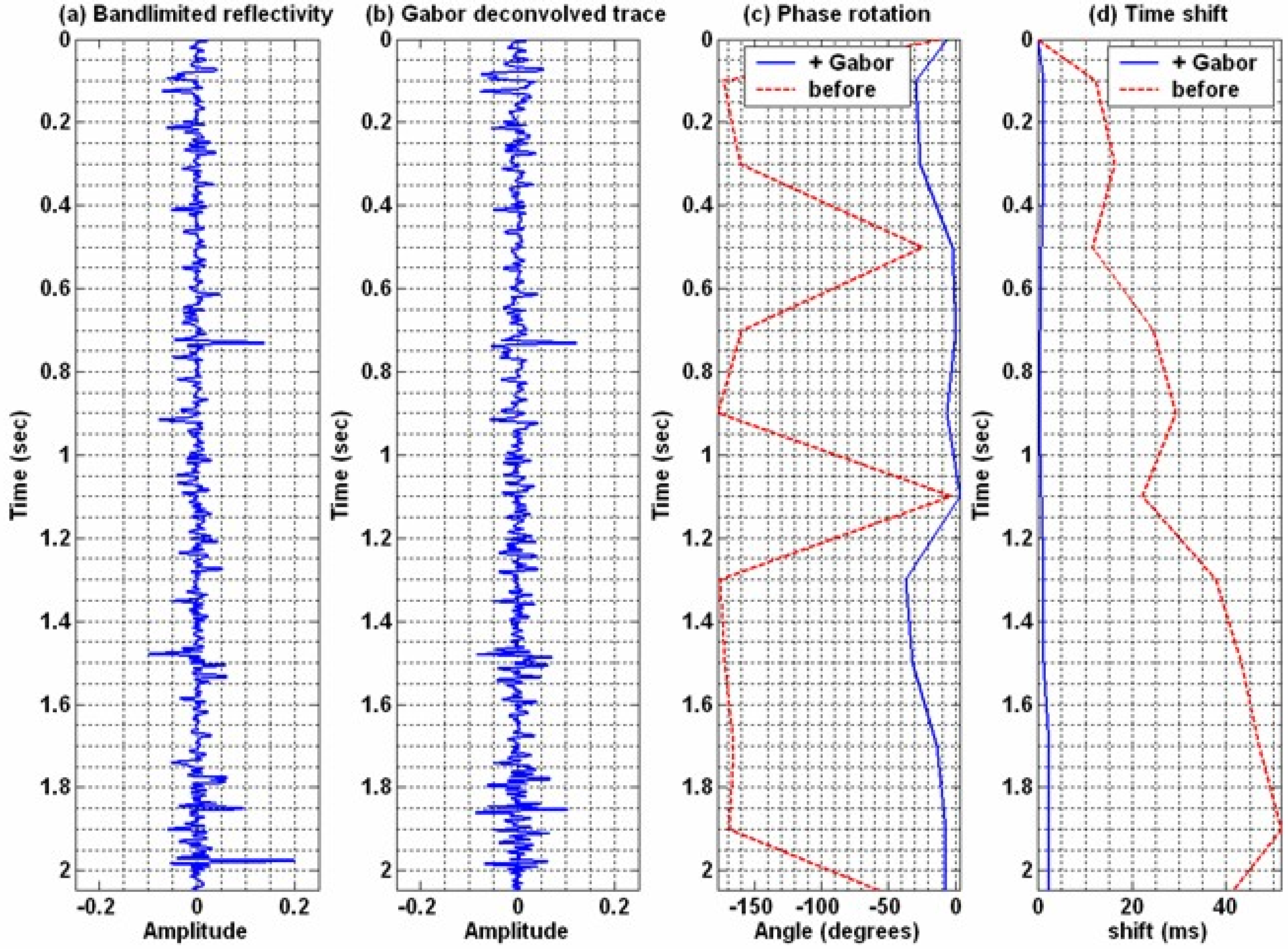
Phase computed using digital Hilbert transform

$$\varphi(f) \approx \sum_{f'=-Nyq}^{Nyq} \frac{\ln|\sigma(f')|}{f-f'} \Delta f'$$



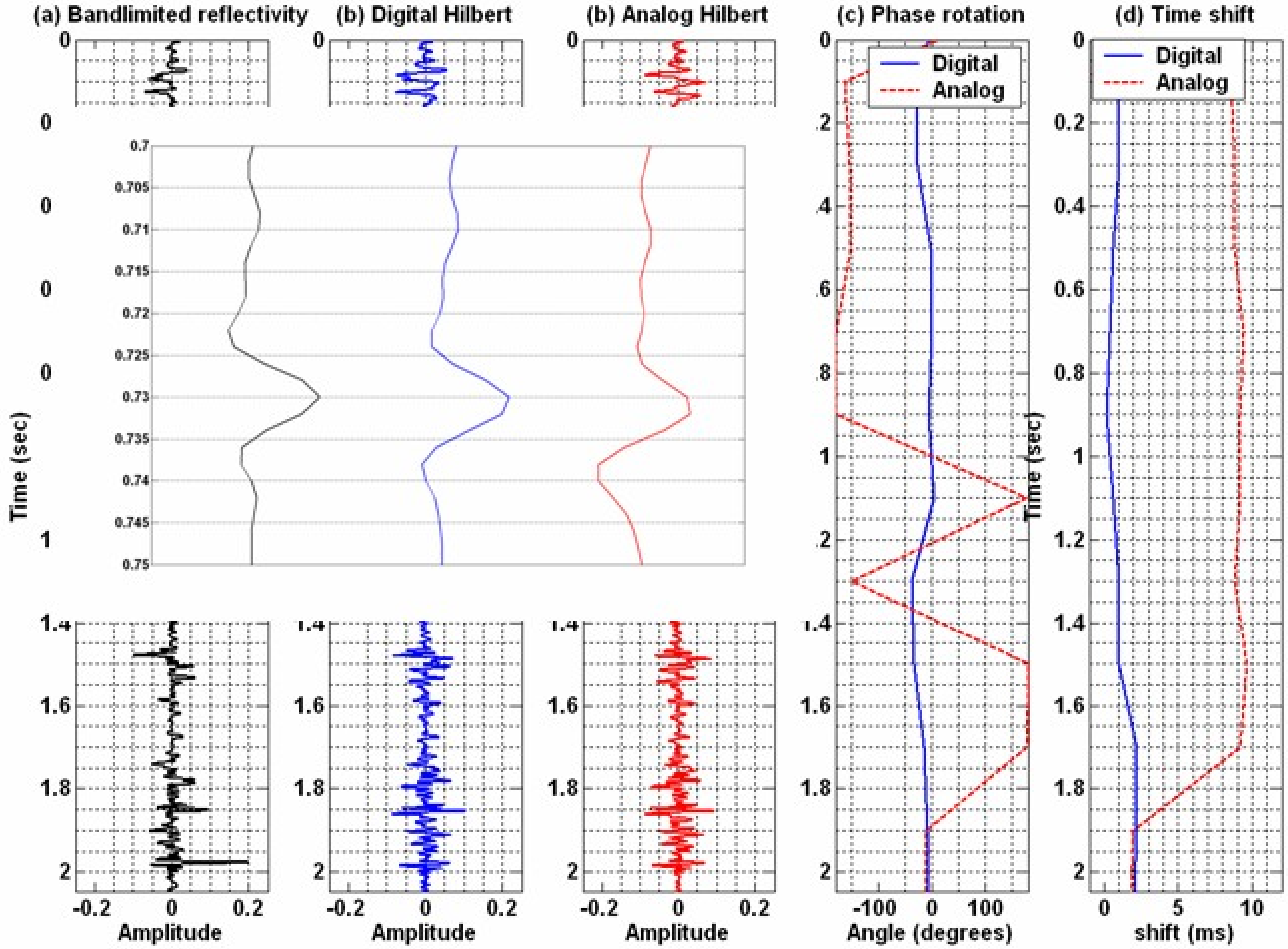
After Gabor + phase correction

$$\varphi_c(t,f) = H\left(\ln\left|\sigma(\tau,f)\right|\right) + \frac{t}{Q}\left(a + bf + cf^2\right)$$



Analog vs. digital phase correction

$$\varphi_c(t,f) = H\left(\ln\left|\sigma(\tau,f)\right|\right) + \frac{t}{Q}\left(a + bf + cf^2\right)$$

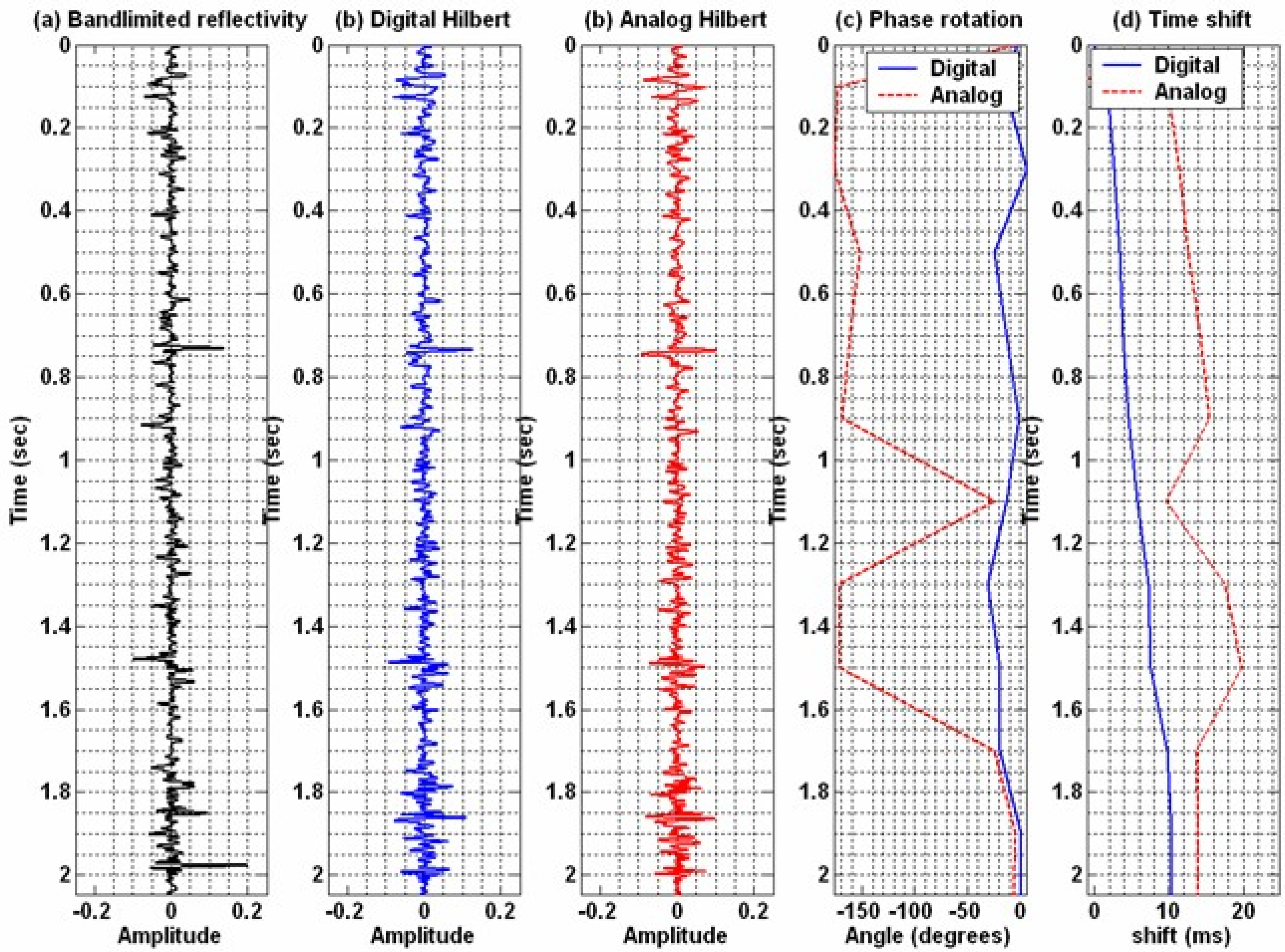


$$\varphi(f) = \frac{f}{V(f)} - \frac{f}{V_\infty}$$

Accurate Q

Analog vs. digital phase correction

$$\varphi_c(t,f) = H\left(\ln\left|\sigma(\tau,f)\right|\right) + \frac{t}{Q}\left(a + bf + cf^2\right)$$

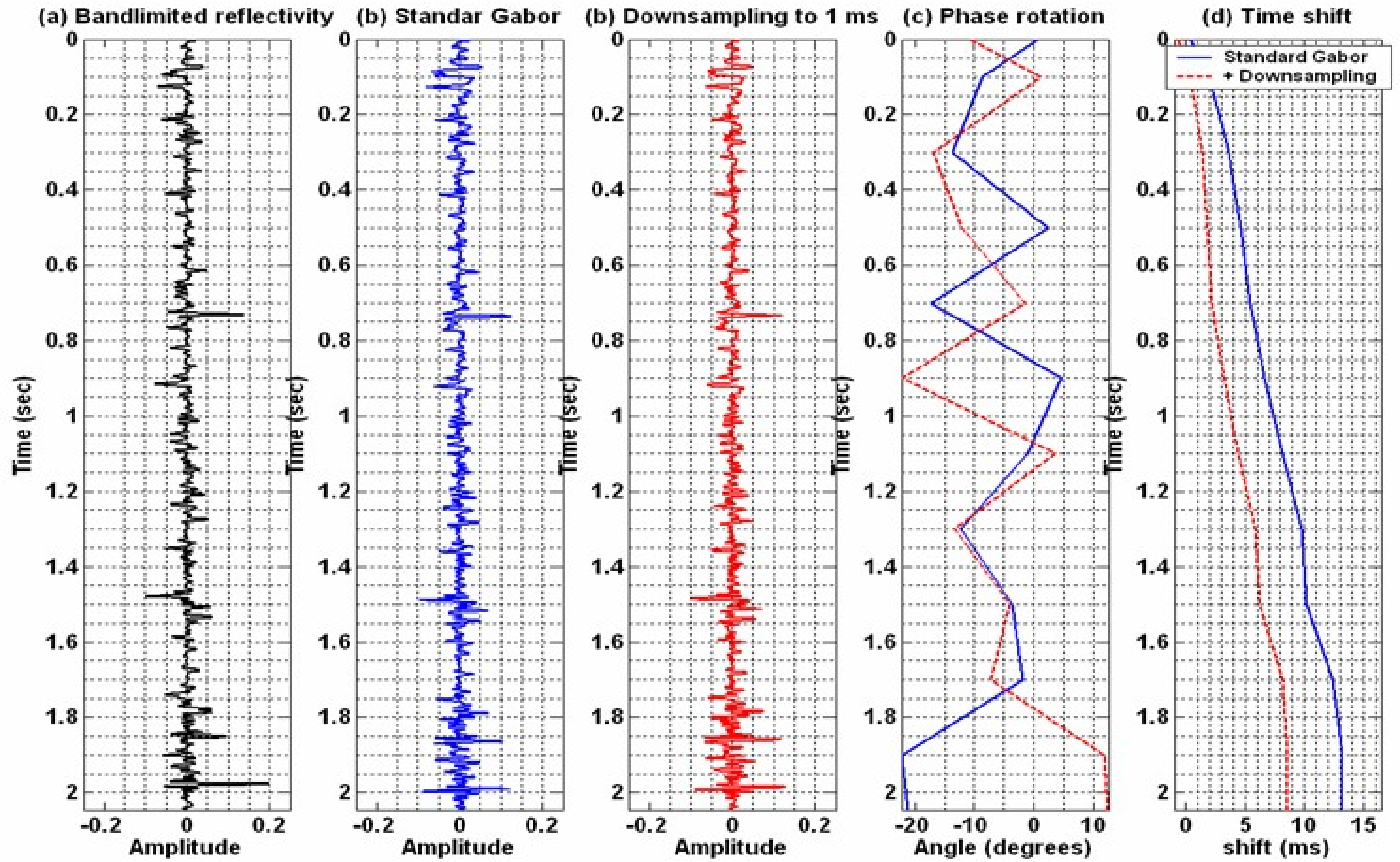


$$\varphi(f) = \frac{f}{V(f)} - \frac{f}{V_\infty}$$

Inaccurate
Q=150

Phase through digital Hilbert + downsampling

$$\varphi(f) \approx \sum_{f'=-Nyq}^{Nyq} \frac{\ln|\sigma(f')|}{f - f'} \Delta f'$$



Conclusions

- Gabor deconvolution is based on minimum phase assumptions, therefore it applies the Hilbert transform for computing the phase spectrum
- A residual phase error remains after Gabor deconvolution is applied due to the inaccuracy of the digital implementation of the Hilbert transform

Conclusions

- The error in the phase can be corrected by adding a correction term, linear in time and quadratic in frequency to the digital Hilbert transform

Work in progress

- Phase correction without explicit knowledge of Q
- Consideration of nonwhite reflectivity
- Surface consistent Gabor deconvolution

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Questions?

Analog vs. digital Hilbert transform