Circular Wavefront Assumptions for Gridded Traveltime Computations

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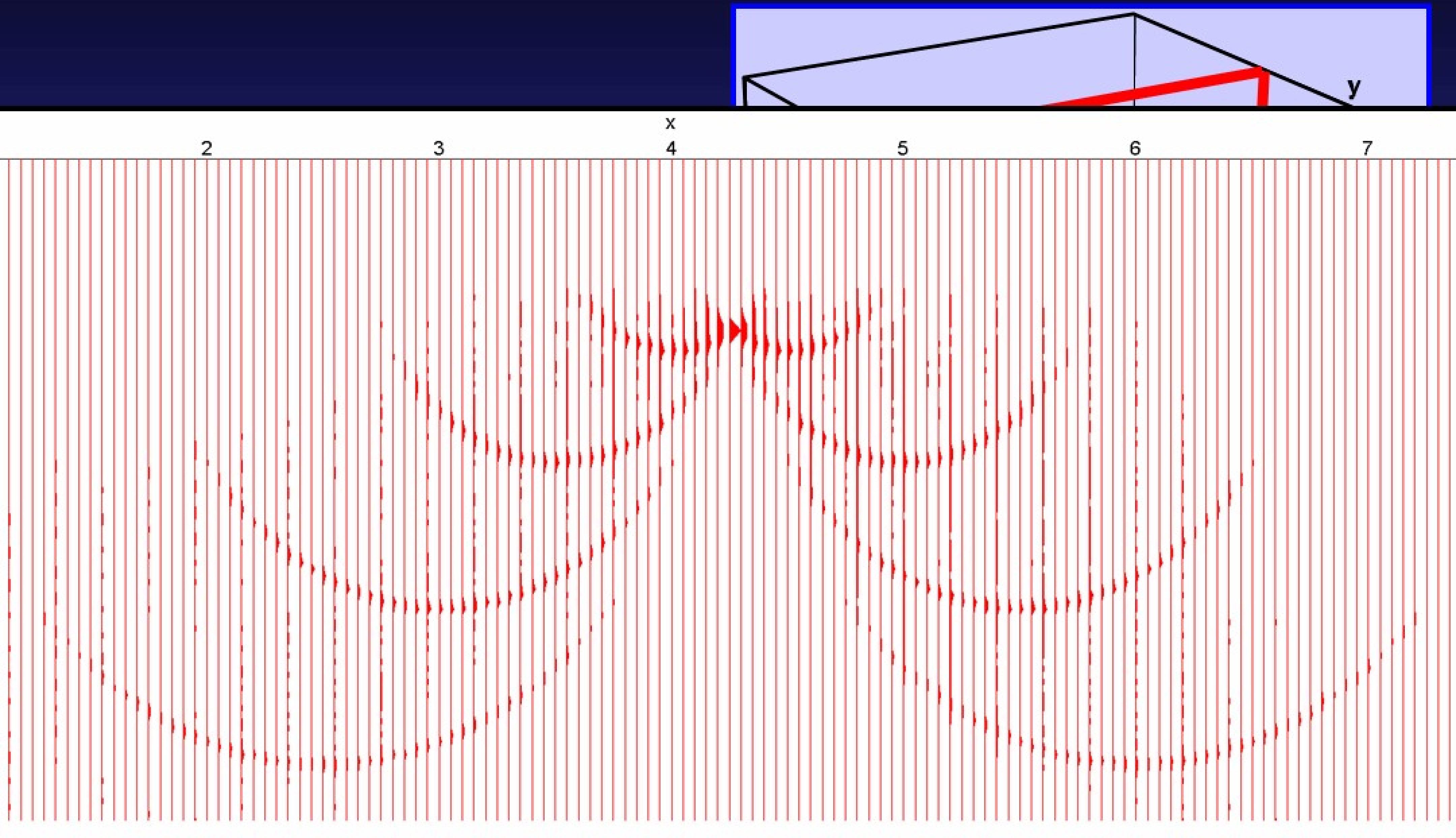
Synopsis

- GPR
- Fast-filtering and the Gabor transform
- Multigrid… statics
- Circular wavefront
- Anisotropy

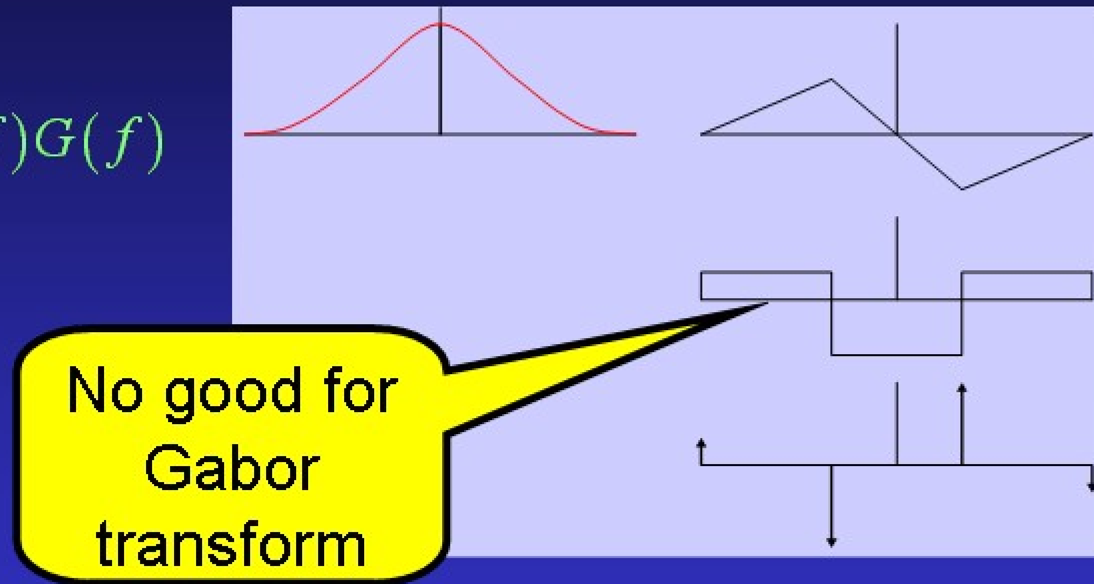
GPR 3D migration of 2D gridded data



Fast-filtering…

$$c(t)=\int_{-\infty}^{\infty} w(t-\tau)g(\tau)\,d\tau \quad C(f)=W(f)G(f)$$

Piecewise quadratic

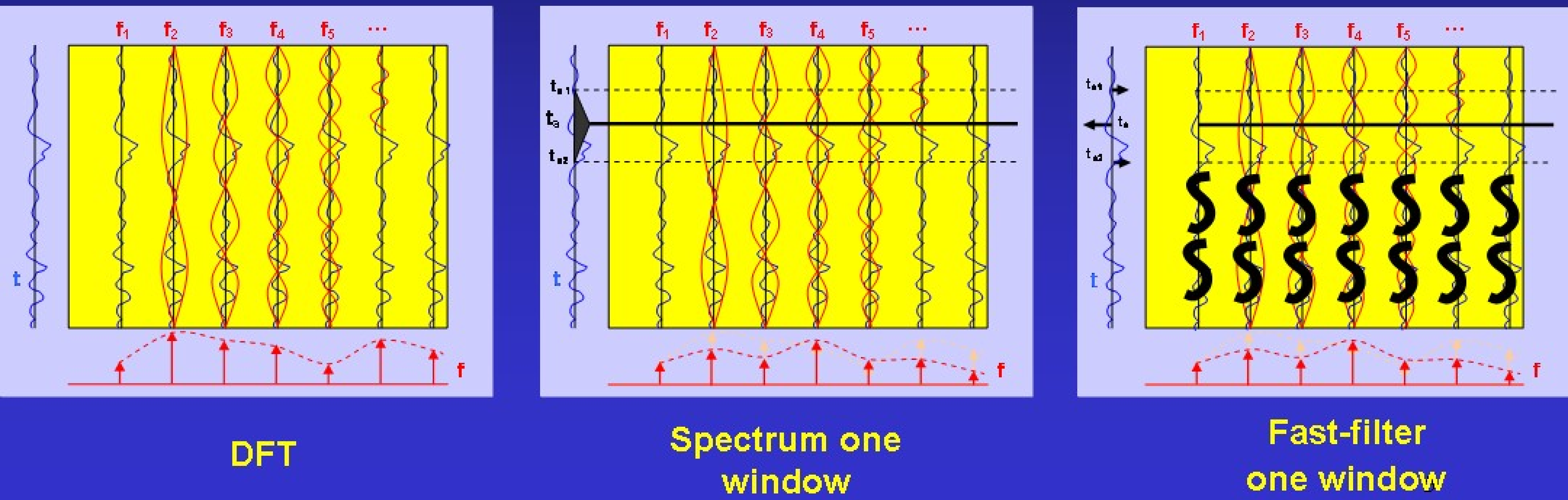


$$C(f)=[j\omega]^2\,W(f)\frac{1}{[j\omega]^2}G(f)$$

$$c(t)=\int_{t-t_1}^{t+t_1}\left(a\delta_{ta}+b\delta_{tb}+c\delta_{tc}+d\delta_{tc}\right)\varsigma(\tau)\,d\tau \quad =a\varsigma(t_a)+b\varsigma(t_b)+c\varsigma(t_c)+d\varsigma(t_c)$$

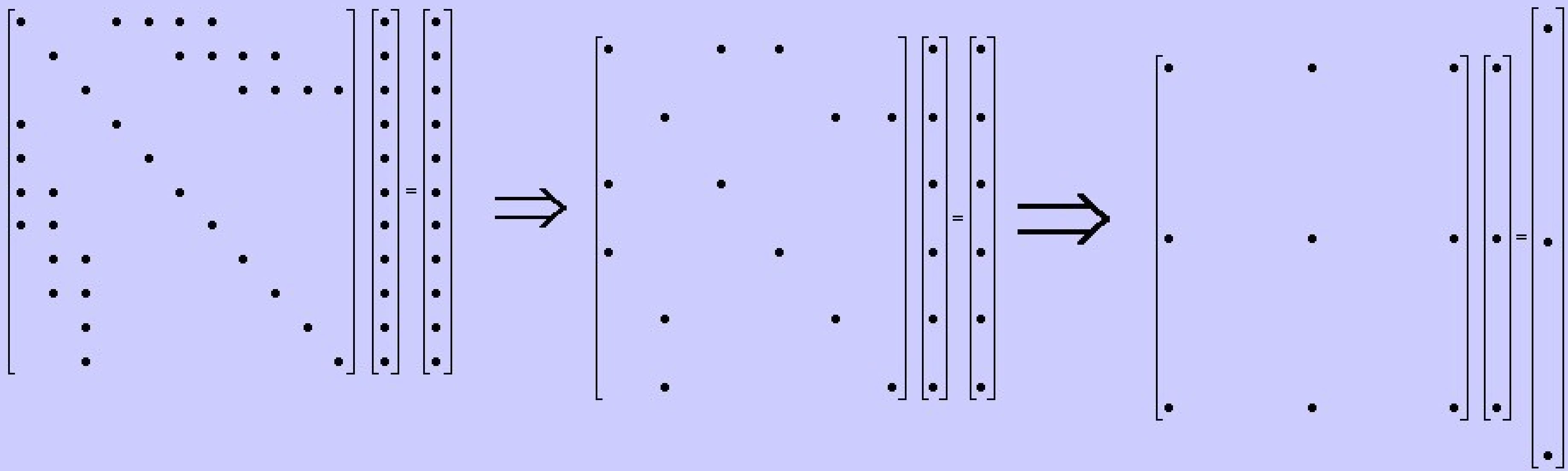
Fast-filtering, Gabor transform

$$S(\omega,t)=\int_{t-t_1}^{t+t_1} w(\tau-t)\,g(\tau)\,e^{-i\omega\tau}\,d\tau$$



Millar multigrid…Gauss-Seidel

Down sample → Solve the lower frequencies first

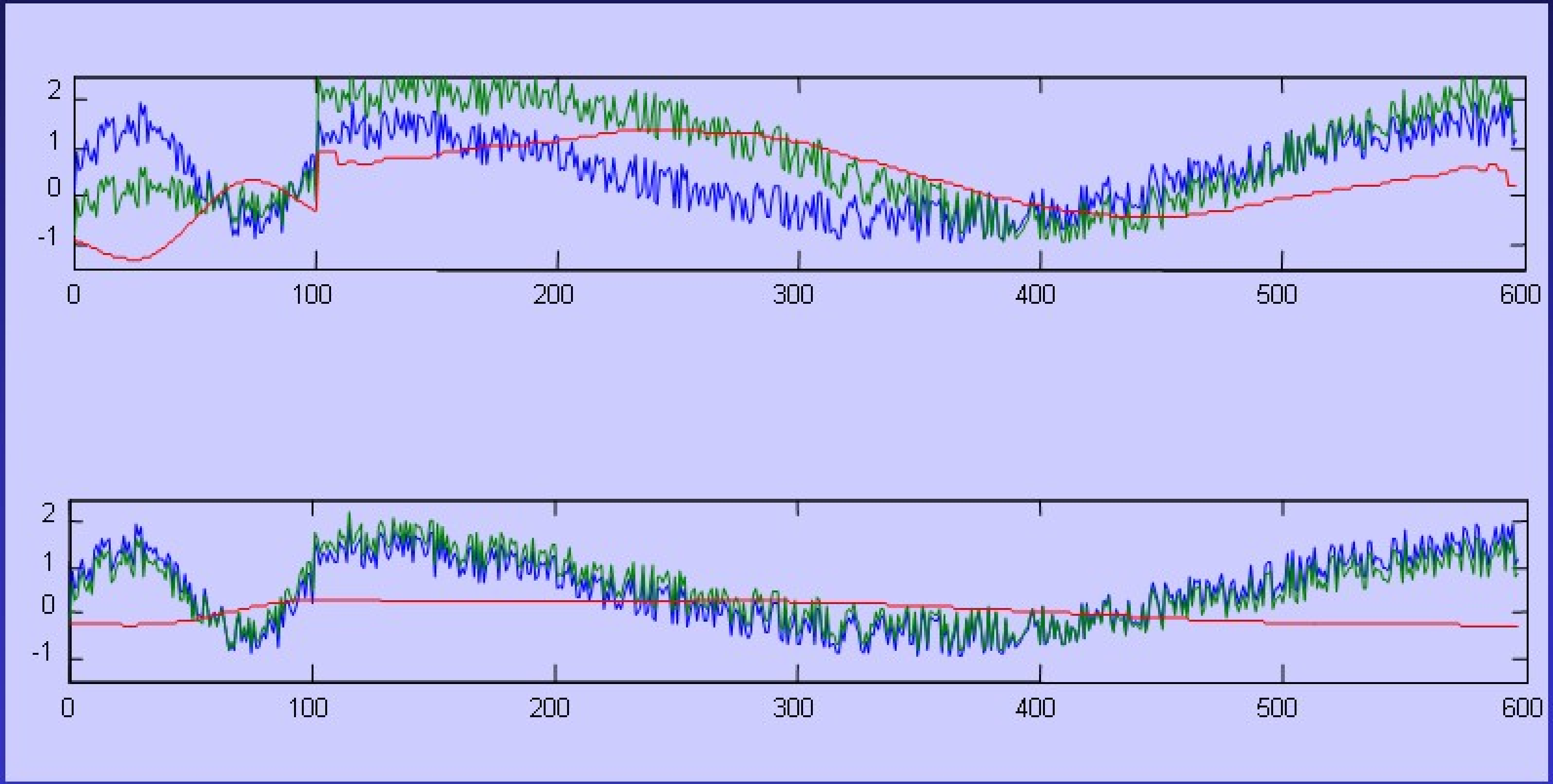


Add in the higher frequencies ← Interpolate

Surface consistent statics

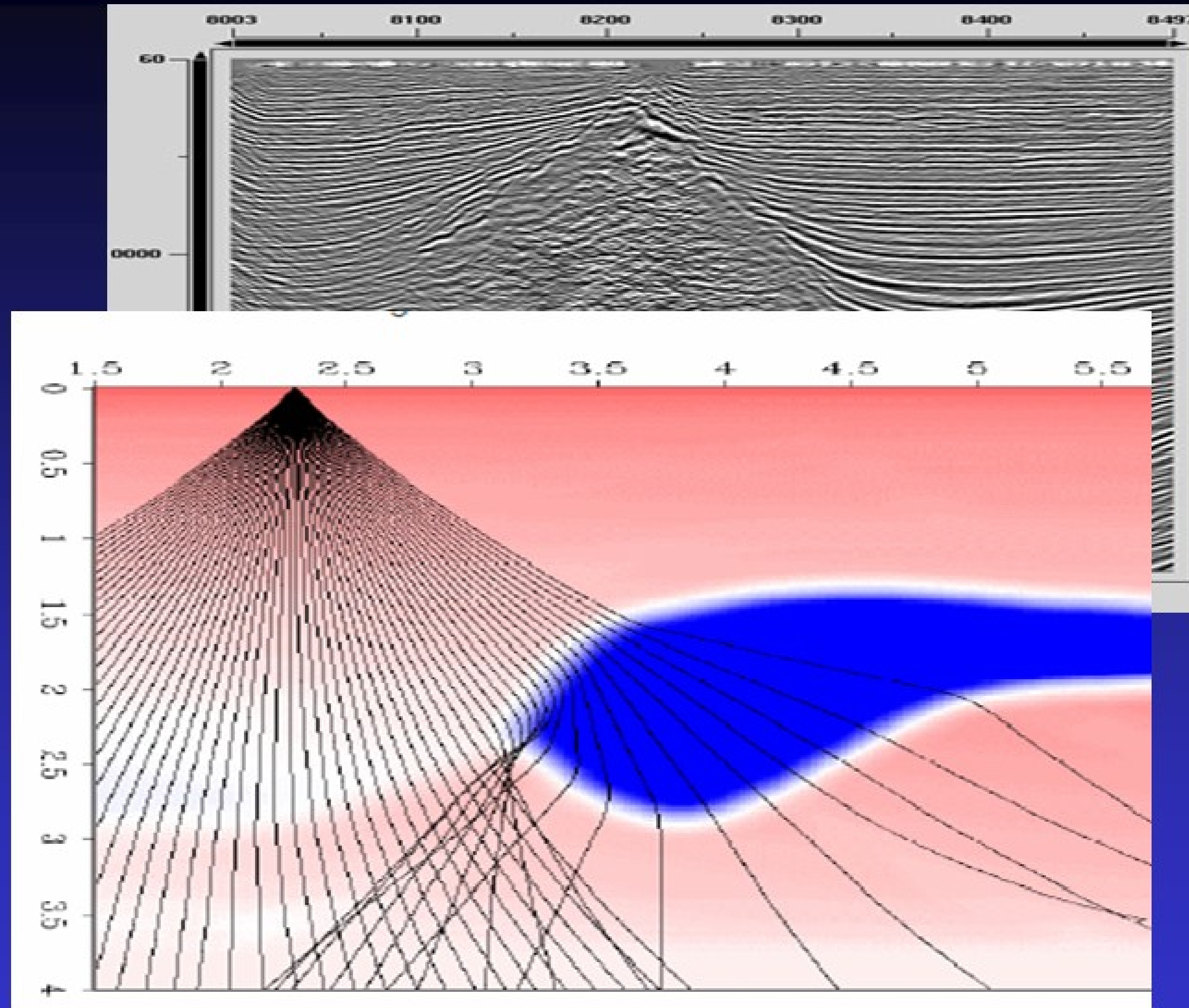
Conventional GS

Multigrid



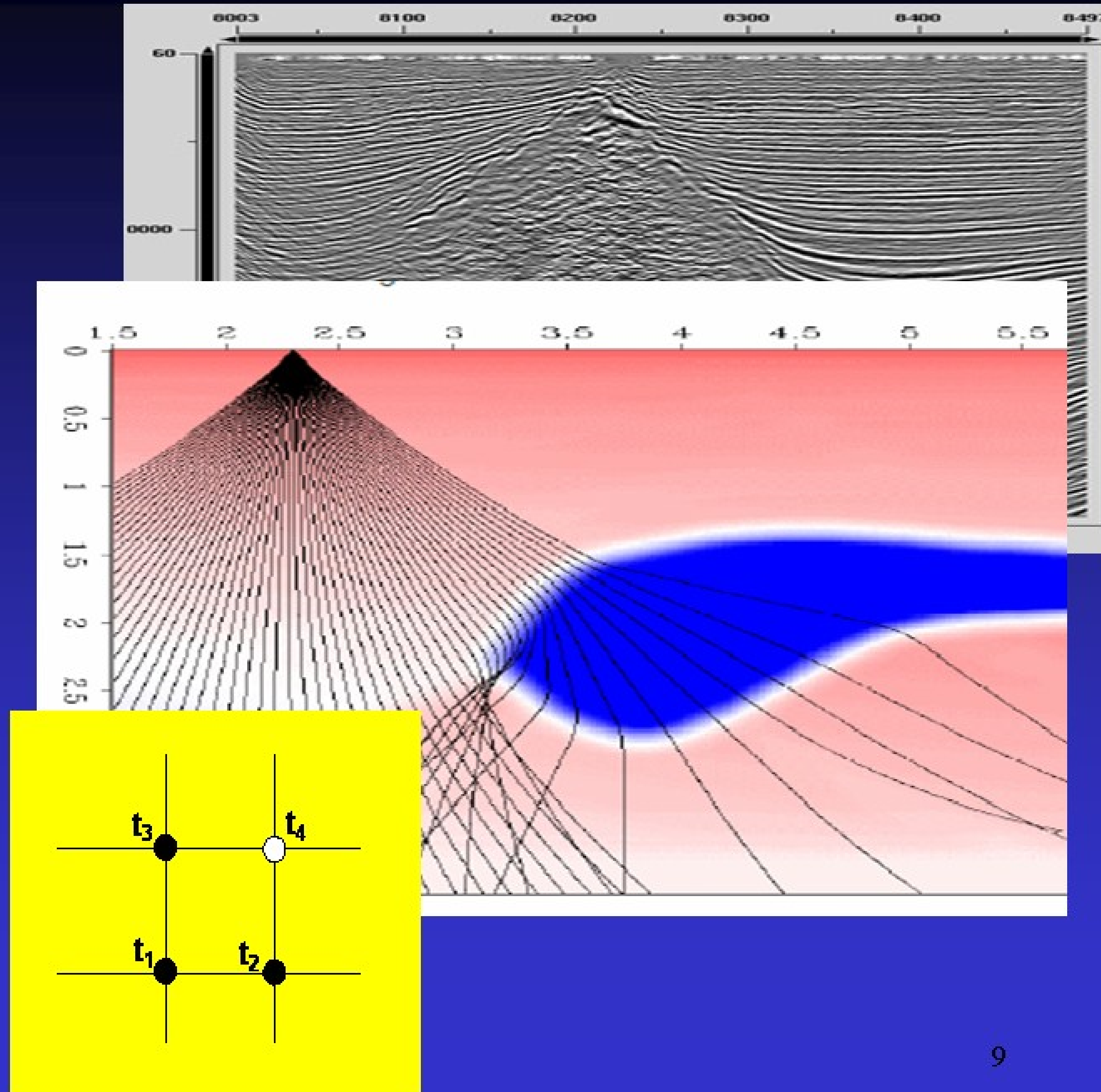
Synopsis

- Depth Migration
  - Kirchhoff migration
  - Ray tracing
  - Gridded traveltimes



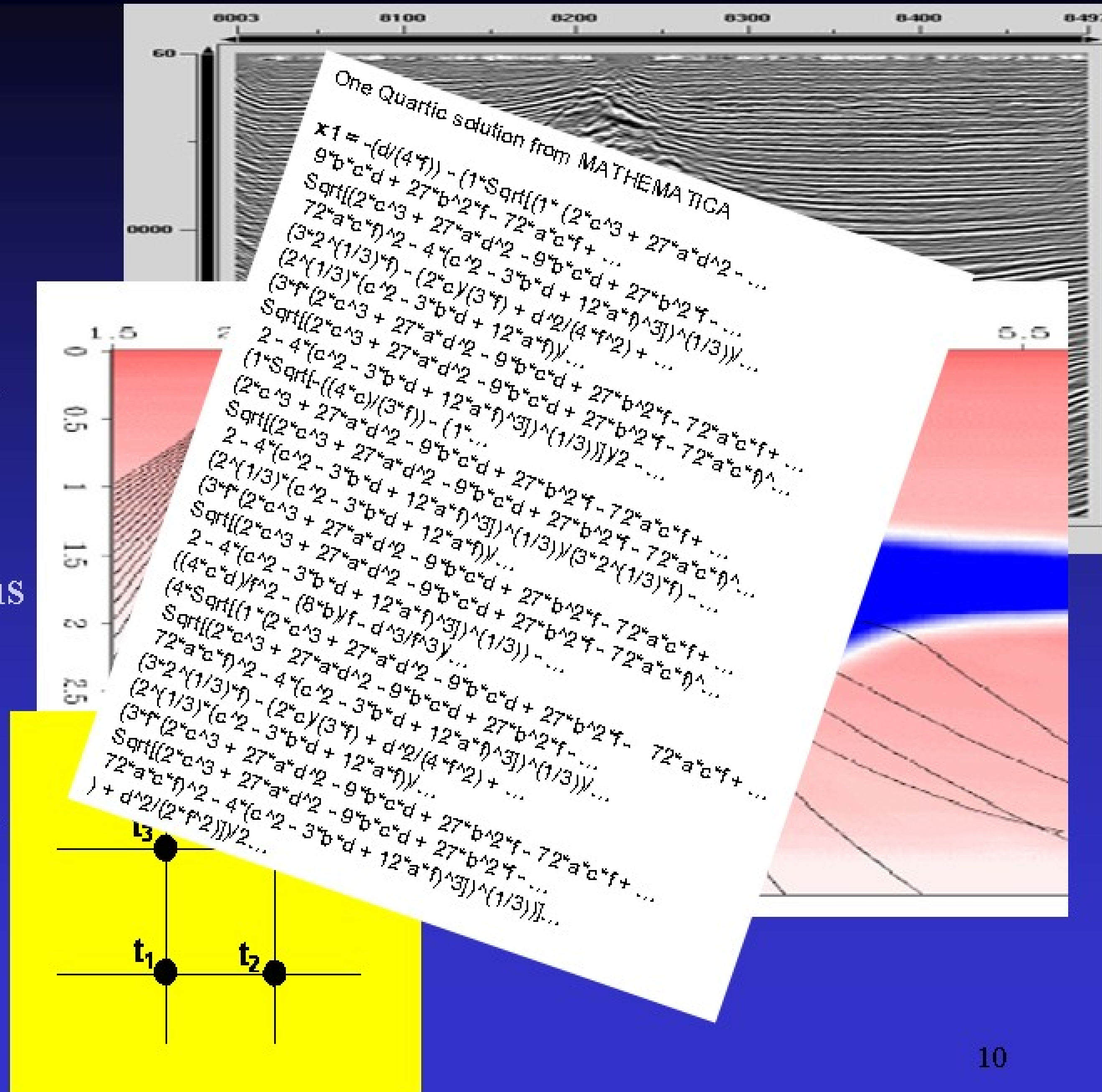
Synopsis

- Depth Migration
  - Kirchhoff migration
  - Ray tracing
  - Gridded traveltimes
- Three point Solutions
  - Plane wave
  - Vidale method
  - Circular assumption



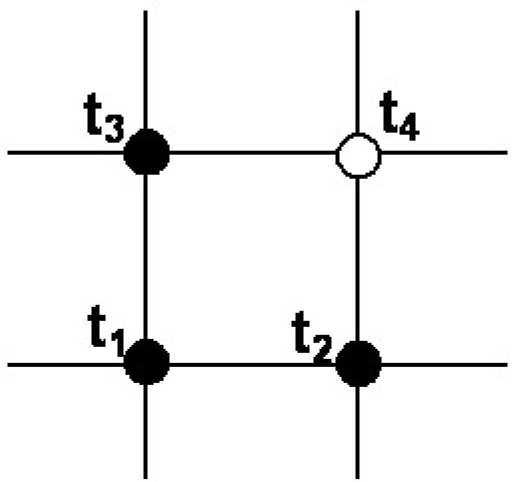
Synopsis

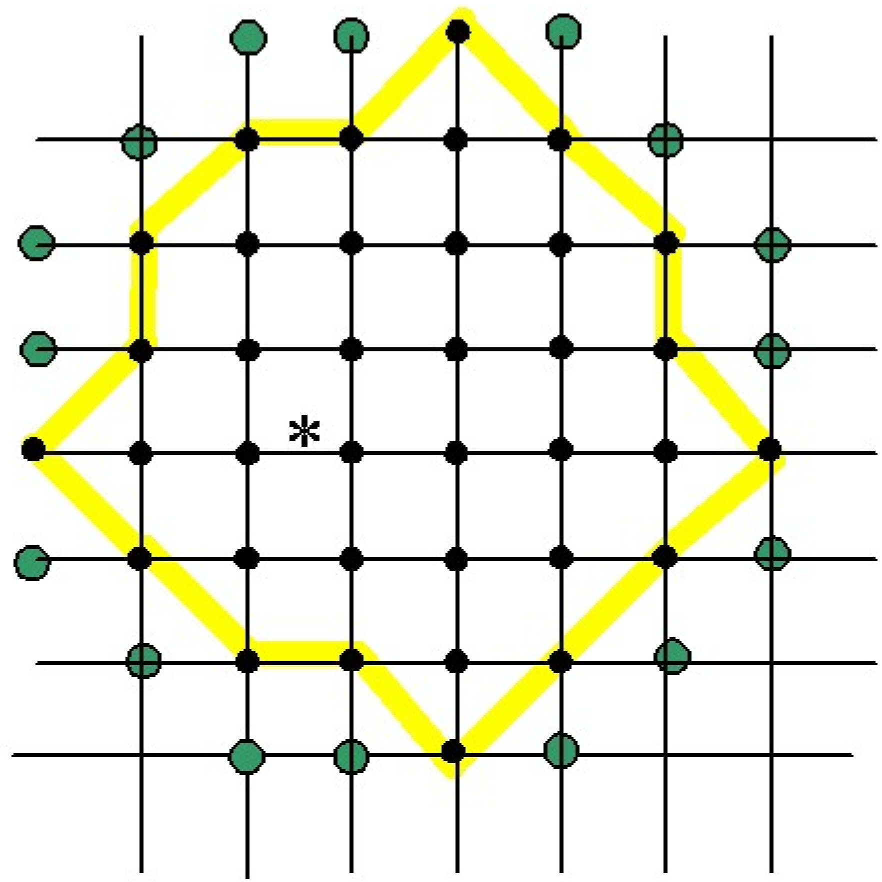
- Depth Migration
  - Kirchhoff migration
  - Ray tracing
  - Gridded traveltimes
- Three point Solutions
  - Plane wave
  - Vidale method
  - Circular assumption
- Computation
  - Quartic solution
  - Iterative solution



Update minimum times

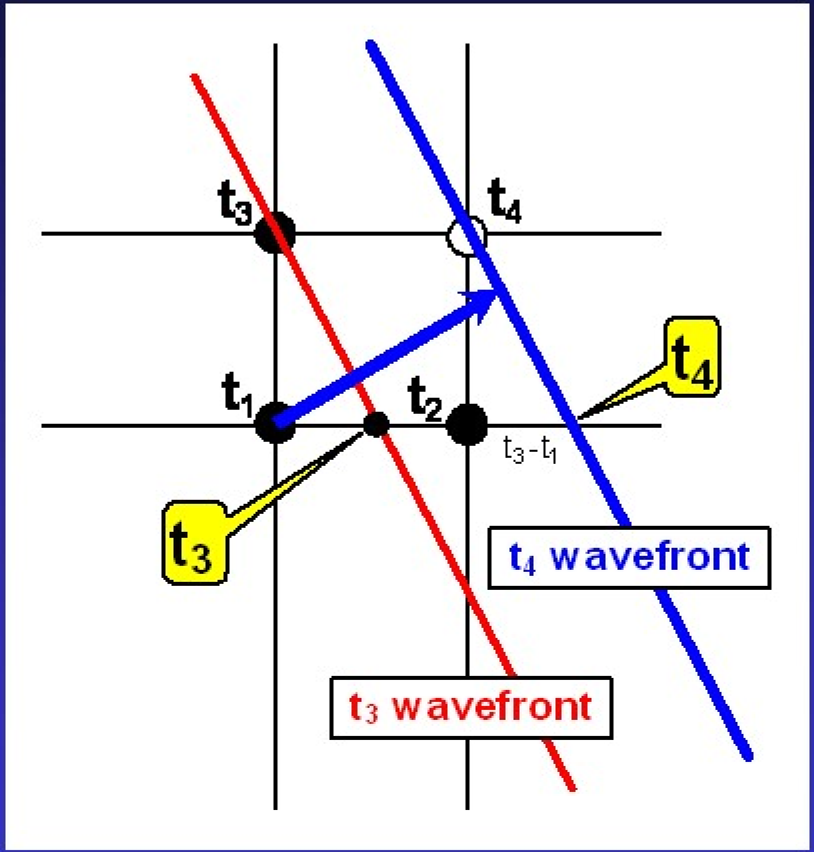
Wavefront tends to circular shape





Plane wave assumptions

$$t_4 = t_2 + t_3 - t_1$$



Vidale's method

<u>Eikonal equation</u>
Finite difference solution of

$$\left(\frac{dt}{dx}\right)^2 + \left(\frac{dt}{dz}\right)^2 = \frac{1}{v^2}$$

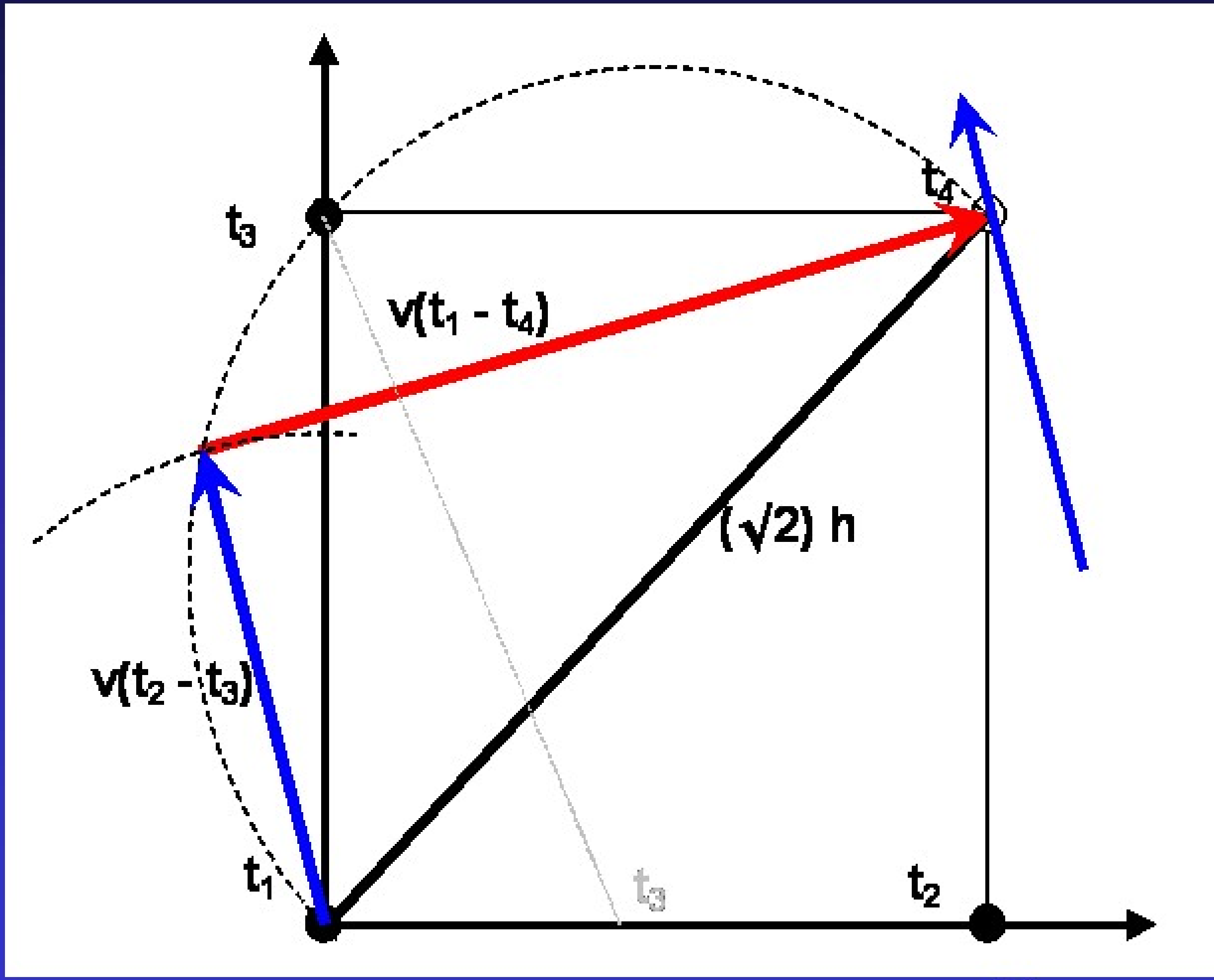
Basic solution in common use

$$t_4 = t_1 + \sqrt{\frac{2h^2}{v^2} - \left(t_3 - t_2\right)^2}$$

Vidale's method

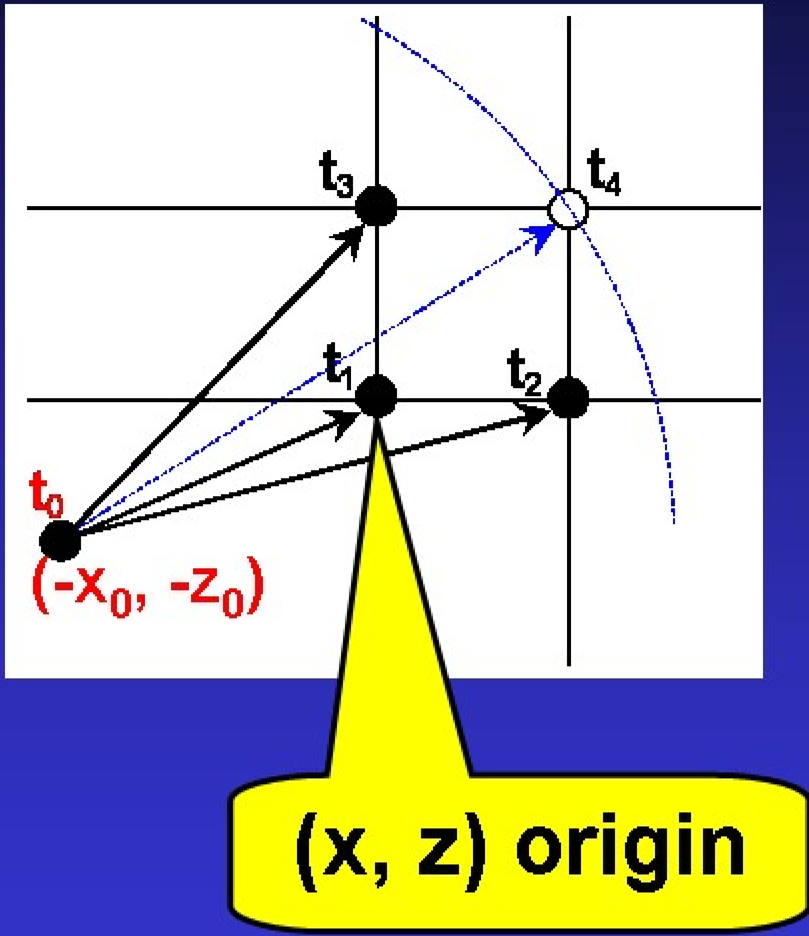
$$t_4 = t_1 + \sqrt{\frac{2h^2}{v^2} - (t_3 - t_2)^2}$$

$$(t_4 - t_1)^2 + (t_3 - t_2)^2 = \frac{2h^2}{v^2}$$



Curved wavefront assumption

1. Constant velocity of the square
2. Estimate center of curvature or apparent source location (x_0, z_0)
3. Define the time t_0
4. Get time t_4



The quartic equation

$$a + xb + x^2c + x^3d + x^4 f = 0$$

$$x^4\left(\frac{b_1^4}{a_1^2} - \frac{2b_1^2b_2^2}{a_1^2a_2^2} + \frac{b_2^4}{a_2^4}\right)$$

$$+x^3\frac{4b_1^2}{a_1^2}\left(\frac{b_2^2}{a_2^2} - b_1^2\right)$$

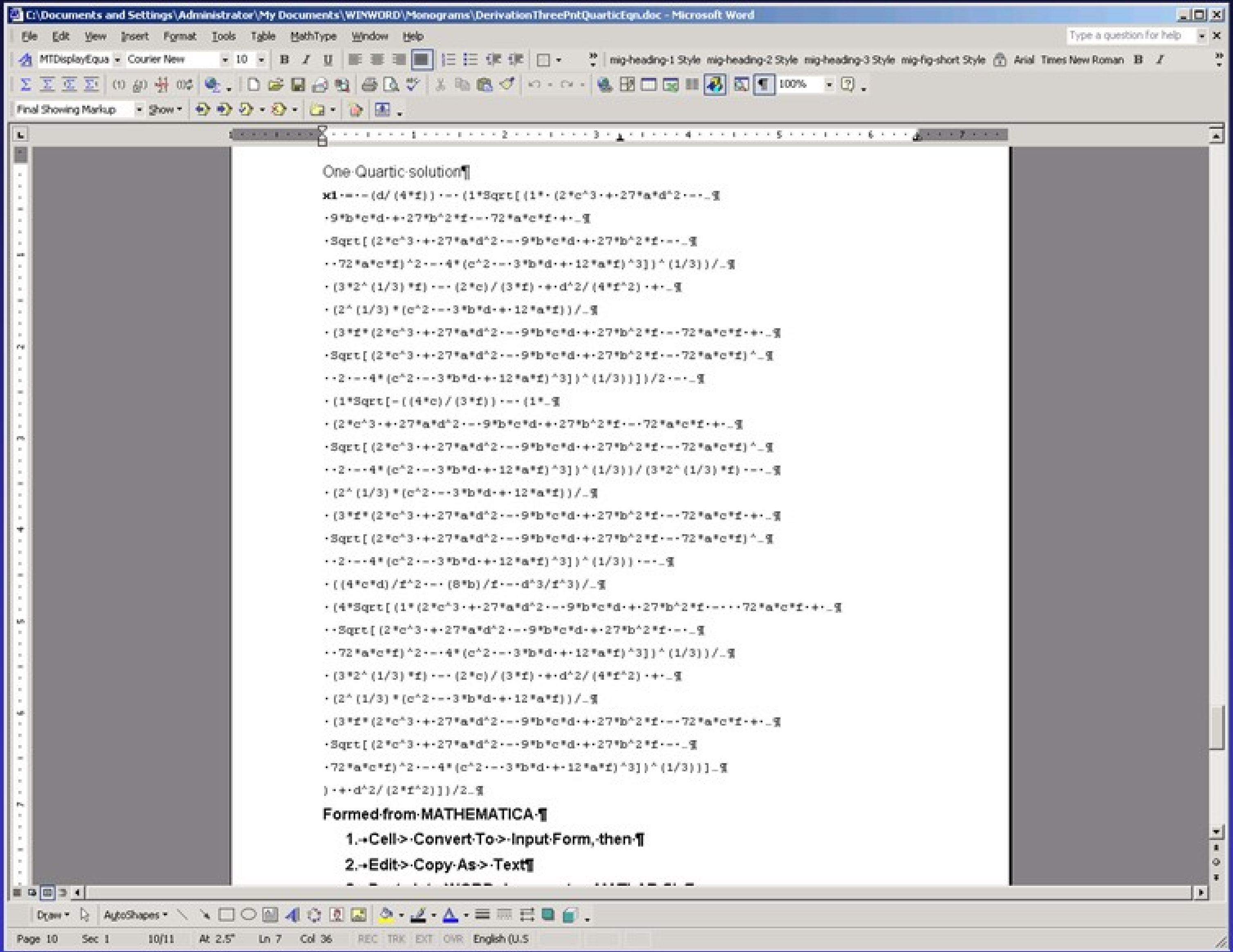
$$+x^2\left(+\frac{6b_1^4}{a_1^4} + \frac{2b_2^4}{a_2^2} - \frac{2b_1^4}{a_1^2} - \frac{2b_1^2b_2^2}{a_1^2} - \frac{2b_1^2b_2^2}{a_1^2a_2^2} + \frac{2b_1^2b_2^2}{a_2^2} - \frac{2b_1^2}{a_1^2} - \frac{2b_2^2}{a_2^2}\right)$$

$$+x\frac{4b_1^2}{a_1^2}\left(1 - \frac{b_1^2}{a_1^2} + b_1^2 + b_2^2\right)$$

$$+\left(b_1^4 + b_2^4 + 2b_1^2 - 2b_2^2 + 2b_1^2b_2^2 + \frac{b_1^4}{a_1^4} - \frac{2b_1^4}{a_1^2} - \frac{2b_1^2}{a_1^2} - \frac{2b_1^2b_2^2}{a_1^2} + 1\right)$$

$$= 0$$

A quartic solution



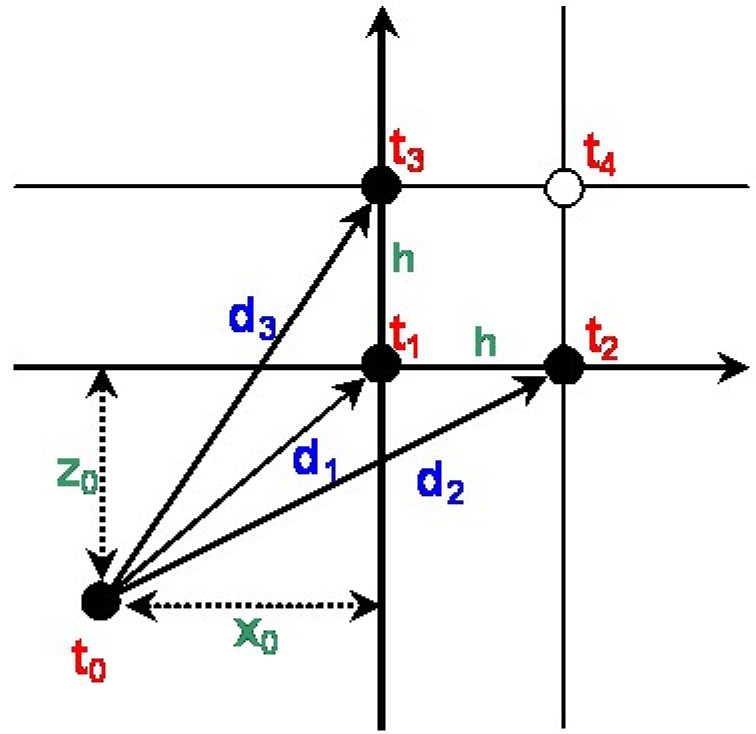
One Quartic solution

```
x1 = -(d/(4*f)) - (1*Sqrt[(1* (2*c^3 + 27*a*d^2 - 
 9*b*c*d + 27*b^2*f - 72*a*c*f + 
 Sqrt[(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 
  72*a*c*f)^2 - 4*(c^2 - 3*b*d + 12*a*f)^3])^(1/3))/
 (3*2^(1/3)*f) - (2*c)/(3*f) + d^2/(4*f^2) + 
 (2^(1/3)*(c^2 - 3*b*d + 12*a*f))/
 (3*f*(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 72*a*c*f + 
 Sqrt[(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 72*a*c*f)^
  2 - 4*(c^2 - 3*b*d + 12*a*f)^3])^(1/3))]])/2 - 
 (1*Sqrt[-((4*c)/(3*f)) - (1*
 (2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 72*a*c*f + 
 Sqrt[(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 72*a*c*f)^
  2 - 4*(c^2 - 3*b*d + 12*a*f)^3])^(1/3))/(3*2^(1/3)*f) - 
 (2^(1/3)*(c^2 - 3*b*d + 12*a*f))/
 (3*f*(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 72*a*c*f + 
 Sqrt[(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 72*a*c*f)^
  2 - 4*(c^2 - 3*b*d + 12*a*f)^3])^(1/3)) - 
 ((4*c*d)/f^2 - (8*b)/f - d^3/f^3)/
 (4*Sqrt[(1*(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 72*a*c*f + 
  Sqrt[(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 
  72*a*c*f)^2 - 4*(c^2 - 3*b*d + 12*a*f)^3])^(1/3))/
 (3*2^(1/3)*f) - (2*c)/(3*f) + d^2/(4*f^2) + 
 (2^(1/3)*(c^2 - 3*b*d + 12*a*f))/
 (3*f*(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 72*a*c*f + 
 Sqrt[(2*c^3 + 27*a*d^2 - 9*b*c*d + 27*b^2*f - 
 72*a*c*f)^2 - 4*(c^2 - 3*b*d + 12*a*f)^3])^(1/3))]
) + d^2/(2*f^2)]])/2
```

Formed from MATHEMATICA

1. Cell> Convert To> Input Form, then
2. Edit> Copy As> Text

$$\begin{cases} t_0 = t_1 - \dfrac{1}{v}\sqrt{x_0^2 + z_0^2} \\ x_0 = \dfrac{v^2}{2h}\left[2t_0\left(t_2 - t_1\right) - t_2^2 + t_1^2\right] + \dfrac{h}{2} \\ z_0 = \dfrac{v^2}{2h}\left[2t_0\left(t_3 - t_1\right) - t_3^2 + t_1^2\right] + \dfrac{h}{2} \end{cases}$$



$$t_0 = t_1 - \frac{1}{v}\sqrt{\left\{\frac{v^2}{2h}\left[2t_0\left(t_2 - t_1\right) - t_2^2 + t_1^2\right] + \frac{h}{2}\right\}^2 + \left\{\frac{v^2}{2h}\left[2t_0\left(t_3 - t_1\right) - t_3^2 + t_1^2\right] + \frac{h}{2}\right\}^2}$$

Only variable is t_0 …implicit

$$t_0 = t_1 - \frac{1}{v}\sqrt{\left\{\frac{v^2}{2h}\left[2t_0\left(t_2 - t_1\right) - t_2^2 + t_1^2\right] + \frac{h}{2}\right\}^2 + \left\{\frac{v^2}{2h}\left[2t_0\left(t_3 - t_1\right) - t_3^2 + t_1^2\right] + \frac{h}{2}\right\}^2}$$

Enter … Newton Raphson

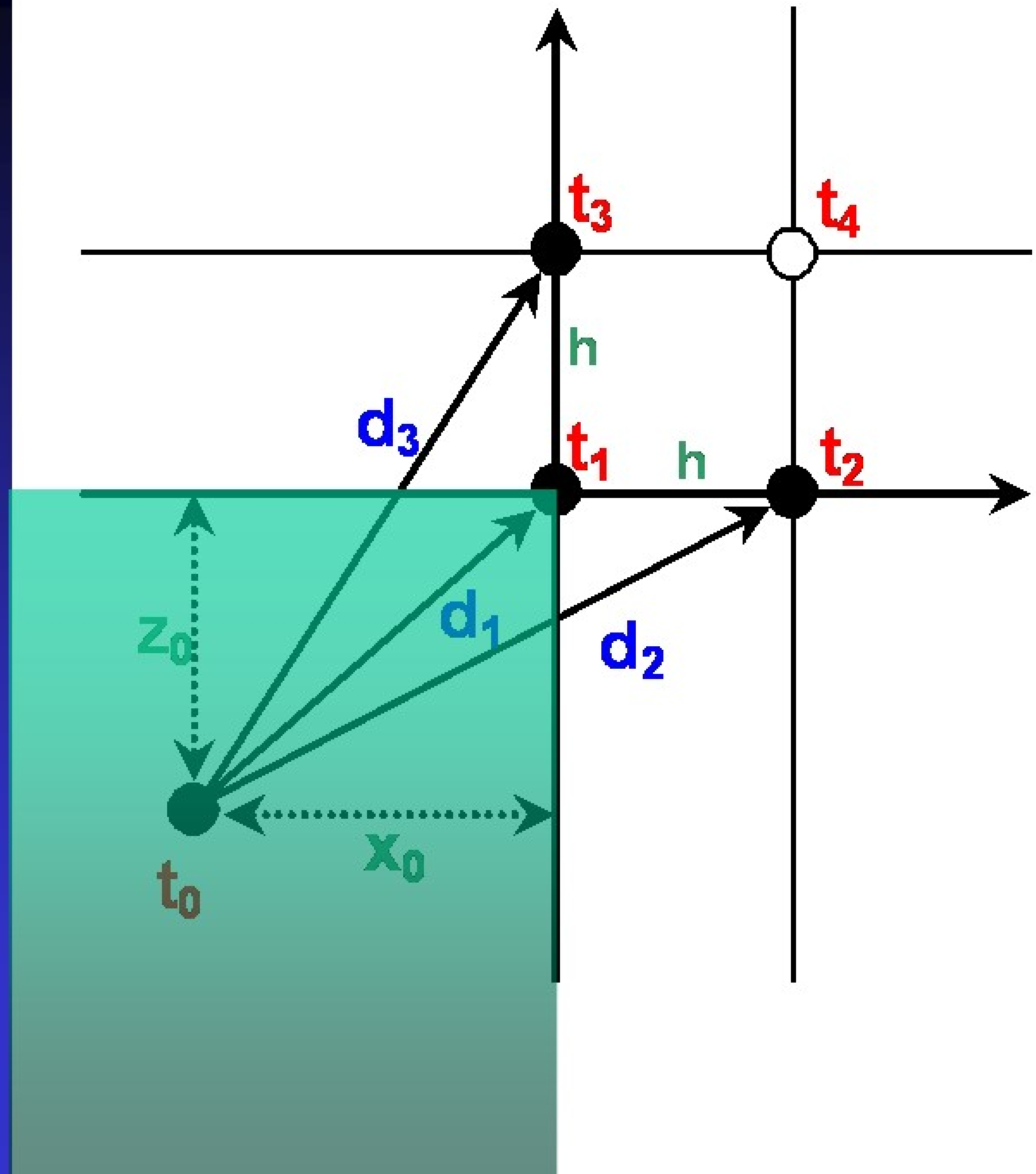
Find t_0

$$f(t_0) \Rightarrow 0$$

$$\frac{d\,f(t_0)}{dt_0} = \quad \sim$$

Solution in third quadrant

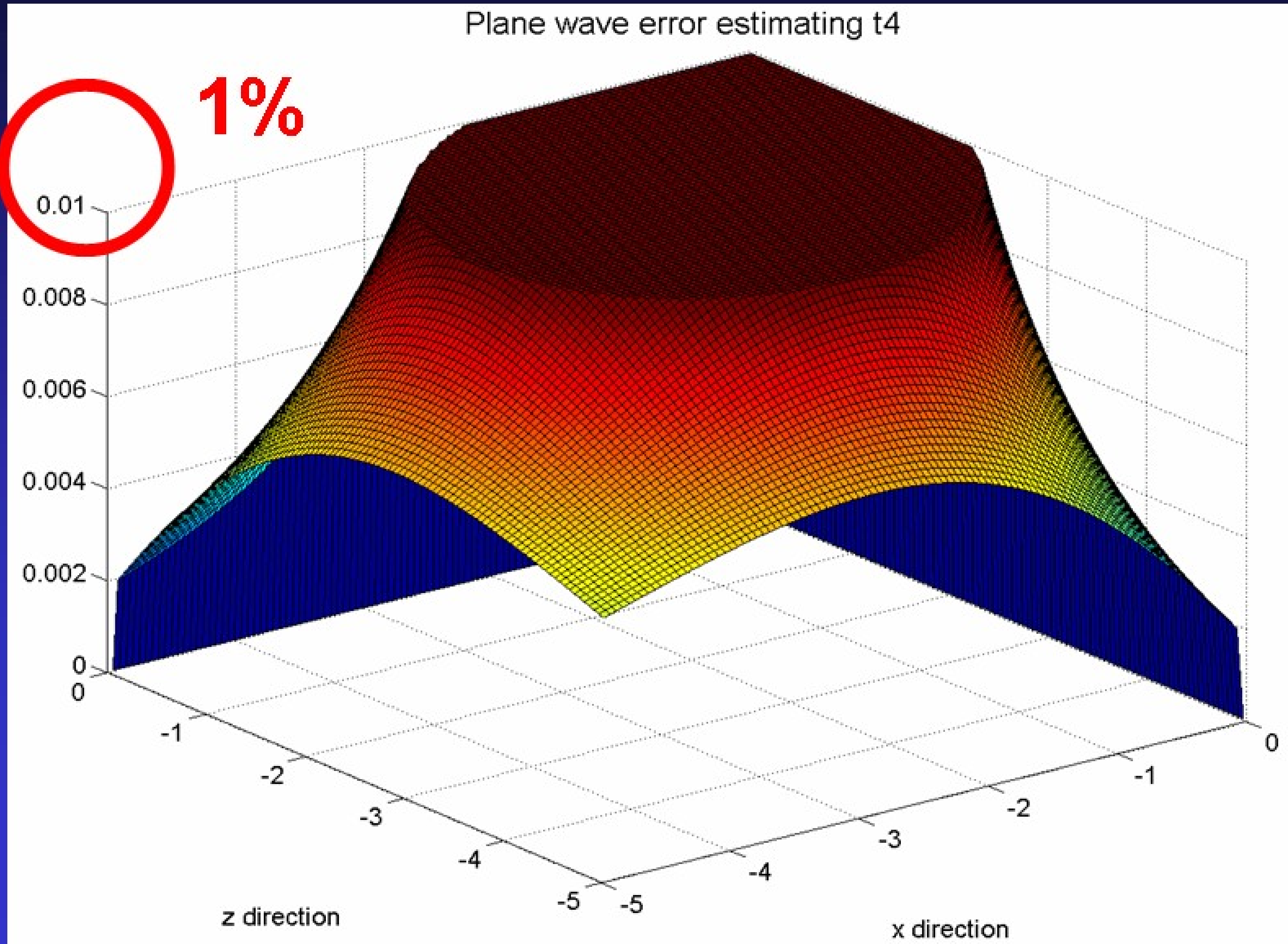
(expanding wavefront)



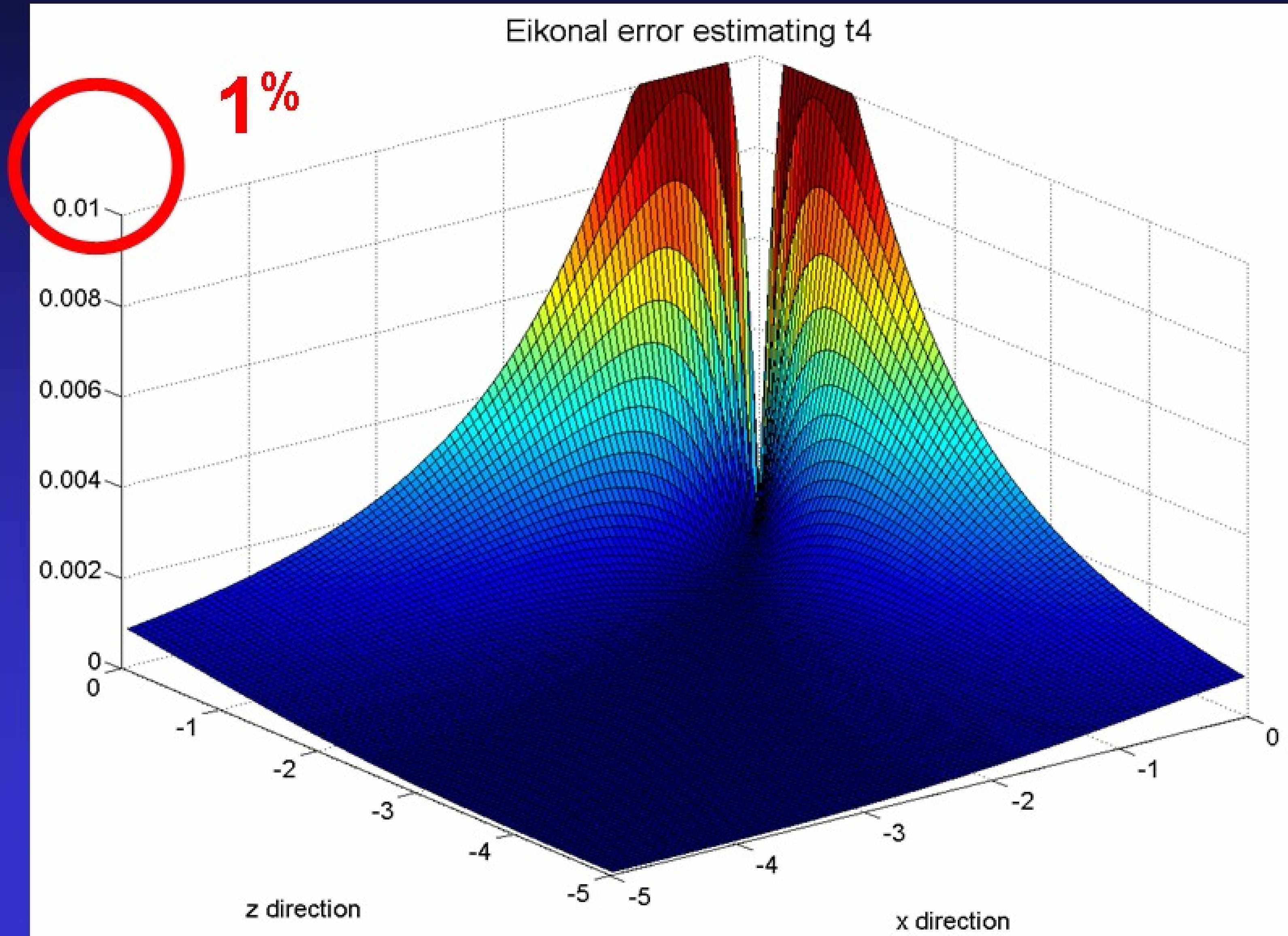
X= -5 to 0

Z = -5 to 0

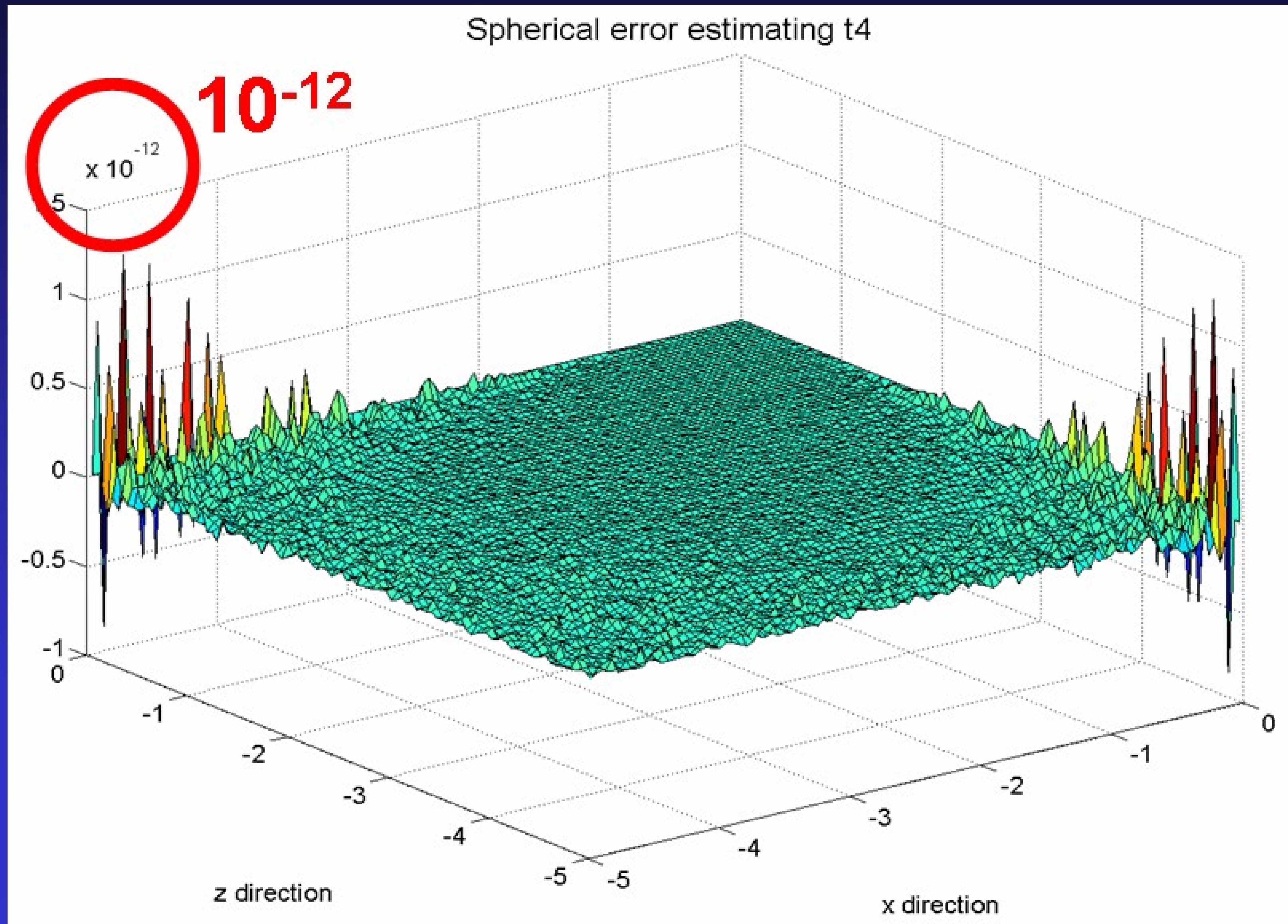
Plane wave solution



Vidale's solution

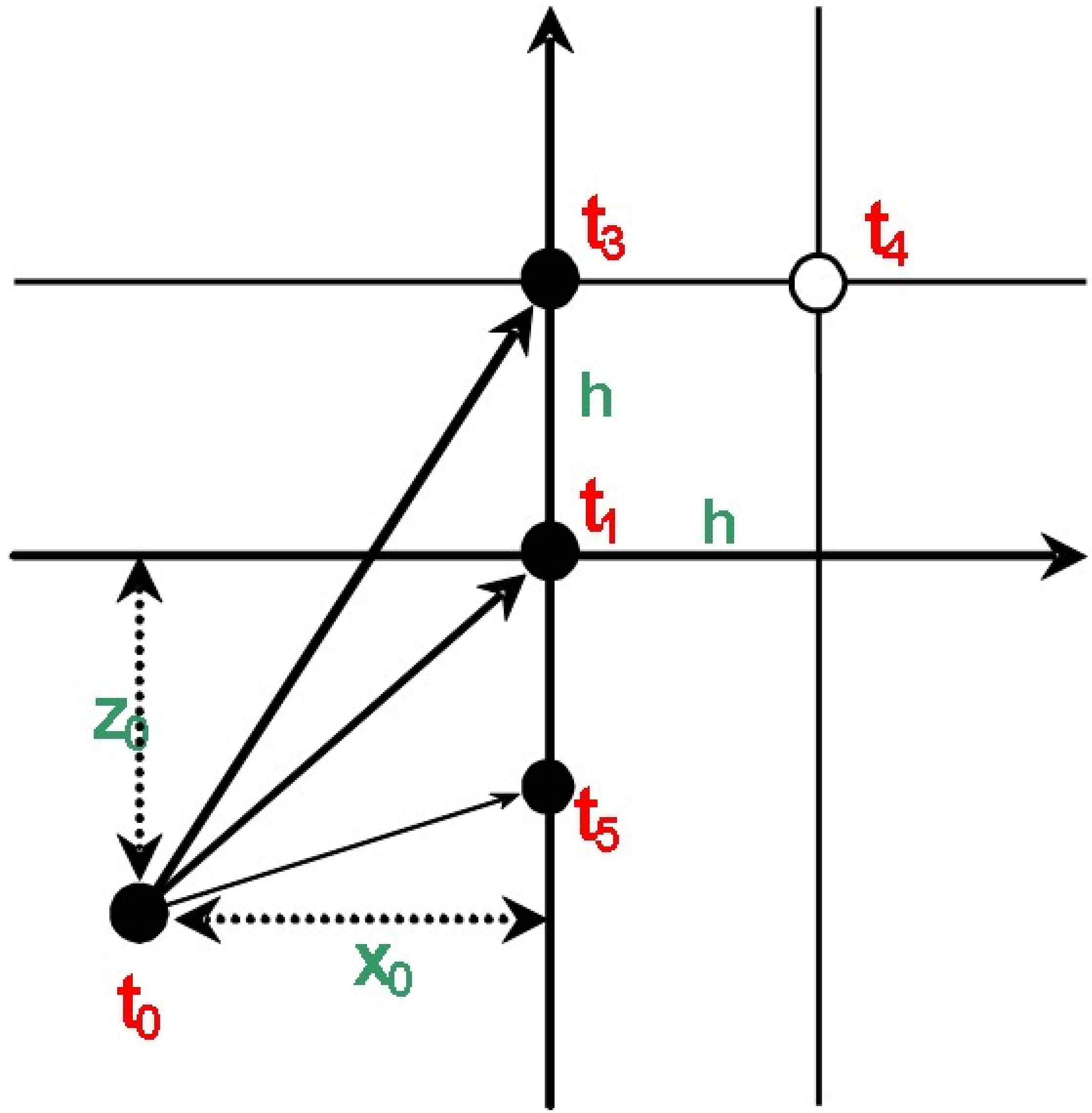


Iterative solution



Alternate solution

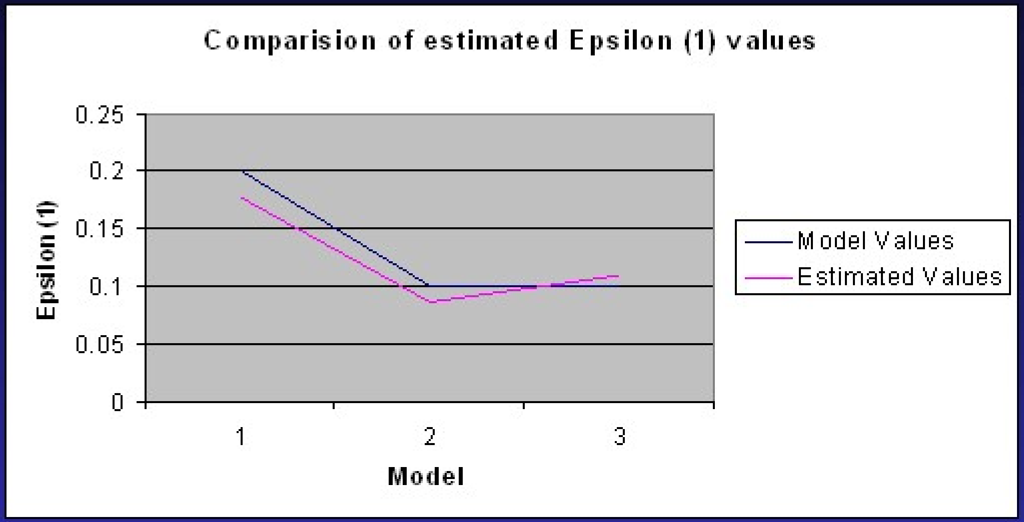
t_0, z_0, x_0

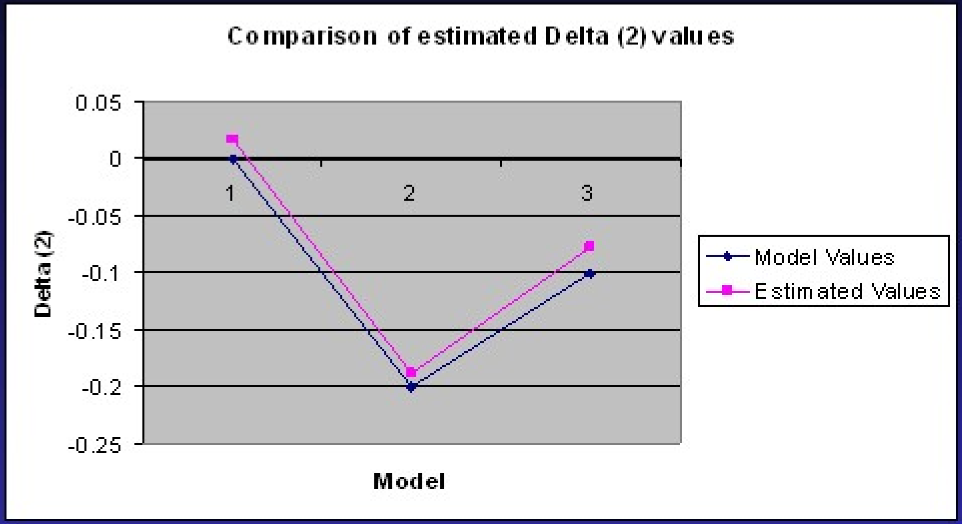


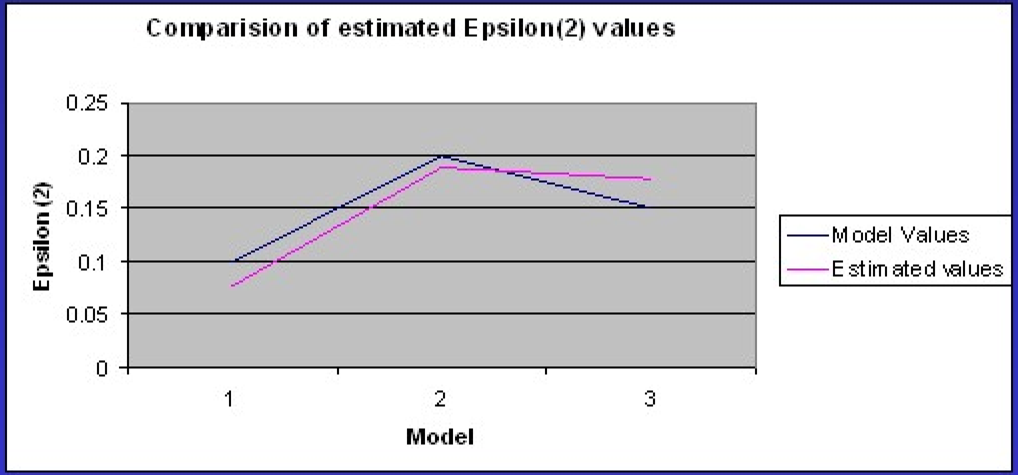
Pavan Elapavuluri:
Estimation of anisotropy parameters

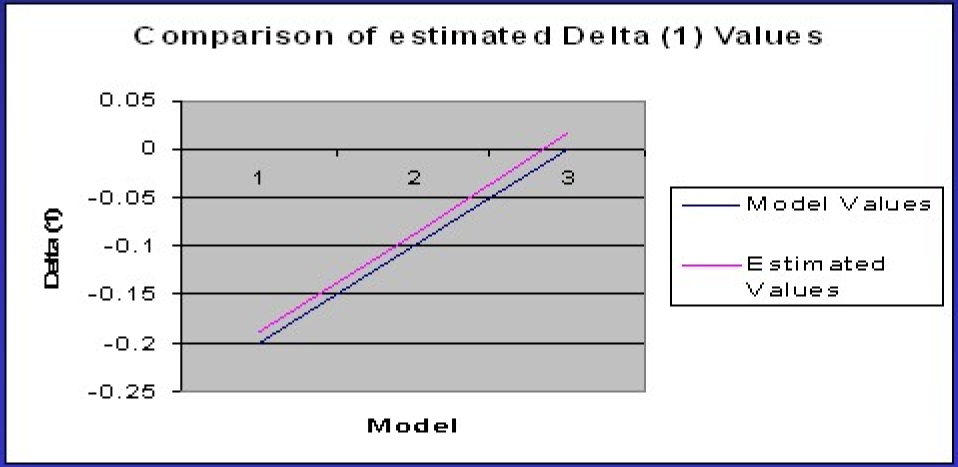
Thanks for the help Pat

Orthorhombic media





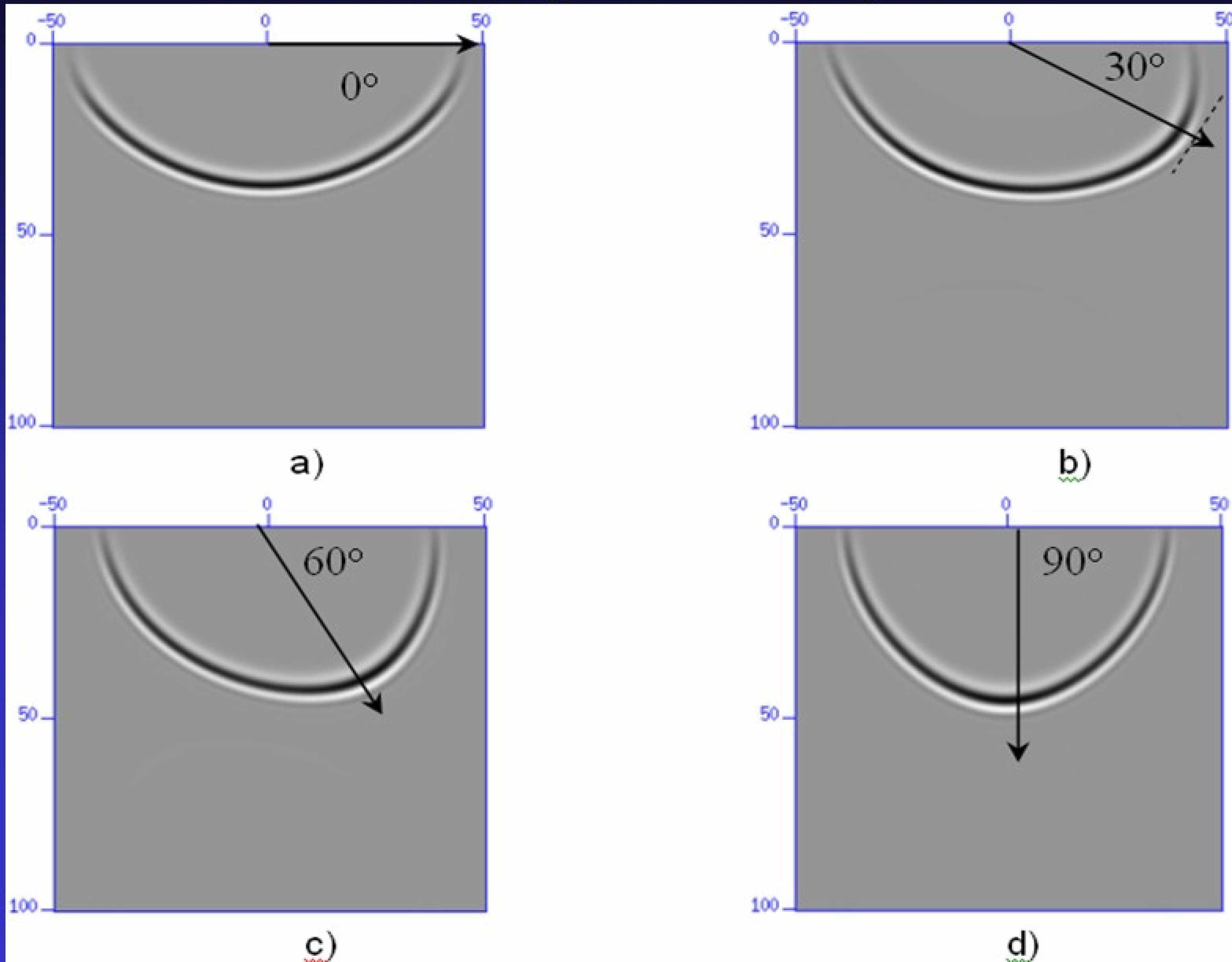




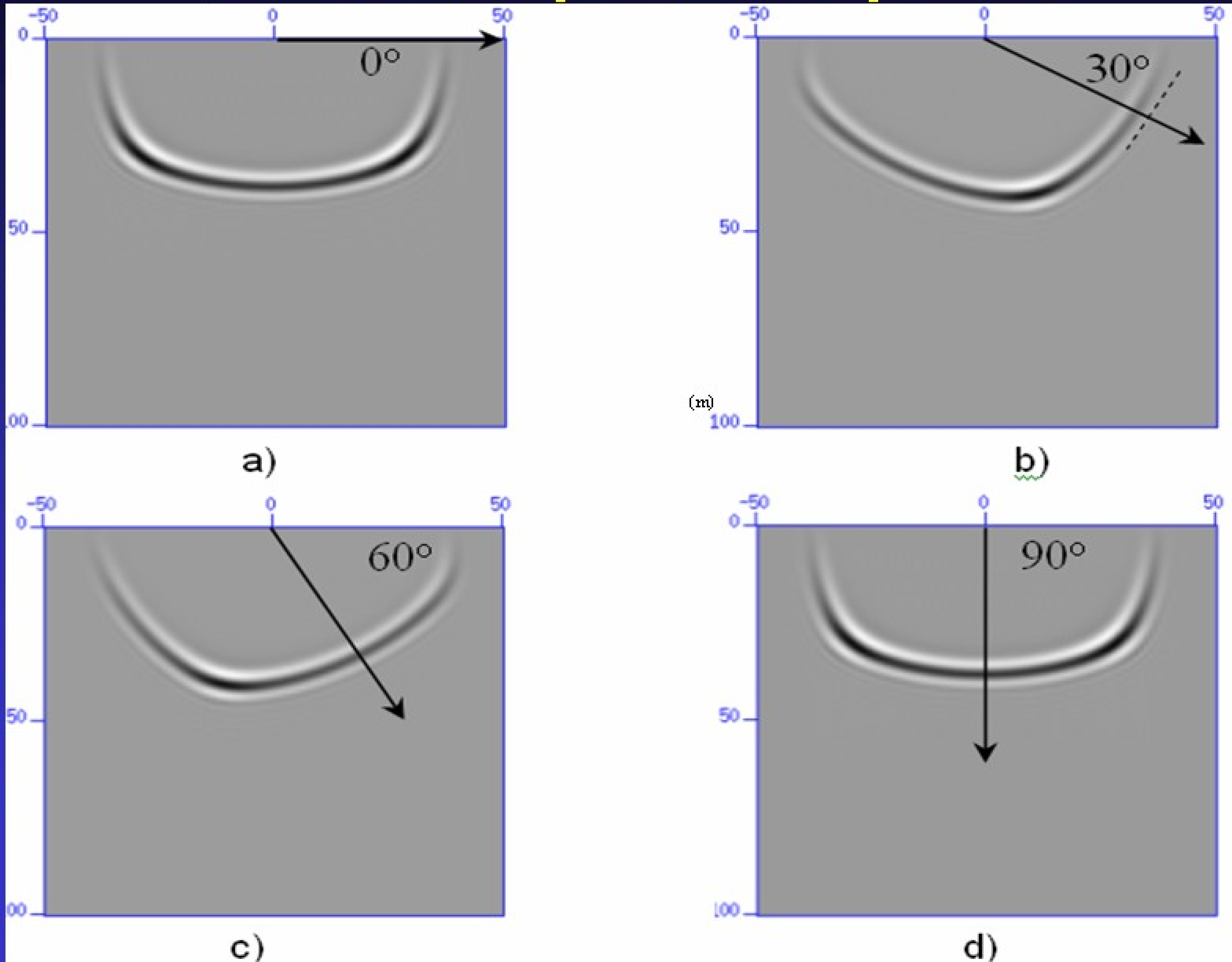
Anisotropic Reverse-Time and PSPI migration for Tilted TI Media

Xiang Du, John C. Bancroft,
Larry R. Lines and Don Lawton

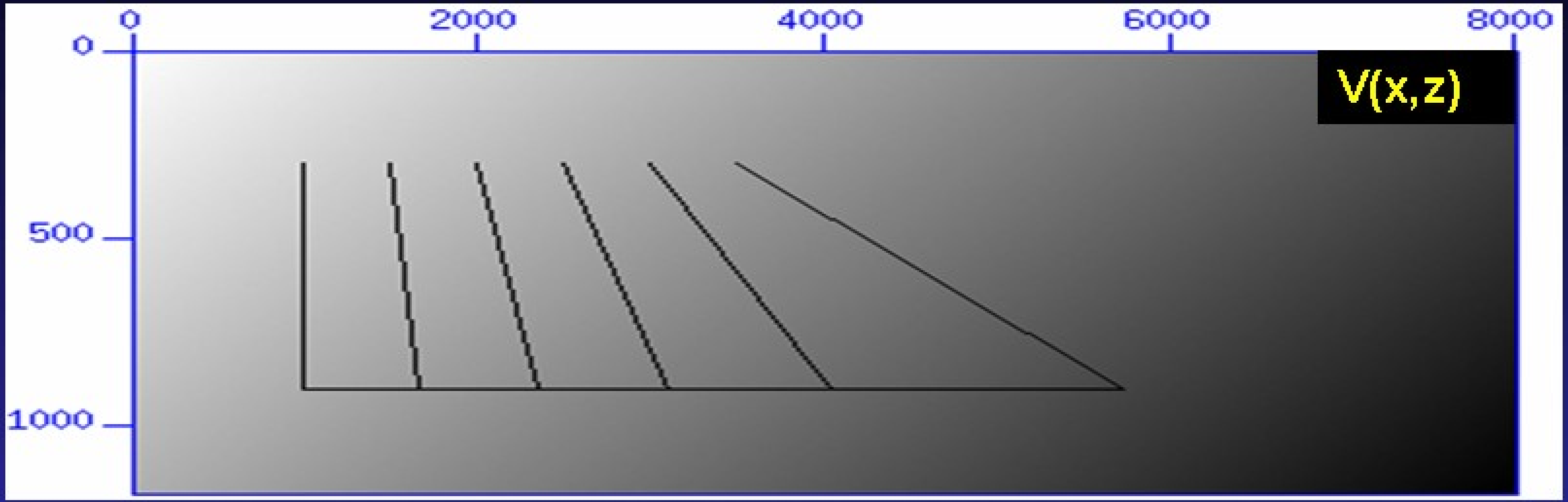
Numerical example: RT migration (x, z)
P wave impulse response

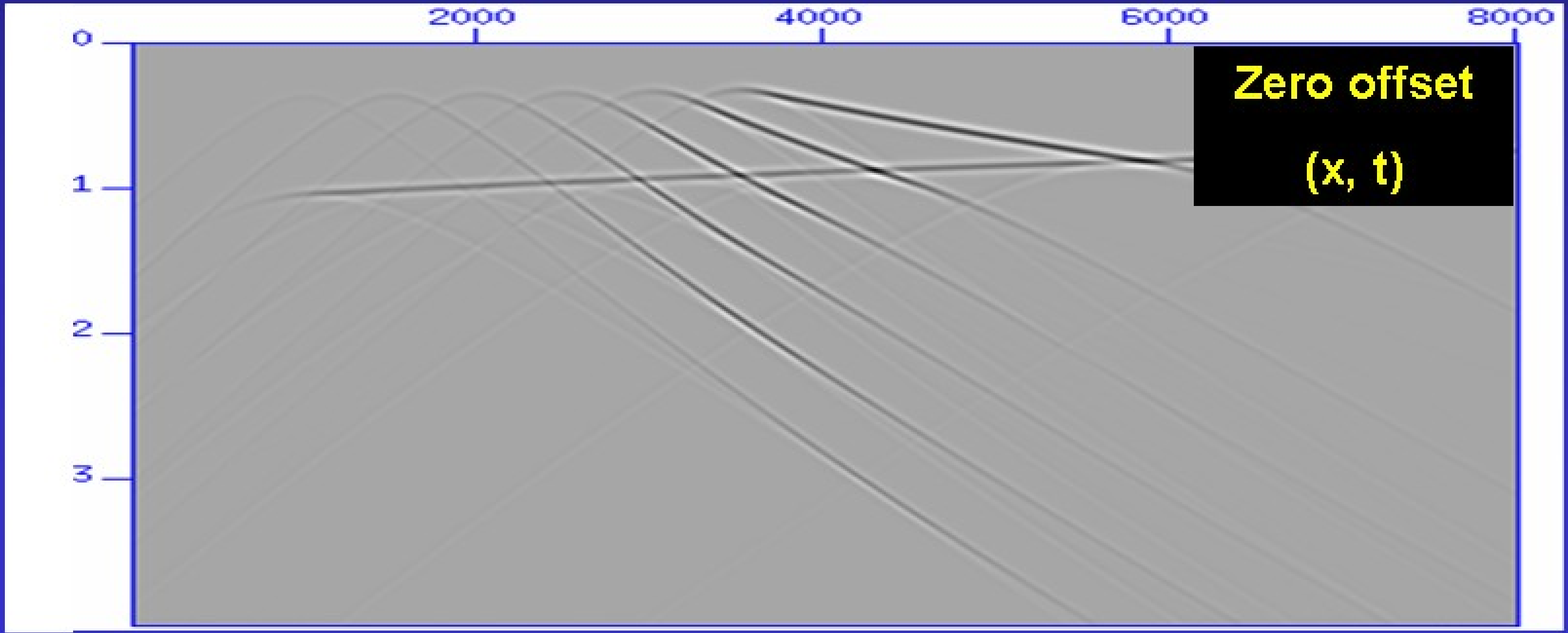


Numerical example: migration (*x*, *z*)
S wave impulse response



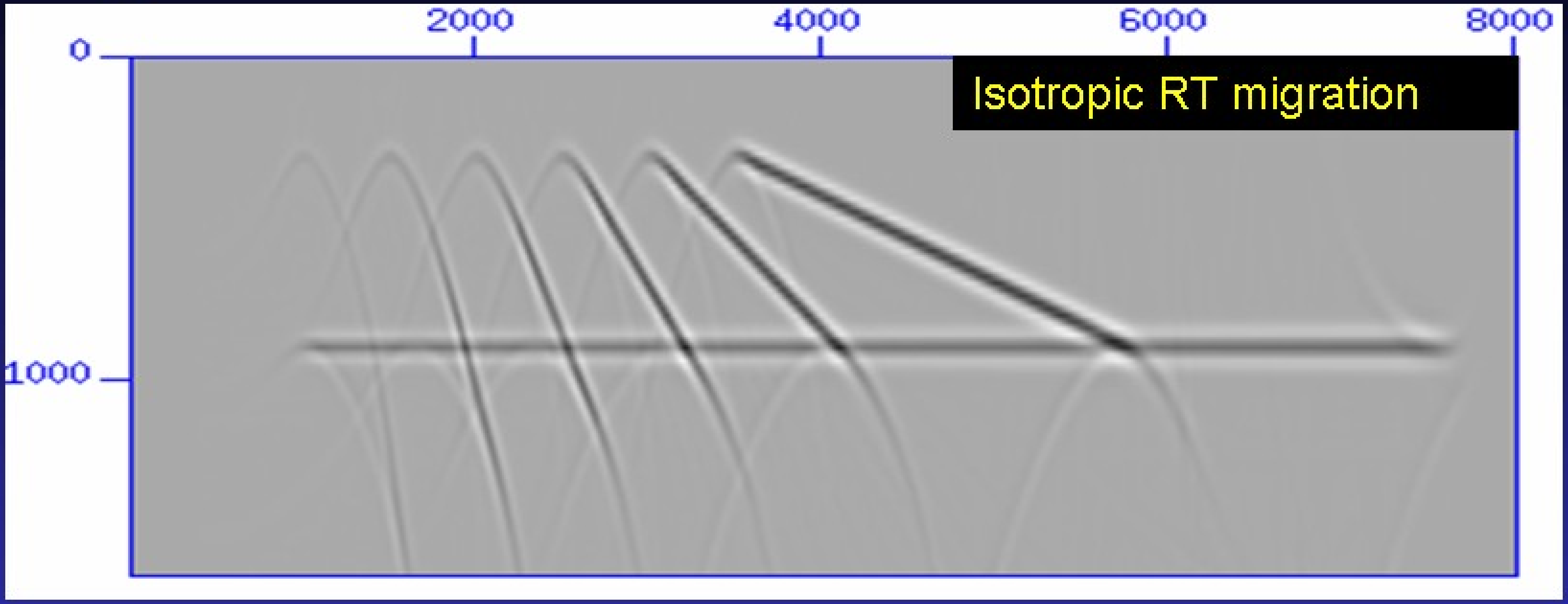
Numerical example: Variable A-velocity model

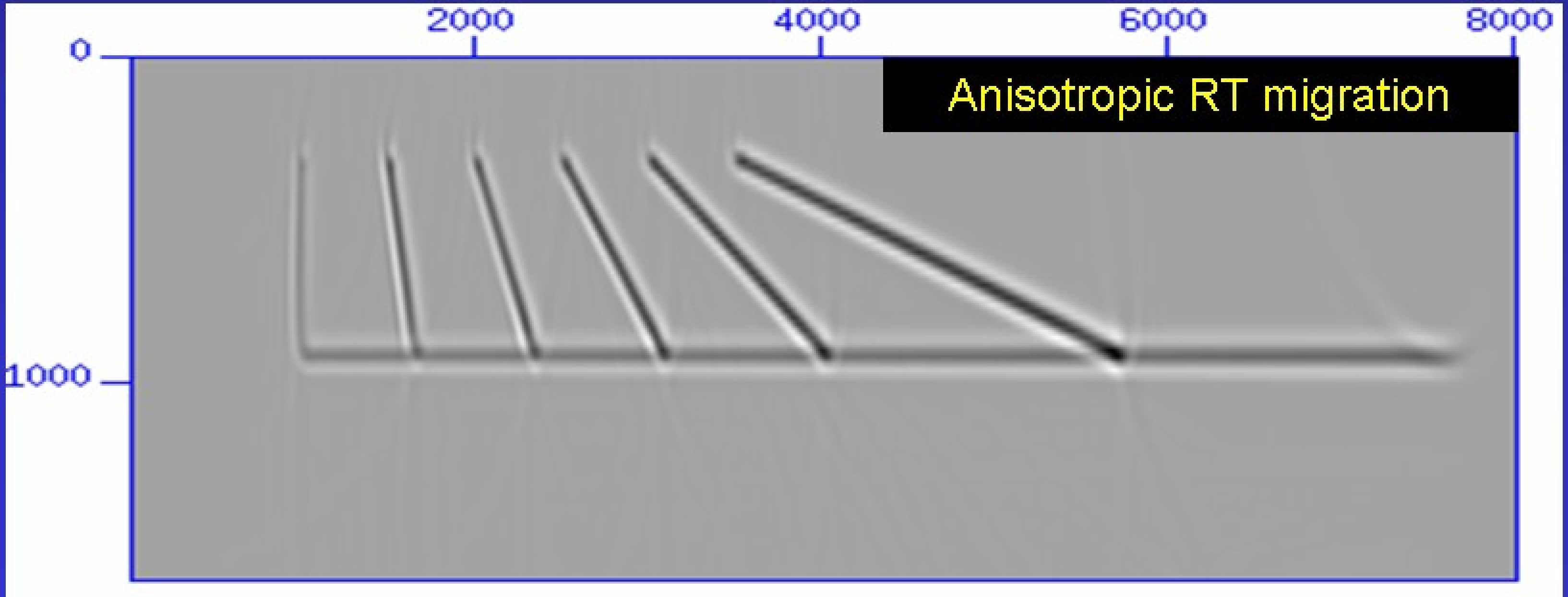




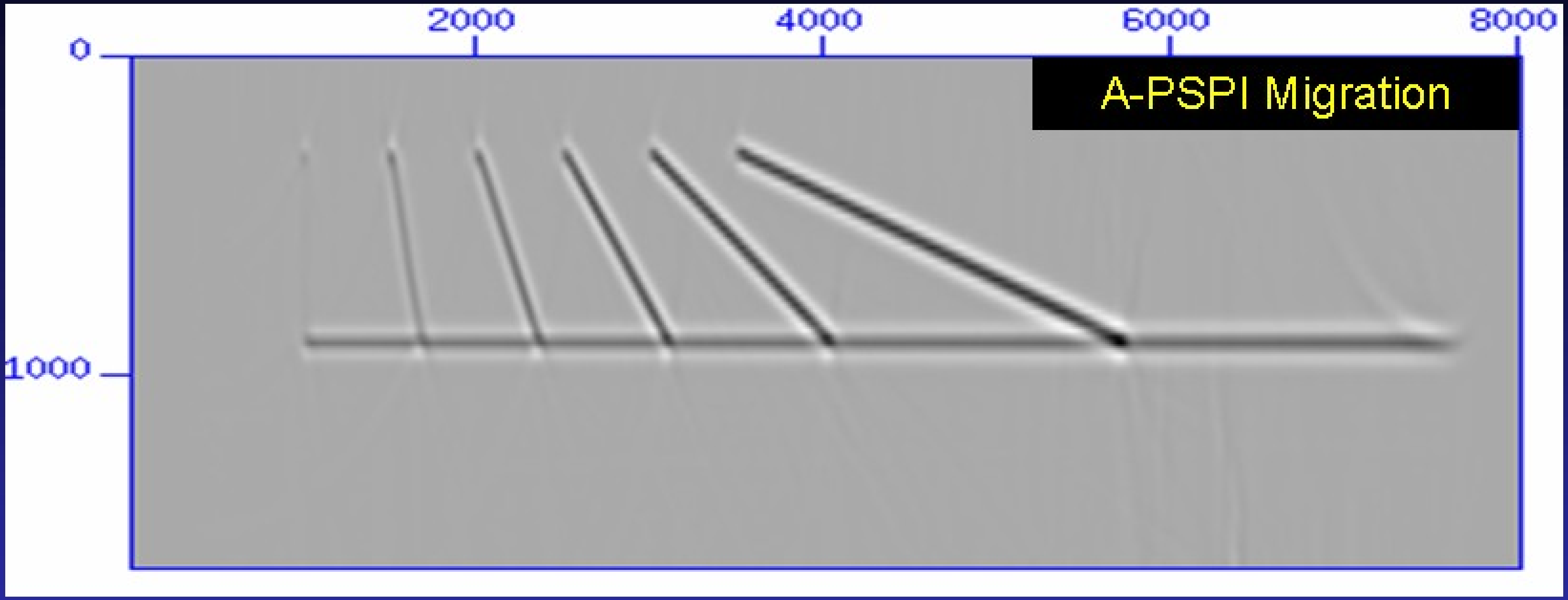
Synthetic zero-offset seismogram obtained using an SU code from Center for Wave Phenomena (CWP).

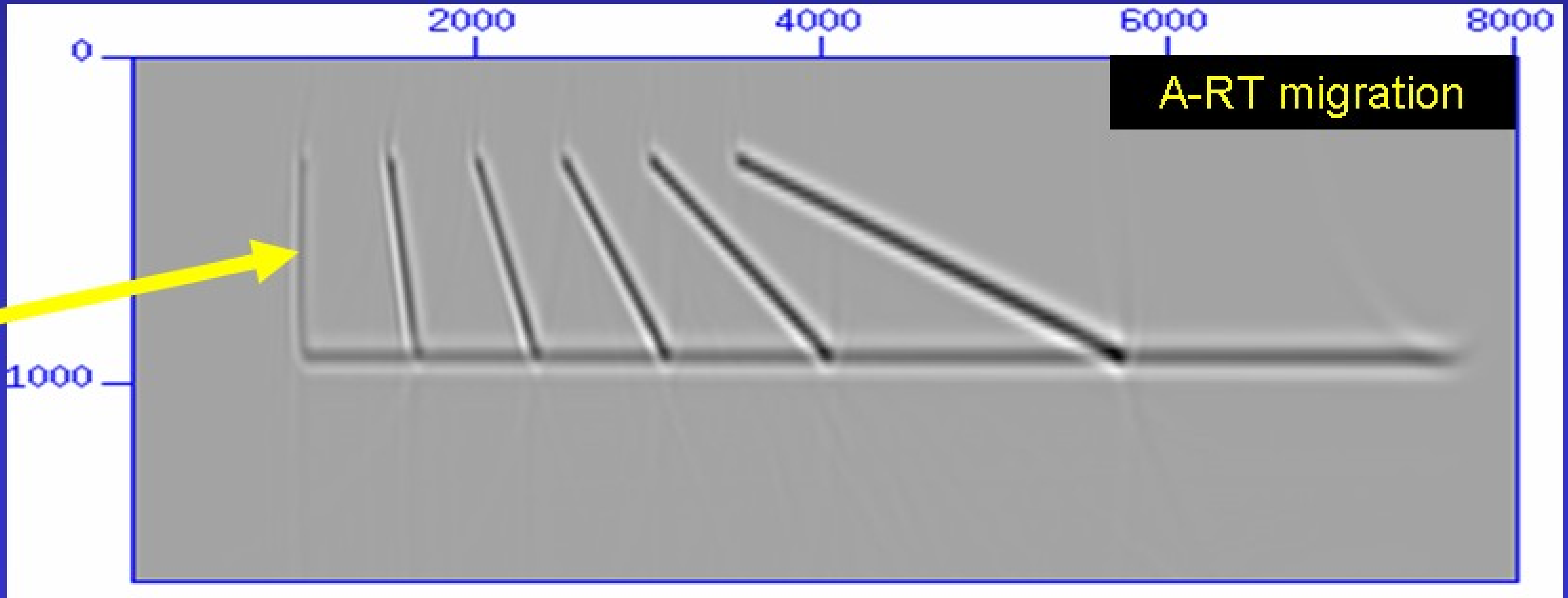
Comparison: Iso. and Aniso. Reverse time mig.



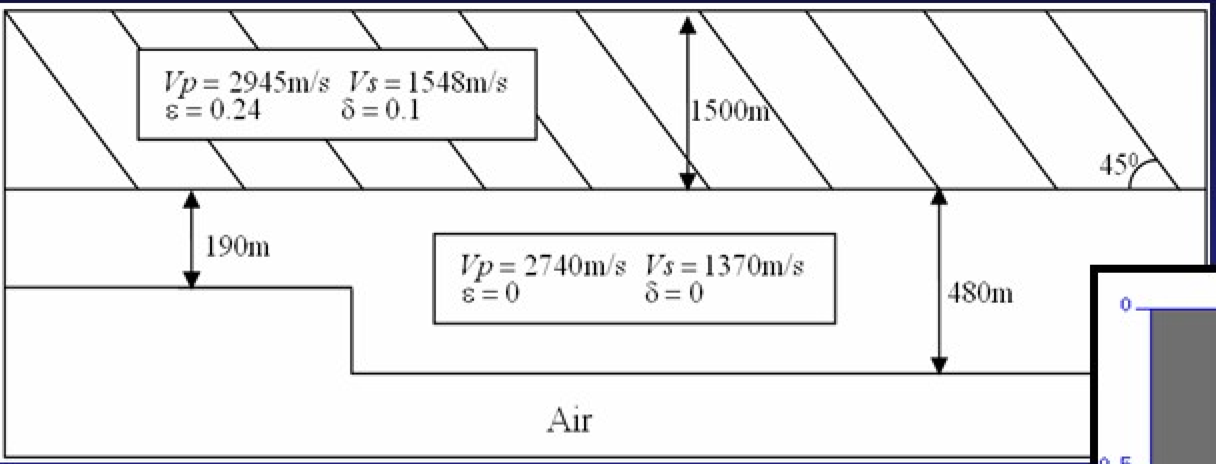


Comparison: A-PSPI A-RT



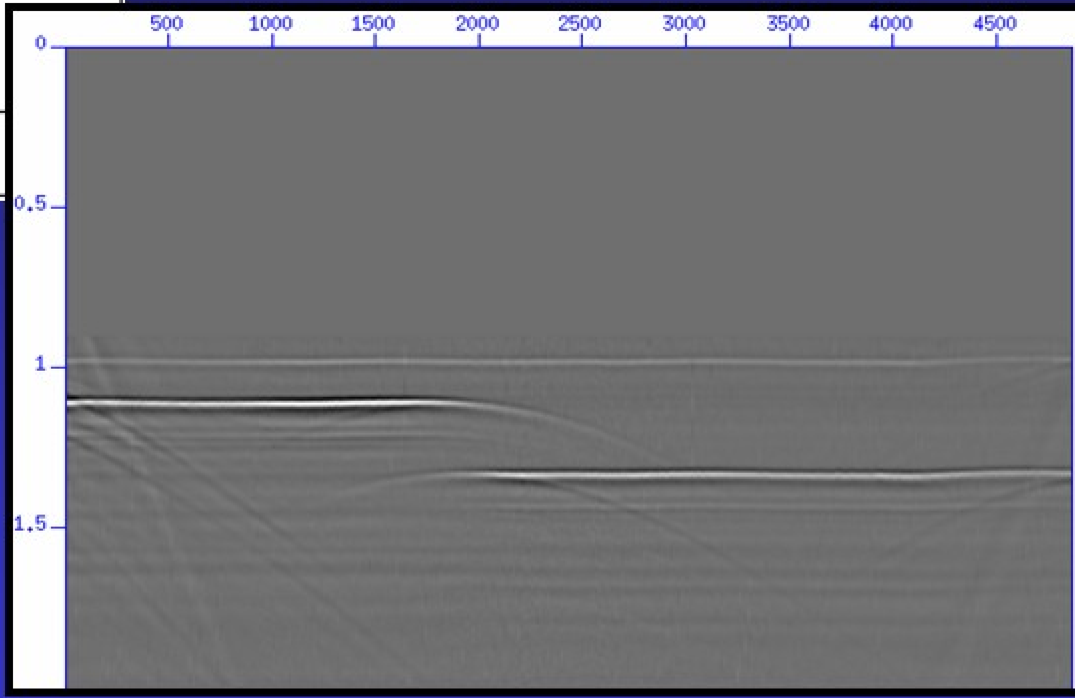


Physical model: reef model with a TTI overburden



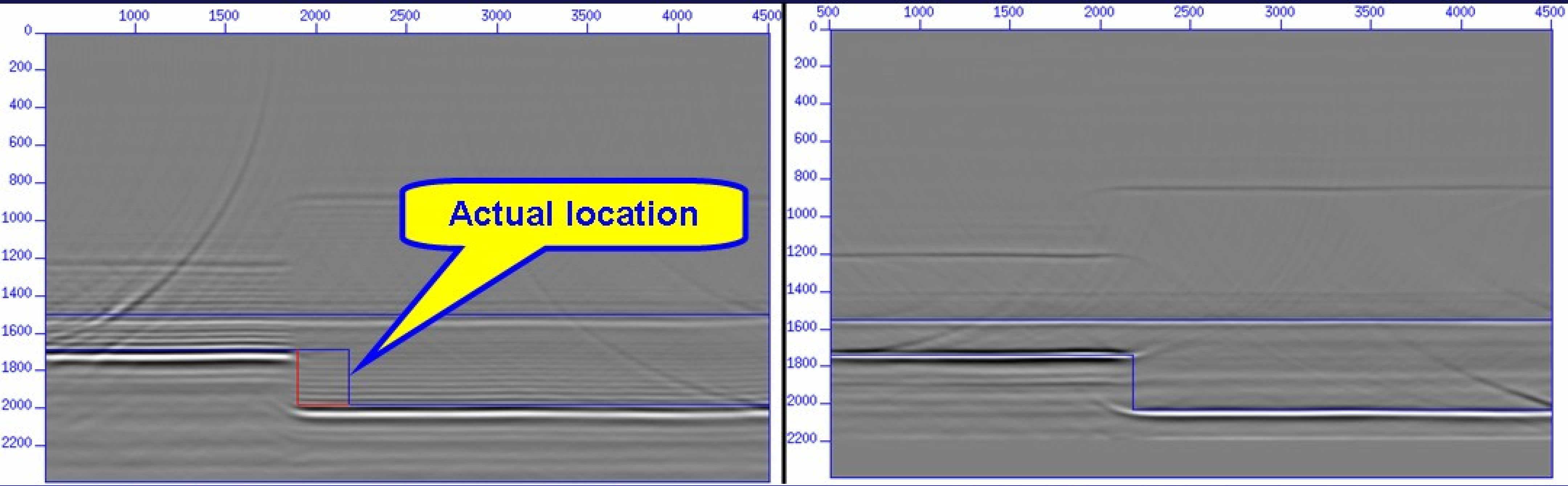
Isotropic “reef” with a TTI overburden

Dr. Don Lawton, Dr. Helen Isaac (models)



Zero-offset seismic section of reef model.

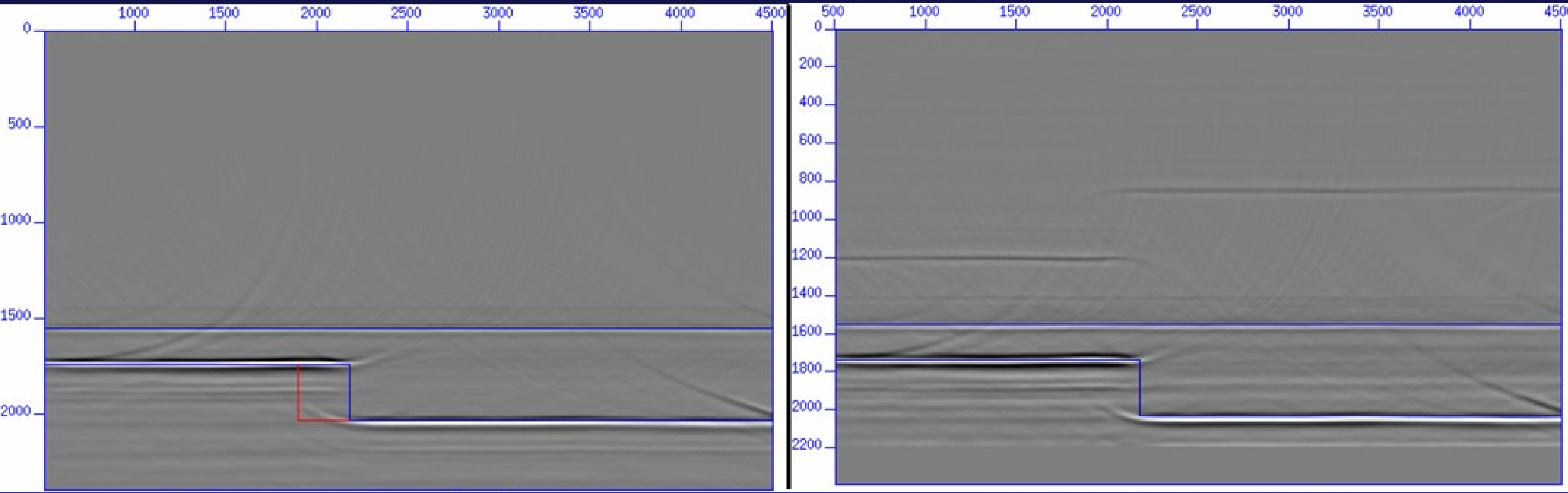
Physical model: Poststack migrations



Isotropic RT migration result of the 6th order accuracy for reef model.

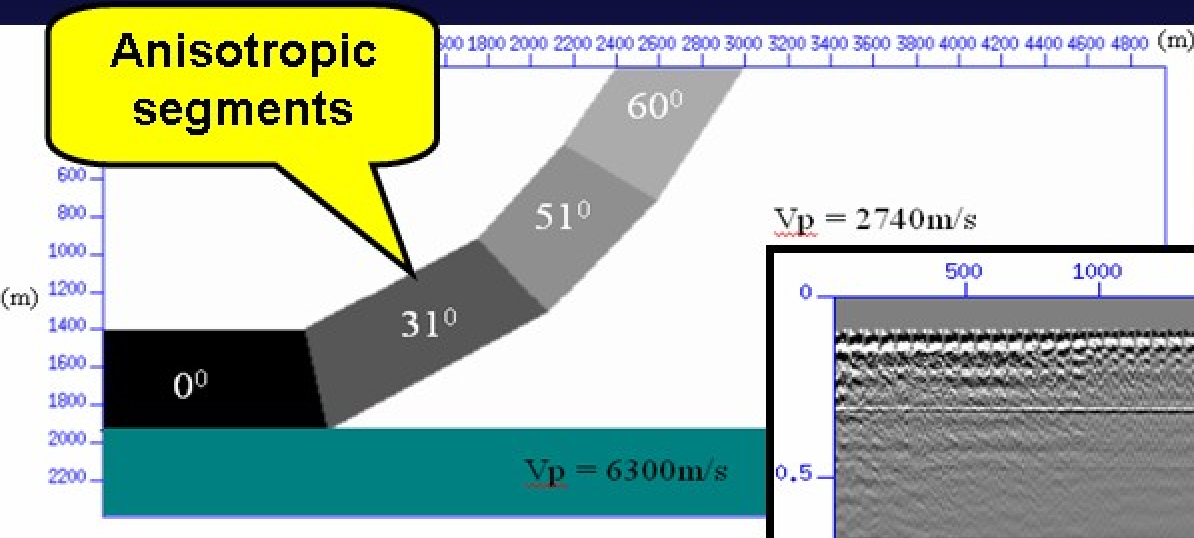
Anisotropic RT migration.

Physical model: Poststack migrations

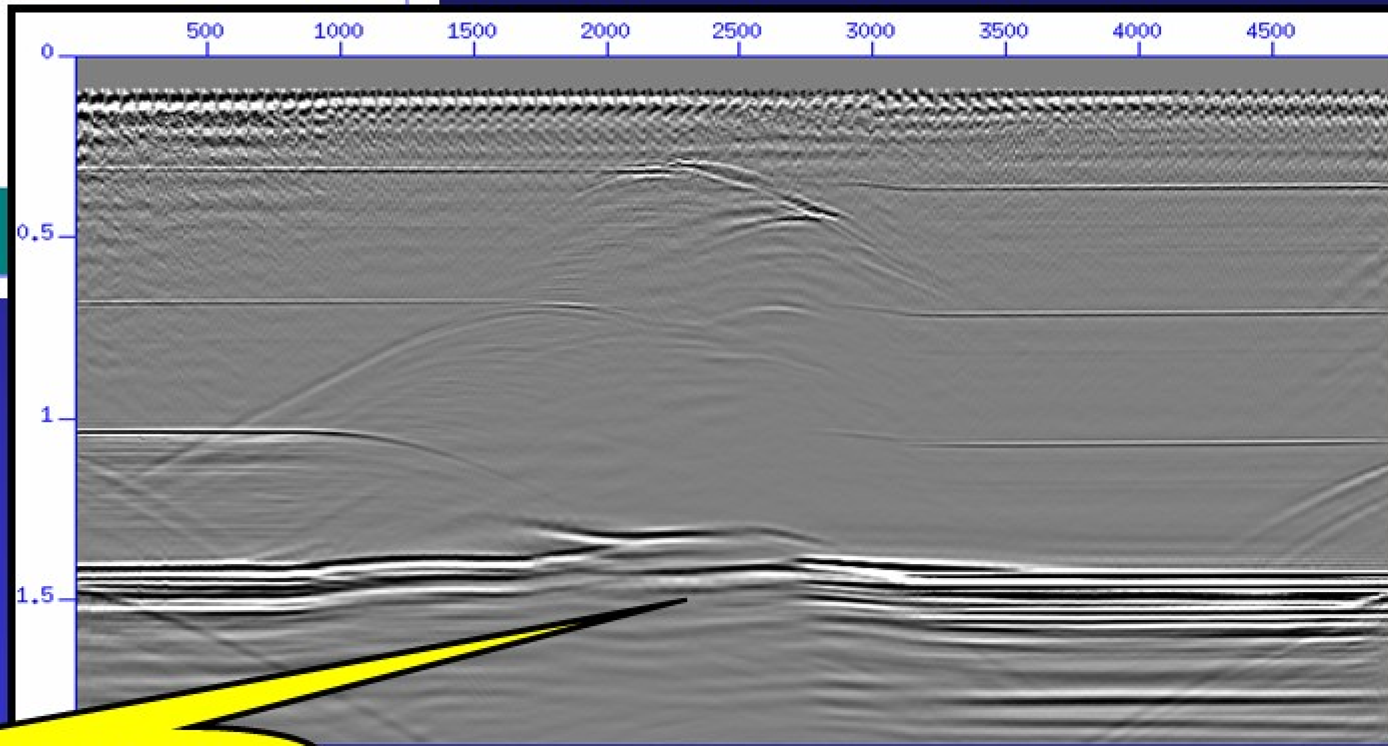


Anisotropic **PSPI** migration.   Anisotropic **RT** migration.

Physical model: Thrust fault



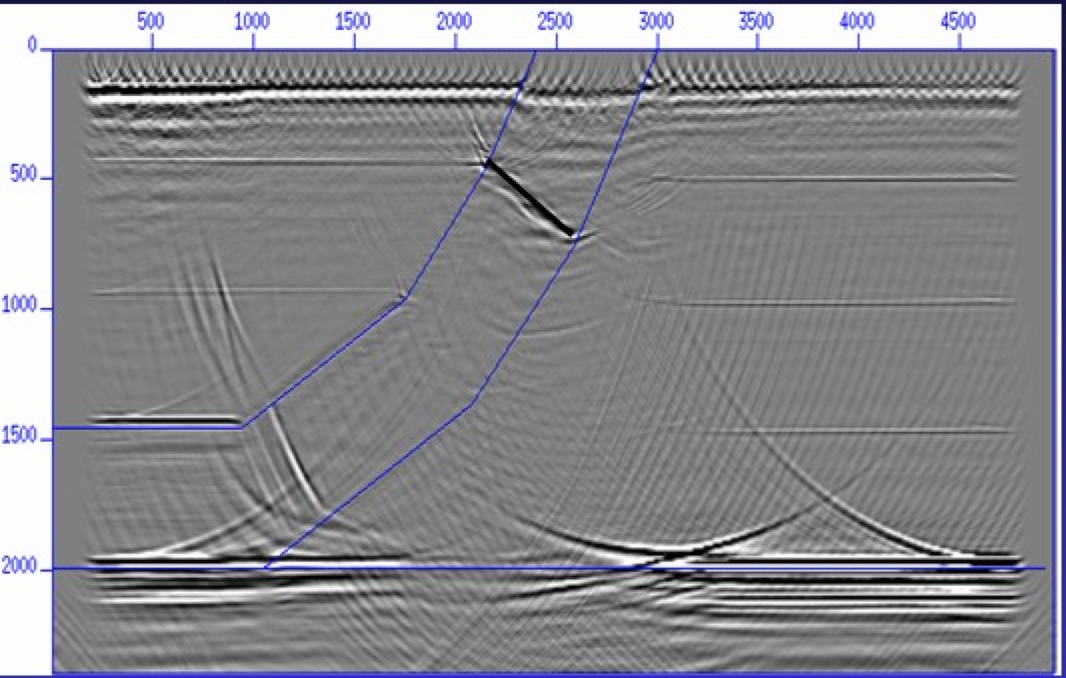
Thrust model.



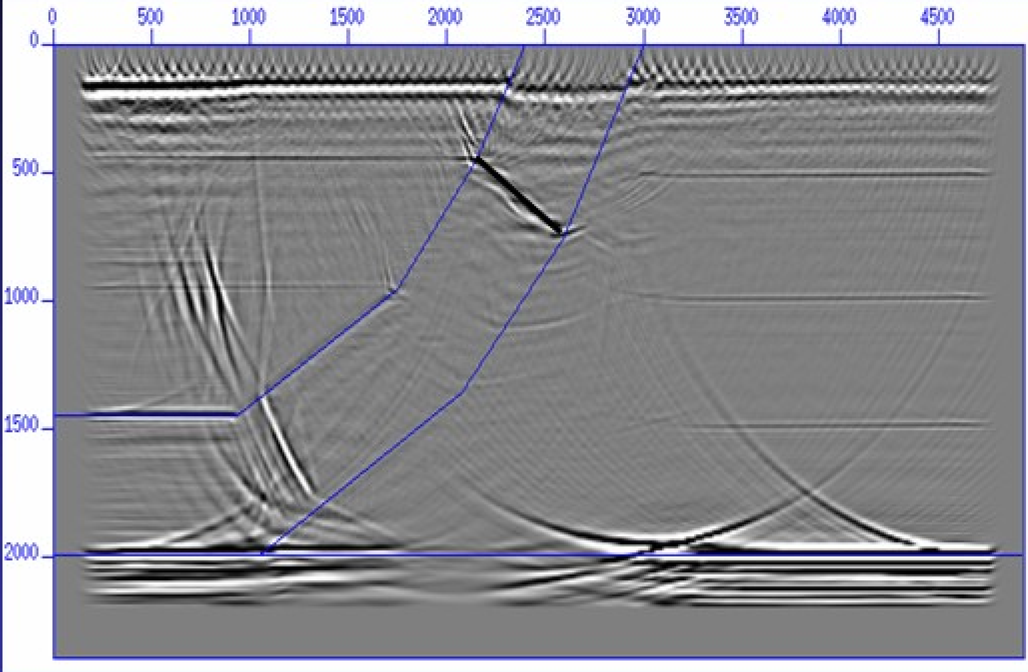
Main area of interest is the apparent pull up

Zero-offset seismic

Physical example: Poststack migration

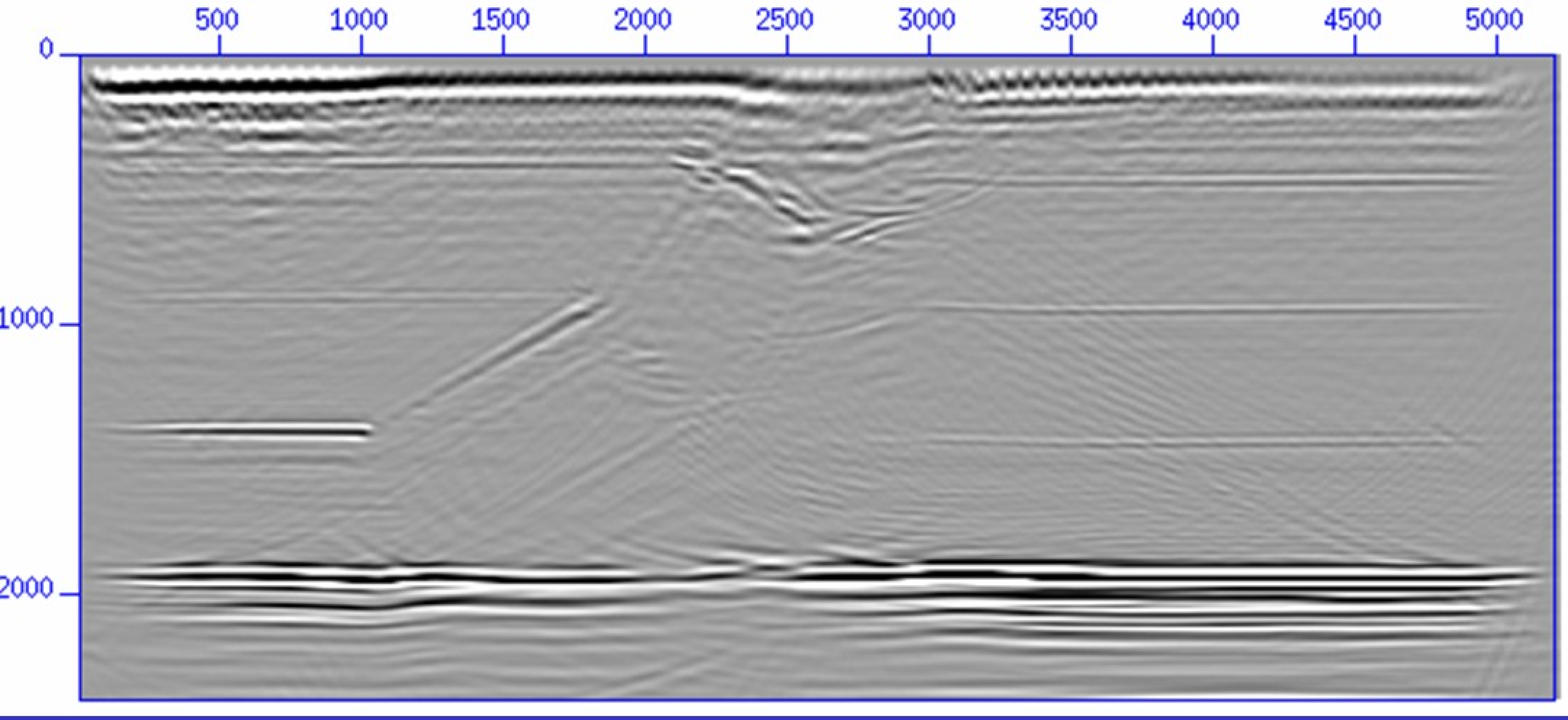


Anisotropic **PSPI** migration .



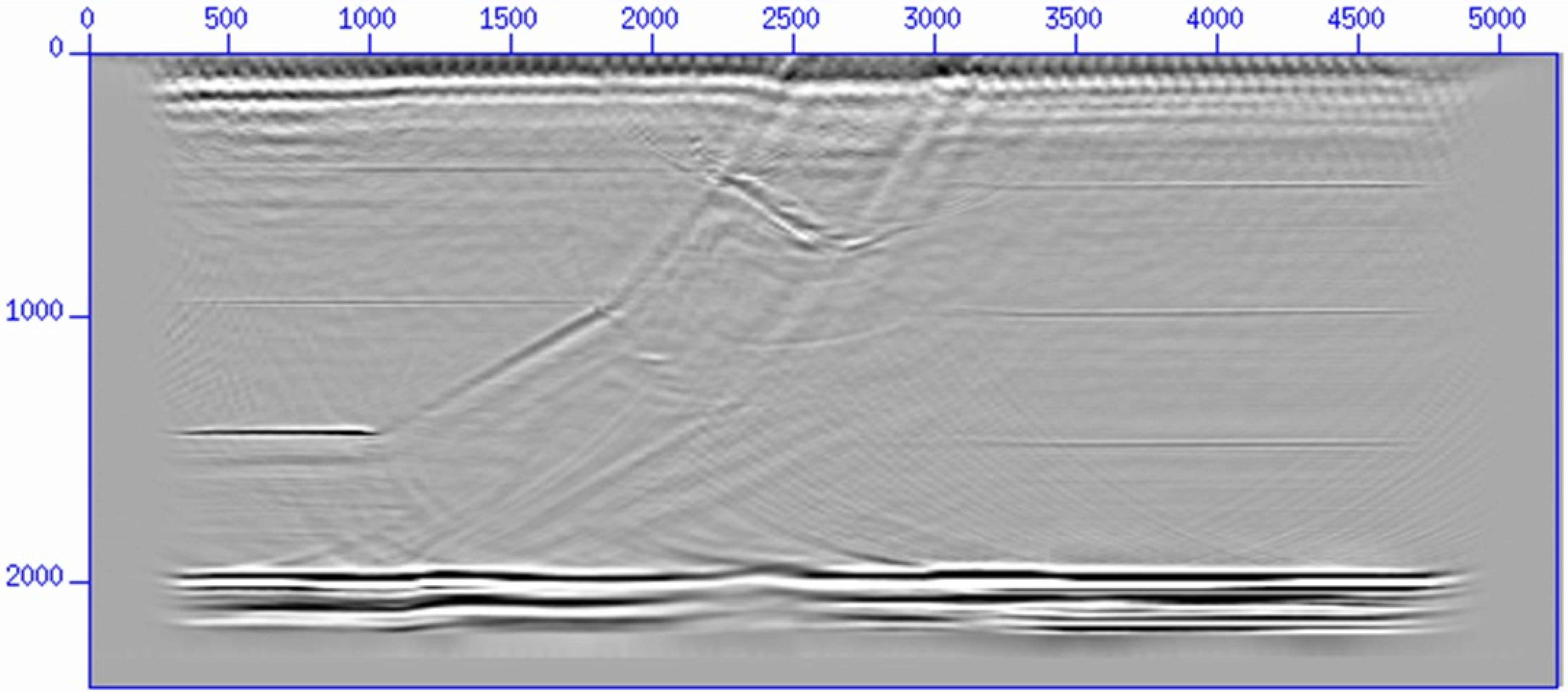
Anisotropic reverse-time migration

Prestack A-PSPI



Prestack **A-PSPI** migration result

Prestack A-RT



Prestack **A-RT** migration result

Some theory: Xiang Du

P and SV phase velocity for TTI media

The VTI phase-velocity equation (Tsvankin, 1996),

$$\frac{V^2(\theta)}{V_{p0}^2} = 1 + \varepsilon \sin^2\theta - \frac{f}{2} \pm \frac{f}{2}\sqrt{\left(1 + \frac{2\varepsilon \sin^2\theta}{f}\right)^2 - \frac{2(\varepsilon - \delta)\sin^2 2\theta}{f}}$$

When we rotate the symmetry axis from vertical to a tilt angle ϕ, the phase velocity in the direction measured from the vertical direction becomes:

$$\frac{V^2(\theta,\phi)}{V_{p0}^2} = 1 + \varepsilon \sin^2(\theta - \phi) - \frac{f}{2} \pm \frac{f}{2}\sqrt{\left(1 + \frac{2\varepsilon \sin^2(\theta - \phi)}{f}\right)^2 - \frac{2(\varepsilon - \delta)\sin^2 2(\theta - \phi)}{f}}$$

Simplified P and SV phase velocity for TTI media

Using a weak anisotropy assumption, the familiar linear approximation can be formed:

$$\frac{V_p^2(\theta,\phi)}{V_{p0}^2} = 1 + 2\delta\sin^2(\theta-\phi)\cos^2(\theta-\phi) + 2\varepsilon\sin^4(\theta-\phi)$$

$$\frac{V_s^2(\theta,\phi)}{V_{p0}^2} = 1 - f + 2(\varepsilon-\delta)\sin^2(\theta-\phi)\cos^2(\theta-\phi)$$

Alkhalifah (2000) acoustic wave equation (By setting f=1) for titled angle ϕ as

$$\frac{V_p^2(\theta,\phi)}{V_{p0}^2} = \frac{1}{2} + \varepsilon\sin^2(\theta-\phi) + \frac{1}{2}\sqrt{(1+2\varepsilon\sin^2(\theta-\phi))^2 - 2(\varepsilon-\delta)\sin^2 2(\theta-\phi)}$$

However this equation has problems with SV wave aliasing.

Typical solution is to include an isotropic layer near the surface.

(Zhang et al., 2004)

For improved accuracy…

Retaining the quadratic terms of exact formula, we could use:

$$\frac{V_p^2(\theta,\phi)}{V_{p0}^2} = 1 + 2\delta\sin^2(\theta-\phi)\cos^2(\theta-\phi) + 2\varepsilon\sin^4(\theta-\phi)$$

$$+\frac{4}{f}(\varepsilon-\delta)[\varepsilon\sin^2(\theta-\phi)+\delta\cos^2(\theta-\phi)]\sin^4(\theta-\phi)\cos^2(\theta-\phi)$$

$$\frac{V_s^2(\theta,\phi)}{V_{p0}^2} = 1 - f + 2(\varepsilon-\delta)\sin^2(\theta-\phi)\cos^2(\theta-\phi)$$

$$-\frac{4}{f}(\varepsilon-\delta)[\varepsilon\sin^2(\theta-\phi)+\delta\cos^2(\theta-\phi)]\sin^4(\theta-\phi)\cos^2(\theta-\phi)$$

Simplified P and SV wave equation for TTI media (Thomsen's equation with tilt)

Using the above linear relationship and Fourier transform, we can get the simplified P wave equation in wavenumber-time domain:

$$\frac{\partial^2 U_p(k_x, k_z, t)}{\partial t^2} = -V_{p0}^2 [k_x^2 + k_z^2 + (2\delta \sin^2\phi \cos^2\phi + 2\varepsilon \cos^4\phi) \frac{k_x^4}{k_x^2 + k_z^2}$$

$$+ \ldots$$

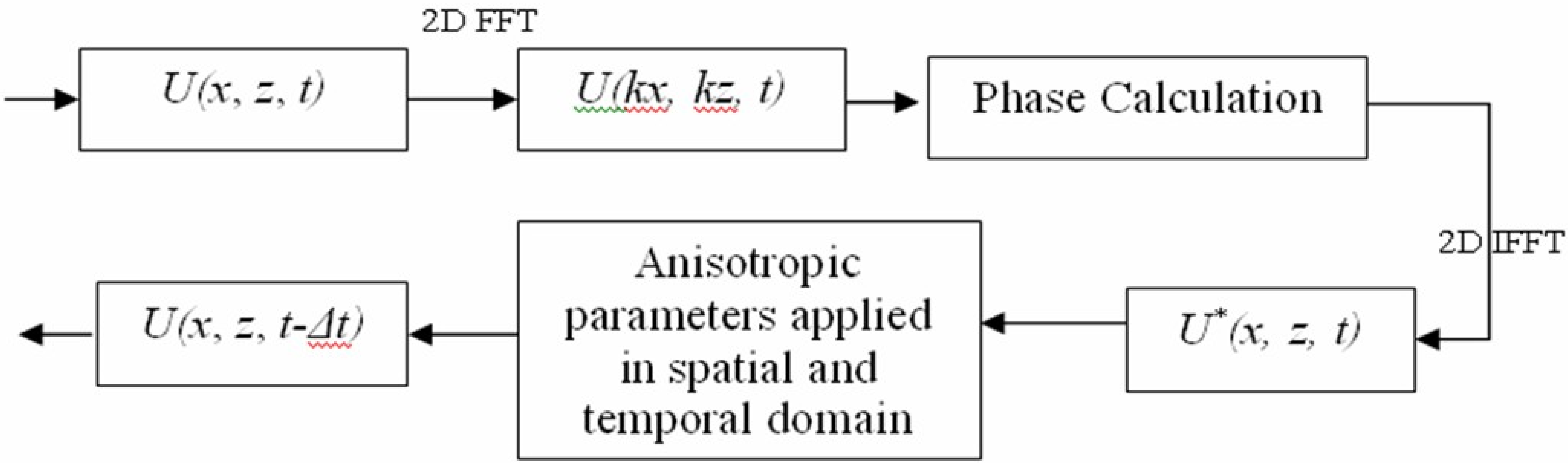
$$+ (-\delta \sin 4\phi - 4\varepsilon \sin 2\phi \sin^2\phi) \frac{k_z^3 k_x}{k_x^2 + k_z^2})] U_p(k_x, k_z, t)$$

Simplified SV wave equation in wavenumber-time domain

$$\frac{\partial^2 U_s(k_x, k_z, t)}{\partial t^2} = -V_{s0}^2 [k_x^2 + k_z^2 + \ldots + 4\varepsilon \sin 2\phi \cos^2\phi) \frac{k_x k_z (k_x^2 - k_z^2)}{k_x^2 + k_z^2}] U_s(k_x, k_z, t)$$

$$\sigma = (\frac{V_{p0}}{V_{s0}})^2 (\varepsilon - \delta)$$

Numerical solution: Rev. time pseudospectral method P and SV wave equations for TTI media



Computational diagram of anisotropic reverse-time scheme corresponding to P- and SV-wave equation

Anisotropic PSPI for TTI media

Considering the frequency-dispersion and phase velocity

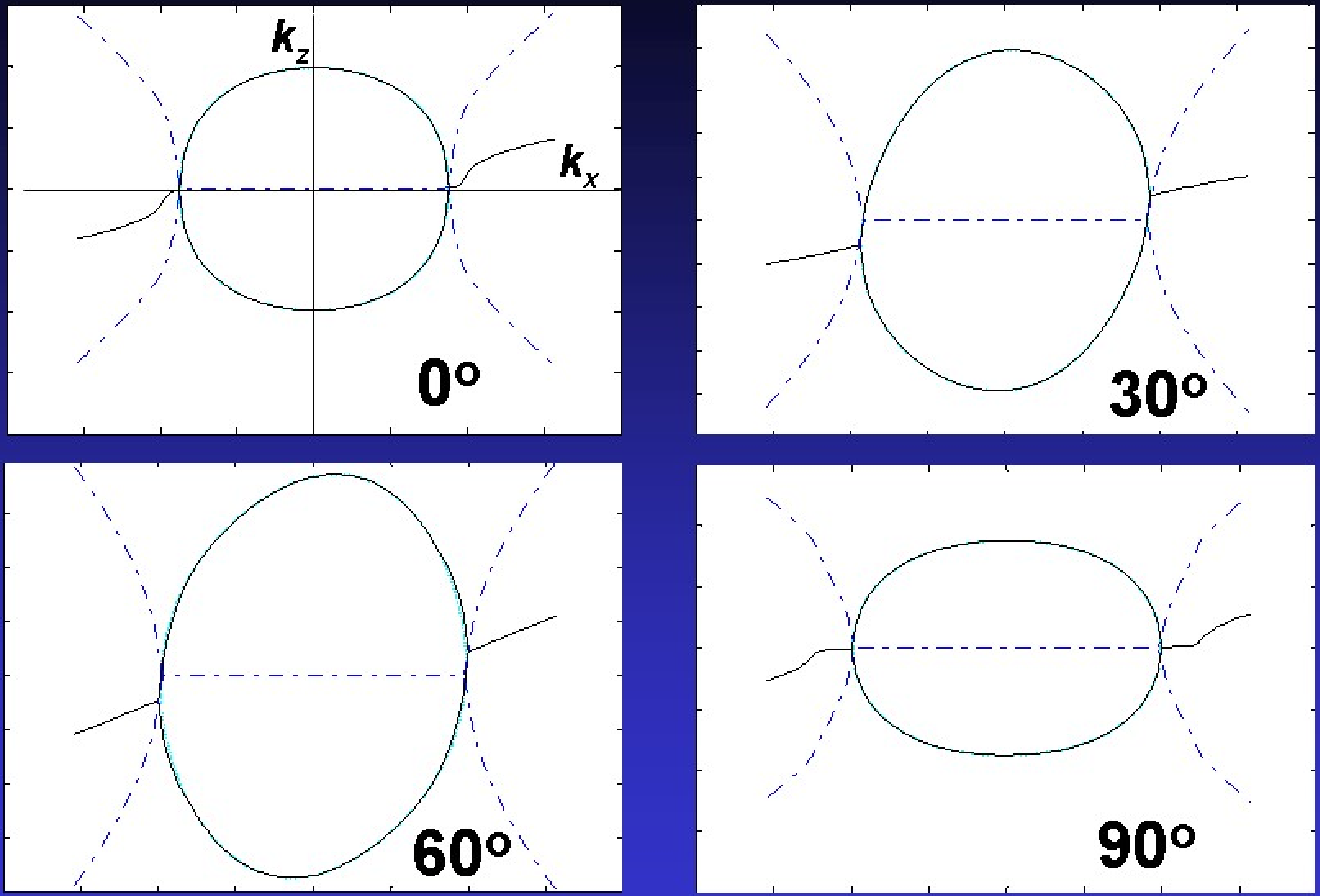
$$k_z = \pm\sqrt{\frac{\omega^2}{v^2(\theta,\phi)} - k_x^2}$$

We can write it as the quartic equation that is different from the table-driven method (1997) and polynomial approximation (2002) .

$$k_z^4 + a_3 k_z^3 + a_2 k_z^2 + a_1 k_z^1 + a_0 = 0$$

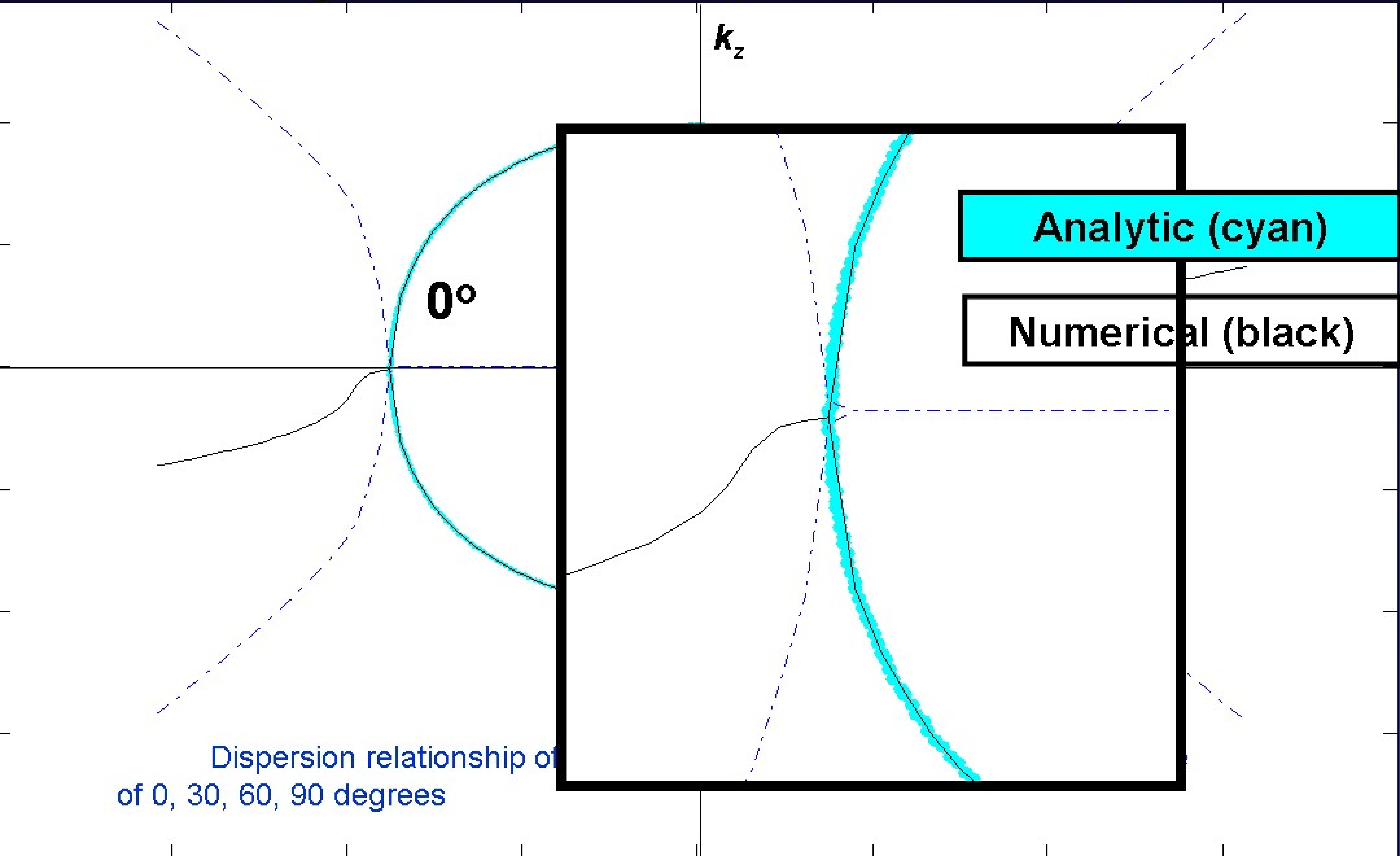
With the quartic equation, we can get the analytic solution for *Kz*

Dispersion relationship *Kx*, *Kz*



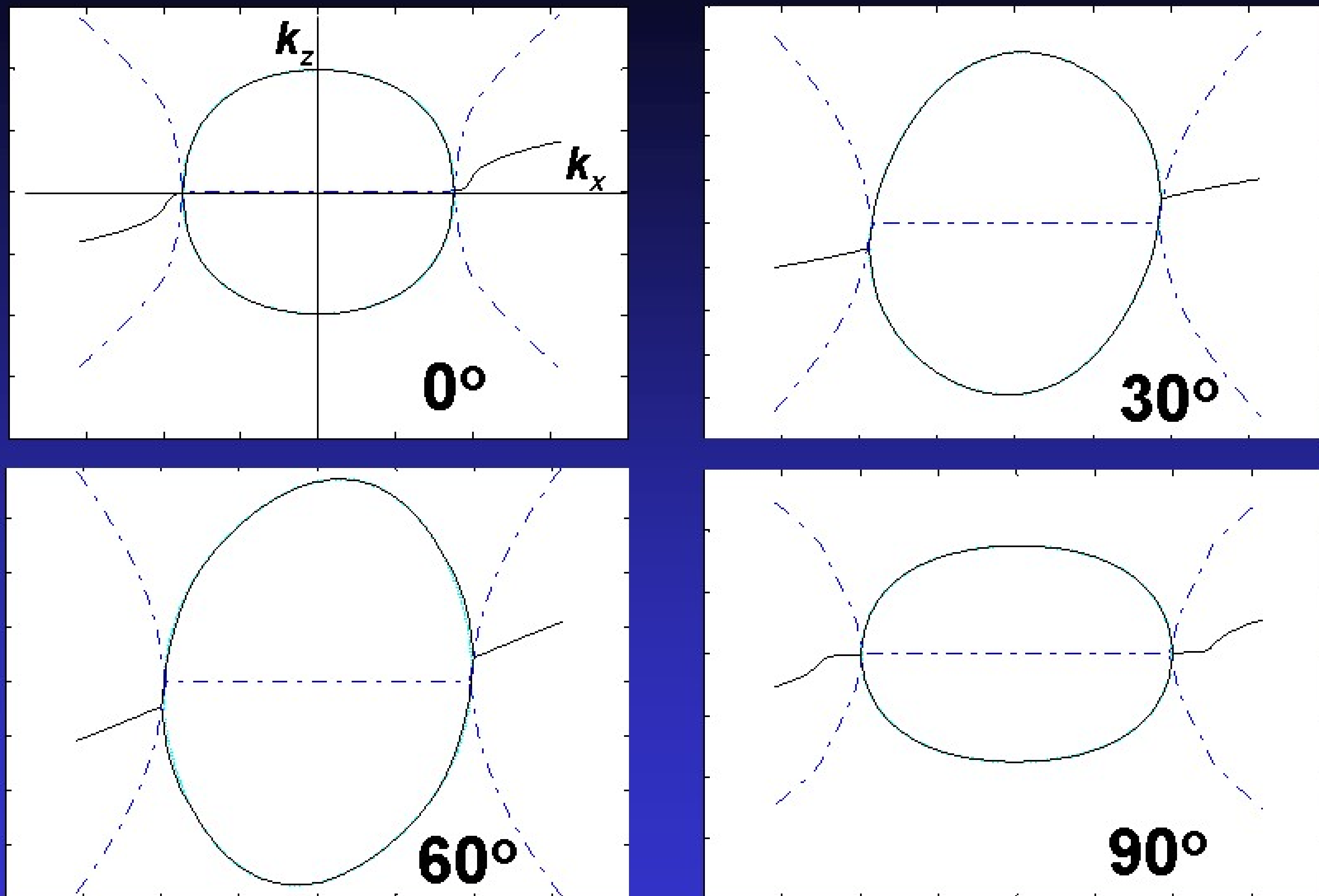
Dispersion relationship of P-wave in a TI medium with tilt angles of 0, 30, 60, 90 degrees

Analytical and Numerical solution for *Kz*



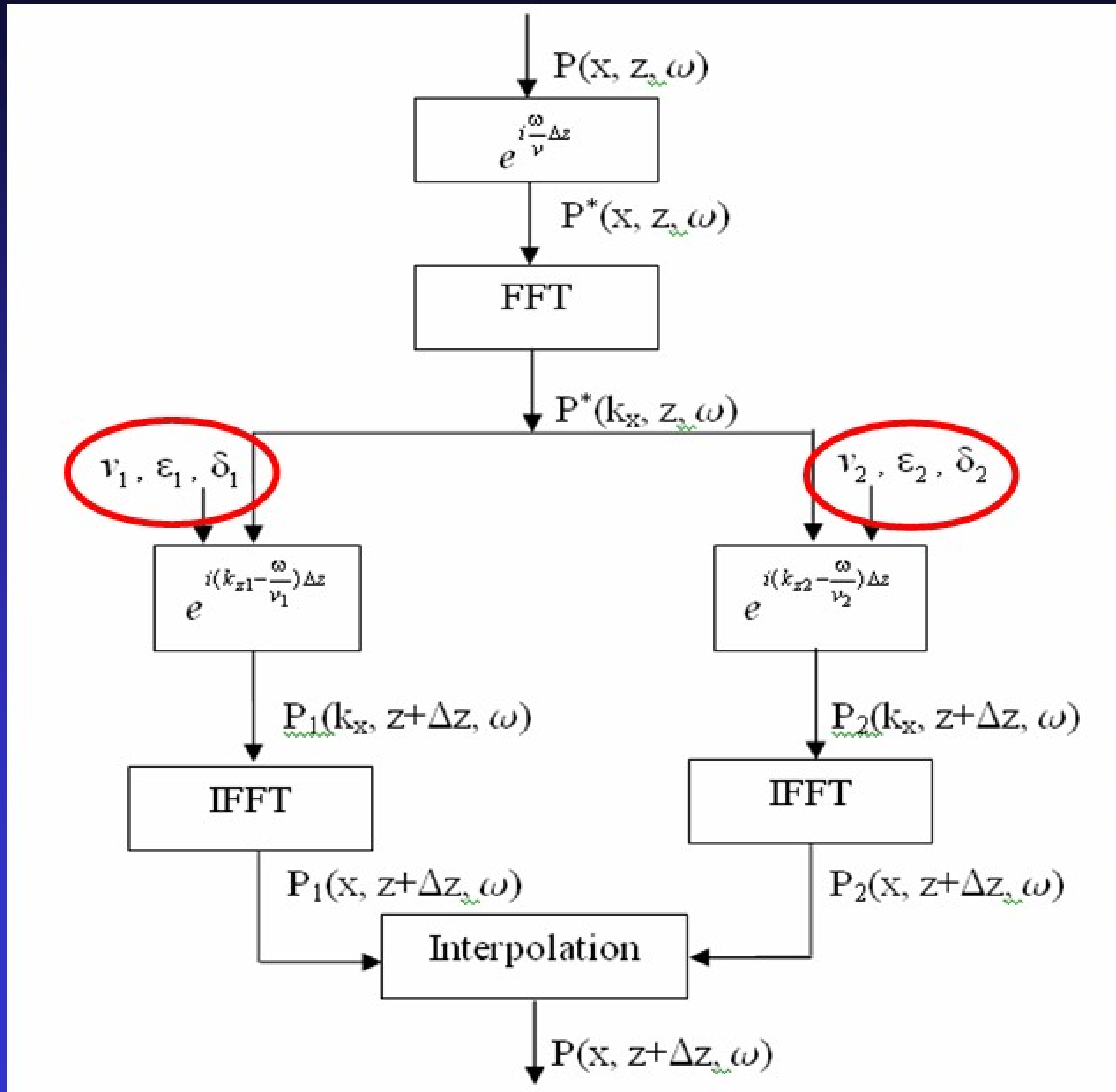
Dispersion relationship of
of 0, 30, 60, 90 degrees

Dispersion relationship *Kx*, *Kz*



Dispersion relationship of P-wave in a TI medium with tilt angles of 0, 30, 60, 90 degrees

Computational diagram of Anisotropic PSPI extrapolation scheme



Conclusions (Du)

- Wave-equations for P- and SV-waves were obtained for use in TTI anisotropy reverse-time migration.
- The pseudo-spectral method was easier to solve for P- and SV-wave equations implementing reverse-time migration.
- Analytical solution for *Kz* is employed for A-PSPI
- Anisotropic PSPI makes a good balance between efficiency and accuracy.
- Anisotropic reverse time migration shows excellent steep angle imaging.

The $\beta\upsilon\tau\tau$ stops here