Linearized AVO and poroelasticity

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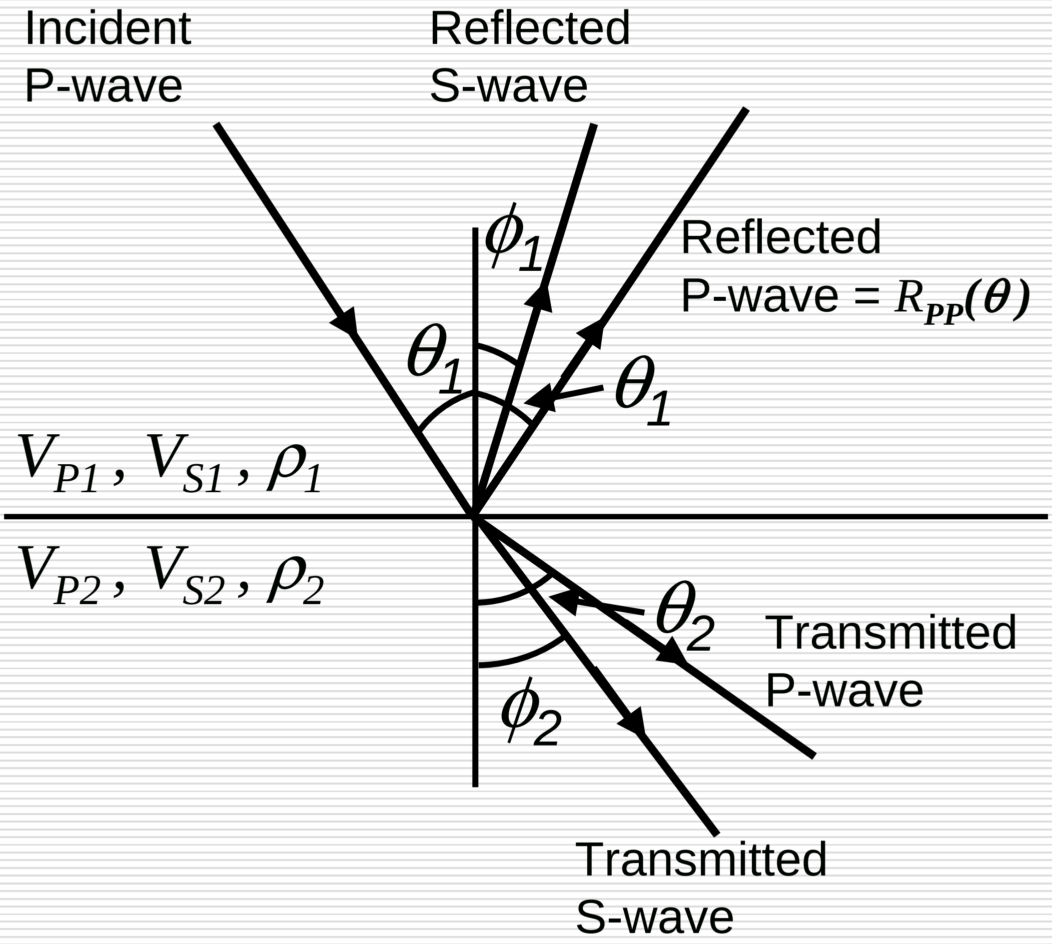
Mode Conversion of an Incident *P*-wave



Consider an interface between two different geological formations, shown on the left.

An incident *P*-wave on the boundary produces *P* and *S* reflected and transmitted waves.

This is called ***mode conversion***, and we wish to compute the amplitudes of each ray.

Linearized approximations to Zoeppritz

- Zoeppritz (1919) solved for the amplitudes of the reflected and transmitted waves, giving a set of four equations with four unknowns.
- Various authors have derived linearized approximations to the Zoeppritz equations which involve the sum of three elastic parameter terms.
- The various combinations are:
  - V_P, V_S and ρ (Aki-Richards, 1980, Wiggins et al., 1983, Fatti et al., 1994)
  - V_P, ρ and σ, or Poisson's ratio (Shuey, 1985)
  - λ, μ (Lamé parameters), and ρ. (Gray et al., 1999)
  - K, μ (Bulk and shear modulus), and ρ. (Gray et al.)

The general linearized equation

- All of the linearized approximations can be written in the same form as:

$$\boxed{R_{PP}(\theta) = a\frac{\Delta p_1}{p_1} + b\frac{\Delta p_2}{p_2} + c\frac{\Delta p_3}{p_3},}$$

where the scaling terms ***a*, *b*,** and ***c*** are functions of $\boldsymbol{\theta}$ and in-situ $\boldsymbol{(V_P/V_S)^2}$, to be called $\gamma_{sat}{}^2$, the $\boldsymbol{p_i}$ terms are the average parameter values across the boundary, and the $\Delta \boldsymbol{p_i}$ terms are the differences of the parameter values across the boundary.

- Let us briefly review the terms in the various equations.

Parameter term summary

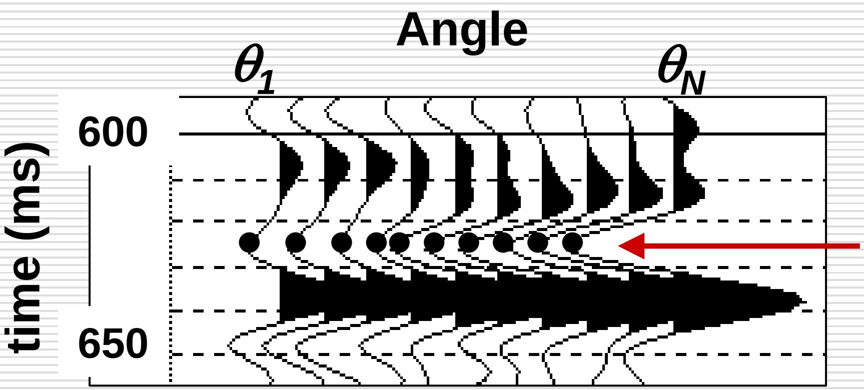
Method	$\Delta p_1 / p_1$	$\Delta p_2 / p_2$	$\Delta p_3 / p_3$
Aki-Richards	$\frac{\Delta V_P}{V_P}$	$\frac{\Delta V_S}{V_S}$	$\frac{\Delta \rho}{\rho}$
Wiggins	$R_{P0} = \frac{\Delta V_P}{2V_P} + \frac{\Delta \rho}{2\rho}$	$\frac{\Delta V_P}{2V_p} - \frac{4}{\gamma_{sat}^2}\frac{\Delta V_S}{V_S} - \frac{2}{\gamma_{sat}^2}\frac{\Delta \rho}{\rho}$	$\frac{\Delta V_P}{2V_P}$
Shuey	$R_{P0} = \frac{\Delta V_P}{2V_P} + \frac{\Delta \rho}{2\rho}$	$\left[\frac{\Delta V_P}{2V_P} - \left(2R_{P0} + \frac{\Delta V_P}{V_P}\right)\frac{1-2\sigma}{1-\sigma}\right] + \frac{\Delta \sigma}{(1-\sigma)^2}$	$\frac{\Delta V_P}{2V_P}$
Fatti	$R_{P0} = \frac{\Delta V_P}{2V_P} + \frac{\Delta \rho}{2\rho}$	$R_{S0} = \frac{\Delta V_S}{2V_S} + \frac{\Delta \rho}{2\rho}$	$\frac{\Delta \rho}{\rho}$
Gray $(\lambda\mu\rho)$	$\frac{\Delta \lambda}{\lambda}$	$\frac{\Delta \mu}{\mu}$	$\frac{\Delta \rho}{\rho}$
Gray $(K\mu\rho)$	$\frac{\Delta K}{K}$	$\frac{\Delta \mu}{\mu}$	$\frac{\Delta \rho}{\rho}$

Scaling term summary

Method	a	b	c
Aki-Richards	$\frac{\sec^2\theta}{2}$	$-\frac{4}{\gamma_{sat}^2}\sin^2\theta$	$0.5-\left[\frac{2}{\gamma_{sat}^2}\sin^2\theta\right]$
Wiggins	1	$\sin^2\theta$	$\sin^2\theta\tan^2\theta.$
Shuey	1	$\sin^2\theta$	$\sin^2\theta\tan^2\theta.$
Fatti	$1+\tan^2\theta$	$\frac{-8\sin^2\theta}{\gamma_{sat}^2}$	$\frac{2\sin^2\theta}{\gamma_{sat}^2}-\frac{1}{2}\tan^2\theta$
Gray $(\lambda\mu\rho)$	$\left(\frac{1}{4}-\boxed{\frac{1}{2\gamma_{sat}^2}}\right)\sec^2\theta$	$\boxed{\frac{1}{2\gamma_{sat}^2}}\sec^2\theta-\frac{2}{\gamma_{sat}^2}\sin^2\theta$	$\frac{1}{2}-\frac{1}{4}\sec^2\theta$
Gray $(K\mu\rho)$	$\left(\frac{1}{4}-\boxed{\frac{1}{3\gamma_{sat}^2}}\right)\sec^2\theta$	$\boxed{\frac{1}{3\gamma_{sat}^2}}\sec^2\theta-\frac{2}{\gamma_{sat}^2}\sin^2\theta$	$\frac{1}{2}-\frac{1}{4}\sec^2\theta$

Applying the various equations

- These equations can be used either in modeling or to extract parameter estimates from seismic data.



To extract parameters, we pick the amplitudes at a constant time on an angle gather, compute the *a, b, c* terms and solve the following equation:

$$\begin{bmatrix} R_{PP}(\theta_1) \\ R_{PP}(\theta_2) \\ \vdots \\ R_{PP}(\theta_N) \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ \vdots & \vdots & \vdots \\ a_N & b_N & c_N \end{bmatrix} \begin{bmatrix} \Delta p_1 / p_1 \\ \Delta p_2 / p_2 \\ \Delta p_3 / p_3 \end{bmatrix}$$

$$\text{or}: R = MP \Rightarrow P = (M^T M)^{-1} M^T R$$

Some observations (1)

- The Aki-Richards formulation was the first to be derived (the “mother” of linearized AVO!).
- The Wiggins and Fatti formulations are simply algebraic re-formulations of Aki-Richards and give the same value for a given model.
- The Wiggins and Shuey formulations are well known and can be written:

$$\boxed{R_{PP}(\theta) = A + B\sin^2\theta + C\sin^2\theta\tan^2\theta}$$

where A is the intercept (or zero-offset reflectivity R_{P0}), B is the gradient, and C is the curvature. A and B can be cross-plotted to reveal fluid anomalies.

Some observations (2)

- The Aki-Richards formulations involve only V_P, V_S and ρ, but the other formulations use elastic constants which are nonlinearly related by the equations:

$$\frac{V_P}{V_S} = \gamma = \sqrt{\frac{2\sigma - 2}{2\sigma - 1}} \Rightarrow \sigma = \frac{\gamma^2 - 2}{2\gamma^2 - 2}$$

$$V_S = \sqrt{\frac{\mu}{\rho}} \Rightarrow \mu = \rho V_S^2$$

$$V_P = \sqrt{\frac{\lambda + 2\mu}{\rho}} \Rightarrow \lambda = \rho V_P^2 - 2\rho V_S^2$$

$$V_P = \sqrt{\frac{K + 4/3\mu}{\rho}} \Rightarrow K = \rho V_P^2 - \frac{4}{3}\rho V_S^2$$

Some observations (3)

- Thus, instead of simply using algebra to re-arrange terms, Shuey (1984) and Gray et al. (1999) made use of the differential forms given by:

$$\Delta V_S = \frac{\partial V_S}{\partial V_P}\Delta V_P + \frac{\partial V_S}{\partial \sigma}\Delta\sigma$$

$$\Delta\lambda = \frac{\partial \lambda}{\partial V_P}\Delta V_P + \frac{\partial \lambda}{\partial \mu}\Delta\mu + \frac{\partial \lambda}{\partial \rho}\Delta\rho$$

$$\Delta K = \frac{\partial K}{\partial V_P}\Delta V_P + \frac{\partial K}{\partial \mu}\Delta\mu + \frac{\partial K}{\partial \rho}\Delta\rho$$

- This means that these equations will give slightly different values than the Aki-Richards expressions when applied to a model.

A generalized formulation

- It was noted that the two formulations by Gray et al. (1999) ($\lambda\mu\rho$ and $K\mu\rho$) differed only by the constants 1/2 and 1/3.
- Russell et al. (2003) asked the question: “For the porous reservoir rock, which term is more applicable, λ or K ?”
- As we showed, it doesn’t matter when each term is expanded for porous media.
- We thus replaced these terms with a more general term f, which reduces to either λ or K.
- The theory was initially developed by Biot (1941) and Gassmann (1951). A good summary is found in Krief et al. (1990).

General equation for *P*-wave velocity

- By equating Biot and Gassmann's formulations, the general equation for saturated *P*-wave velocity can be written:

$$\boxed{V_{P\_sat} = \sqrt{\frac{f+s}{\rho_{sat}}},}$$

where:

$f = \alpha^2 M$, a fluid/porosity term in which α is the Biot coefficient and M is the fluid modulus, and

$s = K_{dry} + 4/3\,\mu = \lambda_{dry} + 2\mu$ = a dry skeleton term.

Also: the shear modulus μ is independent of the fluid.

The fluid term

- Using the seismic velocities and density, we can extract the fluid term using the equation:

$$\boxed{f = \rho V_P^2 - c(\rho V_S^2) = f + s - c\mu}$$

- The constant c must be chosen so that the term $s - c\mu$ is equal to zero. This gives us the following relationship:

$$\boxed{c = (V_P / V_S)^2_{dry} = \gamma^2_{dry}}$$

- Noting that $\rho V_S^2 = \mu$ and dividing both sides of the first equation through by this term, we find:

$$\boxed{\frac{f}{\mu} = \left(\frac{V_P}{V_S}\right)^2_{sat} - \left(\frac{V_P}{V_S}\right)^2_{dry} = \gamma^2_{sat} - \gamma^2_{dry}}$$

Table of values

Here is a table of values for the various ratios:

	γdry^2	γdry	σdry	$Kdry/\mu$	$\lambda dry/\mu$
	4.000	2.000	0.333	2.667	2.000
(4)	3.333	1.826	0.286	2.000	1.333
	3.000	1.732	0.250	1.667	1.000
	2.500	1.581	0.167	1.167	0.500
(3)	2.333	1.528	0.125	1.000	0.333
	2.250	1.500	0.100	0.917	0.250
	2.233	1.494	0.095	0.900	0.233
(2)	2.000	1.414	0.000	0.667	0.000
(1)	1.333	1.155	-1.000	0.000	-0.667

In the above table note that (1) corresponds to $K\mu\rho$, (2) to $\lambda\mu\rho$, (3) to a clean sand and (4) to a shale.

A generalized formulation

Using this equation: $$\Delta f = \frac{\partial f}{\partial V_P}\Delta V_P + \frac{\partial f}{\partial \mu}\Delta \mu + \frac{\partial f}{\partial \rho}\Delta \rho$$

we can re-formulate the Aki-Richards equation as:

$$R_{PP}(\theta) = a\frac{\Delta f}{f} + b\frac{\Delta \mu}{\mu} + c\frac{\Delta \rho}{\rho}$$

where: $$a = \left(\frac{1}{4} - \boxed{\frac{\gamma_{dry}^2}{4\gamma_{sat}^2}}\right)\sec^2\theta, \quad b = \boxed{\frac{\gamma_{dry}^2}{4\gamma_{sat}^2}}\sec^2\theta - \frac{2}{\gamma_{sat}^2}\sin^2\theta$$

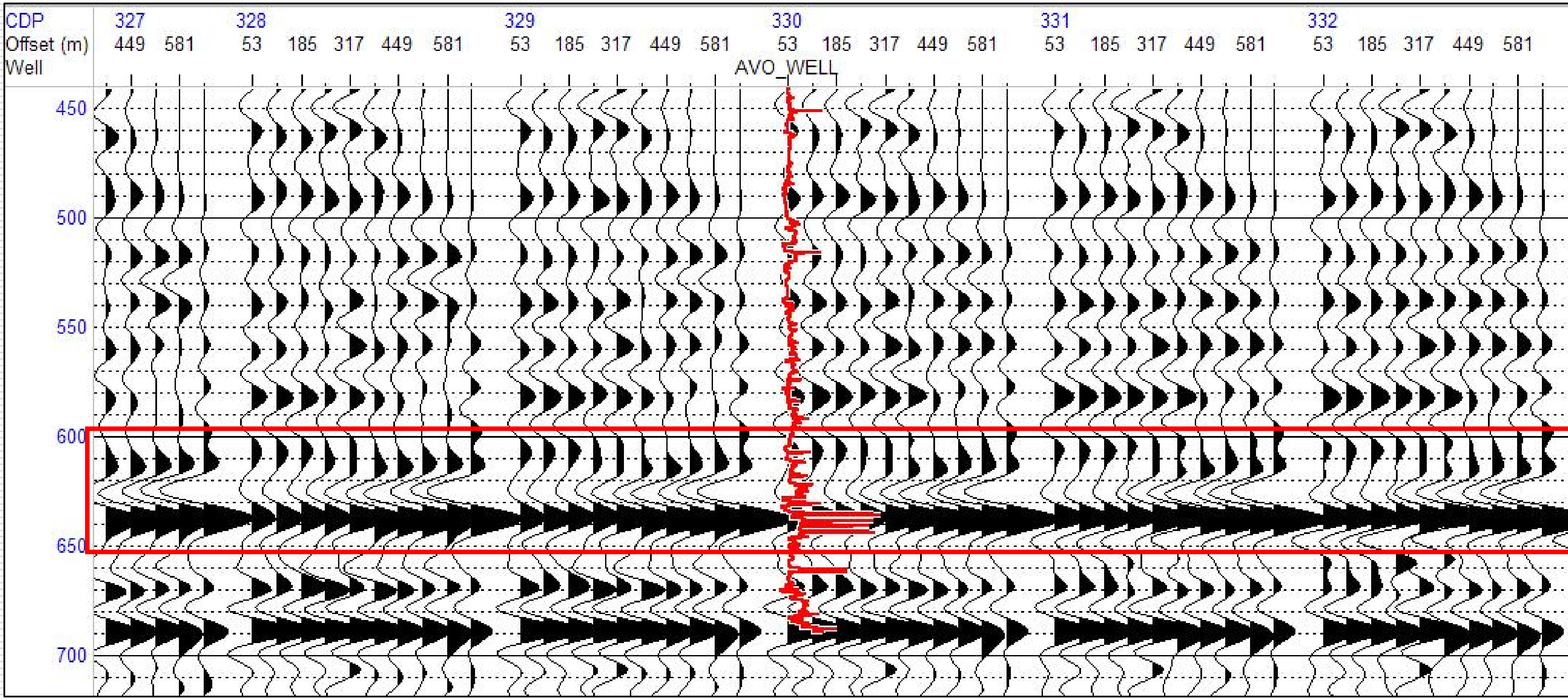
$$c = \frac{1}{2} - \frac{1}{4}\sec^2\theta, \quad \gamma_{sat}^2 = \left[\frac{V_S^2}{V_P^2}\right]_{sat} \text{ and } \gamma_{dry}^2 = \left[\frac{V_S^2}{V_P^2}\right]_{dry}$$

Some observations

Note the following points:

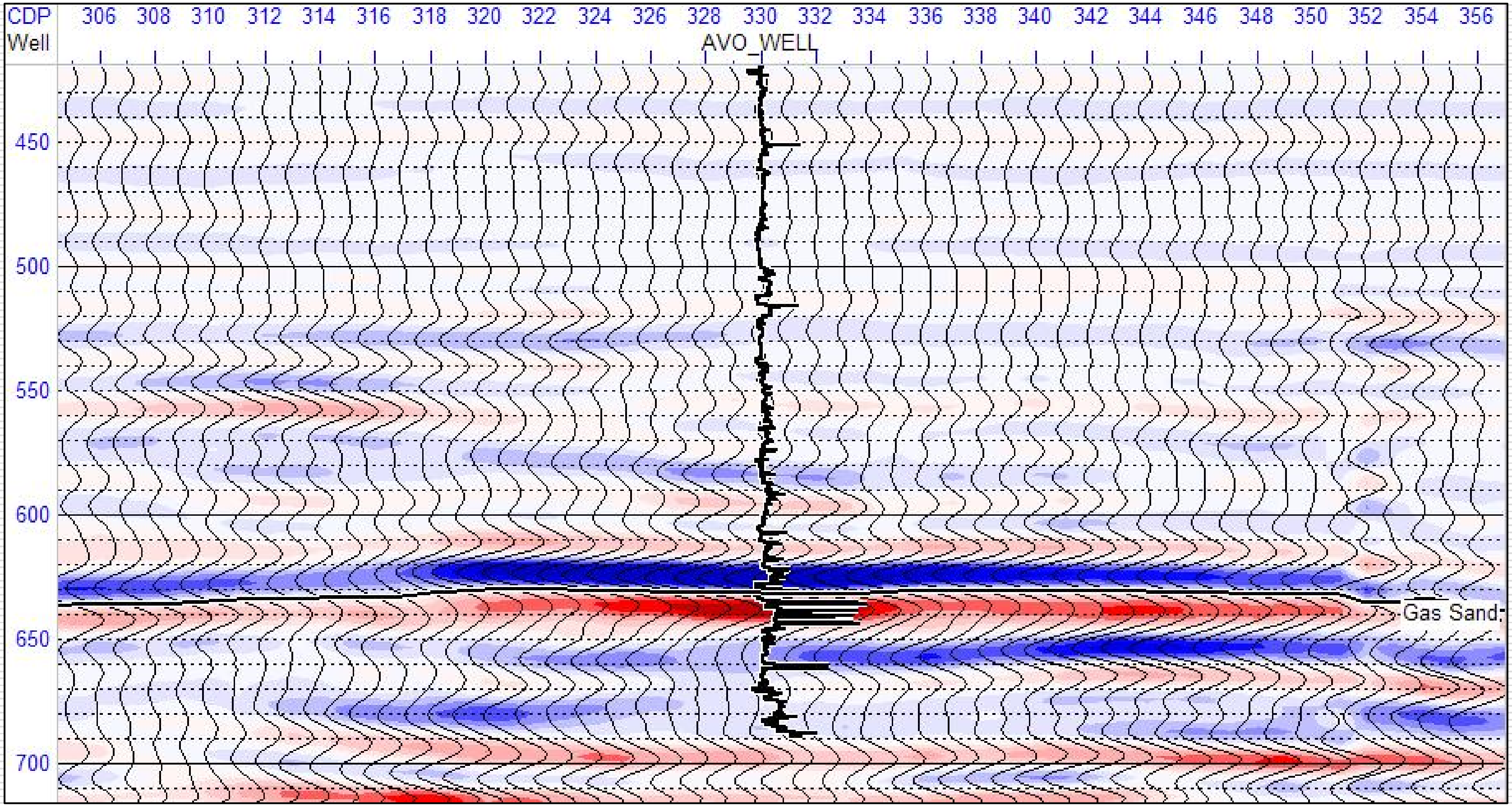
- If we use $\gamma_{dry}{}^2$ = 2, we obtain the Gray et al. (1999) expression for λ, μ, ρ.
- If we use $\gamma_{dry}{}^2$ = 4/3, we obtain the Gray et al. (1999) expression for K, μ, ρ.
- For a clean sandstone, $\gamma_{dry}{}^2$ = 2.333 ($K_{dry}/\mu = 1$)
- For a shale, $\gamma_{dry}{}^2$ = 3.333 ($K_{dry}/\mu = 2$, Tad Smith, personal communication)
- Since we never have a situation in which $\gamma_{dry} / \gamma_{sat} > 1$, the scaling coefficient for the fluid term will always be positive or zero.
- The fluid term equals zero if we are dealing with a dry or non-porous rock.

Real data study – Input gathers



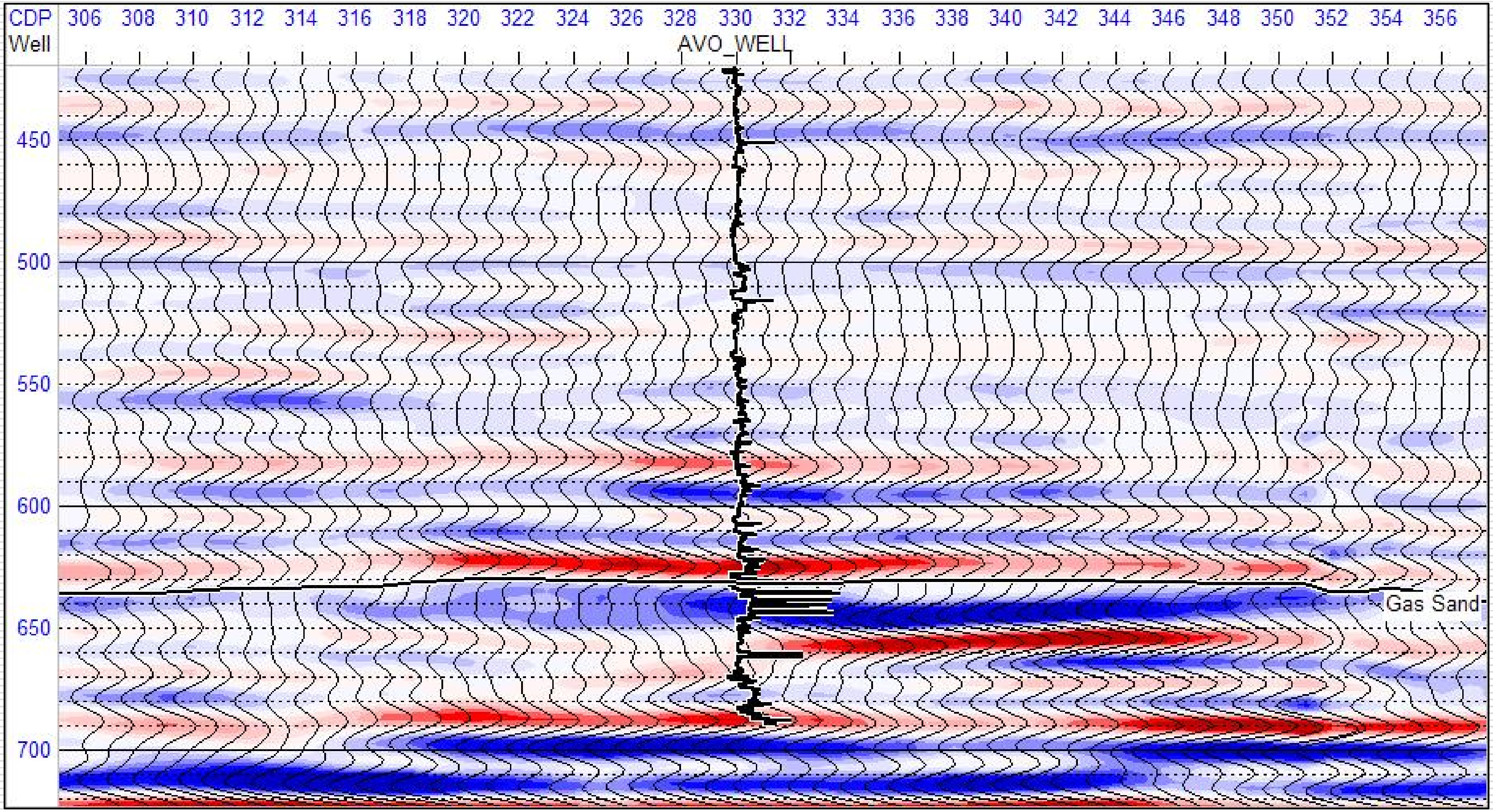
We applied the *f-μ-ρ* method to a Class 3 gas sand from Alberta. The super-gathers are shown above, with the zone of interest highlighted. Since the far angle is at 30^{o}, the density term extraction is considered unreliable.

Real data study – Fluid result



Here is the fluid extraction ($\Delta f/f$) with a picked event at the zero-crossing of the gas sand. We used a dry velocity ratio squared of 2.333.

Real data study – rock skeleton result



Here is the rock skeleton extraction ($\Delta\mu/\mu$) with a picked event at the zero-crossing of the gas sand.

Conclusions

- In this talk, we combined the linearized Amplitude Variations with Offset (AVO) technique with the Biot-Gassmann theory of poroelasticity.
- This gave us a way to extract fluid and skeleton effects from a reservoir using prestack angle gathers, from a knowledge of the dry and saturated velocity ratios.
- One caution is that it is not clear what “dry” means for rocks such as shales and fractured carbonates. More research is needed.